Caringbah High School

2014

Trial HSC Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using a blue or black pen. Black pen is preferred.
- Board approved calculators may be used
- A table of standard integrals is provided at the back of the paper
- In Questions 11 – 16, show all relevant mathematical reasoning

Total Marks – 100

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow approximately 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11 – 16
- Allow approximately 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

For Questions 1–10, use the multiple-choice answer sheet provided.

---

1 Which of the following represents e^{-3} correct to three significant figures?

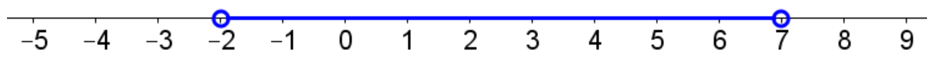
A. 0.049

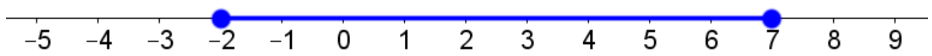
B. 4.98

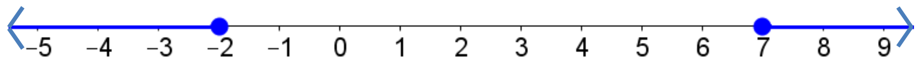
C. 0.0498

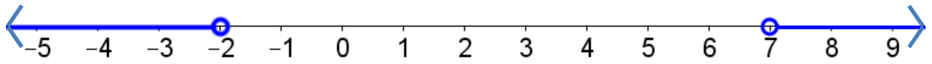
D. 0.0497

2 Which of the following represents the solution to the inequation $|5 - 2x| < 9$?

A. 

B. 

C. 

D. 

3 Which of the following represents the domain of the function:

$$f(x) = \sqrt{9 - x^2} + \frac{1}{x - 3}$$

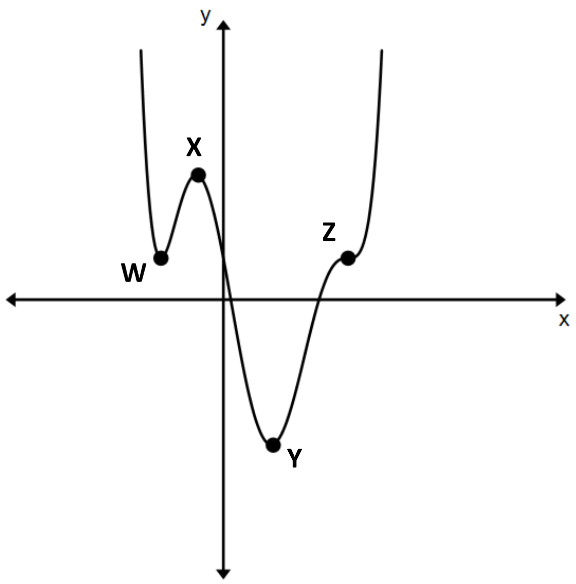
A. $x \neq 3$

B. $x < 3$

C. $-3 < x \leq 3$

D. $-3 \leq x < 3$

4 The diagram below shows the graph of $y = f(x)$. Points W, X, Y and Z are stationary points and Z is a point of inflexion on $f(x)$.



Which of the points on $y = f(x)$ corresponds to the description:

$$y > 0 \qquad \frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} < 0$$

A. W

B. X

C. Y

D. Z

5 For what values of m will the geometric series $1 + 2m + 4m^2 + 8m^3 + \ldots$ have a limiting sum?

A. $-1 \leq m \leq 1$

B. $-\frac{1}{2} \leq m \leq \frac{1}{2}$

C. $-\frac{1}{2} < m < \frac{1}{2}$

D. $m < \frac{1}{2}$

Questions 6 and 7 refer to the parabola $(x - 2)^2 = - 2(y - 3)$

6 What are the co-ordinates of the focus of this parabola?

A. $\left(-2\,,2\,\frac{1}{2}\right)$

B. $\left(2\,,\ 2\,\frac{1}{2}\right)$

C. $(2\,,3)$

D. $(-2\,,-3)$

7 What are the co-ordinates of the point of intersection of the axis of symmetry and the directrix of the parabola?

A. $\left(2\,,\ 3\,\frac{1}{2}\right)$

B. $\left(2\,,-3\,\frac{1}{2}\right)$

C. $\left(-2\,,\ 3\,\frac{1}{2}\right)$

D. $(2\,,\ 2)$

8 Which of the following is a primitive of the function $5 - \dfrac{1}{e^x}$?

A. $5x + \dfrac{1}{e^x} + \mathrm{C}$

B. $5x - \dfrac{1}{e^x} + \mathrm{C}$

C. $5x - \ln x + \mathrm{C}$

D. $5 - \dfrac{1}{e^{2x}} + \mathrm{C}$

9 Is the quadratic function $f(x) = 3x^2 - 2x + 1$:

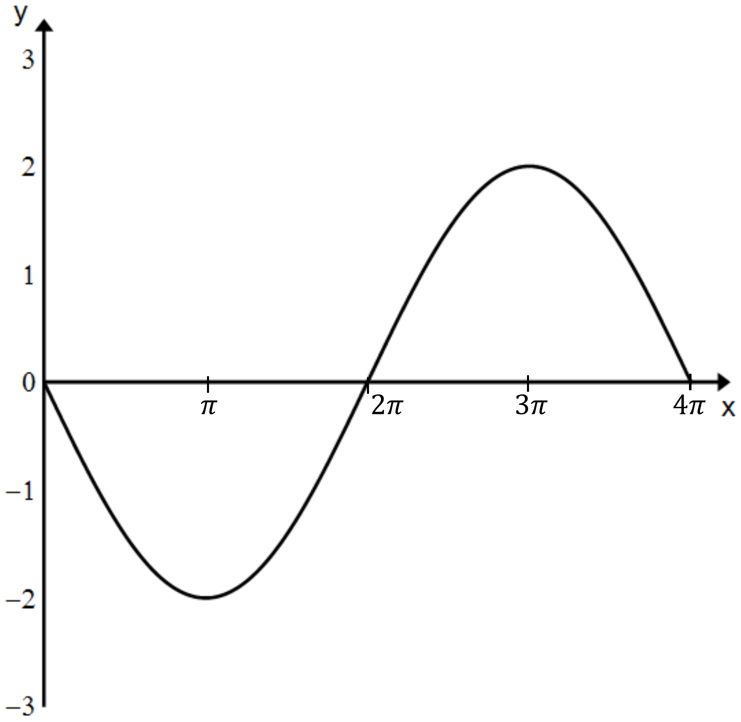
A. Positive definite

B. Negative definite

C. Indefinite

D. None of the above

10 The curve below represents one period of a sine or cosine curve.



Which of the following is a possible equation of the curve?

A. $y = -2\sin 2x$

B. $y = -2\sin\left(\frac{x}{2}\right)$

C. $y = 2\cos\left(x + \frac{\pi}{2}\right)$

D. $y = -2\cos\left(\frac{x}{2}\right)$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

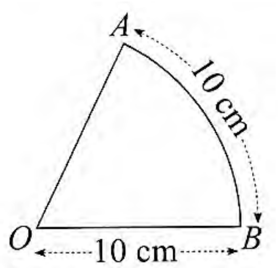
a) Factorise $8x^3 - 1000$ **2**

b) Express $\dfrac{4}{2-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are rational. **2**

c) Differentiate $\dfrac{1}{2x^3}$ **2**

d) Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 + 2x - 15}{x - 3}$ **2**

e) *AB* is a circular arc, centre *O*, length 10 cm. The radius is also 10 cm. **2**



Find the area of sector *AOB*.

f) The first three terms of an arithmetic sequence are $-1 + 4 + 9 + \ldots$

i) Find the 50th term. **1**

ii) Find the sum of the first fifty terms. **1**

g) Solve $4\cos^2\theta - 3 = 0$ for $0 \le \theta \le 2\pi$ **3**

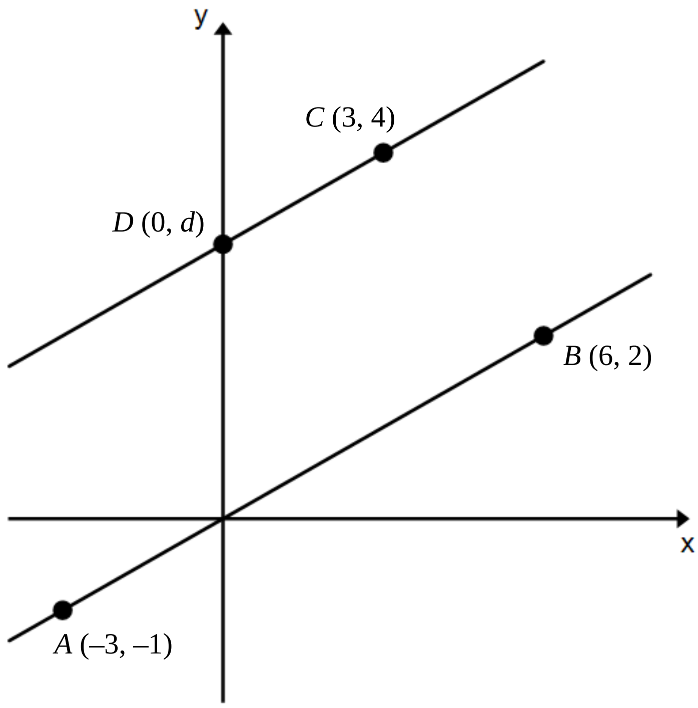
Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Differentiate the following with respect to *x*:

i) $(1 + \sin x)^3$ **2**

ii) $\ln(x^2 - 4)$ **2**

b)



i) Find the gradient of *AB*. **1**

ii) Find the value of *d* if *CD* is parallel to *AB*. **1**

iii) Show that the equation of *AB* is $x - 3y = 0$ **1**

iv) Find the perpendicular distance between *C* and the line *AB*. **1**

v) Prove that $AB = 3CD$ **2**

vi) Find the area of the quadrilateral *ABCD*. **2**

c) Find the exact value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2\left(\frac{x}{2}\right)\, dx$ **3**

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Consider the function $f(x) = |x - 4|$

i) Sketch the function. **1**

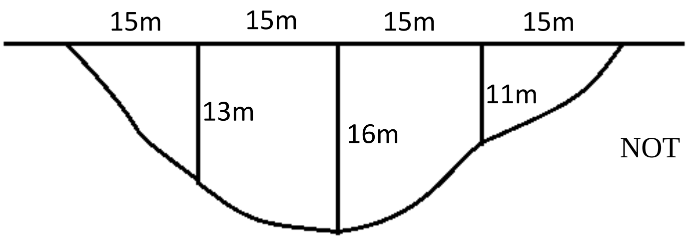
ii) Hence, or otherwise, evaluate $\int_0^6 |x-4| \, dx$ **2**

b) Penny decides to save money by putting some coins in a jar every day. On the first day she will put in 10c, on the second, 15c, and so on, with the amount she puts into the jar increasing by 5c each day.

i) How much will she put in on the 15^{th} day? **1**

ii) For how many days must she save before the jar holds $37.00? **3**

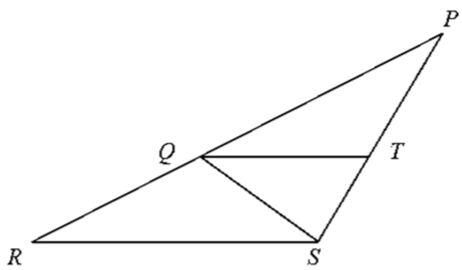
c) The diagram below shows a cross section of a river. The river is 60 metres wide and its depth is recorded at 15 metre intervals across its width, as shown.



NOT TO SCALE

Use Simpson's Rule to approximate the cross-sectional area of the river. **3**

d) In the diagram, $QT \parallel RS$ and TQ bisects $\angle PQS$.



NOT TO SCALE

Copy the diagram into your answer booklet, showing this information.

i) Explain why $\angle TQS = \angle QSR$. **1**

ii) Prove that ΔQRS is isosceles. **2**

iii) Hence show that $PT : TS = PQ : QS$ **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) Consider the function $f(x) = x\,e^{-x}$

i) Show that $f\,'(x) = e^{-x}(1-x)$ **1**

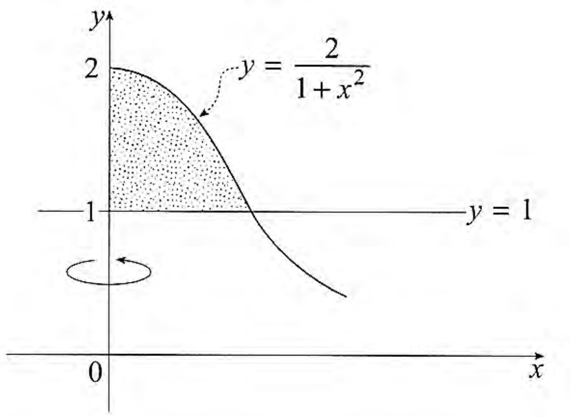
ii) Show that $f\,''(x) = e^{-x}(x-2)$ **1**

iii) Find the coordinates of any stationary points and determine their nature. **3**

iv) Show that there is a point of inflexion at $\left(2, \frac{2}{e^2}\right)$ **2**

v) Sketch $y = f(x)$, indicating all important features. **2**

b) The shaded region on the graph below is rotated about the y axis.



i) Show that the volume of the solid formed is given by **2**

$$V = \pi \int_1^2 \left(\frac{2}{y} - 1\right) dy$$

ii) Hence find its volume. **1**

c) A sum of \$2000 is deposited at the start of each year in an account that earns 10% interest per annum. **3**

Find the total value of the investment at the end of the 15th year, correct to the nearest dollar.

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) i) Sketch the locus of a point that is always two units from the line $y = 3$. **1**

ii) Write down the equation of the locus. **1**

b) A quantity of radioactive material decays at a rate proportional to the amount, M, present at any time, t.

i) Given that $M = M_0 e^{-kt}$ represents the mass of material in grams at any time t years after the material was produced, show that **1**

$$\frac{dM}{dt} = -kM$$

ii) If initially there was 3500 g of material and after 4 years the mass had decayed to 2300 g, calculate k, correct to 4 significant figures. **2**

iii) Determine the number of years needed for the material to decay to 25% of its original quantity. **2**

iv) Calculate the rate at which the material is decaying after 10 years. **1**

c) Solve $2\log_a x = \log_a 2 + \log_a (x + 4)$ **3**

d) A particle is moving moves in a straight line so that it is x metres to the right of a fixed point O at a time of t seconds. The acceleration of the particle is given by

$$\ddot{x} = -\frac{2\pi}{3}\sin\left(\frac{\pi}{3}t\right)$$

Initially the particle is travelling with a velocity of 3m/s.

i) Find an expression for the velocity, v, as a function of t. **1**

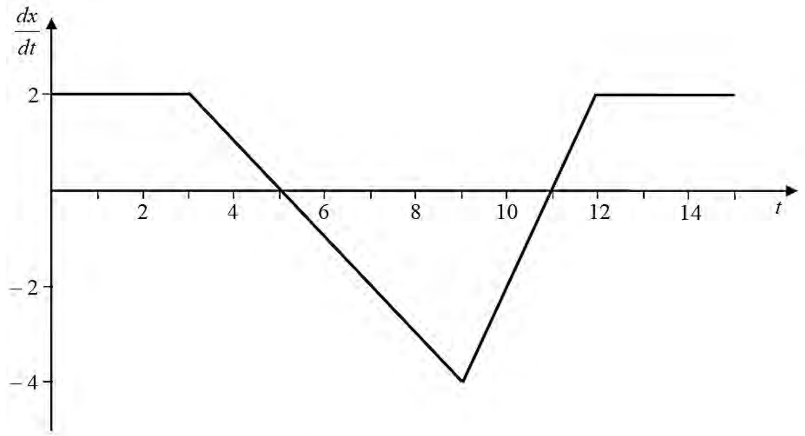
ii) Find the first two times when the particle is stationary. **1**

iii) How far does the particle travel in the third second? **2**

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) Show that $\frac{d}{d\theta}\left[\frac{1}{cos\theta}\right] = \sec\theta\tan\theta$ **2**

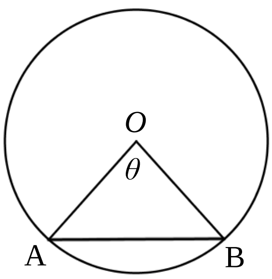
b) The graph below shows the velocity, $\frac{dx}{dt}$, of a particle as a function of time, in its first 15 seconds of motion. Initially the particle is at the origin.



i) At what time does the particle first return to the origin? Justify your answer. **2**

ii) Draw a sketch of the particle's displacement, x, as a function of time, for the first 9 seconds of motion. **2**

c) The chord AB divides the area of the circle of radius r into two parts. AB subtends an angle of θ at the centre, as shown.



i) Show that the ratio of areas of major to minor segment is given by **3**

$$\frac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$$

ii) The chord AB divides the area of the circle in the ratio 15 : 7. **2**

Using $\pi = \frac{22}{7}$, prove that $\theta = 2 + \sin\theta$.

Question 16 continues on page 12

d) On a factory production line a tap opens and closes to fill containers with liquid. As the tap opens, the rate of flow of liquid, R litres per second, increases for the first 10 seconds according to:

$$R = \frac{6t}{50}$$

After 10 seconds, the rate of flow remains constant until the tap begins to close. As the tap closes, the rate of flow decreases at a constant rate for 10 seconds, after which the tap has fully closed and the rate of flow is zero.

i) Show that, while the tap is fully open, the volume in the container, in litres, at any time, t, is given by **2**

$$V = \frac{6}{5}(t - 5)$$

ii) For how many seconds must the tap remain fully open in order to exactly fill a 120 L container with no spillage? **2**

END OF EXAMINATION

2014 2U Trial HSC Solutions

1) C
2) A
3) D
4) B
5) C
6) B
7) A
8) A
9) A
10) B

Question 11

a) $8x^3 - 1000 = 8(x^3 - 125)$

$= 8(x - 5)(x^2 + 5x + 25)$

b)

$$\frac{4}{2-\sqrt{3}} = \frac{4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{8+4\sqrt{3}}{4-3}$$

$$= 8 + 4\sqrt{3}$$

c)

$$\frac{d}{dx}\left[\frac{1}{2x^3}\right] = \frac{d}{dx}\left[\frac{1}{2}x^{-3}\right]$$

$$= -\frac{3}{2}x^{-4}$$

$$= -\frac{3}{2x^4}$$

d)

$$\lim_{x\to 3}\frac{x^2+2x-15}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+5)}{x-3}$$

$$= \lim_{x\to 3} x+5$$

$$= 8$$

e)

Length of arc AB = $r\theta$

$10 = 10\theta$

$\theta = 1$

Area of sector = $\frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 10^2 \times 1$

= 50 units^2

f) $a = -1$ $d = 5$

i) $T_{50} = a + 49d$

$= -1 + 49(5)$

$= 244$

ii) $S_{50} = \frac{50}{2}(a + L)$

$= 25(-1 + 244)$

$= 6075$

g)

$4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 12

a)

i)

$$\frac{d}{dx}[(1+sinx)^3] = 3(1+\sin x)^2 \times \cos x$$

$$= 3\cos x(1 + \sin x)^2$$

ii)

$$\frac{d}{dx}[\ln(x^2-4)] = \frac{2x}{x^2-4}$$

b)

i) $m_{\mathrm{AB}} = \frac{2-(-1)}{6-(-3)} = \frac{1}{3}$

ii) Gradient of CD must equal gradient of AB

$$\frac{4-d}{3-0} = \frac{1}{3}$$

$$d = 3$$

iii) $y - 2 = \frac{1}{3}(x - 6)$

$3y - 6 = x - 6$

$x - 3y = 0$

iv)

$$d_p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 - b^2}}$$

$$d_p = \frac{|1(3) + (-3)(4) + 0|}{\sqrt{(1)^2 - (-3)^2}}$$

$$= \frac{|-9|}{\sqrt{10}}$$

$$= \frac{9}{\sqrt{10}}$$

v)

$$\text{Distance}_{AB} = \sqrt{(2-(-1))^2 + (6-(-3))^2}$$

$$= \sqrt{9+81}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10}$$

$$\text{Distance}_{AB} = \sqrt{(4-3)^2 + (3-0)^2}$$

$$= \sqrt{1+10}$$

$$= \sqrt{10}$$

Therefore $AB = 3CD$

vi) *ABCD* is a trapezium since AB $\parallel$ CD

The perpendicular height is $\frac{9}{\sqrt{10}}$ from part (iv)

$$\text{Area} = \frac{h}{2}(a+b)$$

$$= \frac{9}{2\sqrt{10}}\left(\sqrt{10} + 3\sqrt{10}\right)$$

$$= \frac{9}{2\sqrt{10}}\left(4\sqrt{10}\right)$$

$$= \frac{36}{2}$$

$$= 18 \text{ units}^2$$

c)

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2\left(\frac{x}{2}\right)\,dx = \left[2\tan\left(\frac{x}{2}\right)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

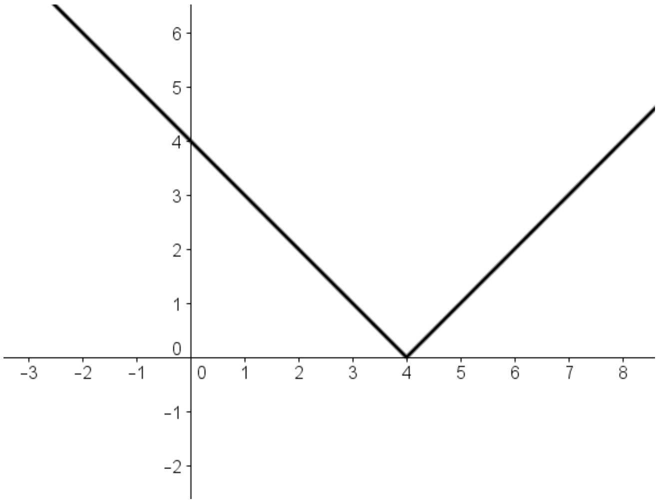
$$= 2\tan\frac{\pi}{4} - 2\tan\frac{\pi}{6}$$

$$= 2\left(1 - \frac{1}{\sqrt{3}}\right)$$

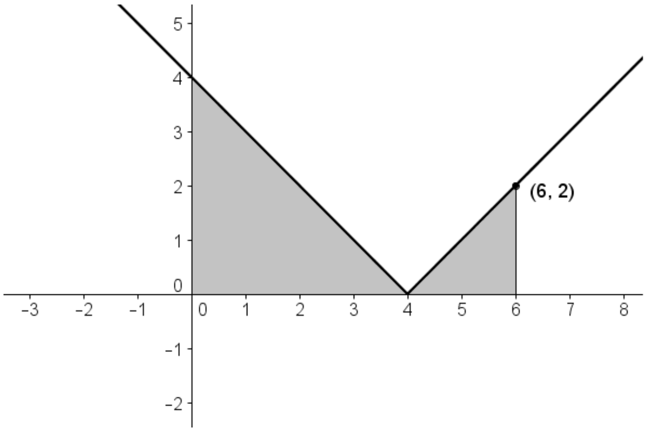
$$= \frac{2(\sqrt{3}-1)}{\sqrt{3}} = \frac{6 - 2\sqrt{3}}{3}$$

Question 13

a) i)



ii)



$$\int_0^6 |x-4|\ dx = \text{area under the curve}$$

$$= \frac{1}{2}(4 \times 4) + \frac{1}{2}(2 \times 2)$$

$= 10$

b) $a = 10$ $d = 5$

i) $T_{15} = a + 14d$

$= 10 + 14(5)$

$= 80$c

ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$3700 = \frac{n}{2}(20 + (n-1)5)$

$7400 = n(15 + 5n)$

$5n^2 + 15n - 7400 = 0$

$n^2 + 3n - 1480 = 0$

$$n = \frac{-3 \pm \sqrt{9 - 4(1)(-1480)}}{2(1)}$$

$= 37$ $(n > 0)$

c)

$$\text{Area} = \frac{\text{h}}{3}[y_{1st} + y_{last} + 2(y_{odds}) + 4(y_{evens})]$$

$$= \frac{15}{3}[0 + 0 + 2(16) + 4(13 + 11)]$$

$= 640 \text{ m}^2$

d) i) $\angle TQS = \angle QSR$ (alternate angles, $QT \parallel RS$)

ii) Let $\angle SQT = x$

Then $\angle PQT = x$ (since TQ bisects $\angle PQS$)

Also, $\angle QRS = x$ (corresponding to $\angle PQT$, $QT \parallel RS$)

$\therefore \angle QRS = \angle QSR$

$\therefore \Delta QRS$ is isosceles (base angles equal)

iii) Construct a line through P parallel to QT

Now $PT : TS = PQ : QR$

(family of parallel lines cuts transversals in same ratio)

But $QR = QS$ (sides opp. equal sides in isos. triangle)

$\therefore PT : TS = PQ : QS$

Question 14

a)

i) $f'(x) = e^{-x} \times 1 + x \times (-e^{-x})$

$= e^{-x} - x e^{-x}$

$= e^{-x}(1 - x)$

ii) $f''(x) = (1 - x) \times -e^{-x} + e^{-x}(-1)$

$= -e^{-x} + x e^{-x} - e^{-x}$

$= e^{-x}(x - 2)$

iii) For stationary points, $f'(x) = 0$

$e^{-x}(1 - x) = 0$

$x = 1$ (since e^{-x} cannot be zero)

When $x = 1$,

$f(1) = (1)(e^{-1})$

$= \frac{1}{e}$

So there is a stationary point at $(1, \frac{1}{e})$

Consider stationary point when $x = 1$

$f''(x) = e^{-(1)}(1 - 2)$

< 0

So the point $(1, \frac{1}{e})$ is a maximum turning point

iv) Possible points of inflexion when $f''(x) = 0$

$e^{-x}(x - 2) = 0$

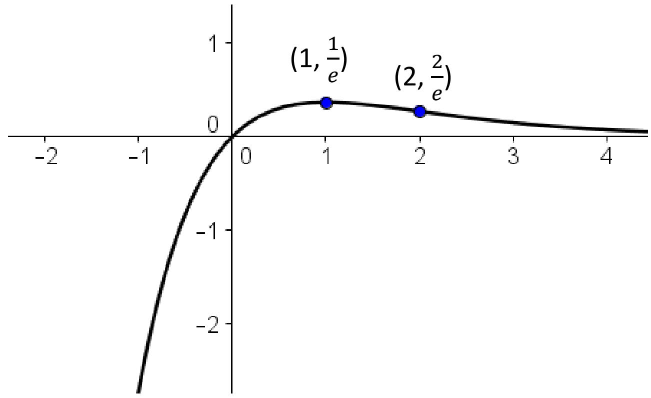
$x = 2$ (since e^{-x} cannot be zero)

When $x = 0$, $f(2) = (2)(e^{-2}) = \frac{2}{e^2}$

When $x < 0$, $f''(x) < 0$ and when $x > 0$, $f''(x) > 0$

Thus concavity changes and there is a point of inflexion at $(2, \frac{2}{e^2})$

v)



b) i) $y = \frac{2}{1+x^2}$

$y(1 + x^2) = 2$

$1 + x^2 = \frac{2}{y}$

$x^2 = \frac{2}{y} - 1$

$$\text{Volume} = \pi \int_1^2 \frac{2}{y} - 1 \, dy$$

ii)

$$\text{Volume} = \pi \int_1^2 \frac{2}{y} - 1 \, dy$$

$$= \pi \left[2\ln y - y\right]_1^2$$

$= \pi(2\ln 2 - 2 - (2\ln 1 - 1))$

$= \pi(2\ln 2 - 1)$

$= 1.21$ units^3 (2 d.p.)

c) 1st investment grows to $= 2000(1.1)^{15}$

2nd grows to $= 2000(1.1)^{14}$

Final investment $= 2000(1.1)^1$

Total $= 2000(1.1)^{15} + 2000(1.1)^{14} + \ldots + 2000(1.1)^1$

$= 2000(1.1)[1 + 1.1 + 1.1^2 + \ldots + 1.1^{14}]$

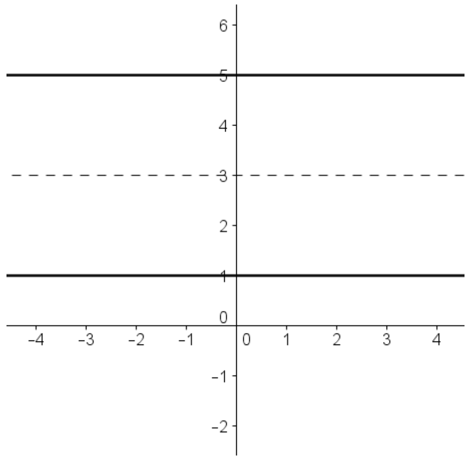
$$= 2000(1.1)\left(\frac{1(1.1^{15} - 1)}{1.1 - 1}\right)$$

= \$69 899 (nearest dollar)

Question 15

a)

i)



ii) Locus has equation $y = 5$ and $y = 1$

b) i) $M = M_0 e^{-kt}$

$$\frac{dM}{dt} = -k\, M_0 e^{-kt}$$

$= -kM$ (since $M = M_0 e^{-kt}$)

ii) When $t = 0$, $M = 3500$

$3500 = M_0 e^{-k(0)}$

$M_0 = 3500$

When $t = 4$, $M = 2300$

$2300 = 3500\, e^{-k(4)}$

$$\frac{23}{35} = e^{-4k}$$

$$-4k = \ln\left(\frac{23}{35}\right)$$

$k = 0.1050$ (4 sig. fig.)

iii) 25% of 3500g = 875g

$875 = 3500\, e^{-0.1050t}$

$$\frac{1}{4} = e^{-0.1050t}$$

$$-0.1050t = \ln\left(\frac{1}{4}\right)$$

$t = 13.2$

Thus it will decay to 25% during the 14th year

iv)

$$\frac{dM}{dt} = -0.1050\,(3500)e^{-0.1050t}$$

$$= -0.1050\,(3500)e^{-0.1050(10)}$$

$= -128.60$ (2 d.p.)

Thus after 10 years, the rate of decay is 128.60 g/year

c) $2\log_a x = \log_a 2 + \log_a(x+4)$

$2\log_a x = \log_a[2(x+4)]$

$\log_a x^2 = \log_a(2x+8)$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = -2, 4$

However, $x \neq -2$, thus $x = 4$ only

d) i)

$$\ddot{x} = -\frac{2\pi}{3}\sin\left(\frac{\pi}{3}t\right)$$

$$\dot{x} = -\left(-\frac{2\pi}{3} \div \frac{3}{\pi}\right)\cos\left(\frac{\pi}{3}t\right) + C$$

$$\dot{x} = 2\cos\left(\frac{\pi}{3}t\right) + C$$

When $t = 0$, $\dot{x} = 3$

$$3 = 2\cos(0) + C$$

$C = 1$

$$\dot{x} = 2\cos\left(\frac{\pi}{3}t\right) + 1$$

ii) $2\cos\left(\frac{\pi}{3}t\right) + 1 = 0$

$\cos\left(\frac{\pi}{3}t\right) = -\frac{1}{2}$

$\frac{\pi}{3}t = \frac{2\pi}{3}, \frac{4\pi}{3}$

$t = 2$ and 4 seconds

iii)

$$\text{Distance} = \text{Area under velocity curve}$$

$$= \left| \int_2^3 2\cos\left(\frac{\pi}{3}t\right) + 1 \; dt \right|$$

$$= \left| \left[\frac{6}{\pi}\sin\left(\frac{\pi}{3}t\right) + t\right]_2^3 \right|$$

$$= \left| \left[\frac{6}{\pi} \times 0 + 3 - \left(\frac{6}{\pi} \times \frac{\sqrt{3}}{2} + 2\right)\right] \right|$$

= 0.65 metres

Question 16

a)

$$\frac{d}{d\theta}\left[\frac{1}{\cos\theta}\right] = \frac{d}{d\theta}[\cos\theta]^{-1}$$

$$= -(\cos\theta)^{-2} \times -\sin\theta$$

$$= \frac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$$

b)

i) After 9s because at this time, the area above the velocity graph equals the area below

Area above = rectangle + triangle

= 3×2 + ½ × 2 × 2

= 8

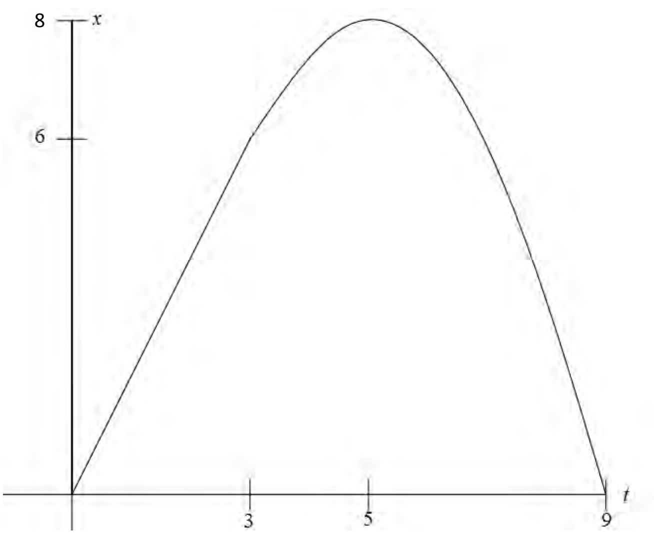
Area below = triangle

= ½ × 4 × 4

= 8

Thus the particle returns to the origin after 8s

b) ii)



c) i)

$$Area_{\,minor} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$Area_{major} = \pi r^2 - \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)$$

$$Ratio = \frac{\pi r^2 - \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)}{\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta}$$

$$= \frac{2\pi r^2 - r^2\theta + r^2\sin\theta}{r^2\theta - r^2\sin\theta}$$

$$= \frac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$$

ii)

$$\frac{15}{7} = \frac{2\pi - \theta + \sin\theta}{\theta - \sin\theta}$$

$$15\theta - 15\sin\theta = 14\pi - 7\theta + 7\sin\theta$$

$$22\theta - 15\sin\theta = 14\left(\frac{22}{7}\right) + 7\sin\theta$$

$$22\theta = 44 + 22\sin\theta$$

$$\theta = 2 + \sin\theta$$

d) i)

$$\frac{dV}{dt} = \frac{6t}{50} \quad \text{for first 10 seconds}$$

$$V = \frac{3t^2}{50} + C$$

When $t = 0$, $V = 0$, so $C = 0$

$$V = \frac{3t^2}{50}$$

Now when $t = 10$

$$V = \frac{3(10)^2}{50} \quad = 6L$$

So, after the tap has finished opening (after 10s) there is 6L in the tank.

The rate of flow from 10s onward is constant and is equal to the rate when $t = 10$. That is,

$$\frac{dV}{dt} = \frac{6(10)}{50} = \frac{6}{5} \quad L/s$$

Thus, the volume in the tank when $t > 10$ is given by

$$V = \frac{6t}{5} + C$$

Now when $t = 10$, $V = 6$

$$6 = \frac{6(10)}{5} + C$$

$$C = -6$$

$$V = \frac{6t}{5} - 6 \quad = \quad \frac{6}{5}(t - 5)$$

ii)

Volume that flows into the container as the while tap is closing is 6 litres.

Therefore the volume that must flow while the tap is fully opened is 120 – 6 = 114L

i.e.

$114 = \frac{6}{5}(t - 5)$

$t - 5 = 95$

$t = 100$

However, for the first 10s the tap is opening.

So the tap must be fully opened for 90 seconds.