Blacktown Boys' High School

2019

HSC Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- All diagrams are not drawn to scale
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 13)
- Attempt Questions 11 - 16
- Allow about 2 hours 45 minutes for this section

Assessor: Villanueva

Student Name: ______________________________________

Teacher Name: ______________________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2019 Higher School Certificate Examination.

Office Use Only

Question	Mark
Q1	/1
Q2	/1
Q3	/1
Q4	/1
Q5	/1
Q6	/1
Q7	/1
Q8	/1
Q9	/1
Q10	/1
Q11 a)	/1
Q11 b)	/2
Q11 c)	/2
Q11 d)	/2
Q11 e)	/2
Q11 f)	/2
Q11 g)	/2
Q11 h)	/2

Question	Mark
Q12 a)	/1
Q12 b)	/5
Q12 c)	/2
Q12 d)	/2
Q12 e)	/5
Q13 a)	/4
Q13 b)	/4
Q13 c)	/7
Q14 a)	/6
Q14 b)	/3
Q14 c)	/6
Q15 a)	/5
Q15 b)	/6
Q15 c)	/4
Q16 a)	/3
Q16 b)	/4
Q16 c)	/8
TOTAL	**/100**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet provided for Questions 1–10.

Q1. Which one of the following is equivalent to 3.507×10^{-3}?

A. 3507×1000

B. 35.07×100

C. $0.3507 \div 100$

D. $3507 \div 100$

Q2. What is the angle of inclination to the nearest minute, of the line $3x + 2y = 7$ with the positive direction of the x axis?

A. $33°41'$

B. $56°19'$

C. $123°41'$

D. $146°19'$

Q3. The first and last terms of an arithmetic series are 10 and 60. If the sum of the series is 3535, how many terms are there in the series?

A. 11

B. 101

C. 110

D. 51

Q4. A tennis player has two serves. If the first serve is out of court, the player has a second serve. The first serve has a probability of 0.5 of going into play, and the second serve has a probability of 0.9 of going into play. The probability of getting one of the serves into play is:

A. $\frac{1}{20}$

B. $\frac{9}{20}$

C. $\frac{11}{20}$

D. $\frac{19}{20}$

Q5. The table below shows the values of a function $f(x) = \sqrt{25 - x^2}$ for six values of x.

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	5.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25 - x^2}\, dx$ using the trapezoidal rule with these six function values.

A. 10.74

B. 12.65

C. 19.98

D. 37.96

Q6. What is the derivative of $\frac{4}{3x^3}$?

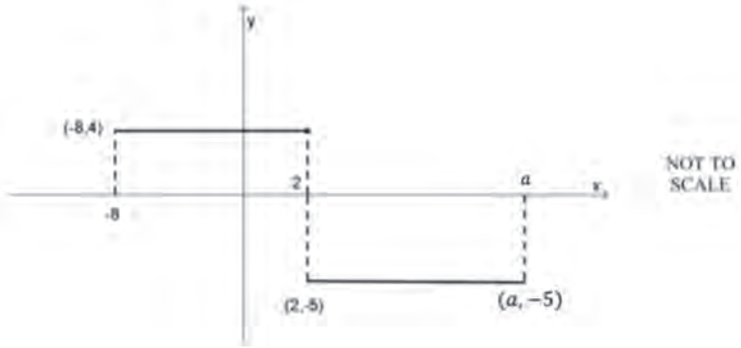
A. $-\frac{4}{x^4}$

B. $-\frac{2}{3x^2}$

C. $-\frac{4}{x^2}$

D. $-\frac{36}{x^4}$

Q7. Using the graph of $y = f(x)$ below, determine the value of a which satisfies the condition: $\int_{-8}^{a} f(x)\,dx = 0$. $(a, -5)$



A. 12

B. 10

C. 8

D. 4

Q8. What is the value of $\int (\sec^2 \pi x)\,dx$?

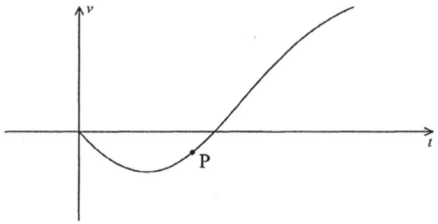
A. $\frac{1}{\pi}\tan \pi x + C$

B. $\tan \pi x + C$

C. $\pi \tan \pi x + C$

D. $\tan^2 \pi x + C$

Q9. The graph shows the velocity of a particle moving along a straight line as a function of time.



Which statement describes the motion of the particle at point P.

A. The particle is moving left at increasing speed.

B. The particle is moving left at decreasing speed.

C. The particle is moving right at increasing speed.

D. The particle is moving right at decreasing speed.

Q10. Let α and β be the roots of the equation $2x^2 - 5x - 9 = 0$. The value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

A. $\frac{5}{2}$

B. $-\frac{9}{5}$

C. $-\frac{9}{2}$

D. $-\frac{5}{9}$

End of Section I

Section II

90 Marks
Attempt Questions 11- 16
Allow about 2 hours and 45 minutes for this section.

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Simplify $5x - (7 + 4x)$. **1**

b) Factorise fully $3x^2 - 75$. **2**

c) Express $\dfrac{4}{3 - \sqrt{10}}$ with a rational denominator. **2**

d) Find the limiting sum of the geometric series $-2 + \dfrac{2}{5} - \dfrac{2}{25} + \cdots$. **2**

e) Differentiate $(e^{10x} + 2x)^4$ **2**

f) Differentiate $y = (x + 5)\ln x$. **2**

g) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos x \, dx$. **2**

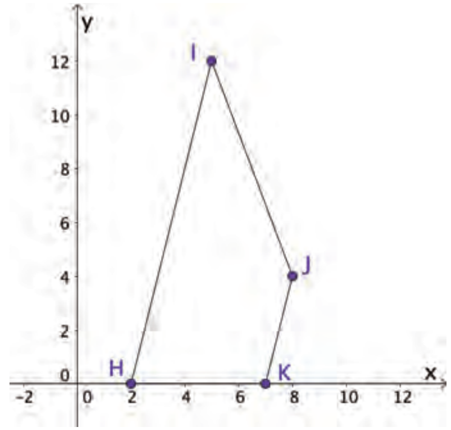
h) Find $\displaystyle\int \frac{x}{x^2 + 2}\, dx$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find the solutions of $2\cos\theta = 1$ for $0 \le \theta \le 2\pi$. **1**

b) The diagram shows the points $H(2, 0)$, $I(5, 12)$, $J(8, 4)$ and $K(7, 0)$ on a number plane.



i) Prove that $HIJK$ is a trapezium. **1**

ii) Show that the equation of HI is $4x - y - 8 = 0$. **1**

iii) Find the perpendicular distance from K to HI. **1**

iv) Hence, or otherwise, find the area of the trapezium $HIJK$. **2**

c) Find $f'(0)$, when $f(x) = \dfrac{x^2 + 2}{x - 1}$. **2**

d) For what values of k does the quadratic equation does the equation $x^2 + (k + 3)x + 9 = 0$, have distinct real roots? **2**

e) The equation of a parabola is given by $y = x^2 - 2x + 5$.

i) Find the coordinate of its vertex. **2**

ii) State the focal length of the parabola. **1**

iii) Find the equation of the normal to this parabola at the point $P(2, 5)$. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Andrew (A) and Bill (B) leave from point O at the same time. Andrew travels at $20km/h$ along a straight road in the direction $085°T$. Bill travels at $25km/h$ along another straight road in the direction $340°T$.

Draw a diagram to represent this information.

i) Show that $\angle AOB$ is $105°$ where $\angle AOB$ is the angle between the directions taken by Andrew and Bill. **1**

ii) Find the distance Andrew and Bill are apart to the nearest kilometre after two hours. **3**

b) i) Find the domain and range for the function $f(x) = -\sqrt{16 - x^2}$. **2**

ii) On a number plane, shade the region where it satisfies the inequalities $y \geq -\sqrt{16 - x^2}$, $y \geq x$, and $y \leq 0$. **2**

c) Given the equation

$$y = 2x^3 - 9x^2 - 60x$$

i) Find any stationary points and determine their nature. **3**

ii) Find any points of inflexion. **2**

iii) Hence sketch the curve, clearly indicating the y-intercepts, stationary points and points of inflexion. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) The velocity in m/s of a particle travelling in a straight line is given by $v = 4t^3 - 6t$ where t is the time taken in seconds. The particle is initially 4 metres to the left of the origin.

i) Find its initial velocity. **1**

ii) Write an expression for the acceleration $a\ m/s^2$ in terms of t. **1**

iii) Write an expression for the displacement $x\ m$ in terms of t. **2**

iv) When is the particle at the origin? **2**

b) There are five candidates, Albert, Ben, Charles, Dennis and Eric standing for the seat of Blacktown in the federal election. Each of their names are written on separate pieces of paper and randomly drawn from a barrel to determine their positions on the ballot paper. The candidate picked first goes at the top of the list.

i) What is the probability that Dennis is drawn first? **1**

ii) What is the probability that the order of the names appear on the ballot paper is as follows? **2**

Albert
Ben
Charles
Dennis
Eric

Question 14 continues on page 11

Question 14 (continued)

c) Jason borrows $750 000 to purchase his first home. He takes out a loan over 30 years, to be repaid in equal monthly instalments. The interest rate is 4.8% per annum, calculated monthly. Let A_n be the amount owing at the end of n months and M be the monthly repayment.

i) Show that $A_2 = 750\,000(1.004)^2 - M(1 + 1.004)$. **1**

ii) Show that $A_n = 750\,000(1.004)^n - M\left(\frac{(1.004)^n - 1}{0.004}\right)$. **1**

iii) Find the monthly repayment required to repay the loan in 30 years. **2**

iv) Jason wants to pay off the loan in less than 30 years. If he can afford to pay $5000 per month, how many months will it take him to pay off the home loan? **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) The number of a type of bacteria grows according to the equation

$$N(t) = Ae^{0.15t}$$

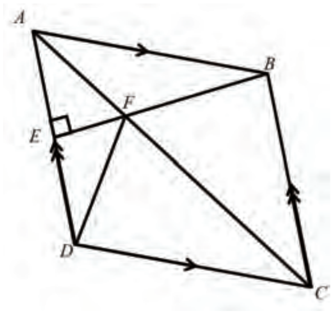
where t is measured in days and A is a constant.

i) Show that the number of bacteria increases at a rate proportional to the number present. **2**

ii) When $t = 3$ the number of bacteria estimated was 1.5×10^8. Evaluate A, correct to 2 significant figures. **1**

iii) The number of bacteria doubles every x days. Find x. Answer correct to 1 decimal place. **2**

b) ABCD is a rhombus, BE is perpendicular to AD and intersects AC at F. AC is one of the diagonals of ABCD. Copy or trace the diagram into your writing booklet.



i) Explain why $\angle BCA = \angle DCA$. **1**

ii) Prove that ΔBFC and ΔDFC are congruent. **3**

iii) Hence, or otherwise, find the size of $\angle FDC$. **2**

Question 15 continues on page 13

Question 15 (continued)

c) An excavation site has been flooded due to recent wet weather. The water is pumped out so that the building can commence. The rate $\frac{dV}{dt}$ at which the water is being pumped out in kilolitres per hour is given by

$5 - \frac{1}{1+2t}$ where $t \geq 0$.

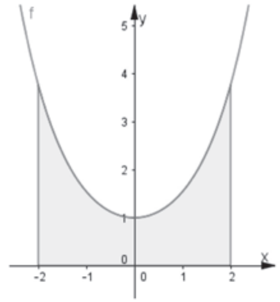
i) Find the initial rate at which the water is being pumped out of the excavation site. **1**

ii) Calculate the total amount of water pumped out during the first 2 hours. Give your answer to the nearest litre. **3**

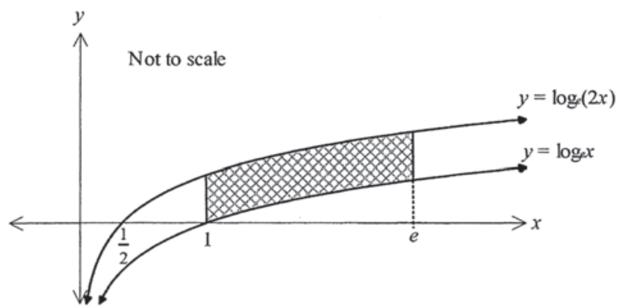
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) Calculate the volume of the solid of the revolution when the function $f(x) = \frac{1}{2}(e^x + e^{-x})$ is rotated about the x-axis between the ordinates $x = -2$ and $x = 2$. **3**



b) The curves $y = \log_e x$ and $y = \log_e(2x)$ are drawn below. **4**

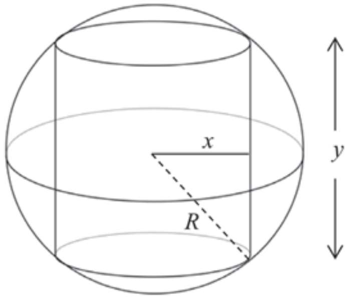


Find the shaded area between the curves $y = \log_e x$ and $y = \log_e(2x)$ and the lines $x = 1$ and $x = e$.

Question 16 continues on page 15

Question 16 (continued)

c) A right cylinder of radius x and height y is inscribed in a sphere of radius R, where R is a constant.



The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

The volume of a right cylinder with radius r and height h is $\pi r^2 h$.

i) Show that the volume of the cylinder V, in the diagram above can be written as **2**

$V = \pi R^2 y - \frac{\pi y^3}{4}$.

ii) Prove that the maximum volume of the cylinder V occurs when **3**

$y = \frac{2R}{\sqrt{3}}$

iii) Find the maximum volume of the cylinder V in terms of R in simplest form. **2**

iv) When the cylinder has a maximum volume, show that the ratio of the volume of the cylinder to the volume of the sphere is $1:\sqrt{3}$. **1**

End of Paper

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Student Name:____

Multiple Choice Answer Sheet

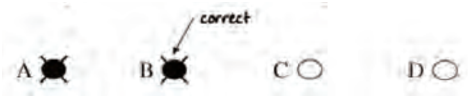
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.



Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

2019 Yr 12 Advanced Trial Solutions

Section I

1) C
2) C
3) B
4) D
5) C
6) A
7) B
8) A
9) B
10) D

Section II

Question 11

a) $5x - (7 + 4x)$

$5x - 7 - 4x$

$x - 7$ ✓

b) $3x^2 - 75$

$3(x^2 - 25)$ ✓

$3(x-5)(x+5)$ ✓

c) $\dfrac{4}{3-\sqrt{10}} \times \dfrac{3+\sqrt{10}}{3+\sqrt{10}}$ ✓ $= \dfrac{4(3+\sqrt{10})}{3^2 - \sqrt{10}^2}$

$= \dfrac{12 + 4\sqrt{10}}{9 - 10}$

$= -12 - 4\sqrt{10}$ ✓

d) $-2 + \frac{2}{5} - \frac{2}{25} + \ldots$

$S_\infty = \dfrac{a}{1-r}$ $\quad r = \dfrac{2/5}{-2} = -\frac{1}{5}$ ✓

$S_\infty = \dfrac{-2}{1 + 1/5}$

$S_\infty = -\frac{5}{3}$ ✓

e) $\frac{d}{dx}(e^{10x} + 2x)^4$

$= 4(10e^{10x} + 2)(e^{10x} + 2x)^3$ ✓ ✓

f) $y = (x+5)\ln x$

$y' = uv' + vu'$

$u = x+5 \qquad v = \ln x$

$u' = 1 \qquad v' = 1/x$ ✓

$y' = \dfrac{x+5}{x} + \ln x$

$y' = \dfrac{x}{x} + \dfrac{5}{x} + \ln x$ ✓

$y' = \dfrac{5}{x} + \ln x + 1$

g) $\displaystyle\int_0^{\pi/4} \cos x \, dx$

$= \left[\sin x\right]_0^{\pi/4}$ ✓

$= \sin\frac{\pi}{4} - \sin 0$

$= \frac{1}{\sqrt{2}}$ ✓

h) $\int \frac{x}{x^2+2}\,dx$

$= \frac{1}{2}\int \frac{2x}{x^2+2}\,dx$ ✓

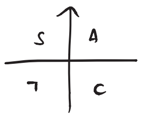
$= \frac{1}{2}\ln|x^2+2| + C$ ✓

Question 12

a) $2\cos\theta = 1$ $0 \le \theta \le 2\pi$

$\cos\theta = 1/2$

$\theta = \pi/3,\ 5\pi/3$ ✓



b) i) HIJK is a trapezium

$m_{HI} = \frac{12-0}{5-2}$ $m_{JK} = \frac{0-4}{7-8}$

$m_{HI} = \frac{12}{3}$ $m_{JK} = \frac{-4}{-1}$

$m_{HI} = 4$ $m_{JK} = 4$ ✓

$\therefore$ HI $\parallel$ JK

$\therefore$ HIJK is a trapezium with one pair of sides parallel to each other.

ii) HI: $4x - y - 8 = 0$ H(2,0)

$y - y_1 = m(x - x_1)$

$y - 0 = 4(x-2)$

$y = 4x - 8$ ✓

$\therefore 4x - y - 8 = 0$

iii) K ⊥ distance to HI K(7,0)

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$d = \frac{|4(7) + (-1)(0) + (-8)|}{\sqrt{4^2 + (-1)^2}}$

$d = \frac{20}{\sqrt{17}}$

$d = \frac{20\sqrt{17}}{17}$ ✓

iv) $\text{Area}_{HIJK} = \frac{h}{2}(a+b)$

$d_{HI} = \sqrt{(5-2)^2 + (12-0)^2}$

$= \sqrt{153}$

$= 3\sqrt{17}$

$d_{JK} = \sqrt{(8-7)^2 + (4-0)^2}$

$= \sqrt{17}$

$$\text{Area} = \frac{1}{2} \times \frac{20}{\sqrt{17}} \times \left(3\sqrt{17} + \sqrt{17}\right)$$

$$= \frac{20 \times \overset{2}{\cancel{4}}\,\cancel{\sqrt{17}}}{\cancel{2}\,\cancel{\sqrt{17}}}$$

$$= 40\ u^2$$

c) $f(x) = \dfrac{x^2+2}{x-1}$

$u = x^2+2 \qquad v = x-1$

$u' = 2x \qquad v' = 1$

$$f'(x) = \frac{(x-1)2x - (x^2+2)}{(x-1)^2}$$

$$f'(x) = \frac{2x^2 - 2x - x^2 - 2}{(x-1)^2}$$

$$f'(x) = \frac{x^2-2x-2}{(x-1)^2}$$

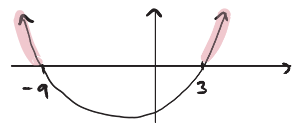
$$f'(0) = \frac{0-0-2}{(-1)^2}$$

$$f'(0) = -2$$

d) $x^2 + (k+3)x + 9 = 0$

For distinct real roots $\Delta > 0$

$$\Delta = b^2 - 4ac$$
$$= (k+3)^2 - 4(1)(9)$$
$$= k^2 + 6k + 9 - 36$$
$$= k^2 + 6k - 27$$
$$= (k-3)(k+9)$$



$(k-3)(k+9) > 0$

$\therefore\ k < -9, \quad k > 3$

e) $y = x^2 - 2x + 5$

i) axis of symmetry

$$x = -\frac{b}{2a}$$

$$x = \frac{2}{2}$$

$$x = 1$$

$$y = 1^2 - 2(1) + 5$$
$$= 1 - 2 + 5$$
$$= 4$$

$\therefore$ Vertex $(1, 4)$

ii) $y = x^2 - 2x + 5$

$y - 5 + 1 = x^2 - 2x + 1$

$y - 4 = (x-1)^2$

$y = (x-1)^2 + 4$

focal length $= 1/4$ ✓

iii) $y = x^2 - 2x + 5$

$dy/dx = 2x - 2$

at $x = 2$, $dy/dx = 2(2) - 2$

$= 2$

$\therefore m_{tangent} = 2$

$m_{tangent} \times m_{normal} = -1$

$\therefore m_{normal} = -1/2$ ✓

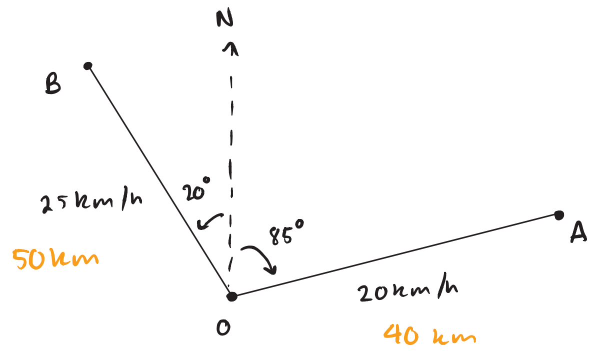
$y - 5 = -\frac{1}{2}(x - 2)$

$2y - 10 = -x + 2$

$x + 2y - 12 = 0$ ✓

Question 13

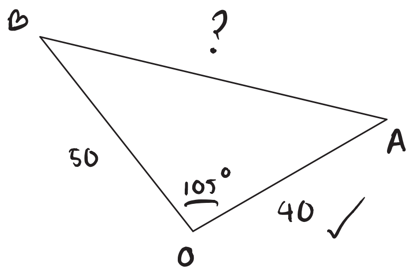
a)



i) $\angle AOB = 85 + 20$

$= 105°$ ✓

ii)



✓

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 40^2 + 50^2 - 2 \times 40 \times 50 \cos 105°$ ✓

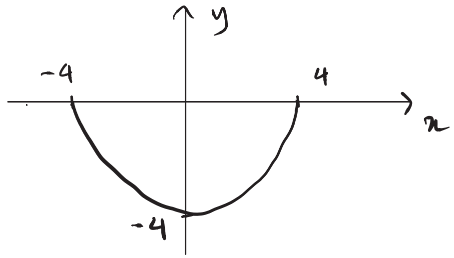
$c^2 = 5135.276$

$c = \sqrt{5135.276}$

$c = 71.66$

$= 72$ km ✓

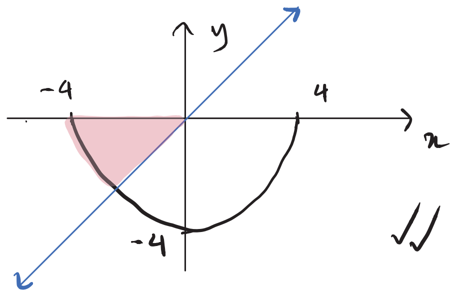
b) i) $f(x) = -\sqrt{16-x^2}$



Domain: $-4 \le x \le 4$

Range: $-4 \le y \le 0$

ii)



c) i) $y = 2x^3 - 9x^2 - 60x$

$y' = 6x^2 - 18x - 60$

$= 6(x^2 - 3x - 10)$

$= 6(x-5)(x+2)$

Stationary at $y' = 0$

$0 = 6(x-5)(x+2)$

at $x = 5$, $y = 2(5)^3 - 9(5)^2 - 60(5)$

$= -275$

at $x = -2$, $y = 2(-2)^3 - 9(-2)^2 - 60(-2)$

$= 68$

$\therefore (5, -275)$ and $(-2, 68)$

$y'' = 12x - 18$

at $(5, -275)$ $y'' > 0$ $\therefore$ concave up, minimum

at $(-2, 68)$ $y'' < 0$ $\therefore$ concave down, maximum

ii) $y'' = 12x - 18$

$= 6(2x-3)$

$0 = 6(2x-3)$

$2x = 3$

$x = 3/2$

at $x = 3/2$, $y = 2(3/2)^3 - 9(3/2)^2 - 60(3/2)$

$= -207/2$

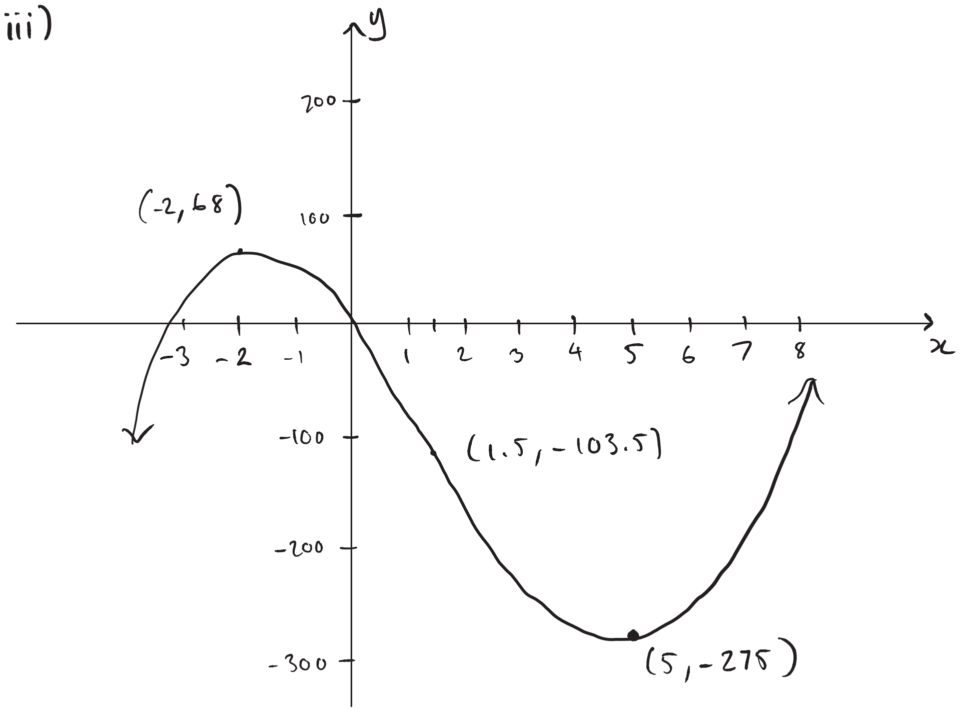
$= -103.5$

x	1	$3/2$	2
y''	-6	0	6
	$\cap$	$\cdot$	$\cup$

$\therefore$ there is a change in concavity.

$(1.5, -103.5)$ is a point of inflexion.

iii)



Question 14

a) i) $t = 0$

$v = 4(0)^3 - 6(0)$

$v = 0$ ✓

ii) $v = 4t^3 - 6t$

$a = 12t^2 - 6$ ✓

iii) $x = \int v \, dt$

$= \int 4t^3 - 6t \, dt$

$= \frac{4t^4}{4} - \frac{6t^2}{2} + c$

$x = t^4 - 3t^2 + c$ ✓

at $t = 0$, $x = -4$

$-4 = (0)^4 - 3(0)^2 + c$

$c = -4$

$\therefore x = t^4 - 3t^2 - 4$ ✓

iv) Particle is at origin when $x = 0$

$0 = t^4 - 3t^2 - 4$

$0 = (t^2 - 4)(t^2 + 1) = 0$ ✓

$t = \pm 2$

$\therefore t = 2$ as time is positive. ✓

b) i) $1/5$ ✓

ii) $1/5 \times 1/4 \times 1/3 \times 1/2 = 1/120$ ✓✓

c) $P = 750\,000$

$r = 4.8\%$ p.a.

$= 0.4\%$ per month

$= 0.004$ per month

$n = 30$ years

$= 360$ months

i) $A_1 = 750\,000(1.004) - M$

$A_2 = A_1(1.004) - M$

$= (750\,000(1.004) - M)(1.004) - M$

$= 750\,000(1.004)^2 - M(1.004) - M$

$= 750\,000(1.004)^2 - M(1 + 1.004)$

ii) $A_3 = A_2(1.004) - M$

$= [750\,000(1.004)^2 - M(1 + 1.004)](1.004) - M$

$= 750\,000(1.004)^3 - M(1.004 + 1.004^2) - M$

$= 750\,000(1.004)^3 - M(1 + 1.004 + 1.004^2)$

$\vdots$

$$A_n = 750\,000(1.004)^n - M(\underbrace{1 + 1.004 + 1.004^2 + \dots 1.004^{n-1}}_{\text{geometric series}})$$

$a = 1 \quad r = 1.004$

$$S_n = \frac{1((1.004)^n - 1)}{1.004 - 1}$$

$$\therefore A_n = 750\,000(1.004)^n - M\left[\frac{1.004^n - 1}{0.004}\right]$$

iii) when loan repaid $A_n = 0$

$$A_{360} = 750\,000(1.004)^{360} - M\left[\frac{1.004^{360} - 1}{0.004}\right]$$

$$0 = 750\,000(1.004)^{360} - M\left[\frac{1.004^{360} - 1}{0.004}\right]$$

$$750\,000(1.004)^{360} = M\left[\frac{1.004^{360} - 1}{0.004}\right]$$

$$M = \frac{750\,000(1.004)^{360}}{\left(\frac{1.004^{360} - 1}{0.004}\right)}$$

$M = \$3934.99$

iv) $A_n = 750\,000(1.004)^n - 5000\left[\frac{1.004^n - 1}{0.004}\right]$

$0 = 750\,000(1.004)^n - 1\,250\,000(1.004^n - 1)$

$0 = 750\,000(1.004)^n - 1\,250\,000(1.004)^n + 1\,250\,000$

$0 = -500000(1.004)^n + 1250000$

$500000(1.004)^n = 1250000$

$$1.004^n = \frac{1\,250\,000}{500\,000}$$

$1.004^n = 2.5$

$\ln(1.004)^n = \ln(2.5)$

$$n = \frac{\ln(2.5)}{\ln 1.004} = 229.53 \text{ months}$$

Question 15

a) i) $N(t) = Ae^{0.15t}$

$\frac{dN}{dt} = 0.15Ae^{0.15t}$ ✓

$\frac{dN}{dt} = 0.15N$ ✓

ii) $t = 3, \quad N = 1.5 \times 10^8$

$1.5 \times 10^8 = Ae^{0.15 \times 3}$

$A = \frac{1.5 \times 10^8}{e^{0.15 \times 3}}$

$A = 95\,644\,222.74$

$A = 96\,000\,000$

$A = 9.6 \times 10^7$ ✓

iii) $2 = e^{0.15x}$ ✓

$\ln 2 = \ln e^{0.15x}$

$\ln 2 = 0.15x$

$x = \frac{\ln 2}{0.15}$ $\qquad x = 4.6$ days ✓

b) i) $\angle BCA = \angle DCA$ (diagonals in a rhombus bisect the angles they pass through) ✓

ii) In $\triangle BFC$ and $\triangle DFC$,

FC is common ✓

$\angle BCA = \angle DCA$ (given in (i))

BC = DC (equal sides in a rhombus) ✓

$\therefore \triangle BFC \equiv \triangle DFC$ (SAS) ✓

iii) $\angle FBC = \angle FEA = 90°$ (alternate angles on parallel lines) ✓

$\angle FBC = \angle FDC = 90°$ (corresponding angles in congruent triangles are equal)

c) i) $\frac{dV}{dt} = 5 - \frac{1}{1+2t}$ ✓

at $t = 0$, $\frac{dV}{dt} = 5 - \frac{1}{1}$

$= 4$

$\therefore$ 4 kL/h ✓

ii) $\int_0^2 5 - \frac{1}{1+2t}\,dt$

$= \left[5t - \frac{1}{2}\ln(1+2t)\right]_0^2$ ✓

$= \left(10 - \frac{1}{2}\ln 5\right) - \left(0 + \ln 1\right)$

$= \left(10 - \frac{1}{2}\ln 5\right)$ kL

$= 9.195$ kL ✓ $\quad = 9195$ L ✓

Question 16

a) $f(x) = \frac{1}{2}(e^x + e^{-x})$

$f(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)})$

$= \frac{1}{2}(e^x + e^{-x})$

$= f(x)$ $\therefore$ even function

$V = \pi \int_{-2}^{2} \left[\frac{1}{2}(e^x + e^{-x})\right]^2 dx$

$= \frac{\pi}{4} \int_{-2}^{2} (e^x + e^{-x})^2 \, dx$

$= \frac{\pi}{2} \int_{0}^{2} (e^x + e^{-x})^2 \, dx$

$= \frac{\pi}{2} \int_{0}^{2} e^{2x} + 2e^x(e^{-x}) + e^{-2x} \, dx$

$= \frac{\pi}{2} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2}\right]_0^2$

$= \frac{\pi}{2} \left(\frac{e^{2(2)}}{2} + 2(2) + \frac{e^{-2(2)}}{-2} - \left(\frac{e^{2(0)}}{2} + 2(0) + \frac{e^{-2(0)}}{-2}\right)\right)$

$= \frac{\pi}{4}(e^4 - e^{-4} + 8) - \left(\frac{1}{2} - \frac{1}{2}\right)$

$= \frac{\pi}{4}(e^4 - e^{-4} + 8)$

$= 49.15$

b) Area $= \int_1^e \log_e(2x) - \log_e x \, dx$

$= \int_1^e \ln 2 + \ln x - \ln x \, dx$

$= \int_1^e \ln 2$

$= [x \ln 2]_1^e$

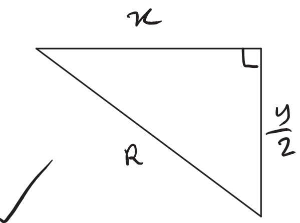
$= e\ln 2 - \ln 2$

$= \ln 2(e - 1)$

$= 1.19$ (2 d.p.)

c) i) $V = \pi r^2 h$

$= \pi x^2 h$



$R^2 = x^2 + \left(\frac{y}{2}\right)^2$

$x^2 = R^2 - \frac{y^2}{4}$

$\therefore V = \pi\left(R^2 - \frac{y^2}{4}\right)y$

$V = \pi R^2 y - \frac{\pi y^3}{4}$

ii) $\frac{dV}{dy} = \pi R^2 - \frac{3\pi y^2}{4}$

$\frac{d^2V}{dy^2} = \frac{-6\pi y}{4}$

$= -\frac{3\pi y}{2}$ ✓

let $\frac{dV}{dy} = 0$

$\pi R^2 - \frac{3\pi y^2}{4} = 0$ ✓

$\frac{3\pi y^2}{4} = \pi R^2$

$3\pi y^2 = 4\pi R^2$

$y^2 = \frac{4\cancel{\pi} R^2}{3\cancel{\pi}}$

$y^2 = \frac{4R^2}{3}$

$y = \frac{2R}{\sqrt{3}}$ $(y>0)$ ✓

Since $\frac{d^2V}{dy^2} < 0$ since $y>0$,

$y = \frac{2R}{\sqrt{3}}$ gives the maximum volume of the cylinder.

iii) Sub $y = \frac{2R}{\sqrt{3}}$

$V = \pi R^2 \times \frac{2R}{\sqrt{3}} - \frac{\pi}{4}\left(\frac{2R}{\sqrt{3}}\right)^3$

$= \frac{2\pi R^3}{\sqrt{3}} - \frac{\pi}{4} \times \frac{8R^3}{3\sqrt{3}}$

$= \frac{2\pi R^3}{\sqrt{3}} - \frac{2\pi R^3}{3\sqrt{3}}$ ✓

$= \frac{6\pi R^3 - 2\pi R^3}{3\sqrt{3}}$

$= \frac{4\pi R^3}{3\sqrt{3}}$

$= \frac{4\pi\sqrt{3}R^3}{3}$ u^3 ✓

iv) $V_{sphere} = \frac{4}{3}\pi R^3$

$\frac{V_{cylinder}}{V_{sphere}} = \frac{4\sqrt{3}\pi R^3}{9} \div \frac{4}{3}\pi R^3$

$= \frac{\cancel{4}\sqrt{3}\cancel{\pi R^3}}{\cancel{9}\,3} \times \frac{\cancel{3}\,1}{\cancel{4\pi R^3}}$

$= \frac{\sqrt{3}}{3}$

$= \frac{1}{\sqrt{3}}$

$\therefore$ The ratio of $V_{cylinder} : V_{sphere} = 1:\sqrt{3}$ ✓