Student Name: ……………………………..

2005
TRIAL HIGHER SCHOOL CERTIFICATE
Sample Examination paper

MATHEMATICS

CARINGBAH HIGH

OMNIA VINCIT DILIGENTIA

General Instructions

Reading Time: 5 minutes
Working Time: 3 hours

- Attempt all questions
- Start each question on a new page
- Each question is of equal value
- Show all necessary working.
- Marks may not be awarded for careless work or incomplete solutions
- Standard integrals are printed on the last page
- Board-approved calculators may be used

Question 1 (12 marks) **MARKS**

(a) Express $\frac{1}{\sqrt{5}-2}$ with a rational denominator **2**

(b) The thickness of a cat's whisker is 0.0000598m. Write this in scientific notation correct to 2 significant figures. **2**

(c) Simplify: $\frac{3}{x-1}-\frac{2}{x+1}$ **2**

(d) Solve the pair of simultaneous equations: **2**

$$x-2y=9$$
$$2x+y=8$$

(e) Find $\frac{dy}{dx}$ given $y=(5-2x)^3$ **2**

(f) Solve: $x^3=4x$ **2**

Question 2 (12 marks) ***Start a new page*** **MARKS**

(a) (i) Find: $\int \frac{\cos 2x}{\sin 2x}\,dx$ **2**

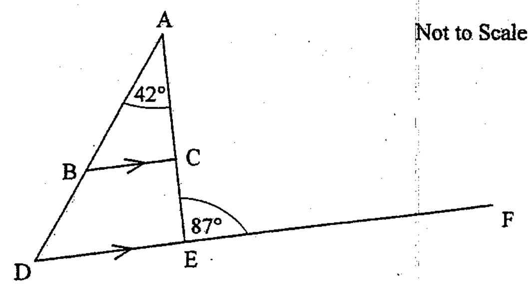
(ii) Evaluate: $\int_0^{\frac{\pi}{3}} \cos 3x\,dx$ **2**

(b) Differentiate with respect to x:

(i) $\frac{x^2}{x+1}$ **2**

(ii) $x^3 \cos x$ **2**

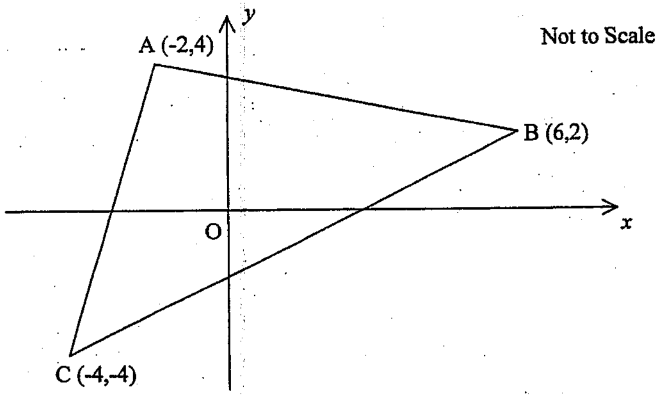
(c) In the diagram below, ADE is a triangle. B and C lie on AD and AE respectively such that BC is parallel to DE. Line DE is produced to F. **3**
$\angle AEF=87^\circ$ and $\angle DAE=42^\circ$
Find the size of $\angle ABC$, giving reasons for your answer.



(d) Evaluate: $\sum_{r=1}^{4} 2^{1-r}$ **1**

Question 3 (12 marks) ***Start a new page*** **Marks**

(a) The diagram below shows the points A (-2, 4), B (6, 2) and C (-4, -4). Copy or trace the diagram onto your worksheet.



(i) Calculate the length of the interval BC. 1

(ii) Find the gradient of BC. 1

(iii) Find the coordinates of M, the midpoint of BC. 1

(iv) Show that the equation of l, the perpendicular bisector of BC, is $5x+3y-2=0$. 2

(v) Show that l passes through A 1

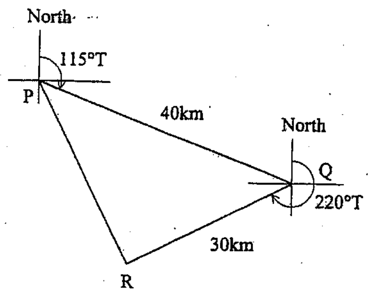
(vi) Hence or otherwise find the area of triangle ABC. 2

(b) Solve: $\sqrt{3}\tan x=-1$ for $0\le x\le 2\pi$ 2

(c) Solve: $|3-2x|\le 5$ 2

Question 4 (12 marks) ***Start a new page*** **MARKS**

(a) From P the bearing of a Lighthouse Q, 40 kilometres distant from P, is $115°T$. From Q the bearing of a headland R, 30 kilometres from Q, is $220°T$. This is illustrated in the diagram below.



(i) Find the size of $\angle PQR$ 1

(ii) Use the Cosine Rule to find the length of PR. Give your answer correct to 2 decimal places. 1

(iii) Find the bearing of R from P. Give your answer to the nearest whole degree. 2

(b) For the parabola: $4x=8y-y^2$

(i) Find the co-ordinates of the vertex. 2

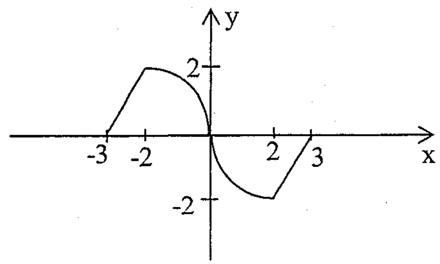
(ii) Find the co-ordinates of the focus. 1

(iii) Sketch the curve, labeling the focus and vertex 2

(c) Find the value of 'k' if the sum of the roots of $x^2-(k-1)x+2k=0$ is equal to the product of the roots. 2

(d) The graph of $y=f(x)$ is shown below. It consists of quadrants of a circle and line segments.

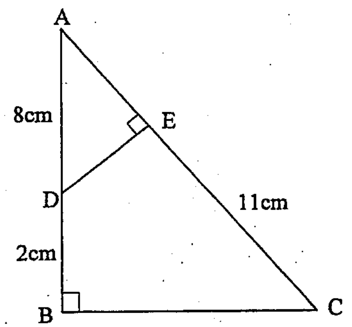
Find $\int_{-3}^{3} f(x)\,dx$ 1



Marks

Question 5 (12 marks) *Start a new page*

(a) ABC is a right-angled triangle in which $\angle ABC = 90°$. Points D and E lie on AB and AC respectively such that AC is perpendicular to DE.
$AD = 8cm$, $EC = 11cm$ and $DB = 2cm$.



Not to Scale

(i) Prove that ΔABC is similar to ΔAED. **3**

(ii) Find the length of AE. **1**

(b) Tom is an enthusiastic gardener. He planted a silky oak tree three years ago when it was 80 centimetres tall. At the end of the first year after planting, it was 130 centimetres tall, that is it grew 50 centimetres. Each years growth was then 90% of the previous years.

(i) What was the growth of the silky oak in the second year? **1**

(ii) How tall was the silky oak after three years? **1**

(iii) Assuming that it maintains the present growth pattern, explain why it will never reach a height of 10 metres. **2**

(iv) In which year will the silky oak reach a height of 5 metres? **2**

(c) For what values of k does $x^2 - (2+k)x + 4 = 0$ have real roots? **2**

Question 6 (12 marks) *Start a new page* **MARKS**

(a) For the function: $f(x) = 8x^3 - 8x^2$

(i) Find the stationary point(s) and determine their nature. **3**

(ii) Find the co-ordinates of any points of inflexion. Confirm that your answer does provide a point of inflexion. **2**

(iii) Sketch the graph of the function $y = f(x)$, showing any stationary Points, points of inflexion and intercepts with the x- and y- axes. **3**

(iv) For what values of x is the curve concave down <u>and</u> decreasing? **2**

(b) For what values of x does the geometric series **2**

$$1 + \ln x + (\ln x)^2 + \ldots$$

have a limiting sum?

Question 7 (12 marks) *Start a new page* **MARKS**

(a) A normal is drawn to the curve $y = \sin x$ at the point $P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$.

The normal cuts the x-axis at Q.

(i) Show that the equation of the normal is: **2**

$$2x + y = \frac{\sqrt{3}}{2} + \frac{2\pi}{3}$$

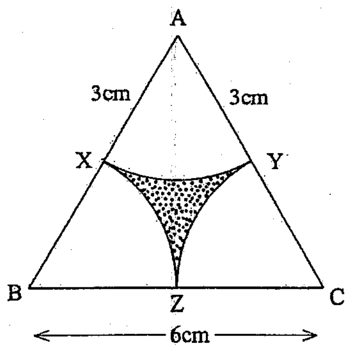
(ii) Find the co-ordinates of Q **1**

(iii) On a diagram, shade the region bounded by the curve $y = \sin x$, the normal at P and the x-axis. **2**

Your diagram should be at least $\frac{1}{3}$ page and show all of the above information.

(iv) Find the area of the shaded region. **3**

(b) ABC is an equilateral triangle with sides of length 6cm. An arc, centre A, and radius 3 cm cuts AB and AC at X and Y respectively.
This is repeated at B and C, as shown in the diagram.



Not to Scale

(i) Explain why $\angle ABC = \frac{\pi}{3}$ radians. **1**

(ii) Find the shaded area enclosed by the arcs XY, YZ and ZX **3**

Question 8 (12 marks) *Start a new page* **MARKS**

(a) (i) Copy and complete the following table of values: **1**

x	-2	-1	0	1	2
3^x					

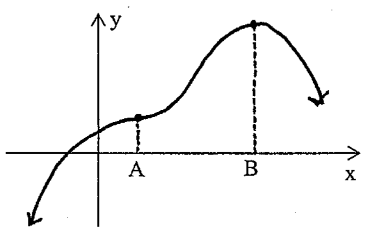
(ii) Use Simpson's Rule with 5 function values to estimate the area enclosed by the curve $y = 3^x$, the x-axis and the ordinates $x = 2$ and $x = -2$ **2**

(b) Find the volume of the solid of revolution formed by rotating the curve $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ about the x-axis from $x = 1$ to $x = 9$. **3**

(c) The graph of $y = f(x)$ is drawn below.

(i) Copy the diagram onto your answer page

(ii) On the same axes, sketch the graph of its gradient function, $y = f^1(x)$ **2**



(d) (i) Sketch the graph of $y = 1 - 2\cos x$ for $0 \le x \le 2\pi$ **3**
Clearly mark on your sketch the endpoints of the curve in the given domain as well as its turning points.

(ii) Use your graph to solve: $1 - 2\cos x > 0$ in the given domain. **1**

Question 9 (12 marks) ***Start a new page*** **Marks**

(a) Ella borrowed \$180 000 to finance an extension on her home. She agreed to pay off the loan in equal monthly instalments of \$P, paid at the end of each month, at an interest rate of 6% per annum, compounded monthly.

(i) Show that after the first instalment is paid, the amount owing on the loan is: **1**

$$\$[180\,000(1.005)-P]$$

(ii) Show that after three months she owes: **2**

$$\$\left[180\,000(1.005)^3-P\left((1.005)^2+(1.005)+1\right)\right]$$

(iii) If the loan is repaid after 8 years, find the value of P, the monthly instalment. **3**

(b) A particle moves in a straight line so that its distance x in metres from a fixed point O is given by:

$$x=2t+e^{-2t} \text{ where } t \text{ is measured in seconds}$$

(i) What is the velocity of the particle when $t=\frac{1}{2}$ sec ? **2**

(ii) Show that initially the particle is at rest. **1**

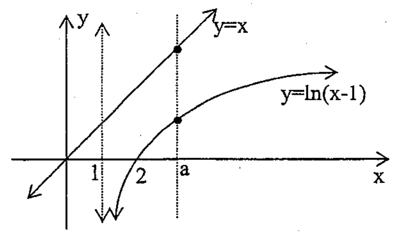
(iii) As t increases, find the limiting velocity of the particle. **1**

(iv) Draw a neat sketch of the graph of the velocity as a function of time. **1**

(v) Using v as the velocity and a as the acceleration, show that $a=4-2v$ **1**

Question 10 (12 marks) ***Start a new page*** **MARKS**

(a) The diagram shows the graphs of $y=\ln(x-1)$ and $y=x$ for $x>0$.



(i) Find an expression for M, the vertical distance between these two curves at any point $x=a$. **1**

(ii) For what value of 'a' is this vertical distance a minimum? Justify your answer. **3**

(iii) Find this minimum distance. **1**

(b) At the beginning of a drought, the number of sheep on a property was 285 000. Six months after the drought commenced this number had reduced to 202 000. Sheep numbers have continued to decreased so that at any time t, the number of sheep, S, is given by the formula:

$$S=A\,e^{-kt}$$

where A and k are constants and t is the number of months since the drought commenced.

(i) Find the values of A and k. **2**

(ii) Show that $\frac{dS}{dt}=-kS$ **1**

(iii) How many sheep will there be 1 year after the drought started? **1**

(iv) When will the flock reach one-third of its original size? **2**

(v) Find the rate of decrease of the number of sheep at this time. **1**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Solutions: 2005 Mathematics TRIAL H.S.C.

① a) $\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5}+2$

b) $0.0000598 = 6.0 \times 10^{-5}$ m

c) $\frac{3}{x-1} - \frac{2}{x+1} = \frac{3(x+1)-2(x-1)}{(x-1)(x+1)}$

$= \frac{3x+3-2x+2}{x^2-1}$

$= \frac{x+5}{x^2-1}$

d) $x-2y = 9$ — ①

$2x+y = 8$ — ②

② × 2 $\quad 2x-4y = 18$ —

$5y = -10$

$y = -2$

$x-2(-2) = 9$

$x = 5 \qquad \therefore (x,y) = (5,-2)$

e) $y = (5-2x)^3$

$\frac{dy}{dx} = 3(5-2x)^2 \times -2 = -6(5-2x)^2$

f) $x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x(x+2)(x-2) = 0$

$x = 0, \pm 2$

d) $\sum_{r=1}^{4} 2^{1-r} = 2^0 + 2^{-1} + 2^{-2} + 2^{-3}$

$= 1\frac{7}{8}$

② a) i) $\frac{1}{2}\int 2\frac{(\cos 2x)}{\sin 2x}\,dx = \frac{1}{2}\ln(\sin 2x) + C$

ii) $\int_0^{\pi/3} \cos 3x\,dx = \left[\frac{\sin 3x}{3}\right]_0^{\pi/3}$

$= \frac{1}{3}\left[\sin\pi - \sin 0\right] = 0$

b) i) $\frac{d}{dx}\frac{x^2}{x+1} = \frac{(x+1).2x - x^2.1}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$

ii) $\frac{d}{dx} x^3.\cos x = x^3.-\sin x + \cos x.3x^2$

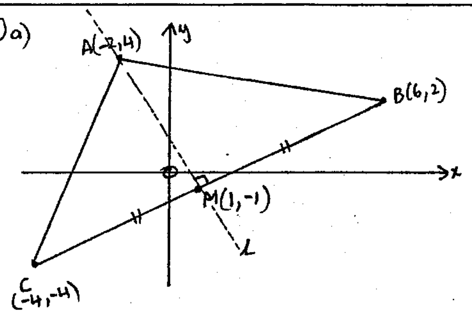
$= x^2(3\cos x - x\sin x)$

c) $\angle ECB = 87^\circ$ (alt $\angle$'s, $BC \parallel AF$)

$\therefore \angle ABC = 87 - 42$ (ext $\angle$ ΔABC)

$= 45^\circ$

③ a)



i) $BC = \sqrt{(6+4)^2 + (2+4)^2} = \sqrt{100+36} = \sqrt{136}$

ii) $m(BC) = \frac{2+4}{6+4} = \frac{6}{10} = \frac{3}{5}$

iii) $M = \left(\frac{6-4}{2}, \frac{-4+2}{2}\right) = (1,-1)$

iv) $m(l) = \frac{-5}{3}$

Eqn l: $\quad y+1 = \frac{-5}{3}(x-1)$

$3y+3 = -5x+5$

$5x+3y-2 = 0$

v) A(−2,4) $\quad$ LHS $= 5x+3y-2$

$= 5(-2)+3(4)-2$

$= -10+12-2$

$= 0 =$ RHS

$\therefore$ A lies on line l.

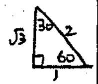
vi) $MA = \sqrt{(1+2)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

$\therefore$ Area $\Delta ABC = \frac{1}{2} \times \sqrt{136} \times \sqrt{34}$

$= 34$ units^2

b) $\sqrt{3}\tan x = -1 \qquad 0 \le x \le 2\pi$

$\tan x = -\frac{1}{\sqrt{3}} \qquad$ Q 2, 4 $\quad \pi/6$



$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6}$

c) $|3-2x| \le 5$

$-5 \le 3-2x \le 5$

$-8 \le -2x \le 2$

$4 \ge x \ge -1 \qquad \therefore -1 \le x \le 4$

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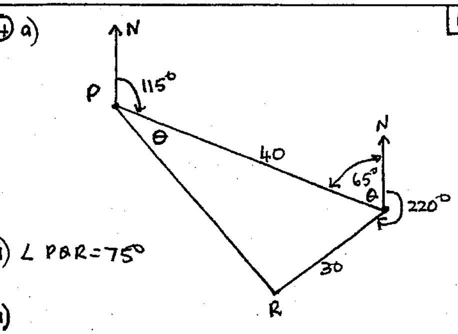
④ a)



i) $\angle PFR = 75^\circ$

ii) $PR^2 = 40^2 + 30^2 - 2.40.30\cos 75$

$= 1878.83$

$PR = 43.35$

iii) $\frac{\sin\theta}{30} = \frac{\sin 75}{43.35}$

$\sin\theta = \frac{30\sin 75}{43.35} = 0.668$

$\theta \doteqdot 42^\circ$

$\therefore$ Bearing $= 115 + 42 = 157^\circ$T

b) $4x = 8y - y^2$

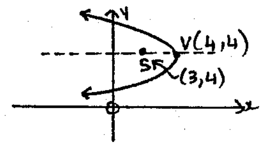
i) $y^2 - 8y + 16 = -4x + 16$

$(y-4)^2 = -4(x-4) \qquad V = (4,4)$

$a = 1$

ii) $S = (3,4)$

iii)



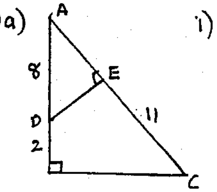
c) $\alpha + \beta = k-1 \qquad \alpha\beta = 2k$

$k-1 = 2k$

$k = -1$

d) $\int_{-3}^{3} f(x)\,dx = 0$

⑤ a)



i) In Δ's ADE, ABC

$\angle A$ is common

$\angle AED = \angle ABC$ (given as 90°)

$\therefore \Delta ADE \equiv \Delta ABC$ (equiangular)

ii) $\frac{AE}{10} = \frac{8}{AE+11}$

$AE^2 + 11AE = 80$

$AE^2 + 11AE - 80 = 0$

$(AE+16)(AE-5) = 0$

$AE = -16, 5$

But $AE > 0 \quad \therefore AE = 5$

b) $80 + 50 + 45 + 40.5 + \ldots$

GP $a = 50 \quad r = 0.9$

i) 45cm

ii) 3 yrs $= 80 + 50 + 45 + 40.5 = 215.5$cm

iii) max height :- $S_\infty = \frac{a}{1-r} = \frac{50}{1-0.9} = 500$cm

$\therefore$ max height $= 80 + 500 = 580$cm

$\therefore$ tree never reaches 10m

iv) 5m → $\quad 500 = 80 + S_n$

$420 = \frac{50(1-0.9^n)}{1-0.9}$

$420 = \frac{50(1-0.9^n)}{0.1}$

$\frac{42}{50} = 1 - 0.9^n$

$0.9^n = 1 - \frac{42}{50} = 0.16$

$\ln(0.9)^n = \ln 0.16$

$n\ln 0.9 = \ln 0.16$

$n = \frac{\ln 0.16}{\ln 0.9} = 17.393\ldots$

$\therefore$ during the 17th year

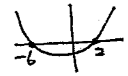
c) Real roots if $b^2 - 4ac \ge 0$

$[(2+k)]^2 - 4(1)(4) \ge 0$

$4 + 4k + k^2 - 16 \ge 0$

$k^2 + 4k - 12 \ge 0$

$(k+6)(k-2) \ge 0$



$\therefore k \le -6$ or $k \ge 2$

⑥ a) $f(x) = 8x^3 - 8x^2$

i) $f'(x) = 24x^2 - 16x$

$f''(x) = 48x - 16$

Stat. pts. when $f'(x) = 0$

$24x^2 - 16x = 0$

$8x(3x-2) = 0$

$x = 0, \frac{2}{3}$

at $x = 0 \quad y = 0 \quad f''(0) = -16 < 0 \quad \therefore$ MAX $(0,0)$

at $x = \frac{2}{3} \quad y = -1\frac{5}{27} \quad f''(\frac{2}{3}) = 16 > 0 \quad \therefore$ MIN $(\frac{2}{3}, -1\frac{5}{27})$

ii) Pt of inflexion when $f''(x) = 0$

$48x - 16 = 0$

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ii) cont

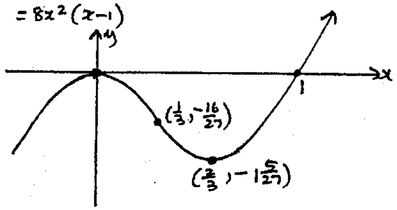
$x = \frac{16}{48} = \frac{1}{3}$

at $x = \frac{1}{3}$, $y = -\frac{16}{27}$

x	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
y''	$-$	0	$+$

Concavity changes $\therefore$ pt inflexion at $(\frac{1}{3}, -\frac{16}{27})$

i) $f(x) = 8x^3 - 8x^2$
$= 8x^2(x-1)$



v) concave down and decreasing:

$0 < x < \frac{1}{3}$

) Limiting sum if $|r| < 1$ $r = \ln x$

$-1 < \ln x < 1$

$\log_e e^{-1} < \log_e x < \log_e e$

$\therefore \frac{1}{e} < x < e$

iv) Area $= \int_0^{\pi/3} \sin x \, dx + \frac{1}{2}bh$

$= [-\cos x]_0^{\pi/3} + \frac{1}{2}(\frac{\sqrt{3}}{4})(\frac{\sqrt{3}}{2})$

$= (-\cos\frac{\pi}{3}) - (-\cos 0) + \frac{3}{16}$

$= -\frac{1}{2} + 1 + \frac{3}{16}$

$= \frac{11}{16}$ units^2

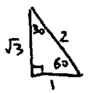
b) i) As ΔABC is equilateral, all angles are $60°$.
Hence $\angle ABC = 60° = \frac{60\pi}{180}$ rads
$= \pi/3$ rads.

ii) Area $\Delta ABC = \frac{1}{2} \times 6 \times 6 \times \sin 60$
$= 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$ cm^2

Area sector $AXY = \frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{3\pi}{2}$ cm^2

$\therefore$ Shaded area $=$ area $\Delta - 3 \times$ area sector
$= 9\sqrt{3} - 3(\frac{3\pi}{2})$
$= 9\sqrt{3} - \frac{9\pi}{2}$ cm^2

7a) $y = \sin x$ $P(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$



i) $\frac{dy}{dx} = \cos x$

at $x = \pi/3$ $m = \cos\frac{\pi}{3} = \frac{1}{2}$ $y = \frac{\sqrt{3}}{2}$

$\therefore m_1 = -2$

Eqn normal: $y - \frac{\sqrt{3}}{2} = -2(x - \frac{\pi}{3})$

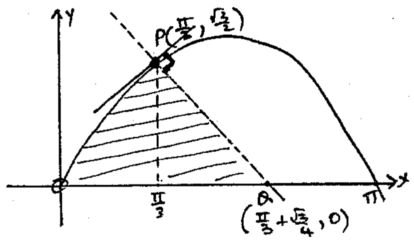
$y - \frac{\sqrt{3}}{2} = -2x + \frac{2\pi}{3}$

$2x + y = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

) $y = 0$ $2x = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3} + \frac{\sqrt{3}}{4}$ $Q[\frac{\pi}{3} + \frac{\sqrt{3}}{4}, 0]$

ii)



⑧ a) i)

x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

ii) $A = \int_{-2}^{2} 3^x \, dx \doteqdot \frac{1}{3}[\frac{1}{9} + 1 + 4 \times \frac{1}{3}] + \frac{1}{3}[1 + 9 + 4 \times 3]$
$\doteqdot 8\frac{4}{27}$

b) $V_x = \pi \int_1^9 y^2 \, dx$ $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$y^2 = x + 2 + \frac{1}{x}$

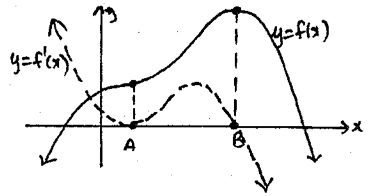
$= \pi \int_1^9 (x + 2 + \frac{1}{x}) dx$

$= \pi [\frac{x^2}{2} + 2x + \ln x]_1^9$

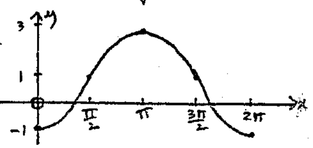
$= \pi [\frac{81}{2} + 18 + \ln 9 - (\frac{1}{2} + 2 + \ln 1)]$

$= \pi [56 + \ln 9]$

c) i)
ii)



d) i) $y = 1 - 2\cos x$



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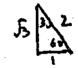
ii) $1 - 2\cos x = 0$



$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$1 - 2\cos x > 0$ for $\frac{\pi}{3} < x < \frac{5\pi}{3}$

⑨ a) \$180 000 \$P/month 6% pa = 0.5% per month

i) $A_1 = 180000 + 180000 \times \frac{0.5}{100} - P$

$= 180000(1 + 0.005) - P$

$= 180000(1.005) - P$

ii) $A_2 = A_1 \times 1.005 - P$

$= [180000 \times 1.005 - P] \times 1.005 - P$

$= 180000 \times 1.005^2 - P \times 1.005 - P$

$= 180000 \times 1.005^2 - P(1.005 + 1)$

$A_3 = A_2 \times 1.005 - P$

$= [180000 \times 1.005^2 - P(1.005 + 1)] \times 1.005 - P$

$= 180000 \times 1.005^3 - P(1.005^2 + 1.005) - P$

$= 180000 \times 1.005^3 - P(1 + 1.005 + 1.005^2)$

iii) $n = 8$ years $= 96$ months

$\therefore A_{96} = 0$

$180000 \times 1.005^{96} - P(1 + 1.005 + 1.005^2 + \ldots + 1.005^{95}) = 0$

$P = \frac{180000 \times 1.005^{96}}{1 + 1.005 + \ldots + 1.005^{95}}$

↳ AP $a = 1$, $r = 1.005$ $n = 96$

$P = \frac{180000 \times 1.005^{96}}{\frac{1(1.005^{96} - 1)}{1.005 - 1}} = \frac{180000 \times 1.005^{96} \times 0.005}{1.005^{96} - 1}$

$P = \$2365.46$

b) $x = 2t + e^{-2t}$

i) $v = \frac{dx}{dt} = 2 - 2e^{-2t}$

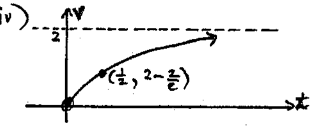
$t = \frac{1}{2}$ $v = 2 - 2e^{-1} = 2 - \frac{2}{e}$ m/sec

ii) $t = 0$ $v = 2 - 2e^0 = 2 - 2 = 0$ $\therefore$ at rest

iii) $v = 2 - \frac{2}{e^{2t}}$ as $t \to \infty$ $e^{2t} \to \infty$ $\frac{2}{e^{2t}} \to 0$

$\therefore$ velocity $\to 2$ m/sec

iv)



v) $a = \frac{dv}{dt} = 4e^{-2t}$ $v = 2 - 2e^{-2t}$

$2e^{-2t} = 2 - v$

$e^{-2t} = \frac{2-v}{2}$

$\therefore a = 4e^{-2t} = 4(\frac{2-v}{2}) = 2(2-v)$
$= 4 - 2v$

⑩ a) i) $M = a - \ln(a-1)$

ii) $\frac{dM}{da} = 1 - \frac{1}{a-1} = 1 - (a-1)^{-1}$

$\frac{d^2M}{da^2} = (a-1)^{-2}$

Stat pts when $\frac{dM}{da} = 0$

$1 - \frac{1}{a-1} = 0$

$a - 1 = 1$

$a = 2$

at $a = 2$, $\frac{d^2M}{da^2} = (2-1)^{-2} > 0$ $\therefore$ min

iii) $M = 2 - \ln(2-1) = 2 - \ln 1 = 2$

b) $S = 285000$ $t = 0$
$S = 202000$ $t = 6$

i) $S = Ae^{-kt}$

$t = 0$, $S = 285000$ } $285000 = Ae^0$
$\therefore A = 285000$

$S = 285000e^{-kt}$

$S = 202000$, $t = 6$ } $202000 = 285000e^{-6k}$

$\frac{202000}{285000} = e^{-6k}$

$\ln(\frac{202}{285}) = \ln e^{-6k} = -6k$

$k = \frac{\ln\frac{202}{285}}{-6}$

$\doteqdot 0.05737$

ii) $S = 285000e^{-kt}$

$\frac{dS}{dt} = -k \cdot 285000e^{-kt}$
$= -k(S)$

iii) $t = 1$ year $= 12$ months

$S = 285000e^{-k \times 12}$
$= 143171$

iv) $\frac{1}{3}$ original size $= 95000$

$95000 = 285000e^{-kt}$

$\frac{1}{3} = e^{-kt}$

$\ln(\frac{1}{3}) = \ln e^{-kt} = -kt$

$t = \frac{\ln\frac{1}{3}}{-k} = 19.15$ mths

v) $\frac{dS}{dt} = -kS = -k \times 95000 = -5450$

$\therefore$ decreasing at 5450 sheep/month.