Caringbah High School



2011 Trial Higher School Certificate Examination

Mathematics

General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

- Attempt Questions 1 -10
- All questions are of equal value

Total marks – 120 Attempt Questions 1–10 All questions are of equal value

Start	t each question in a SEPARATE booklet. Extra booklets are available.	Marks
Que	stion 1 (12 marks) Use a SEPARATE writing booklet.	
(a)	Evaluate $\sqrt{\frac{4^2+11^2}{321-11^2}}$ correct to three significant figures.	2
(b)	Find a primitive of $4 + \sec^2 x$.	2
(c)	Factorise $x^3 + 27$.	2
(d)	Solve $ x-5 = 8$.	2
(e)	Simplify $\frac{x}{x^2 - 9} + \frac{3}{x + 3}.$	2
(f)	Solve $4x = x^2$.	2

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to *x*:

(i)	$x^{5} + 7$				2
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(ii)
$$\frac{x^2}{x+1}$$
 2

(iii)
$$x \cos x$$
 2

Question 2 (continued)

(b) Find

(i)
$$\int \frac{12x^3}{x^4 + 2} dx$$
 2

(ii)
$$\int_{0}^{1} (e^{5x} - 1) dx$$
 2

(c) Find the equation of the tangent to $y = e^{2x}$ at the point $(2, e^4)$. 2





(b) The lengths of the sides of a triangle are 8 cm, 9 cm and 14 cm. Find the size of the angle opposite the smallest side.

(c) Evaluate
$$\sum_{n=2}^{4} (1-3n)$$

Question 4 (12 marks) Use a SEPARATE writing booklet.



AOB is a sector of a circle, centre O and radius 6 cm. The length of the arc AB is 5π cm.

	(i)	Find the exact size of $\angle AOB$	1
	(ii)	Calculate the exact area of the shaded segment.	2
(b)	Consid	der the function $f(x) = 3x^2 - x^3$.	
	(i)	Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature.	3
	(ii)	Sketch the curve showing where it meets the axes.	2
	(iii)	Find the values for which the curve is concave down.	2
(c)	Sally i compo years?	invests \$3000 in a term deposit that earns 6.5% per annum punded annually. What is the value of her investment at the end of 15	2

Question 5 (12 marks) Use a SEPARATE writing booklet.

Madison is learning to drive. Her first lesson is 10 minutes long. Her second (a) lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson. How long will Madison's fifteenth lesson be? (i) How many hours of lessons will Madison have completed after (ii) her fifteenth lesson? During which lesson will Madison have completed a total of 40 (iii) hours of driving lessons? Find the values of *m* for which the expression below is always positive. (b) $x^{2} + 2mx + (3m - 2)$ Find the amplitude and period if $y = -3\cos \pi x$ (c)

1

2

2

2

2

(d) (i) Differentiate $\log_e(\sin x)$ 1

(ii) Hence, or otherwise, find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$ 2

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Find all values of , where $0^{\circ} \le \theta \le 360^{\circ}$, that satisfy the equation . $\cos \theta - \frac{2}{5} = 0$ 2

Give your answer(s) to the nearest degree.

(b) Solve |x-2| > 5 and graph your solution on the number line. 2

Question 6 (continued)

(c) (i) Find the limiting sum of the geometric series

$$3 + \frac{3}{\sqrt{3}+1} + \frac{3}{(\sqrt{3}+1)^2} + \dots$$

(ii) Explain why the geometric series

$$3 + \frac{3}{\sqrt{3}-1} + \frac{3}{(\sqrt{3}-1)^2} + \dots$$

does NOT have a limiting sum.

- (d) A council worker accidently spread a toxic chemical on a local soccer field. The concentration of the chemical in the soil was initially measured at 4 kg/ha. One year later the concentration was found to be 2.6 kg/ha. It is known that the concentration, *C*, is given by $C = C_0 e^{-kt}$, where C_0 and *k* are constants, and *t* is measured in years.
 - (i) Evaluate C_0 and k.
 - (ii) It is safe to use the soccer field when the concentration is below
 0.1 kg/ha. How long must the soccer players wait after the accident before the soccer field can be used? Give your answer in years, correct to one decimal place.

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a)



3

In the diagram, *CD* is parallel to *AB*, *PB* = *QB*, $\angle BPD = 50^{\circ}$ and $\angle BQR = x^{\circ}$. Copy or trace this diagram.

Find the value of *x*, giving complete reasons.

2

1

3

2

Question 7 (continued)

(b) Let α and β be the roots of the equation

$$x^2 - 5x + 2 = 0$$

Find the values of;

(b) (i)
$$\alpha + \beta$$
 1

(ii)
$$\alpha\beta$$
 1

(iii)
$$(\alpha + 1)(\beta + 1)$$
 2

(c) The equation of a parabola is
$$y = \frac{x^2}{8} - 3$$

	(i)	Find the coordinates of the vertex of the parabola.	2
	(ii)	Find the equation of the directrix of the parabola.	1
	(iii)	Sketch the curve clearly labelling the vertex and directrix.	2
Que	stion 8	(12 marks) Use a SEPARATE writing booklet.	
(a)	Use Si	mpson's rule, with three function values to find an	2
	approx	simution for $\int_{0.5}^{1.5} (\log_e x)^3 dx$.	
	Give y	your answer correct to three decimal places.	
(b)	(i)	Write down the discriminant of $2x^2 + (k-2)x + 8$, where <i>k</i> is a constant.	1
	(ii)	Hence, or otherwise, find the values of k for which the parabola	2

 $y = 2x^2 + kx + 9$ does not intersect the line y = 2x + 1.

Question 8 (continued)

(c)	Durin	g a storm, water flows into a 7000 litre tank at a rate of $\frac{dV}{dt}$ litres per			
	minute, where $\frac{dV}{dt} = 120 + 26t - t^2$ and t is the time in minutes since the storm began.				
	(i)	At what time is the tank filling at twice the initial rate?	2		
	(ii)	Find the volume of water that has flowed into the tank since the start of the storm as a function of t .	1		
	(iii)	Initially, the tank contains 1500 litres of water. When the storm finishes, 30 minutes after it began, the tank is overflowing. How many litres of water have been lost?	2		
(d)	Solve	the equation $2 \ln x = \ln(7x - 12)$	2		

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a)



The graphs of y = 2x and $y = 6x - x^2$ intersect at the origin and point B(4,8).

Find the shaded area bounded by y = 2x and $y = 6x - x^2$.

3





The part of the curve $\frac{x^2}{2} + y^2 = 8$ that lies in the first quadrant is rotated about the *x*-axis. Find the volume of the solid of revolution.

3

(c) Mr Smith borrows \$80 000 to purchase a new car. The interest rate is calculated monthly at the rate of 1% per month, and is compounded each month.

Mr Smith intends to repay the loan, with interest, in two equal annual instalments of M at the end of the first and second years.

- (i) How much does Mr Smith owe at the end of the first month?
 (ii) Write an expression involving *M* for the total amount owed by Mr Smith after 12 months, just after the first instalment of \$*M* has been paid.
- (iii) Find an expression for the amount owed at the end of the second year 2 and deduce that

$$M = \frac{80000 \times (1 \cdot 01)^{24}}{(1 \cdot 01)^{12} + 1}$$

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the following equation for *x* :

$$2e^{2x} - e^x = 0$$

(b) Show that
$$f(x) = \frac{x^4 - 8}{x^3}$$
 is an odd function.

- (c) Let $f(x) = \sqrt{25 x^2}$
 - (i) Copy the following table of values into your writing booklet and supply the missing values.

2

1

1

x	0	1	2	3	4	5
f(x)	5.000		4.583			0.000

(ii) Use these six values of the function and the trapezoidal rule to find 2 the approximate value of

$$\int_0^5 \sqrt{25 - x^2} dx$$

(iii) Draw the graph of $x^2 + y^2 = 25$ and shade the region whose area is represented by the integral 2

$$\int_0^5 \sqrt{25 - x^2} \, dx$$

(iv) Use your answer to part (iii) to explain why the exact value of the integral is $2\frac{25\pi}{4}$

(v) Use your answers to part (ii) and part (iv) to find an approximate value of π 2

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x$, x > 0

$$\frac{k_{11}}{(1)} (115 \text{ THSC} \text{ Additional constructs} (115 \text{ Additional constructs} (115 \text{ THSC} \text{ Additional constrults} (115 \text{ THSC} \text{ Additional constrults$$

$$\begin{aligned} |2n| & CHS THSC MATHEMATICS SOLUTIONS \\ &= \frac{3}{3}(5+1) & J_{S} \\ &= \frac{3}{5+1} & J_{S} \\ &= \frac{3}{5}(5+1) & J_{S} \\ &=$$

(i)

$$x = 115^{\circ}$$
(ii)

$$x = 115^{\circ}$$
(iii)

$$x = 115^{\circ}$$
(i)

$$x = 55^{\circ}$$
(i)

$$x = 6 = 5^{\circ}$$
(ii)

$$x = 6 = 5^{\circ}$$
(i)

$$x = 8 = 5^{\circ}$$
(i)

$$x = 8 = 5^{\circ}$$
(i)

$$x = 8 = 5^{\circ}$$
(ii)

$$x = 8 = 5^{\circ}$$
(iii)

$$x = 6^{\circ}$$
(iii)

$$x = (10^{\circ})^{\circ}$$
(iv)

$$x$$

$$20|| CHS THSC MATHEMAtics SOLUTIONS
:.. (k-10)(k+6) < 0
:.. (k-10)(k+6) < 0
:.. (k-10)(k+6) < 0
:.. (k-10)(k+6) < 0
:.. (k-10)(k+6) < 0 = 120 L/min
:.. 240 = 120 + 0 - 0 = 120 L/min
:.. 240 = 120 + 26t - t2
:.. 240 = 120 + 26t - t2
:.. 240 = 120 + 26t - t2
:.. (t-6)(t-20) < 0
:.. (t-$$

B)
$$V_{x} = \pi \int_{1}^{4} (s - \frac{x^{2}}{2}) dx$$

 $\int_{1}^{2} \frac{x^{2}}{2} y^{2} = 8 = \pi \left[8x - \frac{x^{3}}{6} \right]_{0}^{4}$
 $\int_{1}^{1} \frac{x^{2}}{2} y^{2} = 8 = \pi \left[8x - \frac{x^{3}}{6} \right]_{0}^{4}$
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 $\int_{1}^{1} \frac{x^{2}}{2} y^{2} = 8 = \pi \left[(3x - \frac{x^{3}}{6}) \right]_{0}^{4}$
 $\int_{1}^{1} \frac{x^{2}}{2} \frac{1}{2} \frac{x^{2}}{8} \frac{80000}{2} = 1001 + \frac{1}{2} \frac{x^{2}}{4} \frac{80000}{2} = 1001 + \frac{1}{2} \frac{x^{2}}{4} \frac{x^{2}}{8} \frac{80000}{2} = \frac{1}{101} \frac{1}{2} \frac{x^{2}}{4} \frac{x^{2}}{8} \frac{80000}{2} = \frac{1}{101} \frac{1}{2} \frac{x^{2}}{4} \frac{x^{2}}{8} \frac{80000}{1} = \frac{1}{101} \frac{1}{2} \frac{x^{2}}{4} \frac{x^{2}}{8} \frac{80000}{1} = \frac{1}{101} \frac{1}{2} \frac{x^{2}}{4} \frac{x^{2}}{8} \frac{80000}{1} = \frac{1}{101} \frac{1}{2} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{101} \frac{1}{2} \frac{1}{101} \frac{1}{$



(a)

$$\frac{2011}{5} \frac{(-x)}{115} \frac{(-x)^4 - 8}{(-x)^3}$$

$$= \frac{x^4 - 8}{-x^3}$$

$$= -\left(\frac{x^4 - 8}{-x^3}\right)$$

$$= -\int (x) = \int f(x) \text{ is an odd} for other
for other
(c)
(1)
$$\frac{1}{2} \frac{2}{5} \frac{3}{\sqrt{24}} \frac{4}{4} \cdot 583 \frac{4}{3} \frac{3}{0}$$

$$\sqrt{24} = \frac{4 \cdot 899}{4 \cdot 583} \frac{4}{3} \frac{3}{0}$$

$$\sqrt{24} = \frac{4 \cdot 899}{5} \frac{1}{\sqrt{25 - x^2} dx} = \frac{1}{2} \begin{cases} 5 + 2x (\sqrt{24} + 4 \cdot 583 + 4 + 3) \\ -5 \sqrt{25 - x^2} dx = \frac{1}{2} \end{cases}$$

$$(11)$$

$$= \frac{18 \cdot 98197949}{\sqrt{x^2 + y^2} = 25}$$$$

(iii)

$$x^{4}y^{2}=25$$
Area= $\int \sqrt{25-x^{2}} dx$

$$7x 0$$
(iv)

$$\int \sqrt{25-x^{2}} dx = area of \frac{1}{4} of a circle centre (0,0) redives 5 mit = \frac{11.5^{2}}{4}$$

$$= \frac{2577}{4}v^{2}$$

(i)

$$25T = 18.98...$$

 $T = 4 \times 18.98...$
 $= 3.037116718$