CARINGBAH HIGH

2007

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

MATHEMATICS

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using blue or black pen
- Board approved calculators may be used
- Write your name on each page
- Each question is to be started on a new page.
- This examination paper must NOT be removed from the examination room

- There is a total of ten questions.
- Each question is worth 12 marks.
- Marks may be deducted for careless or badly arranged work.

Question One

Marks

a) Evaluate $e^{1\cdot4}$ correct to 3 significant figures. 2

b) Factorise $2x^2+7x-4$ 2

c) Simplify $\tan 30° \cos 60°$ leaving your answer in surd form. 2

d) Differentiate $x^2 \ln x$. 2

e) Evaluate $\lim_{x\to3} \frac{x^2-9}{x-3}$ 2

f) Solve $(x+2)^2=9$. 2

Question Two (Start a new page) **Marks**

a) Solve $3x^2 - x - 5 = 0$ leaving your answer in surd form. 2

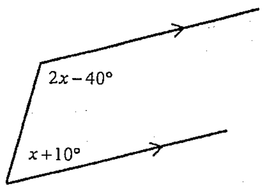
b) Differentiate $\left(4x^3 - 5\right)^6$ 2

c) Find a primitive of $x^2 + \cos 3x$ 2

d) Evaluate $\int_0^1 e^{2x}\,dx$ 2

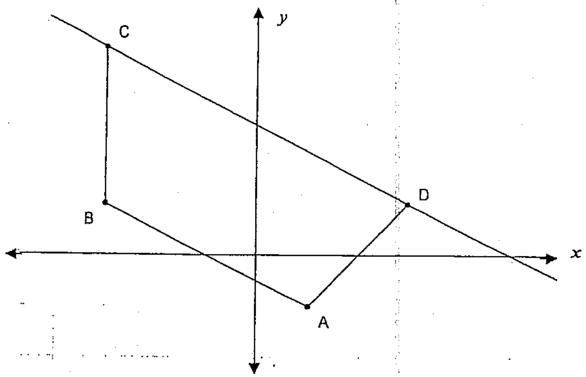
e) Given that $\frac{12}{\sqrt{6}} = \sqrt{a}$, find the value of '$a$'. 2

f) Find the value of 'x' giving a reason for your answer. 2



Question Three (Start a new page) **Marks**

a) A (1, -1), B (-3, 1), C (-3, 4) and D (3, 1) are points on the Cartesian Plane.



i) Find the distance CD. 1

ii) Show that the equation of the line CD is $x + 2y - 5 = 0$. 2

iii) Find the perpendicular distance of A from CD. 2

iv) Hence or otherwise find the area of the triangle ACD. 1

v) What type of quadrilateral is ABCD? Explain carefully. 2

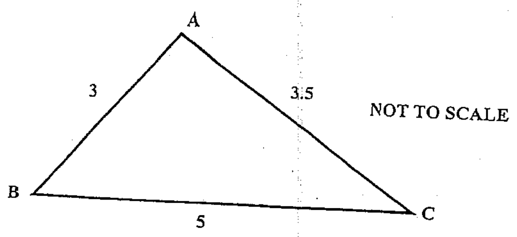
b) Simplify fully $\frac{\sin(180 - \theta) \times \cot\theta}{\sec\theta}$. 2

c) Factorise and simplify $\frac{8x^3 - 27}{4x - 6}$. 2

Question Four (Start a new page)

Marks

a)



In the diagram above AB = 3cm, BC = 5cm and AC = 3.5cm. Find the size of the smallest angle, correct to the nearest degree. 2

b) Find the equation of the tangent to the curve $y = e^x + x$ at the point where $x = 0$. 3

c) State the centre and radius of the circle with equation $(x+3)^2 + y^2 = 16$ 2

d) i) Sketch on the same diagram.

$y = |x - 2|$ and $y = 2x$, showing the 'x' and 'y' intercepts. 2

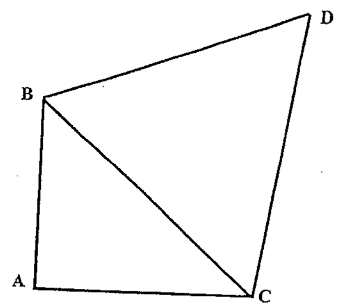
ii) Hence or otherwise solve $|x - 2| = 2x$ 2

iii) Using (i) or otherwise, find $\int_0^4 |x-2|\, dx$ 1

Question Five (Start a new page)

Marks

a)



In the above diagram, ABC is an isosceles triangle in which $\angle BAC = 90°$. BCD is an equilateral triangle.

Copy the diagram onto your answer sheet and mark in the given information.

i) Find the size of $\angle ACD$ giving reasons. 2

ii) If BC = 3cm, find the perimeter of ABDC in exact form. 2

b) The quadratic equation $2x^2 - 3x + 6 = 0$ has roots α and β. Find the value of :

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

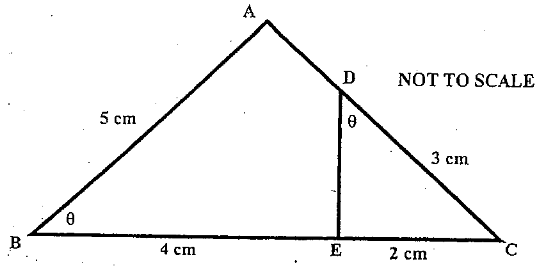
iii) $a^2 + \beta^2$ 2

c) Use Simpson's rule with 3 function values to find an approximation for the value of $\int_0^1 10^x\, dx$. Give your answer to 3 decimal places. 2

d) Evaluate $\int_0^{\frac{\pi}{3}} \sec^2 x\, dx$. Give your answer in exact form. 2

Question Six(Start a new page) — Marks

a)



In the above diagram, ABC and DEC are two triangles in which $\angle ABC = \angle EDC$.
Also AB = 5cm, BE = 4cm, EC = 2cm and CD = 3cm.

Copy the diagram onto your answer sheet.

i) Prove that the two triangles are similar. **2**

ii) Hence, giving reasons find the length of DE. **2**

b) Farmer Brown has hired a driller to drill a borehole to gain access to the underground water on his property. The cost is $260 for the first 3 metres drilled, $280 for the next 2 metres, $300 for the next 2 metres and so on. The price increases by the same amount for each successive 2 metres drilled.

i) Show that the cost of drilling the portion from a depth of 25 metres to to 27 metres is $500. **2**

ii) Calculate the total cost of drilling to a depth of 27 metres. **1**

iii) The cost of drilling the borehole to reach water was $12500. Find the total depth drilled to give access to the water. **3**
[To gain full marks all working needs to be shown.]

c) Solve for x: $\log_a 3 = 2\log_a 6 - \log_a x$ **2**

Question Seven(Start a new page) — Marks

a) A function $f(x)$ is defined by $f(x) = x^3 - 3x^2 - 9x$.

i) Find $f'(x)$ and $f''(x)$. **1**

ii) Find the turning points for the curve and determine their nature. **3**

iii) Show that there is one point of inflexion and find its coordinates. **2**

iv) Sketch the graph of $y = f(x)$ showing the turning points and the point of inflexion. **2**

v) Find the values of 'x' for which the function $f(x)$ is decreasing. **1**

b) For what values of 'k' does the quadratic equation $x^2 - (k+3)x + 4k = 0$

i) have one root equal to 2? **1**

ii) have no real roots? **2**

Question Eight(Start a new page)

Marks

a) A particle is moving in a straight line and its velocity v metres/second at time t seconds is given by:

$$v = \frac{dx}{dt} = 1 - 2\sin 2t, \quad t \geq 0$$

Initially the particle is at the origin.

i) Express the displacement x, as a function of t. 2

ii) Find the position of the particle when $t = \frac{\pi}{6}$. 1

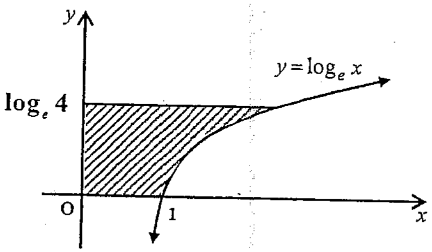
iii) Find an expression for the acceleration in terms of t. 1

iv) Sketch the graph of the acceleration for $0 \leq t \leq \pi$. 2

v) What is the maximum acceleration of the particle? 1

b) In the diagram below, the shaded region bounded by the curve $y = \log_e x$, the x and y axes and the line $y = \log_e 4$ is rotated about the y-axis. 3

Find the exact volume of the solid of revolution formed.



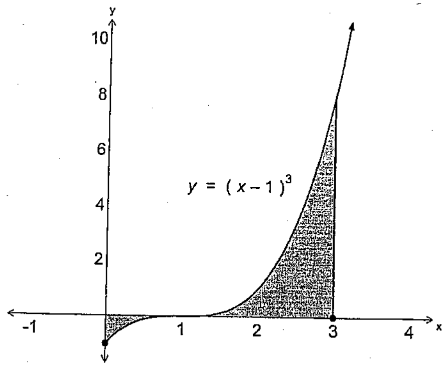
c) The geometric series $1 - x + x^2 - x^3 + \ldots$ has a limiting sum of 4. 2

Find the value of x.

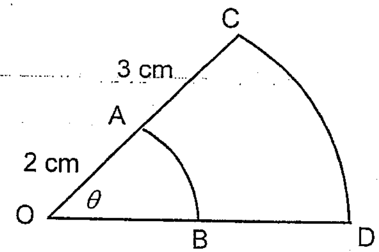
Question Nine(Start a new page)

Marks

a) The shaded area in the diagram below is the region bounded by the curve $y = (x-1)^3$, the x and y axes and the line $x = 3$. Find the shaded area. 4



b)



The arcs AB and CD are parts of concentric circles with centre O. OA = 2 centimetres and AC = 3 centimetres.

i) Find an expression for the area of the sector AOB. 1

ii) Find the ratio of the area of sector AOB : the area of ABDC 2

Question 9 continued over/

Question 9 continued

Marks

c) An industrial plant produces vacuum cleaners. The annual production, P cleaners, at time t years, is given by:

$P = P_0 e^{kt}$ where P_0 and k are constants

Initially the production of the plant was 2500 cleaners per annum. Five years later it had increased to 4000 cleaners per annum.

i) Find the values of P_0 and k. 3

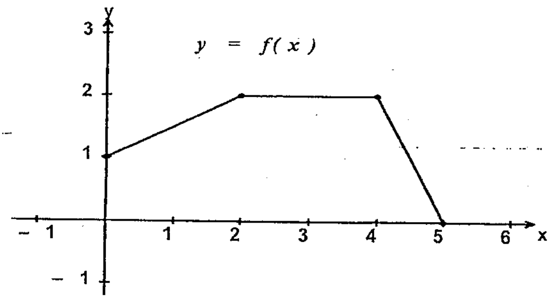
ii) What is the predicted production after 10 years? 1

iii) Find the rate of increase in production after 5 years. 1

Question Ten(Start a new page)

a) The graph of $y = f(x)$ is drawn below.

On a number plane sketch the graph of the derivative function $y = f'(x)$. 2



b) Let A be the point (-2, 0) and B be the point (6, 0).
At the point P(x, y), PA meets PB at right angles.

i) Show that the gradient of PA is $m_1 = \frac{y}{x+2}$. 1

ii) Hence find an equation for the locus of P. 2

Question 10 continued over/

Question 10 continued

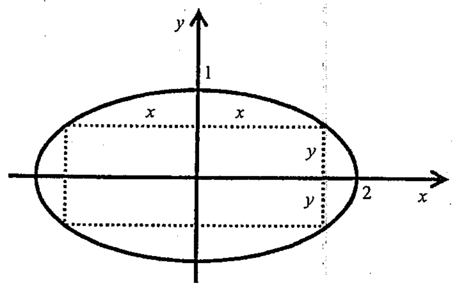
Marks

c) i) Given that $\frac{x^2}{4} + y^2 = 1$,

Show that $y = \frac{\sqrt{4-x^2}}{2}$ for $y \geq 0$ 1

ii) Show that $\frac{dy}{dx} = \frac{-x}{2\sqrt{4-x^2}}$ 1

d) i) The ellipse with equation $\frac{x^2}{4} + y^2 = 1$ is drawn below.



A rectangle of length $2x$ and width $2y$ is to be constructed inside the ellipse with its vertices on the ellipse as shown.

Using part (c) or otherwise show that an expression for the area of the rectangle is given by $A = 2x\sqrt{4-x^2}$. 1

ii) Hence find the value of x so that the area of the rectangle is a maximum. 4

END OF EXAM

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1.a) 4.05519967 = 4.06 {3 Sig Fig} ☑☑

b) $(2x-1)(x+4)$ ☑☑

c) $\frac{1}{\sqrt{3}}\times\frac{1}{2}=\frac{1}{2\sqrt{3}}$ ☑☑

d) $x^2\times\frac{1}{x}+2x\times\ln x = x+2x\ln x$ ☑☑

e) $\lim_{x\to3}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to3}(x+3)=6$ ☑☑

f) $x+2=\pm3 \rightarrow x=1$ and $x=-5$ ☑☑

2.a) $x=\frac{1\pm\sqrt{1^2-4\times3\times-5}}{2\times3}=\frac{1\pm\sqrt{61}}{6}$ ☑☑

b) $6(4x^3-5)^5\times12x^2$ ☑

$=72x^2(4x^3-5)^5$ ☑

c) $\frac{x^3}{3}+\frac{1}{3}\sin3x$ ☑☑

d) $\int_0^1 e^{2x}\,dx=\frac{1}{2}e^{2x}\Big]_0^1$ ☑

$=\frac{1}{2}(e^2-e^0)=\frac{1}{2}(e^2-1)$ ☑

e) $=\frac{12}{\sqrt6}\times\frac{\sqrt6}{\sqrt6}=2\sqrt6$ ☑

$=\sqrt{24} \Rightarrow a=24$ ☑

f) $(2x-40)+(x+10)=180$ { Co-interior ∠'s and || lines } ☑

$3x=210 \Rightarrow x=70°$ ☑

3.a)i) $CD=\sqrt{6^2+(-3)^2}=\sqrt{45}=3\sqrt5$ ☑

ii) $m_{CD}=\frac{-3}{6}=-\frac{1}{2}$ ☑

$\therefore eq^n\ CD: y-1=-\frac{1}{2}(x-3)$ ☑

$2y-2=-x+3 \Rightarrow x+2y-5=0$

iii) $a=1, b=2, c=-5, x_1=1, y_1=-1$ ☑

$\therefore d=\frac{|1\times1+2\times-1+(-5)|}{\sqrt{1^2+2^2}}=\frac{6}{\sqrt5}$ ☑

iv) $A=\frac{1}{2}\times\frac{6}{\sqrt5}\times3\sqrt5=9u^2$ ☑

v) $m_{AB}=\frac{1-(-1)}{-3-1}=-\frac{1}{2}$

Hence using (ii) AB || CD ☑

So ABCD is a Trapezium ☑

b) $\sin\theta\times\frac{\cos\theta}{\sin\theta}\times\cos\theta=\cos^2\theta$ ☑☑

c) $\frac{(2x-3)(4x^2+6x+9)}{2(2x-3)}$ ☑

$=\frac{4x^2+6x+9}{2}$ ☑

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4. a) $\cos\theta=\frac{3.5^2+5^2-3^2}{2\times3.5\times5} \Rightarrow \theta\approx36°$ ☑☑

b) $y'=e^x+1$ ☑

when $x=0$, $m=y'=2$, $y=1$ ☑

$\therefore y-1=2(x-0) \Rightarrow y=2x+1$ ☑

c) Centre is (-3, 0) and Radius is 4 ☑☑

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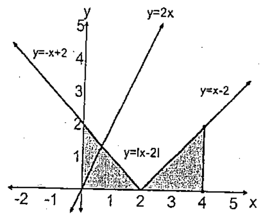
4.d) i)



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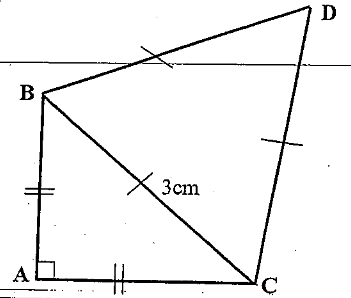
ii) $2x=-x+2$ ☑

$3x=2 \Rightarrow x=\frac{2}{3}$ ☑

iii) Shaded area $=\frac{1}{2}(2\times2)+\frac{1}{2}(2\times2)=4$ ☑

5.a)



i) $\angle ACB=\frac{180-90}{2}=45°$ { ∠ sum isos Δ} ☑

$\angle BCD=60°$ { equilateral Δ } ☑

$\therefore \angle ACD=45°+60°=105°$

ii) Let AB = AC = x; ∴ $x^2+x^2=3^2$

$\therefore 2x^2=9 \Rightarrow x=\frac{3}{\sqrt2}$ ☑

$\therefore P=2\times\frac{3}{\sqrt2}+2\times3=\frac{6}{\sqrt2}+6$ cm ☑

b) i) $\alpha+\beta=\frac{3}{2}$ ☑

ii) $\alpha\beta=3$ ☑

iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ☑

$=\left(\frac{3}{2}\right)^2-2\times3=-3\frac{3}{4}$ ☑

c) $\int_0^1 10^x\,dx=\frac{1-0}{6}\left[f(0)+4f\left(\frac{1}{2}\right)+f(1)\right]$ ☑

$=\frac{1}{6}[1+4\times3.16228+10]\approx3.942$ ☑

d) $\int_0^{\frac{\pi}{3}}\sec^2x\,dx=\left[\tan x\right]_0^{\frac{\pi}{3}}$ ☑

$=\tan\frac{\pi}{3}-\tan0=\sqrt3$ ☑

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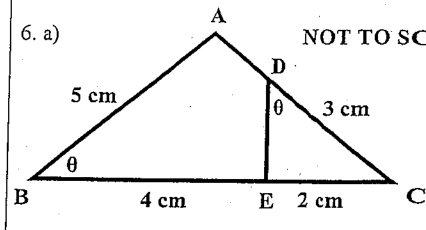
6. a)



i) In the Δ's ABC and EDC

∠ABC = ∠EDC { given }

∠ACB = ∠ECD { common } ☑

∴ Δ ABC ||| Δ EDC { equiangular } ☑

ii) $\frac{DE}{AB}=\frac{DC}{BC}$ {corres. sides in similar Δ's} ☑

$\therefore \frac{DE}{5}=\frac{3}{6} \Rightarrow DE=2.5\text{cm}$ ☑

b) Set up 2 arithmetic sequences:

Depth: 3, 5, 7, …., 25, 27, …. ①

Cost: 260, 280, 300, …. ②

i) Using ①: a = 3, d = 2: $T_n=a+(n-1)d$

$\therefore 27=3+(n-1)\times2 \Rightarrow n=13$ ☑

Using ②: a = 260, d = 20 and n = 13

$T_{13}=260+12\times20=\$500$ ☑

ii) $S_{13}=\frac{13}{2}[260+500]=\$4940$ ☑

iii) $12500=\frac{n}{2}[2\times260+(n-1)\times20]$ ☑

$n^2+25n-1250=0$ {after simplifying} ☑

$(n-25)(n+50)=0 \Rightarrow n=25$

$\therefore T_{25}=3+24\times2=51$ metres drilled ☑

6.c) $\log_a 3 = \log_a 6^2 - \log_a x$

$\log_a 3 = \log_a \dfrac{36}{x} \Rightarrow x = 12$ ☑☑

7. i) $f(x) = x^3 - 3x^2 - 9x$

$f'(x) = 3x^2 - 6x - 9;\ f''(x) = 6x - 6$ ☑

ii) For turning points $f'(x) = 0$

$\therefore 3(x^2 - 6x - 9) = 0$

$3(x-3)(x+1) = 0$

$\therefore x = -1$ and $x = 3$ ☑

When $x = 3$: $y = -7, y'' = 12 > 0$

$\therefore (3, -27)$ is a minimum turning point. ☑

When $x = -1$: $y = 5, y'' = -12 < 0$

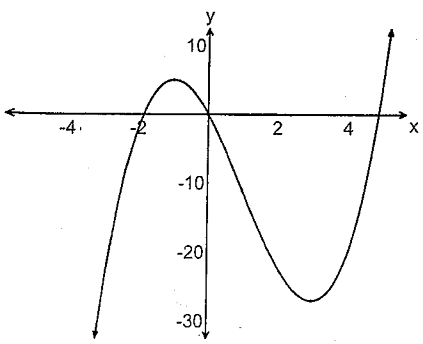
$\therefore (-1, 5)$ is a maximum turning point. ☑

iii) For inflexion points $f''(x) = 0$

$\therefore 6x - 6 = 0 \Rightarrow x = 1$ (Only one point) ☑

It has coordinates $(1, -11)$. ☑

iv) ☑☑



v) For a decreasing function $f'(x) < 0$

Hence from the graph $-1 < x < 3$ ☑

b) i) Substitute $x = 2$ to obtain:

$4 - 2k - 6 + 4k = 0 \Rightarrow k = 1$ ☑

ii) For no real roots $\Delta < 0$, where $\Delta = b^2 - 4ac$

$\therefore (k+3)^2 - 4 \times 1 \times 4k < 0$ ☑

$k^2 - 10k + 9 < 0$

$(k-9)(k-1) < 0 \Rightarrow 1 < k < 9$ ☑

8. a) i) $v = 1 - 2\sin 2t$

$\therefore x = t + \cos 2t + c$ ☑

When $t = 0, x = 0$

$\therefore 0 = 0 + \cos 0 + c$

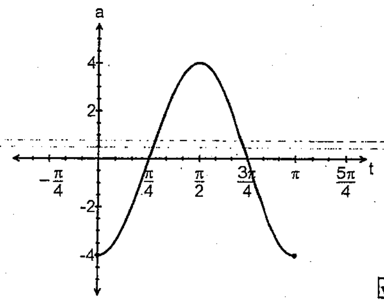
$0 = 1 + c \Rightarrow c = -1$ ☑

$\therefore x = t + \cos 2t - 1$

ii) $x = \dfrac{\pi}{6} + \cos\dfrac{\pi}{3} - 1 = \dfrac{\pi}{6} - \dfrac{1}{2}$ ☑

iii) $a = \ddot{x} - 4\cos 2t$ ☑

iv)



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v) Maximum acceleration is $4\,m/s^2$ ☑

b) $V = \pi \displaystyle\int_0^{\ln 4} x^2\, dy \quad [y = \ln x \Rightarrow x = e^y]$

$V = \pi \displaystyle\int_0^{\ln 4} (e^y)^2\, dy = \pi \int_0^{\ln 4} e^{2y}\, dy$ ☑

$= \dfrac{\pi}{2}\left[e^{2y}\right]_0^{\ln 4} = \dfrac{\pi}{2}\left[e^{2\ln 4} - e^0\right]$ ☑

$\dfrac{\pi}{2}\left[e^{\ln 16} - 1\right] = \dfrac{15\pi}{2}\, u^3$ ☑

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8.c) $S_\infty = \dfrac{a}{1-r}$, with $a = 1, r = -x$ ☑

$\therefore 4 = \dfrac{1}{1+x}$

$4 + 4x = 1 \Rightarrow x = -\dfrac{3}{4}$ ☑

9. a)

$A = \left|\displaystyle\int_0^1 (x-1)^3\, dx\right| + \int_1^3 (x-1)^3\, dx$ ☑☑

$= \left|\dfrac{1}{4}\left[(x-1)^4\right]_0^1\right| + \dfrac{1}{4}\left[(x-1)^4\right]_1^3$ ☑

$= \dfrac{1}{4} + 4 = 4\dfrac{1}{4}$ units^2 ☑

b) i) $A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times 2^2 \times \theta = 2\theta$ ☑

ii) Area sector $OCD = \dfrac{1}{2} \times 5^2 \times \theta = 12.5\theta$

$\therefore$ Area $ABCD = 12.5\theta - 2\theta = 10.5\theta$ ☑

Hence ratio $AOB : ABCD = 2\theta : 10.5\theta$

$= 4 : 21$ ☑

c) i) $P = P_0 e^{kt}$; when $t = 0, P = 2500$

$\therefore 2500 = P_0 e^0 \Rightarrow P_0 = 2500$ ☑

Also when $t = 5, P = 4000$

$\therefore 4000 = 2500 e^{5k} \Rightarrow 1.6 = e^{5k}$ ☑

$\ln(1.6) = 5k \Rightarrow k \approx 0.094$ ☑

ii) $P = 2500 e^{0.094 \cdot 10} \approx 6400$ ☑

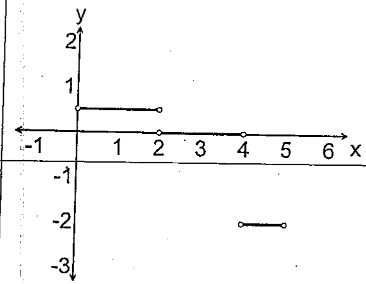
iii) Rate $= \dfrac{dP}{dt}$: $P = 2500 e^{0.094t}$

$\dfrac{dP}{dt} = 0.094 \times 2500 e^{0.094t}$

Hence after 5 years $\dfrac{dP}{dt} = 0.094 \times 2500 e^{0.094 \times 5}$

≈ 376 vacuum cleaners / year ☑

10. a)



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b) i) $m_{PA} = \dfrac{y - 0}{x - (-2)} \Rightarrow m_1 = \dfrac{y}{x+2}$ ☑

ii) $m_{PB} = \dfrac{y - 0}{x - 6} \Rightarrow m_2 = \dfrac{y}{x-6}$

Since PA ⊥ PB then $m_1 \times m_2 = -1$ ☑

$\therefore \dfrac{y}{x+2} \times \dfrac{y}{x-6} = -1$

$y^2 = -(x+2)(x-6)$ ☑

$\therefore x^2 + y^2 - 4x - 12 = 0$

10.c) i) $y^2 = 1 - \frac{x^2}{4} = \frac{4-x^2}{4}$ ☑

$\therefore y = \sqrt{\frac{4-x^2}{4}} = \frac{\sqrt{4-x^2}}{2}$

ii) $y = \frac{1}{2}(4-x^2)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} \times (4-x^2)^{-\frac{1}{2}} \times -2x$ ☑

$= \frac{-x}{2\sqrt{4-x^2}}$

d) i) $A = 2x \times 2y$

$= 2x \times 2\frac{\sqrt{4-x^2}}{2} = 2x\sqrt{4-x^2}$ ☑

ii) $A = 2x\sqrt{4-x^2}$

$\frac{dA}{dx} = \sqrt{4-x^2} \times 2 + 2x \times \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times -2x$ ☑

$= 2\sqrt{4-x^2} - \frac{2x^2}{\sqrt{(4-x^2)}}$

$= \frac{2(4-x^2)}{\sqrt{(4-x^2)}} - \frac{2x^2}{\sqrt{(4-x^2)}}$

$= \frac{8-4x^2}{\sqrt{(4-x^2)}}$ ☑

Stationary points when $\frac{dA}{dx} = 0$

$\therefore 8 - 4x^2 = 0$

$x^2 = 2 \quad \Rightarrow \quad x = \pm\sqrt{2}$ ☑

Since $x > 0$, only need to test $x = +\sqrt{2}$.

x	1.3	$\sqrt{2}$	1.5
A′	0.816	0	-0.756
	/	—	\

☑

$\therefore$ A maximum area occurs when $x = \sqrt{2}$.