Baulkham Hills High School

2019

Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference Sheet is provided at the end of this paper.
- In questions 11-16 show relevant mathematical reasoning and/or calculations.

Total Marks **100**

Section I **10 marks**

- Attempt questions 1 – 10
- Allow about 15 minutes for this section

Section II **90 marks**

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section.

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for questions 1-10

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1. Evaluate $\frac{e}{\sqrt{2.13+0.04^3}}$, correct to four significant figures

(A) 1.871
(B) 0.8504
(C) 1.130
(D) 1.863

2. What is the slope of the line with equation $4x - 2y + 3 = 0$?

(A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2

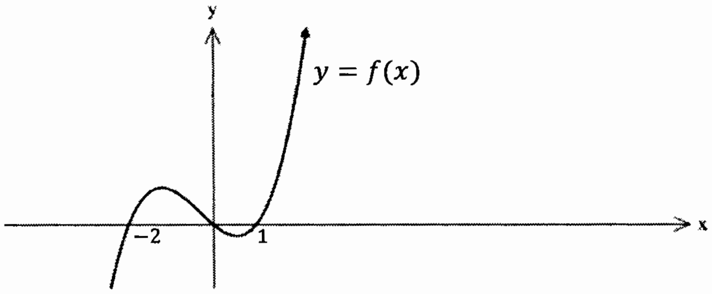
3. The first three terms of an arithmetic series are $-11, -7$ and -3. What is the 15^{th} term of this series?

(A) 59
(B) 255
(C) 45
(D) 495

4. Sixty tickets are sold in a raffle. There are two prizes. Jane buys five tickets. Which expression gives the probability that Jane wins both prizes?

(A) $\frac{5}{60}+\frac{4}{59}$
(B) $\frac{5}{60}+\frac{4}{60}$
(C) $\frac{5}{60}\times\frac{4}{59}$
(D) $\frac{5}{60}\times\frac{4}{60}$

5. The diagram shows the x-intercepts of the curve $y = f(x)$.



Expression to evaluate the area bounded by this curve and the x-axis is

(A) $\int_{-2}^{1} f(x)dx$

(B) $\left| \int_{-2}^{0} f(x)dx \right| + \int_{0}^{1} f(x)dx$

(C) $\left| \int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx \right|$

(D) $\int_{-2}^{0} f(x)dx + \left| \int_{0}^{1} f(x)dx \right|$

6. What is the value of the derivative of $y = 2\sin 2x - 3\tan x$ at $x = 0$?

(A) -1

(B) 1

(C) 3

(D) 0

7. Using Simpson's rule with 4 subintervals, which expression gives the approximate area under the curve $y = xe^x$ between $x = 1$ and $x = 5$?

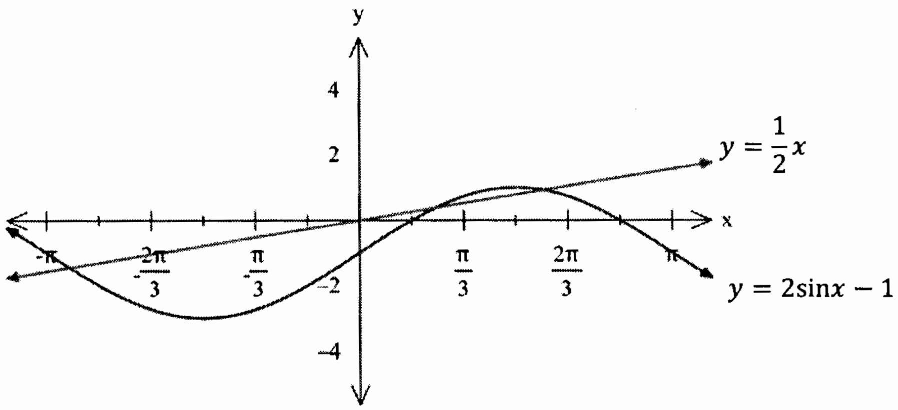
(A) $\frac{1}{3}(e + 6e^2 + 4e^3 + 10e^4 + 3e^5)$

(B) $\frac{2}{3}(e + 6e^2 + 4e^3 + 10e^4 + 3e^5)$

(C) $\frac{2}{3}(e + 8e^2 + 6e^3 + 16e^4 + 5e^5)$

(D) $\frac{1}{3}(e + 8e^2 + 6e^3 + 16e^4 + 5e^5)$

8. The diagram shows a sketch of the graph of $y = 2\sin x - 1$ and $y = \frac{1}{2}x$.



How many solutions are there for the equation $2\sin x - 1 = \frac{1}{2}x$, where $-\pi \leq x \leq \pi$?

(A) 3
(B) 2
(C) 1
(D) 0

9. What are the solution of $\cos(x - \frac{\pi}{3}) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$.

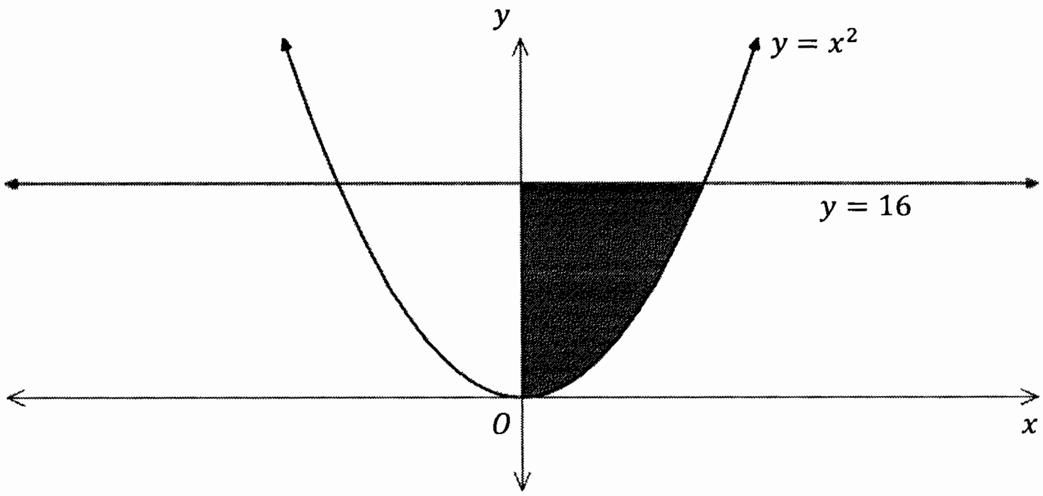
(A) $x = 0, \frac{\pi}{6}$

(B) $x = \frac{2\pi}{3}, 2\pi$

(C) $x = \frac{\pi}{3}, \frac{5\pi}{6}$

(D) $x = \frac{\pi}{3}, 0$

10. The diagram below shows the region bounded by the curve $y = x^2$, the y-axis and the line $y = 16$.



Which of the following gives the volume of the solid of revolution when the region is rotated about the y-axis?

(A) $\pi \int_{0}^{4} x^4 dx$

(B) $\pi \int_{0}^{16} x^4 dx$

(C) $\pi \int_{0}^{4} y dy$

(D) $\pi \int_{0}^{16} y dy$

END OF SECTION I

Section II

90 marks
Attempt questions 11-16
Allow about 2 hours and 45 minutes for this section
Answer each question in the **appropriate page of the writing booklet**. Extra sheets of writing paper are available.
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

QUESTION 11 (15 marks). Answer on the appropriate page.

a) Simplify: $3x - (8 - 13x)$. 1

b) Factorise fully $81 - 3x^3$. 2

c) Express $\frac{4}{2-\sqrt{5}}$ with a rational denominator 2

d) Solve $|3 - 2x| = 4$ 2

e) Differentiate $(x - e^x)^5$. 2

f) Evaluate $\int_0^{\frac{\pi}{3}} \sin 2x dx$ 2

g) Differentiate $\ln\left(\frac{2x}{x-2}\right)$. 2

h) The curve $y = f(x)$ passes through the point $\left(\frac{1}{2}, 0\right)$ and its derivative function is $f'(x) = 2e^{2x-1} - 1$. Find the equation of the curve. 2

End of Question 11

QUESTION 12 (15 marks). Answer on the appropriate page.

a) Find the solutions of $2\sin\theta = \sqrt{3}$ for $0 \leq \theta \leq 2\pi$. 2

b) Find $\int (1 + sec^2 x)dx$ 2

c) Evaluate the limiting sum of the series $4 - 1 + \frac{1}{4} - \frac{1}{16} + \cdots$ 2

d) Find $f'(x)$, where $f(x) = \frac{x^2-3}{3x-4}$. 2

e) For what values of k does the quadratic equation $x^2 - 8x + k = 0$ have real distinct roots? 2

f) $P(1, \frac{1}{4})$ is a point on the parabola $y = \frac{x^2}{4}$.

(i) Find the coordinates of the focus. 1

(ii) Show that the equation of the tangent to the parabola is $2x - 4y - 1 = 0$. 2

(iii) Find the coordinates of the point of intersection of the tangent at P and the directrix of the parabola. 2

End of Question 12

QUESTION 13 (15 marks). Answer on the appropriate page.

a) One of the roots of $x^2 + mx + n = 0$ is three times the other. Prove that $3m^2 = 16n$. 2

b) (i) Find the domain and the range for the function $f(x) = \sqrt{4 - x^2}$ 2

(ii) On the same number plane, shade the region where the points $P(x, y)$ satisfy both of the inequalities $y < \sqrt{4 - x^2}$ and $y \geq x - 5$. 3

c) Consider the curve $y = (1 + x^2)(2 - x)$.

(i) Show that there are stationary points at $x = 1$ and at $x = \frac{1}{3}$. Hence determine their nature. 3

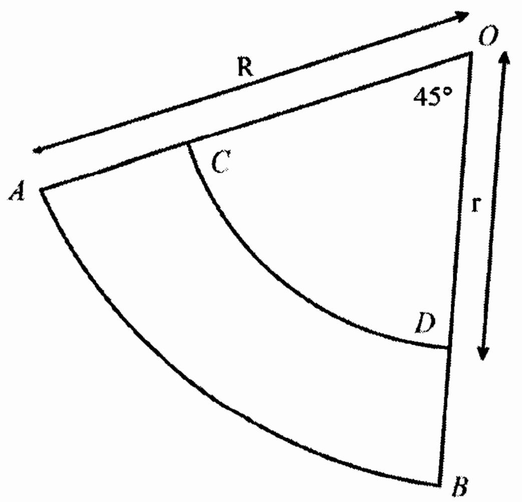
(ii) Find the coordinates of any point of inflexion on the curve. 1

(iii) Sketch the curve, labelling the stationary points, point of inflexion and y-intercept, in the domain $-1 \leq x \leq 3$. 4

End of Question 13

QUESTION 14 (15 marks). Answer on the appropriate page.

(a)



NOT TO SCALE

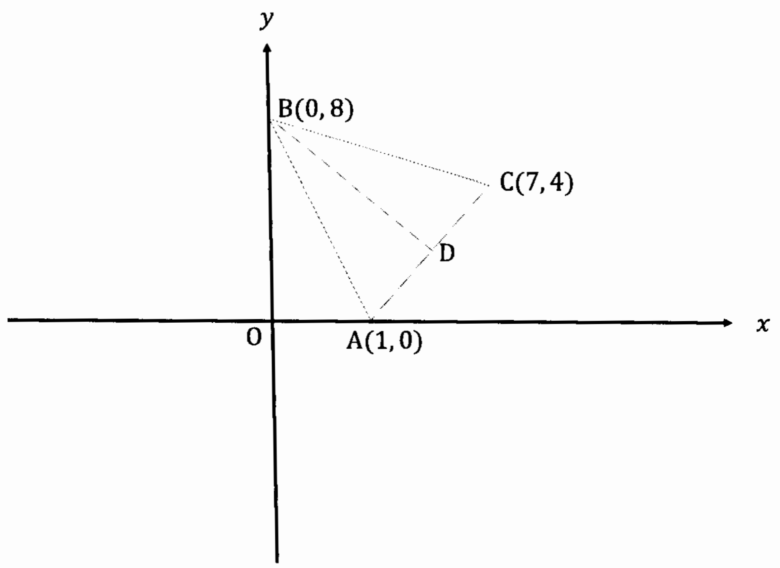
i) Show that the area, S, of the region ACDB is given by $S = \frac{\pi}{8}(R^2 - r^2)$. 2

ii) Find the perimeter of the region ACDB in terms of R and r. 2

Question 14 continues on the following page

Question 14 (continued)

b) In the diagram A, B and C are the points $(1, 0), (0,\ 8)$ and $(7,\ 4)$ respectively.



NOT TO SCALE

(i) Show that the equation of AC is $2x - 3y - 2 = 0$. **2**

(ii) Find the angle between AC and the x-axis. **1**

(iii) Point D lies on AC and $BD \perp AC$. Show that the equation of BD is $3x + 2y - 16 = 0$. **2**

(iv) Show that BD is the perpendicular bisector of AC. **2**

(v) Prove that $\Delta ABD \equiv \Delta CBD$. **2**

(vi) Find the area of ΔABC. **2**

End of Question 14

QUESTION 15 (15 marks). Answer on the appropriate page.

(a) John and Peter play against each other in a tennis match. The champion will be the first person to win two games. John's probability of winning each game is 0.6 and Peter's probability of winning each game is 0.4.

(i) Draw the probability tree diagram and list the sample space for this match. 2

(ii) What is the probability that John wins the match? 2

(b) Joe's employer makes contributions to his superannuation scheme on 1st January each year. Any amount invested in this scheme earns interest at the rate of 7.5% per annum, compounded annually.

(i) Let M be the annual contribution. Show that the first contribution to the scheme is worth $M(1+\frac{7.5}{100})^n$ at the end of the n^{th} year. 1

(ii) Hence show that the value of the investment, A_n, at the end of the n^{th} year is 2

$$A_n = \frac{M\times 1.075\times(1.075^n-1)}{0.075}$$

(iii) The employer makes an annual contribution of \$20526. Joe decides to change jobs when his investment is worth more than \$750 000. What is the least number of whole years that Joe will need to be in this current job? 2

(c) A particle is moving in a straight line. Its velocity, $\dot{x}$, at time t seconds is given by

$$\dot{x} = 2\cos t - 1 \text{ where } t \geq 0.$$

Initially, the particle is at the origin, O, where the displacement is $x = 0$ and x is measured in metres.

(i) Find an expression for x in terms of t. 2

(ii) Describe the initial motion of the particle. 1

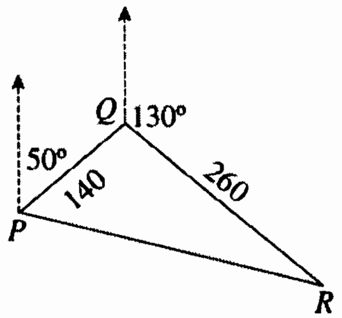
(iii) Determine when the particle first comes to rest. 1

(iv) Calculate the total distance travelled in the first π seconds. 2

End of Question 15

QUESTION 16 (15 marks**).** Answer on the appropriate page.

(a) A truck travelled 140 km from port P to port Q on a bearing of $050°\ T$. It then travelled 260 km from port Q to port R on a bearing of $130°T$.



(i) Explain why angle PQR is $100°$. 1

(ii) Find the distance between ports P and R, correct to the nearest km. 2

(b) A quantity of a radioactive material decays according to the equation

$$\frac{dM}{dt} = -kM,$$

where M is the mass of the material in kg, t is the time in years and k is a constant.

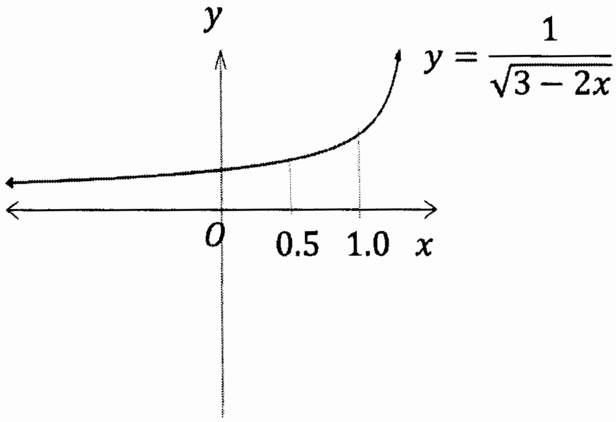
(i) Show that $M = Ae^{-kt}$ is a solution to the equation, where A is a constant. 1

(ii) The half-life period of the material for decay is 300 years. If the initial amount of material is $20kg$, find the amount, correct to two decimal places, remaining after 700 years. 2

Question 16 continues on the following page

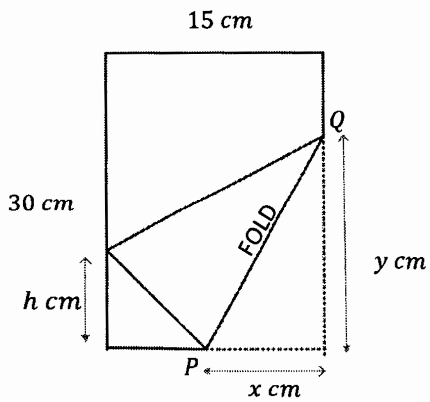
Question 16 (continued)

(c)



The diagram shows the graph of $y = \frac{1}{\sqrt{3-2x}}$. The region bounded by this graph, $x = 0.5$, $x = 1.0$ and the $x-$axis is rotated about the $x-$axis. Find the volume of the solid of revolution formed in exact form. 3

(d) A rectangular metal sheet is 30 cm high and 15 cm wide. The lower right-hand corner of the sheet is folded over along PQ so as to reach the leftmost edge of the sheet. Let x be the horizontal distance and y be the vertical distance folded as shown in the diagram.



NOT TO SCALE

(i) Show that $y = \frac{hx}{2x-15}$ where $h = \sqrt{x^2 - (15-x)^2}$. 2

(ii) Show that L, the length of the FOLD, is given by $L^2 = \frac{2x^3}{2x-15}$. 2

(iii) Hence, find the minimum length of L. 2

End of Question 16

End of Examination

Baulkham Hills High School
Trial HSC Examination 2019
Marking Guideline - Mathematics

Section I (10 marks)

Award 1 mark to each correct answer.

Answers: 1. D 2. D 3. C 4. C 5. D 6. B 7. D 8. A 9. B 10. D

Section II (90 marks)

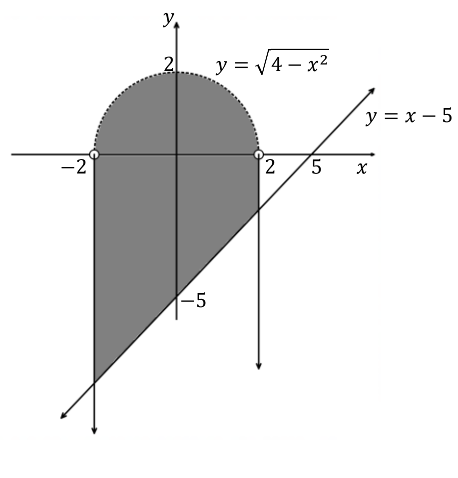
In all questions, award FULL marks for **correct answers** with **necessary working**.

Use this suggested solutions in conjunction with the marking criteria.

Question	Suggested solutions	Marking criteria
11(a)	$-8+16x$	1- correct answer
11(b)	$81-3x^3=3(3^3-x^3)$ $=\mathbf{3(3-x)(9+3x+x^2)}$	2- correct solution 1- HCF & identified the difference of cubes
11(c)	$\frac{4}{2-\sqrt{5}}=\frac{4}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}}$ $=\frac{8+4\sqrt{5}}{4-5}$ $=\mathbf{-8-4\sqrt{5}}$	2- correct solution 1- multiplied by the conjugate surd
11(d)	<u>**Case**</u>: $3-2x\geq 0$ $3-2x=4$ $x=-\frac{1}{2}$ Test: $LHS=\lvert 3+1\rvert=4; RHS=4$ $LHS=RHS.$ <u>**Case**</u>: $3-2x<0$ $-3+2x=4$ $x=\frac{7}{2}$ Test: $LHS=\left\lvert 3-2\times\frac{7}{2}\right\rvert=4; RHS=4$ $LHS=RHS.$ $x=-\frac{1}{2}$ $\boldsymbol{and\ x=\frac{7}{2}}$	2- both values for x 1- one value for x
11(e)	$\frac{d(x-e^x)^5}{dx}=\mathbf{5(x-e^x)^4\times(1-e^x)}$	2- correct solution 1- correct use of **either** the power rule **or** the chain rule.

Question	Suggested solutions	Marking criteria
11(f)	$\int_0^{\frac{\pi}{3}} sin2xdx = [-\frac{1}{2} \times cos2x]_0^{\frac{\pi}{3}}$ $= \mathbf{\frac{3}{4}}$	2- correct solution 1- correct integration.
11(g)	$\frac{d[\ln\left(\frac{2x}{x-2}\right)]}{dx} = \frac{d[\ln(2x)]}{dx} - \frac{d[\ln(x-2)]}{dx}$ $= \frac{1}{x} - \frac{1}{x-2}$ $= \frac{\mathbf{-2}}{\boldsymbol{x^2 - 2x}}$	2- correct solution [not necessarily as a single fraction] 1- correct use of a rule for differentiation.
11(h)	$y = \int (2e^{2x-1} - 1)dx = 2 \times \frac{1}{2} \times e^{2x-1} - x + C$ $\left(\frac{1}{2}, 0\right)$ *lies on the curve.* $\therefore\ C = -\frac{1}{2}$ $\boldsymbol{y = e^{2x-1} - x - \frac{1}{2}}$	2- correct solution 1- integral **without** the integration constant.
12(a)	$2sin\theta = \sqrt{3}$ for $0 \leq \theta \leq 2\pi$. $sin\theta = \frac{\sqrt{3}}{2}$ $\boldsymbol{\theta = \frac{\pi}{3}, \quad \frac{2\pi}{3}}$	2- correct solution 1- reference angle= $\boldsymbol{\frac{\pi}{3}}$.
12(b)	$\int (1 + sec^2x)dx = \int dx + \int sec^2xdx$ $= \boldsymbol{x + tanx + C}$	2- correct solution 1- expressed as sum of $\int dx$ and $\int sec^2xdx$.
12(c)	$4 - 1 + \frac{1}{4} - \frac{1}{16} + \cdots$ is a GP with $a = 1$ *and* $r = -\frac{1}{4}$. As $\lvert r \rvert < 1$, $S_\infty = \frac{4}{1-(-\frac{1}{4})} = \mathbf{\frac{16}{5}} = \mathbf{3\frac{1}{5}}$	2- correct solution 1- $a = 1$ and $r = -\frac{1}{4}$.

12(d)	$f'(x) = \frac{(3x-4)\frac{d(x^2-3)}{dx} - (x^2-3)\frac{d(3x-4)}{dx}}{(3x-4)^2}$ $= \frac{(3x-4) \times 2x - (x^2-3) \times 3}{(3x-4)^2}$ $= \frac{\boldsymbol{3x^2 - 8x + 9}}{\boldsymbol{(3x-4)^2}}$	2- correct solution 1- correct use of the quotient rule.

Question	Suggested solutions	Marking criteria
12(e)	For real distinct roots $\Delta > 0$ where $\Delta = (-8)^2 - 4 \times 1 \times k$ $4k < 64$ $\boldsymbol{k < 16}$	2- correct solution 1- $(-8)^2 - 4 \times 1 \times k > 0$ or equivalent expression
12(f)(i)	$x^2 = 4y \quad and\ P(1, \frac{1}{4})$ $4a = 4 \quad , \quad a = 1\ hence\ the\ \boldsymbol{focus\ is}\ (\mathbf{0}, \mathbf{1})$	1- correct answer
(ii)	Gradient, m, at $P = \frac{2x}{4}\ \ where\ x = 1.\ \therefore m = \frac{1}{2}$ Equation of the tangent to the parabola at P: $y - \frac{1}{4} = \frac{1}{2}(x - 1)$ $\mathbf{2x - 4y - 1 = 0}$	2- correct solution 1- gradient of the tangent at P
(iii)	Equation of the directrix: $y = -1$. substitute $y = -1\ in\ \ 2\boldsymbol{x} - \mathbf{4y} - \mathbf{1} = \mathbf{0}\ \ \therefore\ \ x = \frac{3}{2}$ **Required coordinates:** $(\frac{\mathbf{3}}{\mathbf{2}}, -\mathbf{1})$	2- correct solution 1- equation of the directrix.
13(a)	Let the roots be α and 3α. $\alpha + 3\alpha = -m \quad and \quad \alpha \times 3\alpha = n$ $4\alpha = -m\ \ and\ 3\alpha^2 = n$ $3 \times (\frac{-m}{4})^2 = n \qquad \rightarrow \mathbf{3m^2 = 16n}$	2- correct proof 1- sum **and** product of roots
13(b)(i)	**Domain =** $\{\boldsymbol{x}: -\mathbf{2} \leq \boldsymbol{x} \leq \mathbf{2}\}$ **Range =** $\{\boldsymbol{y}: \mathbf{0} \leq \boldsymbol{y} \leq \mathbf{2}\}$	1- correct answer 1- correct answer
(ii)		3- correct solution [showing all included and excluded points for the region] 2- correct shapes and intercepts (on both axes) for both curves 1- correct graph of ONE of the given functions.

13(c)(i)

At stationary points $\frac{dy}{dx} = 0$ where $y = (2-x)(1+x^2)$

$$\frac{dy}{dx} = -3x^2 + 4x - 1$$

$$= (x-1)(1-3x)$$

$$(x-1)(1-3x) = 0 \quad \rightarrow \quad x = 1 \text{ or } x = \frac{1}{3}$$

Stationary points are $\boldsymbol{A(1,2)}$ ***and*** $\boldsymbol{B\left(\frac{1}{3}, \frac{50}{27}\right)}$.

At point A (1,2)

x	0.9	1	1.1
$\frac{dy}{dx}$	+0.17	0	−0.23

$\therefore \exists$ a maximum at A

At point $B\left(\frac{1}{3}, \frac{50}{27}\right)$

x	0.3	$\frac{1}{3}$	0.34
$\frac{dy}{dx}$	-0.07	0	+0.0132

$\therefore \exists$ a minimum at B

3 - correct solution

2- both x values when $\frac{dy}{dx} = 0$.

1- found $\frac{dy}{dx}$.

13(c)(ii)

$$\frac{d^2y}{dx^2} = -6x + 4$$

$$\frac{d^2y}{dx^2} = 0 \quad \rightarrow \quad x = \frac{2}{3}$$

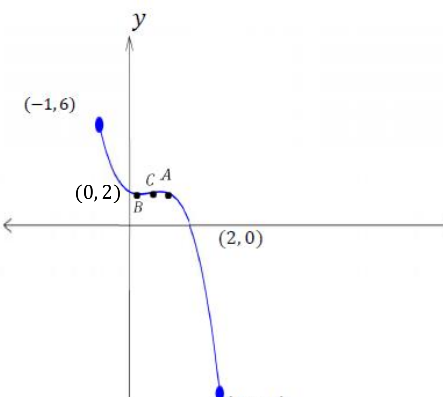
At point $C\left(\frac{2}{3}, \frac{52}{27}\right)$, $\quad \frac{d^2y}{dx^2} = 0$ and $\exists$ *a change in concavity.*

x	0.65	$\frac{2}{3}$	0.67
$\frac{d^2y}{dx^2}$	+0.1	0	−0.02

Point C is a point of inflexion.

1- correct answer

(iii)



4- correct solution[includes the restricted domain and correct shape]

3- stationary points, point of Inflexion AND y −intercept

2- stationary points and point of inflexion

1- y −intercept

14(a)(i)

Area of the region ACDB

$$= Sector\ OAB - Sector\ OCD$$

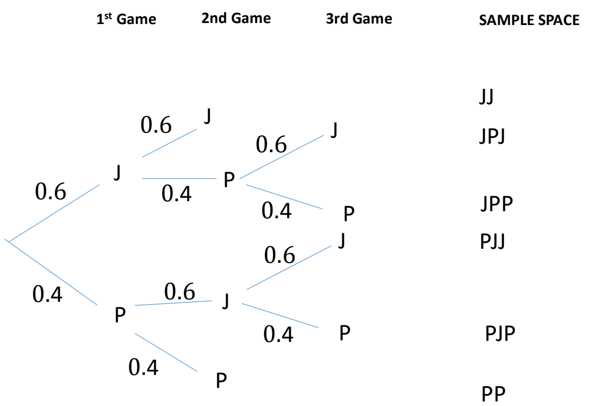
$$= \frac{1}{2} \times R^2 \times \frac{\pi}{4} - \frac{1}{2} \times r^2 \times \frac{\pi}{4}$$

$$S = \frac{\pi}{8}(R^2 - r^2)$$

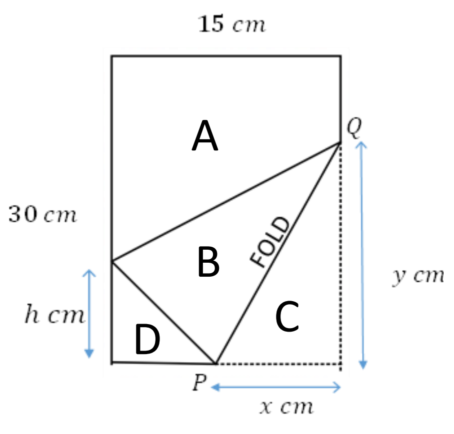
2- correct proof

1- correct algebraic expression for *area of a sector*

14(a)(ii)	$Perimeter\ of\ the\ region\ ACDB$ $= AC + BD + arc\ CD + arc\ AB$ $= (R - r) + (R - r) + \frac{\pi r}{4} + \frac{\pi R}{4}$ $= \mathbf{2(R - r) + \frac{\pi}{4}(R + r)}$	2- correct answer 1- correct algebraic expressions for the arc lengths
14b(i)	$m_{AC} = \frac{4-0}{7-1} = \frac{2}{3}$ Equation of AC: $y - 0 = \frac{2}{3}(x - 1)$ $\mathbf{2x - 3y - 2 = 0}$	2- correct proof 1- correct gradient
(ii)	Angle between AC and the x-axis = $tan^{-1}\left(\frac{2}{3}\right)$ $= \mathbf{33°41'}$ **(or** $\approx \mathbf{34}°$)	1- correct answer
(iii)	$A(1,0)$, $B(0,8)$ and $C(7,4)$. $BD \perp AC$ $m_{BD} \times \frac{2}{3} = -1 \quad \therefore\ m_{BD} = -\frac{3}{2}$ Equation of BD: $y - 8 = -\frac{3}{2}(x - 0)$ $3x + 2y - 16 = 0$	2- correct proof 1- $m_{BD} = -\frac{3}{2}$
(iv)	$M(4,2)$ where M is the midpoint of AC. $Equation\ of\ BD$: $3x + 2y - 16 = 0$ $Equation\ of\ AC$: $2x - 3y - 2 = 0$ Solving simultaneously, we have $D(4,2)$. Hence M and D are one and the same point. $\therefore BD\ is\ the\ perpendicular\ bisector\ of\ AC$.	2- correct proof 1- coordinates of D.
14(b) (v)	$In\ \Delta ABD\ and\ \Delta CBD$, $AD = DC$ [$D\ is\ the\ midpoint\ of\ AC$] $\angle ADB = 90° = \angle CDB$[$BD \perp AC, proven$] $BD\ is\ common$ $\Delta ABD \equiv \Delta CBD$ [SAS]	2- correct proof 1- statements with incorrect ≡ condition.
(vi)	$\lvert\Delta ABC\rvert = \frac{1}{2} \times AC \times BD$ $BD = \sqrt{(4-0)^2 + (2-8)^2} = \sqrt{52}\,units$ $AC = \sqrt{(7-1)^2 + (4-0)^2} = \sqrt{52}\,units$ $\lvert\Delta ABC\rvert = \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$ $= \mathbf{26\ units^2}$	2- correct answer 1- an expression to evaluate $\lvert\Delta ABC\rvert$ $and\ the\ size\ of$ $AC\ or\ BD$.

Question	Suggested solutions	Marking criteria
15(a)(i)		2- correct answer 1- correct **diagram not** showing the probability.
(ii)	$P(John\ is\ the\ champion)$ $= P(win\ in\ 2\ games) + P(win\ in\ 3\ games)$ $= 0.6 \times 0.6 + 0.6 \times 0.4 \times 0.6 + 0.6 \times 0.4 \times 0.6$ $= \mathbf{0.648}$	2- correct answer 1- identified the ways of winning the match.
15(b)(i)	$the\ value\ of\ M\ after\ the\ 1st\ year = M + M \times \frac{7.5}{100}$ $= M\left(1 + \frac{7.5}{100}\right)$ $the\ value\ of\ M\ after\ the\ 2nd\ year$ $= M\left(1 + \frac{7.5}{100}\right)\left(1 + \frac{7.5}{100}\right)$ $= M\left(1 + \frac{7.5}{100}\right)^2$ $Hence, the\ value\ of\ M\ after\ the\ nth\ year$ $= M\left(1 + \frac{7.5}{100}\right)^n$	1- correct proof.
(ii)	The value of the last contribution $= M\left(1 + \frac{7.5}{100}\right)^1$ The value of the 2nd last contribution $= M\left(1 + \frac{7.5}{100}\right)^2$ The value of the 3rd last contribution $= M\left(1 + \frac{7.5}{100}\right)^3$ Note that the above values are the terms of a GP whose first term $a = M\left(1 + \frac{7.5}{100}\right)^1$ with a common ratio $r = 1.075$. $\therefore \quad A_n = \frac{a(r^n - 1)}{(r - 1)}$ $\therefore \quad A_n = \frac{M \times 1.075(1.075^n - 1)}{0.075}$	2- correct solution 1- identified that the final value of each M is a term of a GP.

15b(iii)	Let n be the number of years in the current job to generate \$750 000. $Also\ given\ \ M = \$20526.$ $$750000 = \frac{20526\times1.075(1.075^{n}-1)}{0.075} \quad \_\_\_\_\_\_\text{(A)}$$ $$1.075^{n} = 3.5492$$ $$\therefore \quad n = \frac{ln3.5492}{ln1.075} = 17{,}515\ldots$$ **Joe needs to be in the current job for at least 18 years.**	2- correct answer 1- equation (A)
15(c)(i)	$$\dot{x} = 2cost - 1\ where\ t \geq 0.$$ $$x = \int(2cost - 1)dt \quad = 2sint - t + C$$ Initially, $t = 0\ and\ x = 0.\ \ C = 0.$ $$\boldsymbol{x = 2sint - t}$$	2- correct answer 1- integral expression for x.
15(c)(ii) (iii) (iv)	Initially, $t = 0\ and\ x = 0.\ \ \dot{x} = 1\ m/s.$ **The particle is at O and moves to the right with the speed $\boldsymbol{1\ m/s}$.** The particle comes to rest when $\dot{x} = 0$. $$0 = 2cost - 1 \qquad \rightarrow \quad cost = \frac{1}{2}$$ $$t = \frac{\pi}{3}, \qquad \frac{5\pi}{3}, \ldots..$$ **After $\boldsymbol{\frac{\pi}{3}}$ seconds the particle first come to rest.** Total distance travelled $= \int_0^{\frac{\pi}{3}}(2cost - 1)dt + \mid \int_{\frac{\pi}{3}}^{\pi}(2cost - 1)dt \mid$ $\boldsymbol{= 2\sqrt{3} + \frac{\pi}{3}\ metres}$	1- correct description 1- correct answer 2- correct solution 1- correct expression to calculate the TOTAL distance travelled.
16(a)(i) (ii) 16(b)(i)	Correct explanation for angle $PQR\ = \ 100°$ $$PR^2 = 140^2 + 260^2 - 2 \times 140 \times 260\cos100°$$ $$\approx 316\ km$$ $$M = Ae^{-kt} \ \_\_\_\_\_\ (1)$$ $$\frac{dM}{dt} = A \times (-k)\ e^{-kt}$$ $$= -k(A\ e^{-kt})$$ $$= -kM$$ $\therefore M = Ae^{-kt}$ is a solution to the differential equation (1)	1 – correct answer 2 – correct solution 1 – correct use of the cosine rule 1- correct proof

16(b)(ii)	$t = 0, M = 20 \quad \rightarrow \quad 20 = A$ $A \rightarrow \frac{A}{2} \quad in\ 300\ years$ $\frac{A}{2} = Ae^{-300k} \quad \rightarrow \quad k = \frac{1}{300} \times ln2$ When $t = 700,\ M = 20e^{-\frac{700ln2}{300}}$ $= 20 \times 2^{-\frac{7}{3}}$ $= \mathbf{3.96kg(2dp)}$	2- correct answer 1- formed the equation to calculate k using half-life period.
16(c)	$V = \pi \int_a^b y^2 dx$ $where \quad a = 0.5, b = 1.0 \ and \ y = \frac{1}{\sqrt{3-2x}}$ $V = \pi \int_{0.5}^{1} \frac{1}{3-2x} dx$ $V = [-\frac{\pi}{2}\ln(3-2x)]_{0.5}^{1}$ $\boldsymbol{V = \frac{\pi}{2}\ln 2}$ **cubic units**	3- correct solution 2- correct expression for the definite integral showing limits 1- correct use of the formula for the volume .
16(d)(i)	 Area of the rectangle = Area of A + B + C + D $B = C = \frac{1}{2}xy$ $D = \frac{1}{2}(15-x)h \quad where\ h = \sqrt{x^2 - (15-x)^2}$ $A = \frac{1}{2} \times 15[(30-h) + (30-y)]$ $30 \times 15 = xy + \frac{1}{2}(15-x)h + \frac{1}{2} \times 15[(30-h) + (30-y)]$ $0 = 2xy - hx - 15y$ $y(2x-15) = hx$ $\therefore \quad \boldsymbol{y = \frac{hx}{2x-15}}$	2- correct proof 1- formed an equation in terms of x, y and h.

<table>
<tr><td>16(d)(ii)</td><td>

$$L^2 = x^2 + y^2 \quad \text{------ (A)}$$

$$L^2 = x^2 + \left(\frac{hx}{2x-15}\right)^2$$

$By\ substituting\ for\ h$

$$= \frac{x^2[(2x-15)^2 + (2x-15)15]}{(2x-15)^2}$$

$$\boldsymbol{L^2 = \frac{2x^3}{2x-15}}$$

</td><td>2- correct proof
1- equation (A) to find L</td></tr>
<tr><td>16(d)(iii)</td><td>

$$\frac{d(L^2)}{dx} = \frac{(2x-15)\times 6x^2 - 2x^3 \times 2}{(2x-15)^2}$$

$$\frac{d(L^2)}{dx} = 0 \quad \rightarrow \quad (2x-15)\times 6x^2 - 2x^3 \times 2 = 0$$

$$2x^2(4x-45) = 0$$

$$As\ x \neq 0, \qquad x = \frac{45}{4} = 11.25$$

<table>
<tr><th>x</th><th>11.20</th><th>11.25</th><th>11.30</th></tr>
<tr><td>$\frac{d(L^2)}{dx}$</td><td>−50.176</td><td>0</td><td>+51.076</td></tr>
</table>

Hence for $minimum\ L, x = 11.25$

$$L = \sqrt{\frac{2\times 11.25^3}{2\times 11.25 - 15}} = 19.485..$$

$\boldsymbol{minimum\ length\ of\ the\ FOLD\ is\ 19.5\ cm\ (1dp)}$

</td><td>2- correct answer

1- valid value for x</td></tr>
</table>

End of Marking Guideline