Blacktown Boys' High School

2018

HSC Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- All diagrams are not drawn to scale
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 13)
- Attempt Questions 11 -16
- Allow about 2 hours and 45 minutes for this section

Assessor: E.Efimova

Student Name: ____

Teacher Name: ____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2018 Higher School Certificate Examination.

Office Use Only

Question	Mark
Q1	/1
Q2	/1
Q3	/1
Q4	/1
Q5	/1
Q6	/1
Q7	/1
Q8	/1
Q9	/1
Q10	/1
Q11 a)	/2
Q11 b) (i)	/1
Q11 b) (ii)	/1
Q11 c)	/2
Q11 d)	/2
Q11 e)	/2
Q11 f)	/2
Q11 g) (i)	/2
Q11 g) (ii)	/1
Q12 a) (i)	/1
Q12 a) (ii)	/1
Q12 a) (iii)	/2
Q12 a) (iv)	/1
Q12 a) (v)	/2
Q12 b) (i)	/1
Q12 b) (ii)	/2
Q12 c)	/2
Q12 d) (i)	/2
Q12 d) (ii)	/1
Q13 a) (i)	/1
Q13 a) (ii)	/1
Q13 a) (iii)	/2
Q13 b) (i)	/1
Q13 b) (ii)	/2
Q13 b) (iii)	/1
Q13 c)	/3
Q13 d)	/2
Q13 e)	/2

Question	Mark
Q14 a) (i)	/2
Q14 a) (ii)	/1
Q14 a) (iii)	/2
Q14 a) (iv)	/1
Q14 a) (v)	/1
Q14 b) (i)	/1
Q14 b) (ii)	/1
Q14 b) (iii)	/1
Q14 b) (iv)	/1
Q14 c) (i)	/1
Q14 c) (ii)	/2
Q14 c) (iii)	/1
Q15 a)	/3
Q15 b)	/3
Q15 c)	/2
Q15 d) (i)	/1
Q15 d) (ii)	/1
Q15 d) (iii)	/1
Q15 e) (i)	/1
Q15 e) (ii)	/3
Q16 a)	/3
Q16 b) (i)	/1
Q16 b) (ii)	/2
Q16 b) (iii)	/3
Q16 c) (i)	/4
Q16 c) (ii)	/2
Total	**/100**

Section I:

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

Q1. Which of the following is equal to $\frac{1}{\sqrt{5}-1}$?

(A) $\sqrt{5}-1$

(B) $\frac{\sqrt{5}+1}{4}$

(C) $\frac{\sqrt{5}-1}{4}$

(D) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

Q2. The quadratic equation $x^2+2x-3=0$ has roots α and β.
What is the value of $\alpha\beta+(\alpha+\beta)$?

(A) -1

(B) 5

(C) -5

(D) 3

Q3. Which inequality defines the domain of the function $\frac{3}{\sqrt{x-4}}$?

(A) $x>4$

(B) $x\geq 4$

(C) $x<4$

(D) $x\leq 4$

Q4. What is the derivative of $\frac{x}{\sin x}$?

(A) $\frac{x\cos x-\sin x}{\sin^2 x}$

(B) $\frac{\sin x+x\cos x}{\sin^2 x}$

(C) $\frac{\cos x-x\sin x}{\sin^2 x}$

(D) $\frac{\sin x-x\cos x}{\sin^2 x}$

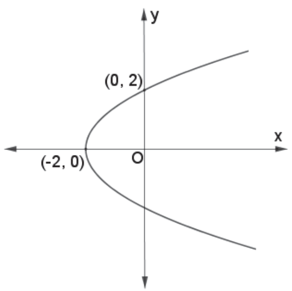
Q5. A parabola passes through the point $(0,2)$ and has its vertex at $(-2,0)$. The equation of the parabola is:

(A) $y^2=-2(x+2)$

(B) $x^2=2(x-2)$

(C) $x^2=-2(x+2)$

(D) $y^2=2(x+2)$



Q6 What is the period of the function $f(x)=\tan\left(\frac{x}{2}\right)$?

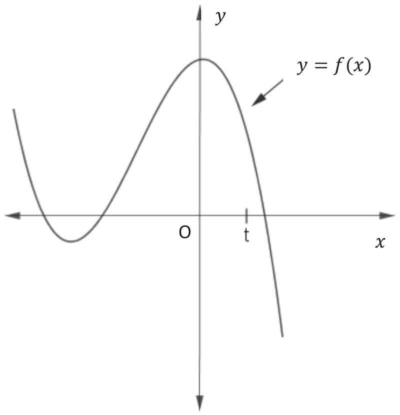
(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 4π

Q7. The diagram shows the graph $y = f(x)$.



Which of the following statements is true for $x = t$?

(A) $f'(t) > 0$ and $f''(t) < 0$

(B) $f'(t) < 0$ and $f''(t) < 0$

(C) $f'(t) > 0$ and $f''(t) > 0$

(D) $f'(t) < 0$ and $f''(t) > 0$

Q8. How many terms are in the series $27 + 40 + 53 + \cdots + 209$?

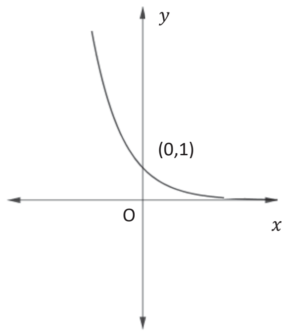
(A) 4

(B) 15

(C) 14

(D) 13

Q9. The graph shown below could be:



(A) $y = 3^x$

(B) $y = (-3)^x$

(C) $y = \left(\frac{1}{3}\right)^x$

(D) $y = \left(-\frac{1}{3}\right)^x$

Q10. A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres.
At a certain time, $\dot{x} = -5\ ms^{-1}$ and $\ddot{x} = 12\ ms^{-1}$.
Which statement describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed.

(B) The particle is moving to the left with increasing speed.

(C) The particle is moving to the right with decreasing speed.

(D) The particle is moving to the left with decreasing speed.

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Evaluate $\dfrac{3\pi(5-\sqrt{2})}{200}$ to three significant figures. **2**

b) Differentiate:

(i) $2e^{2x}$ **1**

(ii) $\log(5x-1)$ **1**

c) Find the exact value of $\log_6 4 + 2\log_6 3$ **2**

d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x\, dx$ **2**

e) Find the equation of the tangent to the curve $y = \ln(2x^2 - 1)$ at the point where $x = 1$ **2**

f) Shade the region denoted by $(x-3)^2 + (y+2)^2 \geq 4$ **2**

g) In a geometric series the 9th term is 96 and the 13th term is 384.

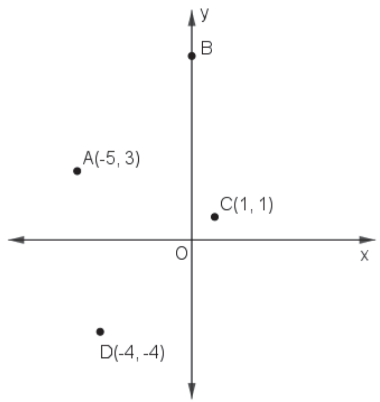
(i) What is the common ratio of the series? **2**

(ii) What is the first term of the series? **1**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) The diagram below shows the points $A(-5,3)$, $C(1,1)$ and $D(-4,-4)$. B is a point on the y-axis.



(i) Find the gradient of AC. **1**

(ii) Find the midpoint M of AC. **1**

(iii) Show that equation of DM is $3x - y + 8 = 0$ **2**

(iv) Find the coordinates of B, given that B lies on the line $3x - y + 8 = 0$ **1**

(v) Hence, show that $ABCD$ is a rhombus. **2**

b) (i) Express the discriminant of $x^2 + (3+k)x + (2k+6) = 0$ in terms of k. **1**

(ii) Hence, find the values of k such that $x^2 - (3+k)x + (2k+6) > 0$ for all values of x. **2**

c) Find the primitive function of $\dfrac{4x}{1+5x^2}$. **2**

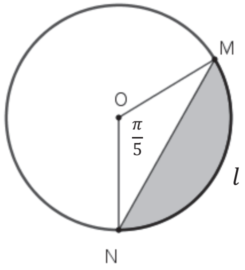
d) (i) Show that $\dfrac{dy}{dx} = -\dfrac{x}{\sqrt{16-x^2}}$, given $y = \sqrt{16-x^2}$ **2**

(ii) Hence, or otherwise, find $\displaystyle\int \frac{4x}{\sqrt{16-x^2}}$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) O is the centre of a circle of radius 20 cm. $\angle \mathrm{MON} = \frac{\pi}{5}$.
l is the minor arc MN as shown in the diagram.

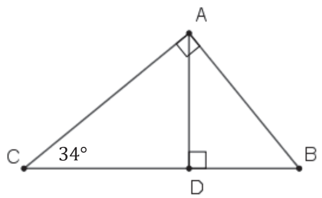


(i) Find the length of l in terms of π. **1**

(ii) Find the area of the minor sector MON. **1**

(iii) Find the shaded area of the segment correct to 2 decimal places. **2**

b)



NOT TO SCALE

Triangle ΔABC has $\angle \mathrm{CAB} = 90°$. D lies on BC where $AD \perp BC$.
Given that $\angle \mathrm{ACD} = 34°$,

(i) Find the size of the angle $\angle \mathrm{CAD}$. Give reasons. **1**

(ii) Prove that ΔDCA and ΔDAB are similar. **2**

(iii) Hence find the length of AD, if $CD = 27cm$ and $BD = 12cm$. **1**

Question 13 continues on page 10

Question 13 (continued)

c) The region bounded by the graph $f(x) = e^x + 3$, the lines $x = 0$, $x = \log_e 3$ and x-axis is rotated about the x-axis to form a solid of revolution. Find the volume generated. **3**

d) A ball is dropped from a height of 4 m and bounces back to a height of 3.6 m. It continues to bounce, each time to a height $\frac{9}{10}$ of the previous height. How far will the ball travel before finally coming to rest. **2**

e) Prove $\dfrac{1 + \tan\theta}{\sec\theta} - \dfrac{\operatorname{cosec}\theta}{\cot\theta + \tan\theta} = \sin\theta$ **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) Consider the curve $f(x) = x^3 - 12x + 5$.

(i) Find any stationary points and determine their nature. **2**

(ii) Find any points of inflexion. **1**

(iii) Sketch this curve showing all key features. **2**

(iv) For what values of x is $f'(x) \leq 0$? **1**

(v) Hence, find the maximum value for $f(x)$ for $-2 \leq x \leq 5$. **1**

b) The displacement of a particle moving along x-axis is given by $x = -3t^2 + 6t + 10$, where t is the time in seconds.

(i) Find the initial displacement of this particle. **1**

(ii) Find the velocity when $t = 1$. **1**

(iii) Show that acceleration of this particle is a constant. **1**

(iv) Describe the motion of this particle. **1**

c) A pool is being drained and volume in litres of water, L, in the pool at time t minutes is given by the equation: $V = 120(40 - t)^2$

(i) Find the initial volume of water in the pool. **1**

(ii) At what rate is the water draining out of the pool when $t = 8$ minutes? **2**

(iii) How long will it take for the pool to be completely empty? **1**

End of Question 14

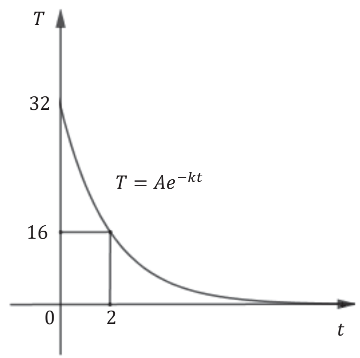
Question 15 (15 marks) Use a SEPARATE writing booklet.

a) Find all solutions of $2cos^2x + \sin x - 2 = 0$, where $0 \leq x \leq 2\pi$ **3**

b) Use Simpson's rule, with five function values, to find an approximate value for $\int_2^6 2^x dx$ round to 2 decimal places. **3**

c) If $y = tan^5x$ show that $\frac{dy}{dx} = 5tan^4x - 5tan^6x$ **2**

d) A ball bearing with initial temperature of $32°C$ is placed in a freezer whose temperature is $0°C$. The temperature of the bearing after t minutes is given by the formula $T = Ae^{-kt}$ and its graph is shown in the diagram.



(i) Find the value of A. **1**

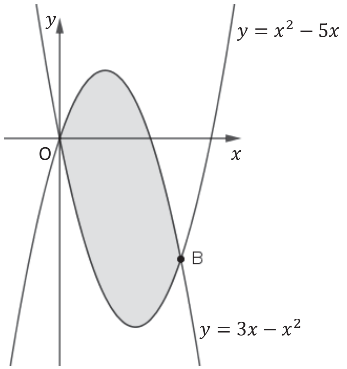
(ii) Find the value of k to 4 decimal places. **1**

(iii) What is the bearing's temperature after 6 minutes? **1**

Question 15 continues on page 13

Question 15 (continued)

e) The diagram shows the parabolas $y = 3x - x^2$ and $y = x^2 - 5x$. The parabolas intersect at the origin and the point B. The region between the two parabolas is shaded.



(i) Find the x-coordinate of the point B. **1**

(ii) Find the area of the shaded region. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) Find the sum of 10 terms of the series $\log_b 5 + \log_b 10 + \log_b 20 + \cdots$ given that $\log_b 5 = 1.46$ and $\log_b 2 = 0.63$. **3**

b) Andrew borrows $\$650000$ from a bank. The loan is to be repaid in 20 years. The interest rate is $6\%\ p.a.$ compounded monthly. There is no repayments for the first three months. Let A_n be the amount owing after n months and M be the monthly repayments.

(i) Find an expression for A_4. **1**

(ii) Show that $A_5 = 650000(1.005)^5 - M(1 + 1.005)$ **2**

(iii) Find the monthly repayments if the loan is to be repaid in 20 years. **3**

c) Two particles A and B start moving along the x-axis at time $t = 0$.
Particle A is initially at $x = 6$ and its velocity at time t is given by $v_A = 3t^2 - 20t + 16$.
Particle B is at $x = 25$ when $t = 2$ and its velocity at time t is given by $v_B = 3t^2 + 1$.

(i) Find expressions for the positions of particles A and B at time t. **4**

(ii) Show that these two particles never meet. **2**

End of Paper

Student Number:____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

correct

A ✗ B ✗ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

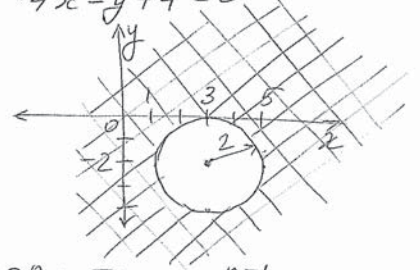
2018 Mathematics Trial Solutions

Section I	Multiple-Choice	
Q1	B $\frac{1}{\sqrt5-1} = \frac{\sqrt5+1}{(\sqrt5-1)(\sqrt5+1)} = \frac{\sqrt5+1}{5-1} = \frac{\sqrt5+1}{4}$	1 mark
Q2	C $x^2+2x-3=0$ $\alpha\beta=-3 \quad \alpha+\beta=-2$ $-3-2=-5$	1 mark
Q3	A $f(x)=\frac{3}{\sqrt{x-4}}$ $x-4>0 \quad \therefore x>4$	1 mark
Q4	D $\left(\frac{x}{\sin x}\right)' = \frac{x'\sin x - x(\sin x)'}{\sin^2 x} =$ $= \frac{\sin x - x\cos x}{\sin^2 x}$	1 mark
Q5	D parabola $(y-k)^2=4a(x-h)$ $y^2=4a(x+2)$ at $(0,2)$ $4=4a\times2 \quad \therefore a=\frac12$ $y^2=2(x+2)$	1 mark
Q6	C $f(x)=\tan(\frac{x}{2})$ period of $f(x)=\tan x$ is π $\frac{x}{2}=\pi \quad \therefore x=2\pi$	1 mark

2018 Mathematics Trial Solutions

Section I	Multiple – Choice	
Q7	B at the point $x = t$ $f'(t) < 0$ $f(x)$ is decreasing as x increases, gradient is negative $f''(t) < 0$ graph is concave down, the gradient of the function is decreasing	1 mark
Q8	B AP: $a = 27$, $d = 13$ $T_n = 209$, $T_n = a + (n-1)d$ $209 = 27 + 13(n-1)$ $n = 15$	1 mark
Q9	C $y = \left(\frac{1}{3}\right)^x = 3^{-x}$ exponential decay	1 mark
Q10	D velocity is negative so particle is moving to the left Acceleration is positive and since particle is moving to the left it is slowing down	1 mark

2018 Mathematics Trial Solutions

Section I	Question 11	
a)	$\frac{3\pi(5-\sqrt{2})}{200} = 0.168976\ldots =$ $= 0.169$	1 mark – correct decimal 2 marks – correct to 3 s.f.
b)(i)	$(2e^{2x})' = 4e^{2x}$	1 mark
(ii)	$(\log(5x-1))' = \frac{5}{5x-1}$	1 mark
c)	$\log_6 4 + 2\log_6 3 = \log_6 4 + \log_6 9 =$ $= \log_6(4 \times 6) = \log_6 36 = 2$	1 mark to get $\log_6(4 \times 9)$ 2 marks – correct answer
d)	$\int_0^{\frac{\pi}{4}} \cos 2x\,dx = \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}} =$ $= \frac{1}{2}\left(\sin\left(2 \times \frac{\pi}{4}\right) - \sin 0\right) =$ $= \frac{1}{2}\left(\sin\frac{\pi}{2} - 0\right) = \frac{1}{2}$	1 mark – correct integration 2 marks – correct solution
e)	$y = \ln(2x^2 - 1)$ at $x = 1$ $\frac{dy}{dx} = \frac{4x}{2x^2-1}$ At $x = 1$ $\frac{dy}{dx} = \frac{4}{1} = 4$ The gradient is 4 $y - 0 = 4(x-1)$ $y = 4x - 4$ $4x - y + 4 = 0$	1 mark – correct gradient 2 marks – correct solution
f)		1 mark – correct graph of circle 2 marks – correct region
g)(i)	GP: $T_n = ar^{n-1}$ $T_9 = a \times r^8 = 96$ $T_{13} = ar^{12} = 384$ $\frac{T_{13}}{T_9} = r^4 = 4$ $\therefore r = \pm\sqrt{2}$	1 mark for correct expressions for both T_9 and T_{13} 2 marks – correct ratio
(ii)	$T_n = ar^{n-1}$ $96 = a(\sqrt{2})^8$ or $96 = a(-\sqrt{2})^8 = a \times 16$ $\therefore a = 6$	correct answer

Section I — Question 12

a) (i) Gradient of $AC = \frac{3-1}{-5-1} = -\frac{2}{6} = -\frac{1}{3}$ — 1 mark

(ii) Midpoint $M = \left(\frac{-5+1}{2}, \frac{3+1}{2}\right) = (-2, 2)$ — 1 mark

(iii) Use $y - y_1 = m(x - x_1)$

$m = \frac{-4-2}{-4-(-2)} = \frac{-6}{-2} = 3, \quad m = 3$

$\therefore y - 2 = 3(x + 2)$

$y - 2 = 3x + 6$

$3x - y + 8 = 0$

1 mark - substitute correct coordinates and gradient
2 marks - correct equation

(iv) Substitute $x = 0$ in equation $3x - y + 8 = 0 \quad \therefore y = 8$

$B(0, 8)$

1 mark correct y-coordinate

(v) Midpoint of $BD = \left(\frac{0+(-4)}{2}, \frac{8+(-4)}{2}\right) = (-2, 2)$ — 1 mark

Since B and D both lie on perpendicular bisector of AC and the midpoint of BD is equal to the midpoint of AC, then the diagonals AC and BD bisect each other at right angles.

$\therefore$ ABCD is a rhombus

2 marks for correct proof.

2018 Mathematics Trial Solutions

Section I — Question 12

b) (i) $x^2 + (3+k)x + (2k+6) = 0$

$\Delta = b^2 - 4ac$

$(3+k)^2 - 4 \times 1(2k+6) =$

$= 9 + 6k + k^2 - 8k - 24 =$

$= k^2 - 2k - 15$

1 mark for correct discriminant

(ii) $\Delta < 0$

$k^2 - 2k - 15 < 0$

$(k-5)(k+3) < 0$

$k = -3, \; k = 5$

$-3 < k < 5$

1 mark for correct factorising
2 marks - correct answer

c) $f(x) = \frac{4x}{1+5x^2}$

$\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + C$

$\int \frac{4x}{1+5x^2} = \frac{4}{10}\log(1+5x^2) + C =$

$= \frac{2}{5}\log(1+5x^2) + C$

1 mark for integration and getting $\log(1+5x^2)$
2 marks - correct solution

d) (i) $y = \sqrt{16-x^2} = (16-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(16-x^2)^{-\frac{1}{2}} \times (-2x) =$

$= -\frac{x}{\sqrt{16-x^2}}$

1 mark for correctly using the chain rule
2 marks - correct solution

(ii) from (i)

$\int \frac{4x}{\sqrt{16-x^2}}\,dx = -4\sqrt{16-x^2} + C$

1 mark - correct solution

2018 Mathematics Trial Solutions

Section I	Question 13	
a) (i)	Arc $l = r\theta = 20 \times \frac{\pi}{5} = 4\pi$ cm	1 mark
(ii)	$A_{MON} = \frac{1}{2}r^2\theta = \frac{1}{2}\, 400 \times \frac{\pi}{5} = 40\pi \text{ cm}^2$	1 mark
(iii)	$A_{segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta =$ $= 40\pi - \frac{1}{2} 400 \times \sin\frac{\pi}{5} = 8.106\ldots =$ $= 8.11 \text{ cm}^2$	1 mark - correct expression for area of segment 2 marks - correct answer
b) (i)	$\angle CAD + \angle ACD + \angle ADC = 180°$ (angle sum of a triangle) $\angle CAD = 180° - 90° - 34° = 56°$	1 mark - correct answer
(ii)	In $\triangle DCA$ and $\triangle DAB$ $\angle DAB + \angle CAD = 90°$ (complementary angles) $\angle DAB = 90° - 56° = 34°$ $\angle DAB = \angle ACD$ $\angle ADC = \angle ADB = 90°$ $\therefore \triangle DCA$ is similar to $\triangle DAB$ (equiangular)	1 mark - to prove $\angle DAB = \angle ACD = 34°$ 2 marks - correct proof and reasoning
(iii)	$\frac{CD}{AD} = \frac{AD}{BD}$ (corresponding sides of similar triangles are in proportion) $AD^2 = 27 \times 12 = 324 \therefore AD = 18$ cm	1 mark - correct answer

2018 Mathematics Trial Solutions

Section I	Question 13	
c)	$f(x) = e^x + 3, \quad x = 0, x = \log_e 3$ $\int_0^{\log 3} \pi (e^x + 3)^2 dx =$ $= \pi \int_0^{\log_e 3} (e^{2x} + 6e^x + 9)\, dx =$ $= \pi \left[\frac{e^{2x}}{2} + 6e^x + 9x\right]_0^{\log_e 3} =$ $= \pi \left[\frac{e^{2\log_e 3}}{2} + 6e^{\log_e 3} + 9\log 3\right] -$ $- \pi\left[\frac{1}{2} + 6\right] =$ $= \pi\left[\frac{9}{2} + 6 \times 3 + 9\log 3\right] - \frac{13}{2}\pi =$ $= \pi\left[\frac{9}{2} + 18 - \frac{13}{2} + 9\log 3\right] =$ $= \pi\left[16 + 9\log_e 3\right]$	1 mark to correct formula (integral) of volume 2 marks for correct integration 3 marks for correct solution
d)	Total distance $=$ $= 4.0 + 2(3.6 + 3.24 + \ldots)$ m Infinite geometric series $a = 3.6, \quad r = 0.9$ $S = \frac{a}{1-r} = \frac{3.6}{0.1} = 36$ m Total distance $= 4 + 2 \times 36 = 76$m	1 mark for correct limiting sum 2 mark - for correct solution
e)	$\frac{1+\tan\theta}{\sec\theta} - \frac{\csc\theta}{\cot\theta + \tan\theta} = \sin\theta$ LHS: $\frac{1 + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} - \frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}} =$ $= \frac{\frac{\cos\theta + \sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} - \frac{\frac{1}{\sin\theta}}{\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}} =$ $= \cos\theta + \sin\theta - \frac{\sin\theta\cos\theta}{\sin\theta} =$	1 mark for substituting $\sin\theta$ and $\cos\theta$ correctly 2 mark for correct proof

$= \cos\theta + \sin\theta - \cos\theta = \sin\theta$ (RHS)

2018 Mathematics Trial Solutions

Section I — Question 14

a)(i) Stationary points — *1 mark for stat. points $(-2, 21)$, $(2, -11)$*

$f'(x) = 0 \quad x^3 - 12x + 5 = 0$

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0 \quad 3(x^2 - 4) = 0 \therefore$

$x = -2, 2$

$f(-2) = (-2)^3 - 12(-2) + 5 = 21$

$f(2) = 2^3 - 12 \times 2 + 5 = -11$

test the nature of stat. points — *2 marks for identifying their nature correctly*

x	-3	-2	-1	2	3
$f'(x)$	15 increas	0	-9	0	15
sketch	/	—	\	—	/

so $(-2, 21)$ - local max

$(2, -11)$ - local min

(ii) $f''(x) = 0$ and $f''(x)$ changes sign — *1 mark for correct point of inflexion*

$f''(x) = 6x$

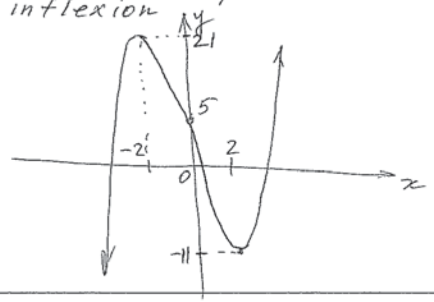
$6x = 0 \therefore x = 0$

x	-1	0	1
$f''(x)$	-6 concave down	0	$+6$ concave up

$f(0) = 5$

So $(0, 5)$ is a point of inflexion

(iii) — *1 mark - correct features; 2 mark - correct sketch*



2018 Mathematics Trial Solutions

Section I — Question 14

a)(iv) $f'(x) < 0$ - curve is decreasing (from graph) — *1 mark*

$\therefore -2 \le x \le 2$

(v) when $-2 \le x \le 5$ — *1 mark*

$f(5) = 5^3 - 12 \times 5 + 5 = 70$

$f(-2) = 21$

$f(5) > f(-2)$ So

$f(5)$ - is a max. value

b) (i) $t = 0 \quad x = 10$ — *1 mark*

(ii) $\dot{x} = -6t + 6 \quad t = 1 \quad \dot{x} = 0$ — *1 mark*

(iii) $\ddot{x} = -6$ - is a constant — *1 mark*

(iv) The particle is initially at a position 10m to the right of O(origin) and travelling to the right. It's slowing down and comes to rest after 1sec. It then moves back to the left. — *1 mark*

c) (i) $V = 120(40 - t)^2$ — *1 mark*

at $t = 0 \quad V = 120 \times 40^2 = 192000L$

(ii) $\frac{dV}{dt} = 120 \times 2(40 - t) \times (-1) = 240t - 9600$ — *1 mark - correct rate of change formula*

$t = 8 \quad \frac{dV}{dt} = 240 \times 8 - 9600 = -7680 \, L/min$ — *2 marks - correct rate*

(iii) $240t - 9600 = 0$ — *1 mark correct time*

$240t = 9600 \quad t = 40$ min

2018 Mathematics Trial Solutions

Section I — Question 15

a)

$2\cos^2 x + \sin x - 2 = 0$

$2(1-\sin^2 x) + \sin x - 2 = 0$

$2 - 2\sin^2 x + \sin x - 2 = 0$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0 \quad \therefore x = 0, \pi, 2\pi$

$\sin x = \frac{1}{2} \quad \therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

1 mark – correct equations of $\sin x$: $\sin x = 0$, $\sin x = \frac{1}{2}$

2 marks – one correct solution.

3 marks – all solutions provided.

b)

$\int_2^6 2^x\,dx = \int_2^4 2^x\,dx + \int_4^6 2^x\,dx =$

$= \frac{4-2}{6}\left[2^2 + 4(2^3) + 2^4\right] +$

$+ \frac{6-4}{6}\left[2^4 + 4(2^5) + 2^6\right] \doteq \frac{260}{3}$

$= 86.67$ (2 d.p.)

1 mark – correct function values

2 marks – correct use of Simpson's rule with correct values

3 marks – correct solution

c)

$y = \tan^5 x$

$\frac{dy}{dx} = \frac{5\tan^4 x}{1} \times \sec^2 x =$

$= 5\tan^4 x \times (1 + \tan^2 x) =$

$= 5\tan^4 x + 5\tan^6 x$

1 mark – for applying the chain rule correctly

2 marks – correct solution

d)(i)

When $t = 0$, $T = Ae^0 = A$, i.e. initial temperature is A degrees

From the graph, $A = 32°C$

1 mark

(ii)

$T = 32e^{-kt}$

When $t = 2$, $T = 16$

$16 = 32e^{-2k}$

$0.5 = e^{-2k}$

$\ln 0.5 = -2k \quad \therefore k = -\frac{1}{2}\ln 0.5 = 0.3466$

1 mark (correct answer)

(iii)

$T = 32e^{-k \times 6} = 4°C$

1 mark

2018 Mathematics Trial Solutions

Section I — Question 15

e)(i)

$3x - x^2 = x^2 - 5x$

$2x^2 - 8x = 0$

$2x(x-4) = 0$

$x = 0, \quad x = 4$

$x = 0$ is origin, so $x = 4$ is a solution

1 mark for correct answer

(ii)

$\text{Area} = \int_0^4 \left((3x - x^2) - (x^2 - 5x)\right)dx$

$= \int_0^4 (-2x^2 + 8x)\,dx =$

$= \left[-\frac{2}{3}x^3 + 4x^2\right]_0^4 =$

$= -\frac{2}{3} \times 64 + 64 = 64 - \frac{128}{3} = \frac{64}{3}$

1 mark for correct formula

2 marks for correct integration

3 marks – correct solution

Section I	Question 16	
a)	$\log_b 5 + \log_b 10 + \log_b 20 + \ldots$ AP: $a = \log_b 5$, $d = \log_b 10 - \log_b 5 = \log_b 2$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $S_{10} = \frac{10}{2}(2\log_b 5 + (10-1)\log_b 2) =$ $= 5(2\times 1.46 + 9\times 0.63) = 42.95$	1 mark - correct 1st term and common difference 2 marks - substituting all values correctly 3 marks correct solution
b)(i)	$A_1 = 650000(1.005)$ $A_2 = 650000(1.005)^2$ $A_3 = 650000(1.005)^3$ $A_4 = 650000(1.005)^4 - M$	1 mark
(ii)	$A_4 = 650000(1.005)^4 - M$ $A_5 = [650000(1.005)^4 - M]\times 1.005 - M =$ $= 650000(1.005)^5 - M\times 1.005 - M =$ $= 650000(1.005)^5 - M(1 + 1.005)$	1 mark 2 marks
(iii)	$A_{240} = 0 = 650000(1.005)^{240} -$ $- M(1 + 1.005 + \ldots + 1.005^{236})$ $0 = 650000(1.005)^{240} - M\left[\frac{1.005^{237} - 1}{1.005 - 1}\right]$ $M\left[\frac{1.005^{237} - 1}{0.005}\right] = 650000(1.005)^{240}$ $M = \frac{650000(1.005)^{240}\times 0.005}{1.005^{237} - 1} =$ $= \$4758.05$	1 mark - showing geometric series 2 marks - correct use of sum to n terms of GP formula 3 marks - correct solution

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Section I	Question 16	
c)(i)	$x_A = \int(3t^2 - 20t + 16)dt =$ $= t^3 - 10t^2 + 16t + C_1$ When $t = 0$ $x = 6$: $6 = 0 - 0 + 0 + C_1$ $\therefore C_1 = 6$ $x_A = t^3 - 10t^2 + 16t + 6$ $x_B = \int(3t^2 + 1)dt = t^3 + t + C_2$ when $t = 2$ $x = 25$ $25 = 8 + 2 + C_2$ $\therefore C_2 = 15$ $x_B = t^3 + t + 15$	1 mark - correct integration to find a position for particle A 2 marks - for correct position of one particle 3 marks - correct substitution of t and x 4 marks for correct expressions for both particles
(ii)	$x_A = x_B$ $t^3 - 10t^2 + 16t + 6 = t^3 + t + 15$ $-10t^2 + 15t - 9 = 0$ $10t^2 - 15t + 9 = 0$ $\Delta = b^2 - 4ac$ $\Delta = 225 - 4\times 10\times 9 = -135$ $\Delta < 0$ Since $10t^2 - 15t + 9 = 0$ has $\Delta < 0$, it has no real roots, i.e. $x_A \neq x_B$ (the particles never meet)	1 mark - Correct quadratic equation 2 marks - negative discriminant and correct reasoning.