## Caringbah High School



# 2011 <br> Trial Higher School Certificate Examination 

## Mathematics

## General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value

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Attempt Questions 1-10
All questions are of equal value
Start each question in a SEPARATE booklet. Extra booklets are available.
Question 1 (12 marks) Use a SEPARATE writing booklet.
(a) Evaluate $\sqrt{\frac{4^{2}+11^{2}}{321-11^{2}}}$ correct to three significant figures.
(b) Find a primitive of $4+\sec ^{2} x$.
(c) Factorise $x^{3}+27$.
(d) Solve $|x-5|=8$.
(e) Simplify $\frac{x}{x^{2}-9}+\frac{3}{x+3}$.
(f) Solve $4 x=x^{2}$.

Question 2 (12 marks) Use a SEPARATE writing booklet.
(a) Differentiate with respect to $x$ :
$\begin{array}{ll}\text { (i) } x^{5}+7 & 2 \\ \text { (ii) } \frac{x^{2}}{x+1} & 2\end{array}$
(iii) $x \cos x$

Question 2 (continued)
(b) Find
(i) $\int \frac{12 x^{3}}{x^{4}+2} d x$
(ii) $\int_{0}^{1}\left(e^{5 x}-1\right) d x$
(c) Find the equation of the tangent to $y=e^{2 x}$ at the point $\left(2, e^{4}\right)$.

Question 3 (12 marks) Use a SEPARATE writing booklet.
(a)


In the diagram, $O A B C$ is a trapezium with $O A \| C B$. The coordinates of $O, A$ and $B$ are $(0,0),(1,1)$ and $(-4,6)$ respectively.
(i) Calculate the length $O A$.
(ii) Write down the gradient of line OA.
(iii) What is the size of $\angle A O C$ ?
(iv) Find the equation of the line $B C$, and hence find the coordinates of $C$.
(v) Show that the perpendicular distance from $O$ to the line $B C$ is $5 \sqrt{2}$.
(vi) Hence, or otherwise, calculate the area of the trapezium $O A B C$.
(b) The lengths of the sides of a triangle are $8 \mathrm{~cm}, 9 \mathrm{~cm}$ and 14 cm . Find the size of the angle opposite the smallest side.
(c) Evaluate $\sum_{n=2}^{4}(1-3 n)$

Question 4 (12 marks) Use a SEPARATE writing booklet.
(a)

$A O B$ is a sector of a circle, centre $O$ and radius 6 cm . The length of the arc $A B$ is $5 \pi \mathrm{~cm}$.
(i) Find the exact size of $\angle A O B$
(ii) Calculate the exact area of the shaded segment.
(b) Consider the function $f(x)=3 x^{2}-x^{3}$.
(i) Find the coordinates of the stationary points of the curve $y=f(x)$ and determine their nature.
(ii) Sketch the curve showing where it meets the axes.
(iii) Find the values for which the curve is concave down.
(c) Sally invests $\$ 3000$ in a term deposit that earns $6 \cdot 5 \%$ per annum compounded annually. What is the value of her investment at the end of 15 years?

Question 5 (12 marks) Use a SEPARATE writing booklet.
(a) Madison is learning to drive. Her first lesson is 10 minutes long. Her second lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson.
(i) How long will Madison's fifteenth lesson be?
(ii) How many hours of lessons will Madison have completed after her fifteenth lesson?
(iii) During which lesson will Madison have completed a total of 40 hours of driving lessons?
(b) Find the values of $m$ for which the expression below is always positive.

$$
x^{2}+2 m x+(3 m-2)
$$

(c) Find the amplitude and period if $y=-3 \cos \pi x$.
(d) (i) Differentiate $\log _{e}(\sin x)$
(ii) Hence, or otherwise, find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x . d x$

Question 6 (12 marks) Use a SEPARATE writing booklet.
(a) Find all values of , where $0^{\circ} \leq \theta \leq 360^{\circ}$, that satisfy the equation $\cos \theta-\frac{2}{5}=0$
Give your answer(s) to the nearest degree.
(b) Solve $|x-2|>5$ and graph your solution on the number line.

Question 6 (continued)
(c) (i) Find the limiting sum of the geometric series

$$
3+\frac{3}{\sqrt{3}+1}+\frac{3}{(\sqrt{3}+1)^{2}}+\ldots
$$

(ii) Explain why the geometric series

$$
3+\frac{3}{\sqrt{3}-1}+\frac{3}{(\sqrt{3}-1)^{2}}+\ldots
$$

does NOT have a limiting sum.
(d) A council worker accidently spread a toxic chemical on a local soccer field. The concentration of the chemical in the soil was initially measured at 4 $\mathrm{kg} / \mathrm{ha}$. One year later the concentration was found to be $2 \cdot 6 \mathrm{~kg} / \mathrm{ha}$. It is known that the concentration, $C$, is given by $C=C_{0} e^{-k t}$, where $C_{0}$ and $k$ are constants, and $t$ is measured in years.
(i) Evaluate $C_{0}$ and $k$.
(ii) It is safe to use the soccer field when the concentration is below $0.1 \mathrm{~kg} / \mathrm{ha}$. How long must the soccer players wait after the accident before the soccer field can be used? Give your answer in years, correct to one decimal place.

Question 7 (12 marks) Use a SEPARATE writing booklet.
(a)


In the diagram, $C D$ is parallel to $A B, P B=Q B, \angle B P D=50^{\circ}$ and $\angle B Q R=x^{\circ}$. Copy or trace this diagram.
Find the value of $x$, giving complete reasons.

Question 7 (continued)
(b) Let $\alpha$ and $\beta$ be the roots of the equation

$$
x^{2}-5 x+2=0
$$

Find the values of;
(b) (i) $\alpha+\beta$
(ii) $\alpha \beta$
(iii) $\quad(\alpha+1)(\beta+1)$
(c) The equation of a parabola is $y=\frac{x^{2}}{8}-3$
(i) Find the coordinates of the vertex of the parabola.
(ii) Find the equation of the directrix of the parabola.
(iii) Sketch the curve clearly labelling the vertex and directrix.

Question 8 (12 marks) Use a SEPARATE writing booklet.
(a) Use Simpson's rule, with three function values to find an
approximation for $\int_{0.5}^{1.5}\left(\log _{e} x\right)^{3} d x$.
Give your answer correct to three decimal places.
(b) (i) Write down the discriminant of $2 x^{2}+(k-2) x+8$, where $k$ is a constant.
(ii) Hence, or otherwise, find the values of k for which the parabola 2 $y=2 x^{2}+k x+9$ does not intersect the line $y=2 x+1$.

## Question 8 (continued)

(c) During a storm, water flows into a 7000 litre tank at a rate of $\frac{d V}{d t}$ litres per minute, where $\frac{d V}{d t}=120+26 t-t^{2}$ and $t$ is the time in minutes since the storm began.
(i) At what time is the tank filling at twice the initial rate?
(ii) Find the volume of water that has flowed into the tank since the start of the storm as a function of $t$.
(iii) Initially, the tank contains 1500 litres of water. When the storm finishes, 30 minutes after it began, the tank is overflowing. How many litres of water have been lost?
(d) Solve the equation $2 \ln x=\ln (7 x-12)$

Question 9 (12 marks) Use a SEPARATE writing booklet.
(a)


The graphs of $y=2 x$ and $y=6 x-x^{2}$ intersect at the origin and point $B(4,8)$.

Find the shaded area bounded by $y=2 x$ and $y=6 x-x^{2}$.

Question 9 (continued)
(b)


The part of the curve $\frac{x^{2}}{2}+y^{2}=8$ that lies in the first quadrant is rotated about the $x$-axis. Find the volume of the solid of revolution.
(c) Mr Smith borrows $\$ 80000$ to purchase a new car. The interest rate is calculated monthly at the rate of $1 \%$ per month, and is compounded each month.
Mr Smith intends to repay the loan, with interest, in two equal annual instalments of $\$ M$ at the end of the first and second years.
(i) How much does Mr Smith owe at the end of the first month?
(ii) Write an expression involving $M$ for the total amount owed by Mr Smith after 12 months, just after the first instalment of $\$ M$ has been paid.
(iii) Find an expression for the amount owed at the end of the second year and deduce that

$$
M=\frac{80000 \times(1 \cdot 01)^{24}}{(1 \cdot 01)^{12}+1}
$$

(iv) What is the total interest over the two-year period?

Question 10 (12 marks) Use a SEPARATE writing booklet.
(a) Solve the following equation for $x$ :

$$
2 e^{2 x}-e^{x}=0
$$

(b) Show that $f(x)=\frac{x^{4}-8}{x^{3}}$ is an odd function.
(c) Let $f(x)=\sqrt{25-x^{2}}$
(i) Copy the following table of values into your writing booklet and supply the missing values.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 5.000 |  | 4.583 |  |  | 0.000 |

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of

$$
\int_{0}^{5} \sqrt{25-x^{2}} d x
$$

(iii) Draw the graph of $x^{2}+y^{2}=25$ and shade the region whose area is represented by the integral

$$
\int_{0}^{5} \sqrt{25-x^{2}} d x
$$

(iv) Use your answer to part (iii) to explain why the exact value of the integral is $\frac{25 \pi}{4}$.
(v) Use your answers to part (ii) and part (iv) to find an approximate value of $\pi$

## End of Examination

## STANDARD INTEGRALS

| $\int x^{n} d x$ | $=\frac{1}{n+1} x^{n+1}, \quad n \neq-1 ; \quad x \neq 0 \text {, if } n<0$ |
| :---: | :---: |
| $\int \frac{1}{x} d x$ | $=\ln x, \quad x>0$ |
| $\int e^{a x} d x$ | $=\frac{1}{a} e^{a x}, \quad a \neq 0$ |
| $\int \cos a x d x$ | $=\frac{1}{a} \sin a x, \quad a \neq 0$ |
| $\int \sin a x d x$ | $=-\frac{1}{a} \cos a x, \quad a \neq 0$ |
| $\int \sec ^{2} a x d x$ | $=\frac{1}{a} \tan a x, \quad a \neq 0$ |
| $\int \sec a x \tan a x d x$ | $=\frac{1}{a} \sec a x, \quad a \neq 0$ |
| $\int \frac{1}{a^{2}+x^{2}} d x$ | $=\frac{1}{a} \tan ^{-1} \frac{x}{a}, \quad a \neq 0$ |
| $\int \frac{1}{\sqrt{a^{2}-x^{2}}} d x$ | $=\sin ^{-1} \frac{x}{a}, \quad a>0,-a<x<a$ |
| $\int \frac{1}{\sqrt{x^{2}-a^{2}}} d x$ | $=\ln \left(x+\sqrt{x^{2}-a^{2}}\right), \quad x>a>0$ |
| $\int \frac{1}{\sqrt{x^{2}+a^{2}}} d x$ | $=\ln \left(x+\sqrt{x^{2}+a^{2}}\right)$ |

NOTE: $\ln x=\log _{e} x, \quad x>0$

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Q
(a) 0.828
(b) $4 x+\tan x+c$
(c) $(x+3)\left(x^{2}-3 x+a\right)$
(d) $x=-3$ or 13
(e) $\frac{x}{(x-3)(x+3)}+\frac{3(x-3)}{(x+3)(x-3)}=\frac{4 x-9}{(x-3)(x+3)}$
(f)

$$
\begin{aligned}
& x^{2}-4 x=0 \\
& x(x-4)=0 \Rightarrow x=0 \text { or } 4
\end{aligned}
$$

Q2 (a) (i)
(ii)

$$
\begin{aligned}
& \frac{5 x^{4}}{2 x(x+1)-1 \cdot x^{2}} \\
& \begin{aligned}
(x+1)^{2} & =\frac{x^{2}+2 x}{(x+1)^{2}} \\
& =\frac{x(x+2)}{(x+1)^{2}}
\end{aligned}
\end{aligned}
$$

(iii) $\cos x-x \sin x$
(b) (i) $3 \int \frac{4 x^{3}}{x^{4}+2} d x=3 \ln \left(x^{4}+2\right)+c$
(ii)

$$
\begin{aligned}
{\left[\frac{e^{5 x}}{5}-x\right]_{0}^{1} } & =\frac{e^{5}}{5}-1-\left(\frac{1}{5}-0\right) \\
& =\frac{e^{5}}{5}-\frac{6}{5} \\
& =\frac{e^{5}-6}{5}
\end{aligned}
$$

(c) $y^{\prime}=2 e^{2 x} \Rightarrow m_{7}(2)=2 e^{4}$

$$
\begin{aligned}
\therefore y-e^{4} & =2 e^{4}(x-2) \\
y & =2 e^{4} x-4 e^{4}+e^{4}
\end{aligned}
$$

$\therefore$ Tangent is $\quad y=2 e^{4} \cdot x-3 e^{4}$
Q3 (a) (i) $O A=\sqrt{(1-0)^{2}+(1-0)^{2}}=\sqrt{2}$
(ii) $\quad m_{O A}=\frac{1-0}{1-0}$

$$
=1
$$

(b)

$$
\begin{aligned}
& \cos \alpha=\frac{9^{2}+14^{2}-8^{2}}{2.9 .14} \\
& \therefore \alpha^{\prime} \div 32.3025^{\circ} \\
& \vdots 32^{\circ} 18^{\prime} \\
& \text { Weed } d_{B C}=\sqrt{(-10--4)^{2}+(6-0)^{2}} \\
& =\sqrt{36+36} \\
& =6 \sqrt{2} \\
& \therefore \text { Area }_{\text {OABC }} \frac{1}{2} \cdot 5 \sqrt{2} \cdot(6 \sqrt{2}+\sqrt{2}) \\
& =350^{2}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\text { Sum } & =(1-12)+(1-9)+(1-6) \\
& =-11-8-5 \\
& =-24
\end{aligned}
$$

Q4(a) (1) $l=r \theta$
(iii) $\angle A O C=135^{\circ}$
(v) $\quad m_{B C}=1 \quad B=(-4,6)$

$$
\begin{gathered}
y-6=1(x--4) \\
y=x+4+6
\end{gathered}
$$

line $B C \equiv \quad y=x+10$
atc $y=0 \therefore x=-10$
(v)

$$
\begin{aligned}
\text { line } B C \equiv & x-y+10=0 \\
\therefore D_{h} & =\frac{|0.1-0.1+10|}{\sqrt{1^{2}+1^{2}}} \\
& =\frac{|10|}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\
& =\frac{10 \sqrt{2}}{2} \\
& =5 \sqrt{2}
\end{aligned}
$$

(vi)

$$
\begin{aligned}
& l=r \theta \\
& 5 \pi=6 \theta \\
& \theta=\frac{5 \pi^{c}}{6}
\end{aligned}
$$

(11) Area $=\frac{1}{2} 6^{2}\left(\frac{5 \pi}{6}-\sin \frac{5 \pi}{6}\right)$ $=15 \pi-18 \sin \pi / 6$

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$$
\begin{aligned}
\text { Q4 }(a)(i i)\left(\text { cont }^{\prime} d\right) & =15 \pi-18 \cdot \frac{1}{2} \\
& =15 \pi-9 \mathrm{v}^{2}
\end{aligned}
$$

(b)
(1)

$$
\begin{aligned}
f^{\prime}(x) & =6 x-3 x^{2} \\
& =3 x(2-x)=0
\end{aligned}
$$

when $x=0$ or 2
If $x=0, y=0 \Rightarrow(0,0)$
If $x=2, y=4 \Rightarrow(2,4)$
To getnature consider $5^{\prime \prime}(x)$

$$
\begin{aligned}
f^{\prime \prime}(x) & =6-6 x \\
& =6(1-x) \\
f^{\prime \prime}(0) & =6>0 \Rightarrow \text { O Mrwimum }
\end{aligned}
$$

$$
a+(0,0)
$$

(II)

(III) coneave doum $\Rightarrow f^{\prime \prime}(x)<0 \Rightarrow 6-6 x<0$
(c) Use $A=P(1+r)^{n}$

$$
\begin{aligned}
& =\$ 3000(1.065)^{15} \\
& =\$ 7715.52
\end{aligned}
$$

Q5 (a) (i)

$$
a=10 \quad d=5, T_{15}=?
$$

$$
\text { ve } T_{n}=a+(n-1) d^{\prime}
$$

$$
\begin{aligned}
T_{15} & =10+14 \times 5 \\
& =80 \text { minvtes }
\end{aligned}
$$

(ii)

$$
\begin{aligned}
S_{15}=? \text { Use } S_{M} & =\frac{1}{2}\{a+l) \\
& =\frac{15}{2}(10+80)
\end{aligned}
$$

(iii) $S_{n}=40 \times 60$ $n=$ ?
Use $S_{n}=\frac{n}{2}(2 a+(n-1) d)$

$$
\begin{gathered}
40 \times 60=\frac{n}{2}(20+5 n-5) \\
4800=15 n+5 n^{2} \\
n^{2}+3 n-960=0 \\
n=\frac{-3 \pm \sqrt{9+4 \times 460}}{2} \\
n=29 \cdot 5
\end{gathered}
$$

$\therefore$ Duning the 30 th lesson
(b) Always positive $\Rightarrow \Delta>0$

$$
\begin{aligned}
& \therefore b^{2}-4 a c=(2 m)^{2}-4(3 m-2)>0 \\
& 4 m^{2}-12 m+8>0 \\
& m^{2}-3 m+2>0 \\
&(m-2)(m-1)>0 \\
& \therefore m<1 \text { ar } m>2
\end{aligned}
$$

(c) amplitude $=3$

$$
\begin{aligned}
\text { period } & =\frac{2 \pi}{\pi} \\
& =2
\end{aligned}
$$

(d)(i) $\frac{\cos x}{\sin x}=\cot x$
(ii)

$$
\begin{aligned}
& {[\sin x} \\
& {[\ln (\sin x)]_{\pi / 4}^{\pi / 2}} \\
& =\ln (\sin \pi / 2)-\ln (\sin \pi / 4) \\
& =\ln (1)-\ln \left(\frac{1}{\sqrt{2}}\right) \\
& =\ln \sqrt{2}
\end{aligned}
$$

Q6 (a)

$$
\begin{aligned}
\cos \theta & =\frac{2}{5} \\
\theta & =66^{\circ} \text { or } 294^{\circ}
\end{aligned}
$$

$$
\begin{array}{rr}
x-2>5 & \text { or }-(x-2)>5  \tag{b}\\
x>7 & x<-3
\end{array}
$$

(c) (i) $a=3 \quad r=\frac{1}{\sqrt{3}+1}$

$$
S_{\infty}=\frac{3}{1-\frac{1}{\sqrt{3}+1}}
$$

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$$
\begin{aligned}
& =\frac{3(\sqrt{3}+1)}{\sqrt{3}+1-1} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
& =\sqrt{3}(\sqrt{3}+1) \\
& =3+\sqrt{3}
\end{aligned}
$$

(ii)

$$
\begin{aligned}
& r\left.=\frac{1}{(\sqrt{3}-1}\right) \cdot(\sqrt{3}+1) \\
&(\sqrt{3}+1) \\
&=\frac{\sqrt{3}+1}{3-1} \\
&=\frac{1}{2}(1+\sqrt{3}) \\
&=1.366>1 \Rightarrow \text { no limiting sum }
\end{aligned}
$$

(d)(i) $C_{0}=4$
and $2.6=4 x e^{-|k|}$

$$
\begin{aligned}
& \frac{2.6}{4}=e^{-k} \\
& k=-\ln \left(\frac{2.6}{4}\right) \div 0.4307829161 \\
& 0.1=4 x e^{-k t} \\
& e^{-k t}=\frac{0.1}{4} \\
&-k t=\ln \left(\frac{0.1}{4}\right) \\
& t=-\frac{\ln \left(\frac{0.1}{4}\right)}{k} \\
&=8.563198113 y s \\
&=8.6 y r s
\end{aligned}
$$

Q7 (a) $\triangle B P Q$ is isosceles ( $B P=B Q$ )

$$
\angle B Q P=180^{\circ}-x \text { (anglesum straight angle }
$$

R $\varphi p$ )
$\angle B P Q=\angle P Q B$ (base angles isosceles trinugle

$$
\begin{gathered}
=180^{\circ}-x \quad B A^{\circ} Q \text { ) } \\
\angle Q B P=\angle B P D=50^{\circ}(\text { atternate angles, } \\
P D \| Q B) \\
\therefore 180-x+180-x+50=180^{\circ}(\text { angle sum, } \triangle B P Q)
\end{gathered}
$$


(b) (i) $\alpha+\beta=-\frac{b}{a}=5$
(ii) $\alpha \beta=\frac{c}{a}=2$
(iii)

$$
\begin{aligned}
(\alpha+1)(\beta+1) & =\alpha \beta+\alpha+\beta+1 \\
& =2+5+1 \\
& =8
\end{aligned}
$$

(c) (i)

$$
\begin{aligned}
8 y & =x^{2}-24 \\
x^{2} & =8 y+24 \\
& =8(y+3) \\
(x-0)^{2} & =4 \times 2 x(y+3) \\
\therefore \text { Verte } x & =(0,-3)
\end{aligned}
$$

(ii) $a=2$
(iii)


Q8 (a)

$$
\begin{aligned}
\int_{0.5}^{1.5}\left(\log _{e} x\right)^{3} d x & \vdots \frac{1}{6}\left\{(\ln (0.5))^{3}+4(\ln (1))^{3}\right. \\
& \left.+(\ln (1.5))^{3}\right\} \\
& \equiv-0.044
\end{aligned}
$$

(b) $(i)$

$$
\begin{aligned}
& \Delta \equiv(k-2)^{2}-4.2 .8 \\
&=k^{2}-4 k-60 \\
&=(k-10)(k+6) \\
& 2 x^{2}+k x+9=2 x+1 \\
& 2 x^{2}+(k-2) x+8=0
\end{aligned}
$$

(ii)
no solutions $\Rightarrow$ no intersedions $\Rightarrow \Delta<0$

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$$
\therefore(k-10)(k+6)<0
$$



$$
\therefore-6<k<10
$$

(c) (i) initial rate $\Rightarrow t=0$

$$
\begin{gathered}
\therefore \frac{d V}{d t}=120+0-0=120 \mathrm{~L} / \mathrm{min} \\
\therefore \quad 240=120+26 t-t^{2} \\
t^{2}-26 t+120=0 \\
(t-6)(t-20)=0 \\
\therefore t=6 \operatorname{ar} 20 \mathrm{~mm}
\end{gathered}
$$

(ii)

$$
\begin{aligned}
& V=\int\left(120+260 t-t^{2}\right) d t \\
& V=120 t+13 t^{2}-\frac{t^{3}}{3} t c
\end{aligned}
$$

(iii) $t=0, v=1500$

$$
\begin{aligned}
& \therefore 1500=0+0-0+c \\
& \therefore V=1500+120 t+13 t^{2}-\frac{t^{3}}{3} \\
& V(30)=1500+120 \times 30+13 \times 30^{2}-\frac{30^{3}}{3} \\
& \\
& =7800 \Rightarrow 800 \mathrm{los})
\end{aligned}
$$

(d)

$$
\begin{aligned}
& \ln \left(x^{2}\right)=\ln (7 x-12) \\
& \therefore x^{2}=7 x-12 \\
& x^{2}-7 x+12=0 \\
& (x-3)(x-4)=0 \\
& \therefore x=3 \operatorname{ar} 4
\end{aligned}
$$

$\frac{p 9}{(a)}$

$$
\begin{aligned}
\text { Area } & =\int_{0}^{4}\left(6 x-x^{2}-2 x\right) d x \\
& =\int_{0}^{4}\left(4 x-x^{2}\right) d x \\
& =\left[2 x^{2}-\frac{x^{3}}{3}\right]_{0}^{4} \\
& \left.=32-\frac{64}{3}-10-0\right) \\
& =\frac{32}{3} v^{3}
\end{aligned}
$$

(b) $\quad V_{x}=\pi \int_{0}^{4}\left(8-\frac{x^{2}}{2}\right) d x$


$$
\therefore x^{2}=16
$$

$$
\begin{aligned}
& =\pi\left[8 x-\frac{x^{3}}{6}\right]_{0}^{4} \\
& =\pi\left(32-\frac{64}{6}-(0-0)\right)
\end{aligned}
$$

$$
x=4=6 \frac{4 \pi}{3} v^{3}
$$

(c) let Amount owing after $n$ months be $A O_{n}$ ( 1 )

$$
\begin{aligned}
\therefore A O_{1} & =\$ 80000+1 \%_{0} \text { of } \$ 80000 \\
& =1.01 \times \$ 80000
\end{aligned}
$$

(II)

$$
\begin{aligned}
A 02 & =1.01 \times A 01 \\
& =1.01^{2} \times \$ 80000 \\
& =1.01 \\
\therefore A_{012} & =1.01^{12} \times \$ 80000-\$ M=Y 1
\end{aligned}
$$

(iii)

$$
\begin{aligned}
A 013 & =1.01 \times Y_{1} \\
\text { A014 } & =1.01^{2} \times Y_{1} \\
A_{024} & =1.01^{12} \times Y_{1}-\$ 7 . \\
& =1.01^{12}\left(1.012^{12} \times \$ 80000-\$ m\right)-\$ m
\end{aligned}
$$

as $\mathrm{AO}_{4}=0$
then $1.01^{24} \times \$ 80000=1.01^{12} m+m$

$$
\begin{aligned}
& =M\left(1+1.01^{12}\right) \\
\therefore \quad M & =\frac{1.01^{24} \times \$ 80000}{1+1.01^{12}}
\end{aligned}
$$

(iv)

$$
\therefore M=\$ 47,760 \cdot 756
$$

$\therefore$ total pard back $=\$ 95,521.51$
$\therefore$ total interest $=\$ 15,521.51$
Q10 (a) let $u=e^{x}, \therefore$ solve $2 u^{2}-u=0$

$$
\begin{array}{r}
u(2 v-1)=0 \\
u=0 \quad \text { or } u=\frac{1}{2} \\
e^{x} \neq 0 \quad \therefore e^{x}=\frac{1}{2} \\
\therefore x=\ln (0.5)
\end{array}
$$

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(b)

$$
\begin{aligned}
& f(-x)=\frac{(-x)^{4}-8}{(-x)^{3}} \\
&=\frac{x^{4}-8}{-x^{3}} \\
&=-\left(\frac{x^{4}-8}{x^{3}}\right) \\
&=-f(x) \Rightarrow f(x) \text { isan odd } \\
& \text { function }
\end{aligned}
$$

(c)
(1)

| $\%$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 5 | $\sqrt{24}$ | 4.583 | 4 | 3 | 0 |

$$
\sqrt{24} \div 4.899
$$

(11)
(III)

(iv)

$$
\begin{aligned}
\int_{0}^{5} \sqrt{25-x^{2}} d x= & \operatorname{area} \text { of } \frac{1}{4} \text { of a cincle } \\
& \text { centre }(0,0) \text { radius } 5 \text { units } \\
= & \frac{\pi .5^{2}}{4} \\
= & \frac{25 \pi}{4} v^{2}
\end{aligned}
$$

(v)

$$
\begin{aligned}
\therefore \quad \frac{25 \pi}{4} & \div 18.98 \ldots \\
\therefore \pi & \div \frac{4}{25} \times 18.98 \ldots \\
& \doteq 3.037116718
\end{aligned}
$$

