CALLAGHAN COLLEGE

2014
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 100
Section 1 Pages 2- 4
- **10 marks**
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section 11 Pages 5-10
- **90 marks**
- Attempt Questions 11 -16
- Allow about 2 hours 45 minutes for this section

Marks

Section 1 : ***Objective Response Questions.***

Answer on the answer sheet provided.

1 What are the coordinates of the *focus* of the parabola $(x-4)^2 = 8(y+3)$?

(A) $(4,-1)$
(B) $(-4,1)$
(C) $(-3,-1)$
(D) $(-3,-4)$

2 What is the equation of the tangent to the curve $y = x^2 - 5x$ at the point $(1,-4)$?

(A) $y = -3x - 1$
(B) $y = -3x - 7$
(C) $y = 3x + 7$
(D) $y = 3x - 7$

3 For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots

(A) $k \geq -3$
(B) $k \leq -34$
(C) $k \geq 3$
(D) $k \leq 3$

4 The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

(A) -5
(B) 5
(C) 13
(D) 42

5 What is the area enclosed between the curves
$y = x^2 + 1$ and $y = 3x + 1$?

(A) $\frac{3}{2}$ square units

(B) $\frac{9}{2}$ square units

(C) $\frac{27}{2}$ square units

(D) $\frac{45}{2}$ square units

6 The graph $y = f(x)$ passes through the point $(1, 4)$ and $f'(x) = 3x^2 - 2$.
Which of the following expressions is $f(x)$?

(A) $x^3 - 2x$

(B) $2x - 1$

(C) $x^3 - 2x + 3$

(D) $x^3 - 2x + 5$

7 What is the value of $\sum_{r=1}^{10}(5x + 2)$?

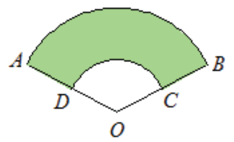
(A) 59

(B) 295

(C) 590

(D) 795

8 A car windscreen wiper traces out the area *ABCD* where *AB* and *CD* are arcs of circles with a centre *O* and radii 40 cm and 20 cm respectively. Angle *AOB* measures 120º.



Not to scale

What is the area of *ABCD*?

(A) 419 cm^2

(B) 1257 cm^2

(C) 1676 cm^2

(D) 2095 cm^2

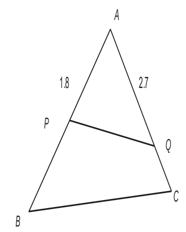
9 If AB = 6, AC = 4 and ΔAPQ ||| ΔACB,
find the length of PQ if BC = 5.

(A) 2.25

(B) 3.375

(C) 1.44

(D) 2.4



10 On the Gill family holiday at Perisher snow fields 4 cm of snow falls on the first day. In each following days the snowfalls increase by 1.5 cm, so on the second day there is 5.5 cm, on the third day there is 7 cm. How much snow falls on the 10th day?

(A) 15 cm

(B) 17.5 cm

(C) 19 cm

(D) 107.5 cm

Marks

Part B

Marks

Question 11: **Start a new Booklet** **(15)**

(a) Evaluate $\left(\frac{1}{e^{2.5}}-1\right)^2$ correct to 3 significant figures. *(2)*

(b) Factorise fully $36x^2 - 16y^2$ *(2)*

(c) Find integers a and b such that $(\sqrt{2}+1)(5\sqrt{2}-3) = a\sqrt{2}+b$ *(2)*

(d) Given that the vertices of triangle ABC are A (2, 8), B (-3, -7) and C (5, -1)

(i) Find the equation of the line through B and C *(1)*

(ii) Determine the length of the altitude from A to BC. *(2)*

(iii) Find the length of the side BC *(1)*

(e) Express $0.4\dot{7}$ as a fraction in its simplest form using an algebraic method. *(2)*

(f) Solve for *x:* $|4x+1| < 5$ and graph the solution set. *(3)*

Marks

Question 12: **Start a new Booklet** **(15)**

(a) Find $\int \frac{4}{x^2}\,dx$ *(2)*

(b) Show that the value of the definite integral $\int_0^2 \frac{3x^2}{x^3+1}dx$ is $2\ln 3$ *(2)*

(c) Find the first derivative of:

(i) $y = \frac{x^3 + 3}{x}$ *(2)*

(ii) $y = e^x ln2x$ *(2)*

(d) A series is $\log(x^{-1}) + \log x + \log(x^3) + \log(x^5) + \ldots$
Is this series arithmetic or geometric? Fully justify your answer. *(2)*

(e) If $f(x) = \log_e 2x$ evaluate $f'(2) + f''(2)$ *(2)*

(f) The first two terms of a geometric series are 15 and 12.

(i) Calculate the next term in the series. *(2)*

(ii) Determine the limiting sum. *(1)*

Marks

Question 13: ***Start a new Booklet*** ***(15)***

(a) For the parabola with equation $x^2 = -8y$.

(i). Find the coordinates of the focus (S) of the parabola. *(1)*

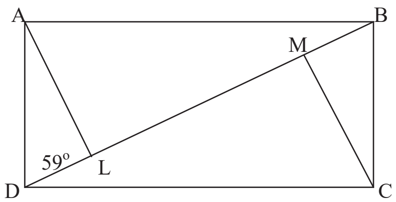
(ii).Find the equation of the directrix of the parabola. *(1)*

(iii). Show that the point A(-8, -8) lies on the parabola. *(1)*

(iv). Find the equation of the focal chord of the parabola which passes through A. *(2)*

(v). Find the equation of the tangent to the parabola at A. *(1)*

(b) Given ABCD is a rectangle, ‹ADL is 59°. AL ⊥ BD and CM ⊥ BD.



(i) Find the size of ‹MBC, give reason. *(1)*

(ii) Prove that ΔADL is congruent to ΔCBM, give reasons. *(2)*

(iii) Hence show that AL=MC, give reasons. *(1)*

(c) If α and β are solutions to the equation $4x^2 + 5x - 1 = 0$, without solving the equation, find the value of:

(i)α +β *(1)*

(ii) αβ *(1)*

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ *(1)*

(d) Find $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$. *(2)*

Marks

Question 14: ***Start a new Booklet*** ***(15)***

(a) A woman walks 120 metres on a bearing of 312°, then turns and walks for a further 96 metres on a bearing of 056°.

(i) Draw a diagram in your answer booklet , labeling all given information. *(2)*

(ii) Determine how far the woman is from her starting point to the nearest kilometre? *(2)*

(iii) Hence find the bearing of the woman from her starting point? *(2)*

(b) For the function $y = 2x^3 - \frac{x^4}{2}$ find:

(i) the first and second derivative. *(2)*

(ii) the two stationary points and determine their nature. *(3)*

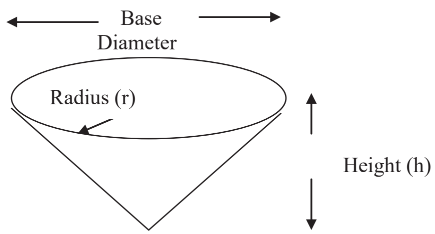
(iii) Sketch the function showing all intercepts on the axes , over the domain -5≤x≤5. *(2)*

(c) Prove that $\frac{\sin^2 x}{1 - \cos x} + \frac{\sin^2 x}{1 + \cos x} = 2$ *(2)*

Marks

Question 15: ***Start a new Booklet*** ***(15)***

(a) The diagram below represents a conical water container. The sum of its base diameter and its height is 60 metres.



(i) Write an expression for the height in terms of the radius (r). *(2)*

(ii) Show that the volume is given by : $V = 20\pi r^2 - \frac{2}{3}\pi r^3$ *(2)*

(iii) Find the radius which makes the volume a maximum. *(3)*

(b) (i) Sketch the graph of $y = e^{-x}$ showing the y – intercept. *(1)*

(ii) Find the exact area of the region bounded by the curve $y = e^{-x}$, the x-axis and the lines $x = 1$ and $x = -\ln 3$. *(3)*

(c) Given $\log_m p = 1.75$ and $\log_m q = 2.25$. Find

(i) $\log_m pq$ *(1)*

(ii) $\log_m \frac{q}{p}$ *(1)*

(iii) $\sqrt[5]{pq^2}$ in terms of m *(2)*

Callaghan College Jesmond Campus
2014 HSC Mathematics – Trial HSC examination

Marks

Question 16: ***Start a new Booklet*** ***(15)***

(a) Use the table

x	3	3.25	3.5	3.75	4	4.25	4.5
fx	1.0	0.8	0.65	0.55	0.5	0.48	0.45

to find an approximation to the value of the definite integral

$$\int_3^{4\bullet 5} f(x)dx,$$

using Simpson's Rule. Give your answer correct to 3 significant figures. *(4)*

(b) Con and Angela want to buy an investment property on the Gold Coast. They decide to borrow $250 000 to buy an apartment. Interest is calculated monthly on the balance still owing, at a rate of 6% per annum. The loan is to be repaid in full at the end of 15 years with equal monthly repayments of $\$M$.

Let $\$A_n$ be the amount owing after the nth repayment.

(i) Derive an expression for A_3. *(1)*

(ii) Find the value of M. *(2)*

(iii) Hence, calculate the amount still owing after 5 years of payments at this rate. *(2)*

(iv) At the end of 5 years, the interest rate is increased to 7.2% per annum and Con and Angela decide to increase their repayments to $2400 per month. How many more months are required to pay off the remainder of the loan? *(2)*

(c) The area bounded by the curve $y = \frac{1}{x}$, the x-axis and the ordinates $x = a$ and $x = 4$ is rotated about the x-axis. If the volume generated is $\frac{\pi}{2}$ unit^3, where $0 < a < 4$, find the value of a. *(4)*

MATHEMATICS - HSC TRIAL, 2014.

SECTION 1 : Multiple Choice

1. focus of parabola $(x-4)^2=8(y+3)$

parabola written in form $(x-h)^2=4a(y-k)$

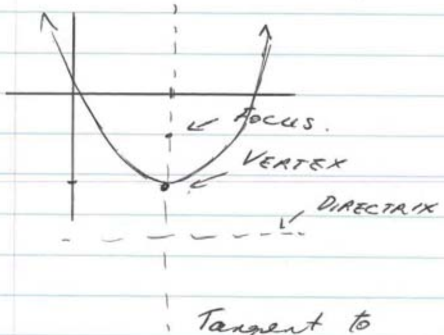
$\therefore$ Vertex $=(4,-3)$

also

$4a=8$

$\therefore a=2$.

Focus at $(4,-3+2)=(4,-1)$ A.



Distance from Focus to VERTEX is always equal to Distance from VERTEX to DIRECTRIX $=a$.

2. Tangent to $y=x^2-5x$ at $(1,-4)$

$\therefore y'=2x-5 \Rightarrow m_T=2\times1-5$
$=-3$

$y--4=-3(x-1)$
$y+4=-3x+3$
$y=-3x-1$ A

3. $x^2-6x-3k=0$ real roots

this means $\Delta\geq0$

$b^2-4ac\geq0$

$a=1$
$b=-6$
$c=-3k$

$b^2-4ac\geq0$

$(-6)^2-4\times1\times(-3k)\geq0$

$36+12k\geq0$

$12k\geq-36$

$\therefore k\geq-3$ A.

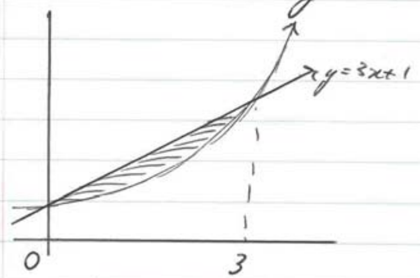
*Only change sign when dividing/multiplying by negative.

4. Arithmetic Series $\therefore T_4=a+3d=27$
$T_7=a+6d=12$

Subtract $3d=-15$
$d=-5$

A.

5. Area enclosed between $y=x^2+1$ and $y=3x+1$



* find intersection

$\therefore x^2+1=3x+1$
$x^2-3x=0$
$x(x-3)=0$
$x=0,3$

$\int_0^3 x^2+1-(3x+1)\,dx=\int_0^3 x^2-3x$

P.T.O.

Q5 ctd.

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3$$

$$= \frac{27}{3} - \frac{3 \times 9}{2}$$

$$= \frac{27}{3} - \frac{27}{2}$$

$$= \frac{54-81}{6}$$

$$= \frac{-27}{6} = \underline{-4\tfrac{1}{2} \text{ units}^2}$$

WHY NEGATIVE?

We got a negative because we subtracted the line from the parabola – it should have been parabola from line. As long as we are aware of this – then we can say $\underline{\underline{A = 4\tfrac{1}{2} u^2}}$

B.

6. finding $f(x)$.

$f'(x) = 3x^2 - 2$ → find indefinite integral

$$\therefore \int (3x^2 - 2)\,dx = \frac{\cancel{3}x^3}{\cancel{3}} - 2x + C.$$

$$\therefore f(x) = x^3 - 2x + C$$

passes throu' $(1,4)$

$$\therefore 4 = (1)^3 - 2(1) + C$$

$$\therefore \underline{C = 5}$$

So, $f(x) = x^3 - 2x + 5$

D.

7. $\sum_{r=1}^{10} (5x+2) = 7 + 12 + 17 \ldots\ldots 52$

Using $S_n = \frac{n}{2}(a + l)$

$$S_{10} = \frac{10}{2}(7 + 52)$$

$$= 5 \times 59$$

$$= \underline{295}$$

B.

8. The windscreen wiper follows the path of an ANNULUS

$$\therefore A = \left(\frac{120}{360}\right) \times (\pi 40^2 - \pi 20^2)$$

$$= \frac{1}{3} \times \pi \times (1600 - 400)$$

$$= \frac{\pi \times 1200}{3}$$

$$= 1256.63$$

$$= \underline{1257 \text{ cm}^2}$$

B.

9. The 2 triangles are congruent – so determine the corresponding sides.

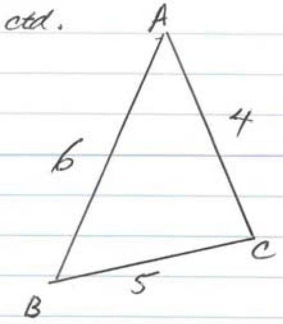
$AB = 6$ $\quad AP = 1.8$

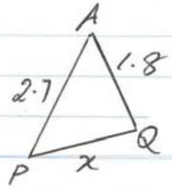
$AC = 4$ $\quad AQ = 2.7$

$\frac{AB}{AC} = \frac{6}{4} = 1.5$ $\quad \frac{AQ}{AP} = \frac{2.7}{1.8} = 1.5$ $\quad \therefore$ AB corresponds to AQ
AC corresponds to AP.

P.T.O.

Q9 ctd.





$5 \to x$
$6 \to 2{\cdot}7$

$\dfrac{5}{6} = \dfrac{x}{2{\cdot}7}$

$x = \dfrac{5 \times 2{\cdot}7}{6}$

$= \dfrac{13.5}{6} = \underline{2{\cdot}25}$

A.

10. $T_1 = a = 4$
$d = 1.5$.
$T_2 = a + d = 4 + 1.5 = 5.5$ etc.

$T_{10} = a + 9d = 4 + (9 \times 1.5)$

$= 4 + 13.5$
$= \underline{17.5\text{cm}}$

B.

SECTION 2.

QUESTION 11.

a) $\left(\dfrac{1}{e^{2.5}} - 1\right)^2 = 0.8425\ldots\ldots$

$= \underline{0{\cdot}843}$ to 3 sf.

b) Factorise fully

$36x^2 - 16y^2 = 4(9x^2 - 4y^2)$ Difference of 2 squares.

$= \underline{4(3x-2y)(3x+2y)}$

c) $(\sqrt{2}+1)(5\sqrt{2}-3) = a\sqrt{2}+b$

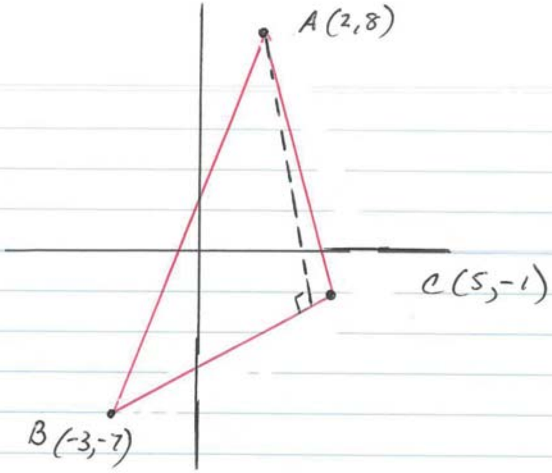
$5 \times (\sqrt{2})^2 - 3\sqrt{2} + 5\sqrt{2} - 3 = a\sqrt{2} + b$

$10 + 2\sqrt{2} - 3 = a\sqrt{2} + b$

$7 + 2\sqrt{2} = a\sqrt{2} + b$ $\therefore$ Equating

$\underline{a = 2,\ b = 7.}$

a)



i) equation of line joining B to C.

$$m_{BC} = \frac{-7--1}{-3-5} = \frac{-6}{-8}$$

$$= \frac{3}{4}$$

$y-y_1 = m(x-x_1)$ Using $m=\frac{3}{4}$

$y--1 = \frac{3}{4}(x-5)$ $(x_1,y_1)=(5,-1)$

$y+1 = \frac{3}{4}(x-5)$

$4y+4 = 3x-15$

$3x-4y-19=0$ (General Form).

ii) Altitude from A to BC.

i.e. Perpendicular Distance.

∴ Perpendicular Distance from $(2,8)$ to $3x-4y-19=0$

from Line

$$\therefore d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$ $a=3$, $b=-4$, $c=-19$

$$= \left|\frac{-4\times2+3\times8-19}{\sqrt{3^2+4^2}}\right|$$ $\left.\begin{matrix} x_1=2 \\ y_1=8 \end{matrix}\right\}$ point

$$= \left|\frac{-8+24-19}{\sqrt{25}}\right|$$

$$= \frac{3}{5}.$$

iii) $d_{BC} = \sqrt{(-1--7)^2+(5--3)^2}$

$= \sqrt{6^2+8^2}$

$= \sqrt{100} = 10$ units

e) Let $x = 0.4\dot{7}$

i.e. $x = 0.477777\ldots$

$10x = 4.77777\ldots$

$100x = 47.7777\ldots$

$\therefore 90x = 43$

$x = \frac{43}{90}$

f)

$$|4x+1| < 5$$

$4x+1<5$

$4x<4$

$\underline{x<1}$

$-(4x+1)<5$

$-4x-1<5$

$-4x<6$

$\underline{x>-\frac{6}{4}}\ \left(-\frac{3}{2}\right)$

$$\underline{-\frac{3}{2}<x<1}$$

Question 12.

a) $\displaystyle\int \frac{4}{x^2}\,dx = 4\int x^{-2}\,dx$

$\displaystyle = \frac{4\times x^{-1}}{-1} + C$

$\displaystyle = \underline{-\frac{4}{x} + C}.$ 2

b) SHOW $\displaystyle\int_0^2 \frac{3x^2}{x^3+1}\,dx = 2\ln 3$

$\therefore$ L.H.S. $= \displaystyle\int_0^2 \frac{3x^2}{x^3+1}\,dx$ ← Top line is differential of bottom. $\therefore$ $\underline{\underline{\log_e \text{ funct.}}}$

$= \left[\log_e(x^3+1)\right]_0^2$

$= \log_e(2^3+1) - \log_e(0+1)$

$= \log_e 9 - \cancel{\log_e 1} = 0$

$= \log_e 9$

$= \log_e 3^2$

$= 2\log_e 3.$ or $\underline{\underline{2\ln 3}}$

$= \underline{\underline{\text{R.H.S.}}}$ 2

c) * Must know PRODUCT RULE
QUOTIENT RULE
CHAIN RULE

* Must be able to identify when to use them.

i) $y = \dfrac{x^3+3}{x}$

dividing, so <u>QUOTIENT</u>

$u = x^3+3 \qquad v = x$

$u' = 3x^2 \qquad v' = 1$

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$\therefore y' = \dfrac{x \times 3x^2 - (x^3+3)\times 1}{x^2}$

$= \dfrac{3x^3 - x^3 - 3}{x^2}$

$= \dfrac{2x^3-3}{x^2}$

2

ii) $y = e^x \ln 2x$

multiplying, so PRODUCT

$u = e^x \qquad v = \ln 2x$

$u' = e^x \qquad v' = \dfrac{1}{2x} \times 2$

$= \dfrac{1}{x}$

$\dfrac{dy}{dx} = uv' + vu'$

$\dfrac{dy}{dx} = e^x \times \dfrac{1}{x} + \ln 2x \times e^x$

$= e^x\left[\dfrac{1}{x} + \ln 2x\right]$

2

d)

$\log x^{-1} + \log x^1 + \log x^3 + \log x^5 + \dots$

Use your log laws.

$\therefore -1\times\log x + 1\times\log x + 3\times\log x + 5\times\log x + \dots$

So, $T_1 = -\log x$

$T_2 = \log x$

$T_3 = 3\log x$

For an ARITHMETIC SERIES, $T_3 - T_2 = T_2 - T_1$

GEOMETRIC $\dfrac{T_3}{T_2} = \dfrac{T_2}{T_1}$

① Try A.S.

$T_3 - T_2 = 3\log x - \log x$

$= 2\log x$

$T_2 - T_1 = \log x - {}^-\log x$

$= 2\log x$

$\therefore$ Proved this is an ARITHMETIC SERIES

2

e) $f(x) = \ln 2x$

$$f'(x) = \frac{1}{2x} \times 2 = \underline{\frac{1}{x}} \qquad (x^{-1})$$

$$f''(x) = -x^{-2} = \underline{\underline{\frac{-1}{x^2}}}$$

So, $f'(2) + f''(2) = \frac{1}{2} - \frac{1}{2^2}$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= \underline{\underline{\frac{1}{4}}} \quad ②$$

f) Told its a Geometric Series

$$\therefore T_1 = a = 15$$
$$T_2 = ar = 12$$

$$\frac{T_2}{T_1} = \frac{\cancel{a}r}{\cancel{a}} = r = \frac{12}{15} = \frac{4}{5} \quad (0.8)$$

i) T_3 (next term) $= ar^2$

$$= 15 \times \left(\frac{4}{5}\right)^2$$

$$= \underline{\underline{\frac{48}{5}}} \quad (9.6) \quad ②$$

ii) Limiting Sum $= S_\infty = \frac{a}{1-r} = \frac{15}{1-\frac{4}{5}}$

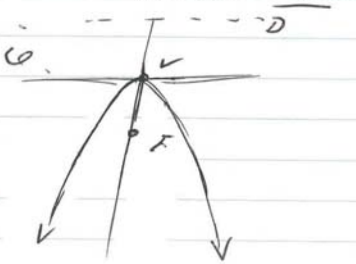
$$= \frac{15}{\frac{1}{5}} = \underline{75} \quad ①$$

13.

$x^2 = -8y$ is of form $x^2 = 4ay$ but what about the $\underline{\underline{\text{negative}}}$?

Vertex $= (0,0)$

Concave Down



$4a = -8$

$$\therefore a = \frac{-8}{4} = \underline{\underline{-2}}$$

i) $\therefore$ Focus at

$$(0, 0-2)$$

$$= \underline{\underline{(0,-2)}} \quad ①$$

ii) Equation of Directrix is $y = -a$

$$\therefore y = -(-2)$$

$$\text{i.e. } \underline{y = 2} \quad ①$$

iii) for $A(-8,-8)$ to lie on parabola it must satisfy equation

$$\therefore \quad L.H.S. = x^2$$
$$= (-8)^2$$
$$= 64$$
$$R.H.S. = -8 \times -8$$
$$= 64 \quad ①$$
$$= L.H.S. \qquad \therefore \underline{\underline{\text{Yes}}}.$$

iv) equation of focal chord passing thro' A.

$F = (0, -2)$

$A = (-8, -8)$

$\therefore m = \frac{-8 - -2}{-8 - 0}$

$= \frac{-6}{-8}$

$= \frac{3}{4}$

$y - -2 = \frac{3}{4}(x - 0)$

$4y + 8 = 3x$

$3x - 4y - 8 = 0$ (2) General Form.

v) equation of tangent to parabola at A.

$x^2 = -8y$

$\therefore y = \frac{-1}{8} x^2$

$\frac{dy}{dx} = \frac{-1}{8} \times 2x$

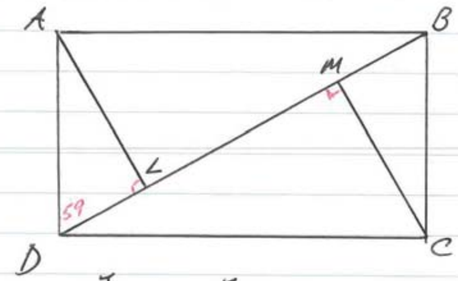
$= -\frac{x}{4}$

at $(-8, -8)$ $\quad m_T = -\frac{-8}{4} = \underline{\underline{2}}$

$y - -8 = 2(x - -8)$

$y + 8 = 2x + 16$ (1) Always General Form unless asked differently

$2x - y + 8 = 0.$

b) 

$\angle ADL = 59^\circ$

$AL \perp BD$

$CM \perp BD$

i) $M\hat{B}C = A\hat{D}L$

$= 59^\circ$, alternate angles of transversal BD are equal, and opposite sides of a rectangle are parallel. (1)

ii)

1) $M\hat{B}C = A\hat{D}L$

$= 59^\circ$ (from above)

2) $A\hat{L}D = B\hat{M}C$

$= 90^\circ$ (given)

3) $AD = BC$ (opposite sides of rectangle are EQUAL)

$\therefore$ By AAS

$\triangle ADL \equiv \triangle CMB.$ (2)

iii) Corresponding sides of congruent triangle

$\therefore$ $AL = MC$ (1) (also $DL = MB$)

c) $4x^2+5x-1=0$

$a=4$
$b=5$
$c=-1$

i) $\alpha+\beta = \dfrac{-b}{a}$ Sum of Roots

$= \dfrac{-5}{4}$

(-1.25) 1

ii) $\alpha\beta = \dfrac{c}{a}$ Product of Roots

$= \dfrac{-1}{4}$

(-0.25) 1

iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta}$

$= \dfrac{\text{Sum of Roots}}{\text{Product of Roots}}$

$= \dfrac{\frac{-5}{4}}{\frac{-1}{4}}$

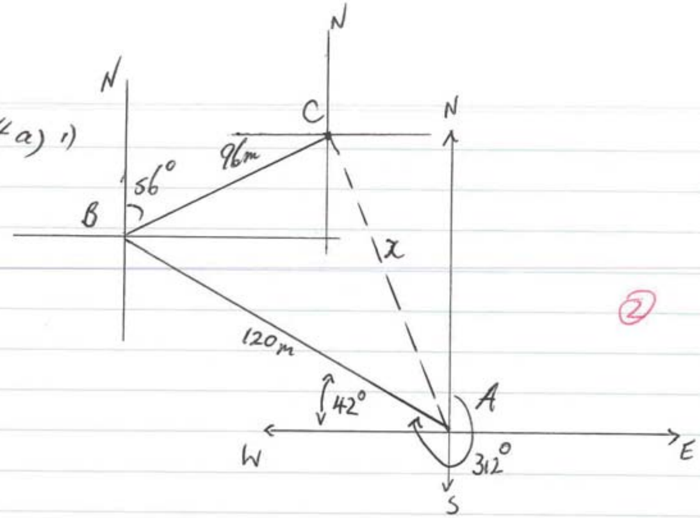
$= \underline{5}$ 1

d) $\lim\limits_{x\to1} \dfrac{x^2+x-2}{x-1} = \lim\limits_{x\to1} \dfrac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}}$

$= \lim\limits_{x\to1} x+2$

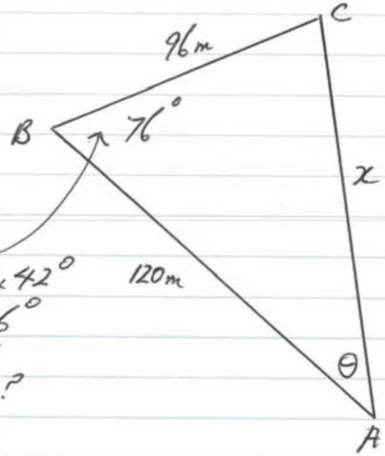
$= \underline{3}$ 2

(14a) i)



2

ii)



$34° + 42°$
$= 76°$
WHY?

Using Cosine Rule

$x^2 = 120^2 + 96^2 - 2\times120\times96\times\cos 76$

$x = 134.32\text{m}$

$\underline{\underline{x = 134\text{m}}}$ 2

iii) Bearing from Starting Point is found by calculating $B\hat{A}C$ and adding this onto $\underline{312}$.

$\dfrac{\sin\theta}{96} = \dfrac{\sin 76}{134}$

$\sin\theta = \dfrac{96\times\sin 76}{134}$

$\therefore A = 44°$ 2

Bearing $= 44+312 = \underline{356}$

14b)

1) $y = 2x^3 - \frac{x^4}{2}$

$y' = 6x^2 - \frac{4x^3}{2}$

$= \underline{\underline{6x^2 - 2x^3}} = 2x^2(3 - x).$

$y'' = \underline{\underline{12x - 6x^2}} = 6x(2 - x).$

ii) S.P. and NATURE

$\therefore$ Put y' or $y'' = 0$.

$y' = 0 \quad \therefore 6x^2 - 2x^3 = 0$

$2x^2(3 - x) = 0$

$\therefore \underline{\underline{x = 0, 3}}$

$\therefore$ Possible S.Points at $x = 0,\ y = 0$

$x = 3,\ y = 2 \times 3^3 - \frac{3^4}{2}$

$= 54 - 40.5$

$= \underline{13.5}$

S.Points at $(0,0)$, $(3, 13.5)$

But need to TEST

$\underline{x = 0}$.

Sub. in $2x^2(3-x)$	-1	0	1
	>0	0	>0

Need to go on with this point and check Inflexion

$\underline{x = 3}$

2	3	4
>0	0	<0

$\therefore$ MAX. at $\underline{(3, 13.5)}$

P.T.O.

what happens at $(0,0)$

$\therefore$ Put $y'' = 0$

$12x - 6x^2 = 6x(2 - x) = 0$

i.e. $x = 0, 2$

we know when $x = 0$, $y = 0$ | $x = 2$, $y = 2 \times 2^3 - \frac{2^4}{2}$

$= 16 - 8 = \underline{\underline{8}}$

$\underline{\underline{(0,0)}}$ $\underline{\underline{(2,8)}}$

TEST

at $x = 0$

Sub. in $12x - 6x^2$ / $6x(2-x)$	-1	0	1
	<0	0	>0

$\therefore$ Change in Concavity

$\therefore$ Point of INFLEXION

But not just Inflexion we have HORIZONTAL INFLEXION

at $x = 2$

1	2	3
>0	0	<0

$\therefore$ Change in CONCAVITY

$\therefore$ Inflexion

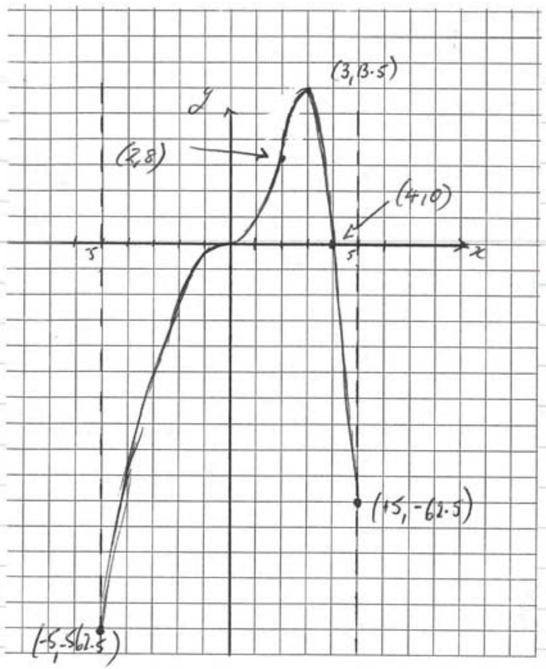
So, what do we have:

1) $(3, 13.5)$ MAXIMA

$(0,0)$ HORIZONTAL INFLEXION

$(2,8)$ INFLEXION

3



14 iii) Sketch the function over $-5 \leq x \leq 5$.

Intercept
at $x=0$
$y=0$

at $y=0$
$x=4$

Need to look at end points at DOMAIN
at $x=5$
$y=-62.5$
$x=-5$
$y=-562.5$

c) Prove

$$\frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos x} = 2$$

$\sin^2 x + \cos^2 x = 1$
$\therefore \sin^2 x = 1-\cos^2 x$

$$\therefore L.H.S. = \frac{\sin^2 x}{1-\cos x} + \frac{\sin^2 x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{1-\cos x} + \frac{1-\cos^2 x}{1+\cos x}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}} + \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{1+\cos x}}$$

$$= 1+\cos x + 1 - \cos x$$

$$= 2$$

$$= \underline{R.H.S}$$

2

15 a)

i) We're told $h+d=60$
but $d=2r$

$\therefore h+2r=60$

$\underline{h=60-2r.}$

ii) $V = A \times h$
$= \pi r^2 \times h$ } This is for a CYLINDER
what happens to a CONE?

$V = \frac{1}{3} \times A \times h$

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (60-2r)$

$\underline{V = 20\pi r^2 - \frac{2}{3}\pi r^3}$

iii) As soon as they talk about MAXIMA, we find $\frac{dV}{dr}=0$.

$\frac{dV}{dr} = 40\pi r - 3 \times \frac{2}{3}\pi r^2$

$= 40\pi r - 2\pi r^2$

$= 0$

Then $40\pi r - 2\pi r^2 = 0$
$2\pi r(20-r) = 0$

P.T.O.

$2\pi r(20-r)=0$

$\therefore \quad \cancel{r=0} \quad \text{or} \quad \underline{r=20}$

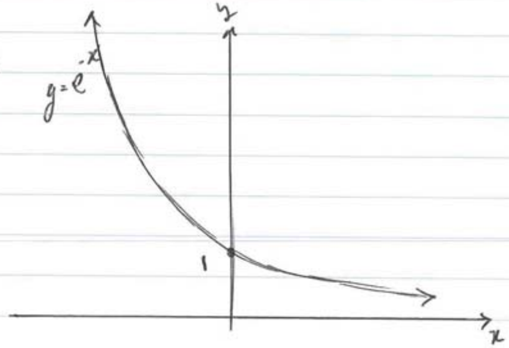
Test $r=20$

19	20	21
>0	0	<0

$\therefore$ MAXIMUM Volume when $\underline{\underline{r=20m}}$

b)

i)



ii) $\displaystyle\int_{-\ln 3}^{1} e^{-x}\,dx = \left[-e^{-x}\right]_{-\ln 3}^{1}$

$= -e^{-1} - -e^{\ln 3}$

$= -\frac{1}{e} + e^{\ln 3}$

$= \left(3-\frac{1}{e}\right) \underline{\text{units}^2}$

By definition $\underline{\underline{e^{\ln 3}=3}}$

c) $\log_m p = 1.75$

$\log_m q = 2.25$

Need LOG LAWS

i) $\log_m pq = \log_m p + \log_m q$

$= 1.75 + 2.25$

$= \underline{\underline{4}}$

ii) $\log_m \frac{q}{p} = \log_m q - \log_m p$

$= 2.25 - 1.75$

$= \underline{\underline{0.5}}$

iii) $\sqrt[5]{pq^2}$

$= \left(pq^2\right)^{\frac{1}{5}}$

$= \left(m^{1.75} \times \left(m^{2.25}\right)^2\right)^{\frac{1}{5}}$

$= \left(m^{1.75} \times m^{4.5}\right)^{\frac{1}{5}}$

$= m^{\frac{6.25}{5}}$

$= \underline{\underline{m^{\frac{27}{20}}}}$

$\log_m p = 1.75$

$\therefore p = m^{1.75}$

$q = m^{2.25}$

16a)

$$\int_3^{4.5} f(x)\,dx = \frac{0.25}{3}\Big[1+0.45+4\times[0.8+0.55+0.48] + 2\times[0.65+0.5]\Big]$$

$$= \frac{0.25}{3}\Big[1.45+7.32+2.3\Big]$$

$$= \underline{\underline{0.9225}}$$

4

b)

i) $A_1 = 250\,000 \times 1.005 - M$

$A_2 = A_1 \times 1.005 - M$

$= [250\,000 \times 1.005 - M] \times 1.005 - M$

$A_2 = 250000 \times 1.005^2 - M(1.005+1)$

$\therefore A_3 = 250000 \times 1.005^3 - M(1+1.005+1.005^2)$

ii) pay off in 15 yrs $= 15 \times 12$
$= \underline{\underline{180 \text{ months}}}$

$$A_{180} = \underbrace{250\,000 \times 1.005^{180}}_{\text{Compound Interest}} - M(1+1.005+\ldots\ldots 1.005^{179})$$

Compound Interest $- M \times \underline{\underline{G.P.}}(S_n)$

don't we want $\underline{\underline{A_{180} = 0}}$?

Sum the G.P.

$\therefore$ S of $1+1.005+1.005^2+\ldots\ldots+1.005^{179}$

$$S = \frac{a(r^n-1)}{r-1}$$

$$S_{180} = \frac{1(1.005^{180}-1)}{1.005-1}$$

$$= \frac{1.005^{180}-1}{0.005}$$

So, $A_{180} = 0 = 250\,000 \times 1.005^{180} - M \times \dfrac{(1.005^{180}-1)}{0.005}$

$$\frac{M \times (1.005^{180}-1)}{0.005} = 250000 \times 1.005^{180}$$

$$M = \frac{250000 \times 1.005^{180} \times 0.005}{(1.005^{180}-1)}$$

$\therefore M = \underline{\$2109.64}$ Monthly Repayment

iii) Amount owing after $\underline{5 \text{ years}}$

5 years $= 5 \times 12$
$= \underline{60 \text{ months}}$

$$A_{60} = 250000 \times 1.005^{60} - \underset{\$2109.64}{M}\underbrace{(1+1.005+\ldots\ldots+1.005^{59})}_{\text{Sum of } \underline{G.P.}}$$

P.T.O.

$$A_{60} = 250000 \times 1.005^{60} - 2109.64 \times \left(\frac{1.005^{60} - 1}{0.005}\right)$$

$= \underline{\$190\ 022.89}$ still owing after $\underline{5}$ years.

16 iv) at end of 5 years — things change.

$r = 7.2\%$ p.a. $= 0.006$ per month

Repayment $= \$2400$.

$A_n = 0$ → when loan paid off.

$$0 = A_n = 190\ 022.89 \times 1.006^n - 2400 \times \left(\frac{1.006^n - 1}{0.006}\right)$$

$$= 190\ 022.89 \times 1.006^n - 400000 \times 1.006^n + 400\ 000$$

$$400\ 000 = 209\ 977.11 \times 1.006^n$$

$$1.006^n = \frac{400\ 000}{209\ 977.11}$$

$1.006^n = 1.9049\ldots$ take logs of both sides

$$n \log 1.006 = \log 1.905$$

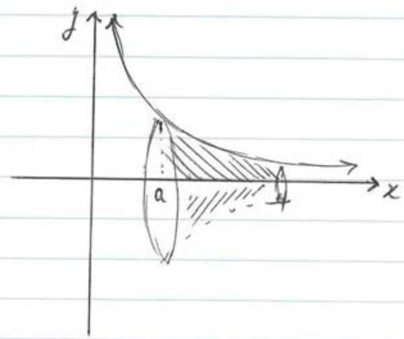
$$n = \frac{\log 1.905}{\log 1.006} = \underline{107.7 \text{ months}}$$

So, it will take another $\underline{108 \text{ months}}$ to pay off the loan.

c) $y = \frac{1}{x}$ from $x = a$ to $x = 4$ where $0 < a < 4$

is $\frac{\pi}{2}$ units^2

$\therefore V = \pi \int y^2\, dx$ $y = x^{-1}$ $\therefore y^2 = x^{-2}$



$$\therefore V = \pi \int_a^4 x^{-2}\, dx$$

$$= \pi \left[\frac{x^{-1}}{-1}\right]_a^4$$

$$= -\pi \left[\frac{1}{x}\right]_a^4$$

$$= -\pi \left(\frac{1}{4} - \frac{1}{a}\right)$$

$$\frac{\pi}{2} = \pi \left(\frac{1}{a} - \frac{1}{4}\right)$$

$$\therefore \frac{1}{2} = \frac{1}{a} - \frac{1}{4}$$

$$\frac{1}{a} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\therefore \frac{1}{a} = \frac{3}{4} \Rightarrow \underline{a = \frac{4}{3}} \quad (1\tfrac{1}{3} = 1.\dot{3})$$