Caringbah High School

Year 12 2018
Mathematics
HSC Course
Trial HSC

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 100

Section I **10 marks**
Attempt Questions 1-10
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **90 marks**
Attempt Questions 11-16
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: ______________________________ **Class:** ___________

Marker's Use Only								
Section I	**Section II**						**Total**	
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**	**Q16**		
/10	/15	/15	/15	/15	/15	/15	/100	

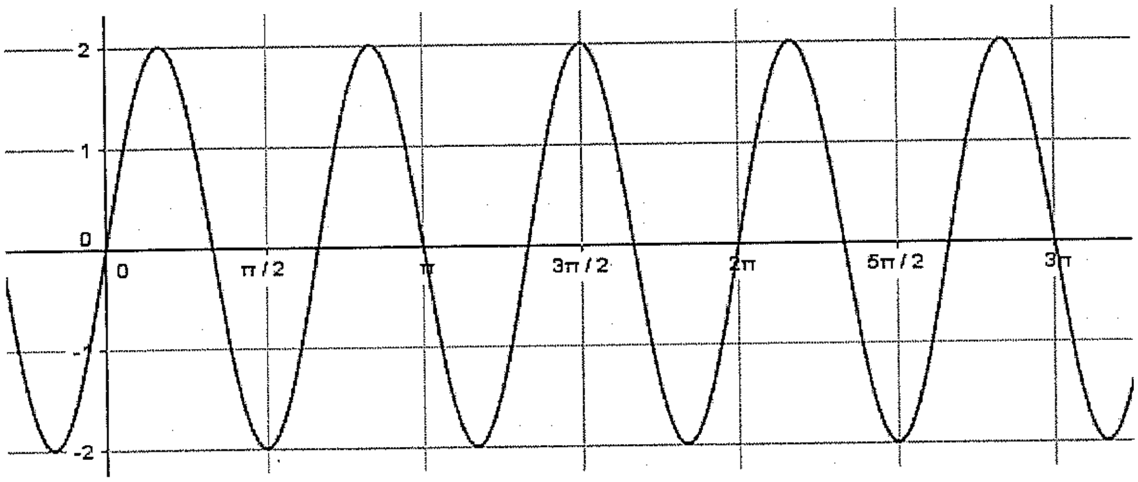
Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

---

1. The graph below shows the function:



(A) $y = 3\sin 2x$

(B) $y = 2\sin 3x$

(C) $y = 3\sin\left(\dfrac{x}{2}\right)$

(D) $y = 2\sin\left(\dfrac{x}{3}\right)$

2. $\displaystyle\int \frac{dx}{3x+1} =$

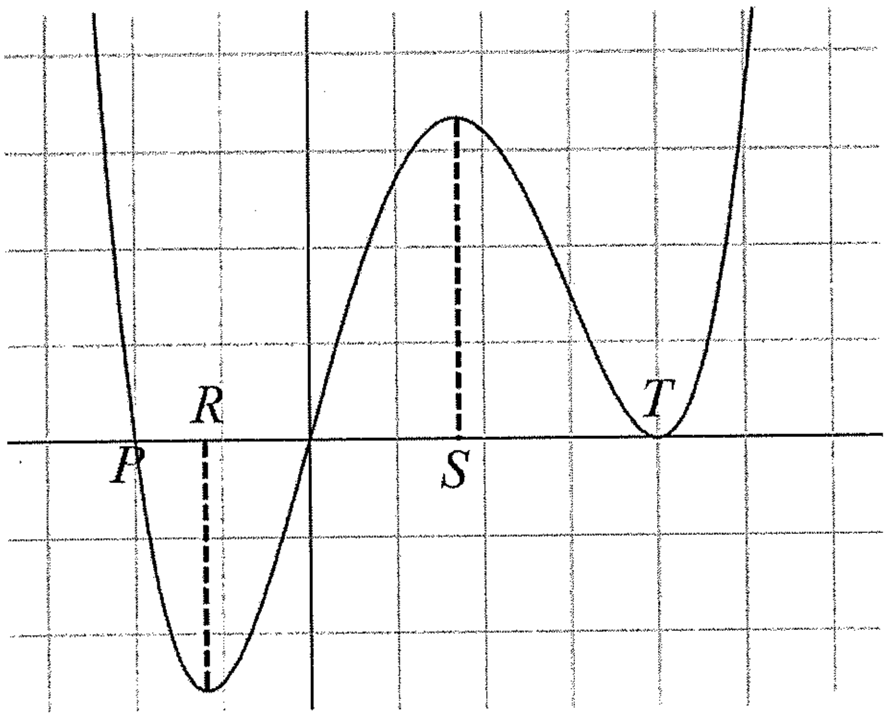
(A) $\dfrac{1}{3}\log_e(3x+1)+c$

(B) $\log_e\left(\dfrac{3x+1}{3}\right)+c$

(C) $3\log_e(3x+1)+c$

(D) $\log_e(3x+1)+c$

3. The diagram below shows the graph of $y = f'(x)$.



Which letter indicates the position of the x-value of the maximum turning point of the function $y = f(x)$?

(A) P

(B) R

(C) S

(D) T

4. A parabola has the equation $x^2 + 2x + 25 = 8y$. The focus of the parabola is:

(A) $(-1,1)$

(B) $(-1,5)$

(C) $(1,1)$

(D) $(-1,3)$

5. The fourth term of a geometric series is 192 and the seventh term is –24.

What is the infinite sum of the series?

(A) 1024

(B) –128

(C) 128

(D) –1024

6. If $\int_2^5 f(x)\, dx = 7$, what is the value of $\int_2^5 1 - f(x)\, dx$?

(A) –6

(B) –4

(C) 8

(D) 10

7. The domain of the function $y = \log_e(2x-1)$ is:

(A) All real x, $x \neq \frac{1}{2}$

(B) $x \geq \frac{1}{2}$

(C) $x > \frac{1}{2}$

(D) $x \leq \frac{1}{2}$

8. A circular metal plate of area A cm^2 is beinmg heated. It is given that

$$\frac{dA}{dt} = \frac{\pi t}{32} \ \text{cm}^2/\text{h}$$

What is the exact area of the plate after 8 hours, if initially the plate had a radius of 6 cm?

(A) $\pi \ \text{cm}^2$

(B) $0.25\pi \ \text{cm}^2$

(C) $36\pi \ \text{cm}^2$

(D) $37\pi \ \text{cm}^2$

9. The series $1+2+4+5+7+8+\ldots\ldots\ldots+199+200$ is obtained by omitting from the first 200 positive integers all the multiples of 3. The sum of this series is:

(A) 20100

(B) 10050

(C) 17688

(D) 13467

10. It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by

$N(t) = \dfrac{A}{1+e^{-t}}$ where A is a constant and t is measured in months.

At time $t = 0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A?

(A) 2×10^5

(B) 2×10^{-5}

(C) 4×10^5

(D) 4×10^{-5}

End of Section I

Section II 60 marks

Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Express $\dfrac{2}{\sqrt{7}-1}$ with a rational denominator in its simplest form. **2**

(b) Express $\log_3 9 + \log_3 27$ as an integer. **2**

(c) Evaluate $\int_0^3 e^{2x+3}\,dx$, expressing your answer as an exact value. **2**

(d) Differentiate the following with respect to x:

(i) $\tan(x^2)$ **1**

(ii) $x^2 e^x$ **2**

(iii) $\dfrac{1}{3x^6}$ **2**

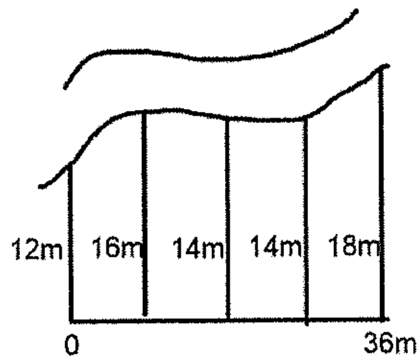
(e) Show that $\sqrt{2}+\sqrt{8}+\sqrt{18}+\ldots$ is an arithmetic series and hence find the exact value of the sum of the first 50 terms in simplest form. **2**

(f) Prove that $\dfrac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta=\tan\theta$ **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram below shows a paddock with one side bounded by a river.



Use Simpson's Rule with the 5 function values shown on the diagram to find the approximate area of the paddock. **1**

(b) A person walks 21 km due east from A. Another person walks 12 km from A on a bearing of $322^\circ 29'$. How far apart are they, correct to the nearest km? **2**

(c) Consider the curve $y = x^3 - 6x^2 - 9x + 4$

(i) Find the stationary points and determine their nature. **4**

(ii) Find the coordinates of any points of inflexion. **2**

(iii) Find the values of x for which the curve is increasing and concave up. **1**

(iv) Hence draw a half page sketch of the curve $y = x^3 - 6x^2 - 9x + 4$ **2**

(d) Given that a parabola has its focus at (2,1) and vertex at (3,1) find:

(i) the focal length **1**

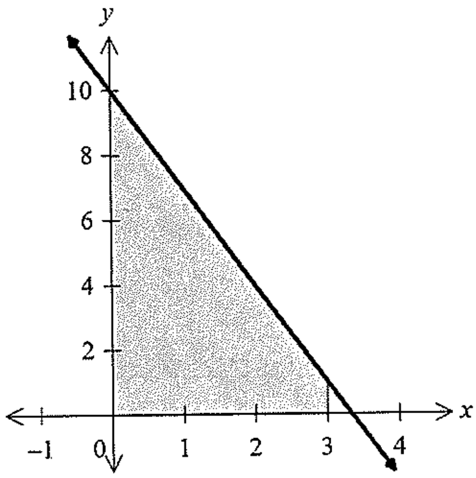
(ii) the equation of the directrix **1**

(iii) the equation of the parabola **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) The shaded region that lies between the x-axis and the line $y = 10 - 3x$ from $x = 0$ to $x = 3$ is rotated about the x-axis to form a solid of revolution. Find the exact volume of this solid. **3**



(b) The graph of a function has the following properties: **2**

- It is continuous
- Passes through the origin and has a minimum turning point at (4,0).
- Concave up for $x > 3$ and $x < 0$.
- Increasing for $-2 < x < 2$ and $x > 4$.

Sketch a possible graph of the function.

(c) Find:

(i) $\displaystyle\int \frac{dx}{\sqrt{x+6}}$ **2**

(ii) $\displaystyle\int_0^2 \frac{x}{x^2+4}\,dx$ leaving your answer as an exact value in simplest form. **2**

(d) **(i)** Sketch the graph of $y = 1 + 2\sin x$ for $0 \le x \le 2\pi$ **2**

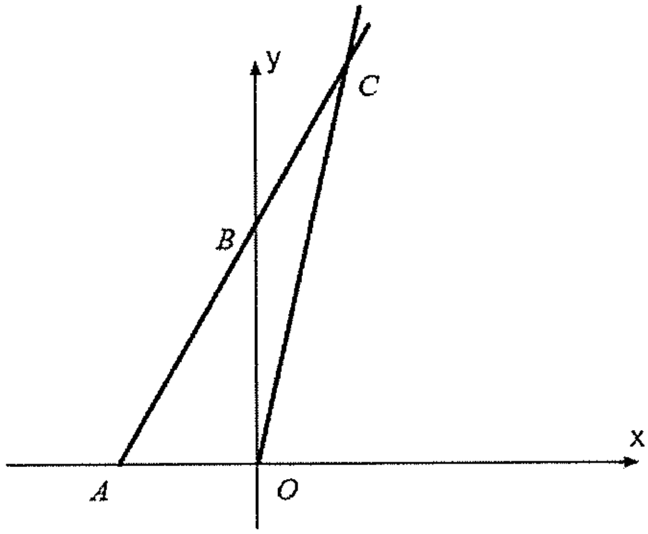
(ii) Find the exact value of the coordinates of the points where the graph of $y = 1 + 2\sin x$ crosses the x-axis in the domain $0 \le x \le 2\pi$ **2**

(e) Factorise fully $16x^2 - 16a^2 + 24a - 9$ **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a)



In the diagram above A and B are points on the x and y axes respectively. The line AB has equation $\sqrt{3}x - y + \sqrt{3} = 0$. The point C lies on AB such that the area of ΔAOC is $3\sqrt{3}$ square units.

(i) Show that the coordinates of A and B are $(-1, 0)$ and $(0, \sqrt{3})$. **1**

(ii) Find the gradient of AB. **1**

(iii) Find the size of $\angle BAO$. **1**

(iv) Hence find the length AC. **2**

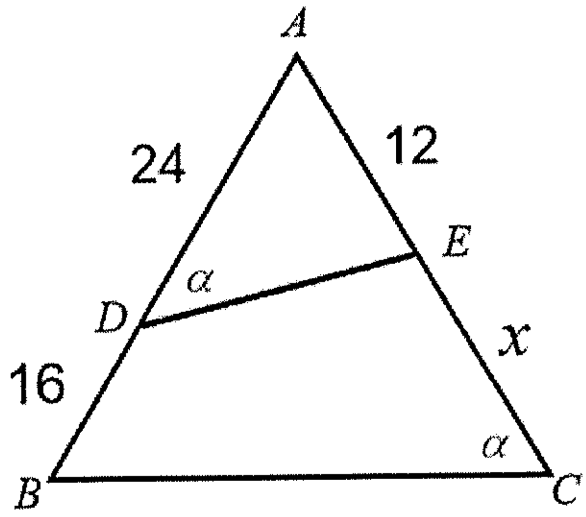
(b) Find the exact value of $\tan\dfrac{\pi}{3} + \operatorname{cosec}\dfrac{\pi}{4}$ **2**

(c) Find the values for x so that $\log_e x = 2\log_e 2x$ **2**

(d) Consider the quadratic function $x^2 - (k+2)x + 4 = 0$ **2**
For what values of k does the quadratic function have real roots?

Question 14 continues on page 10

(e) In the diagram below, $\Delta ABC ||| \Delta ADE$. **2**



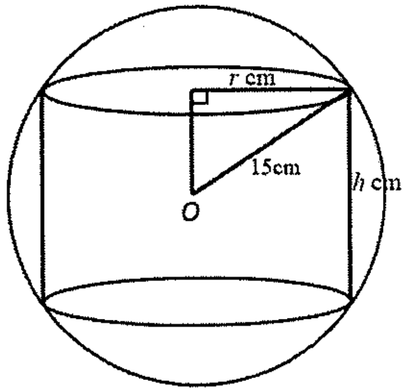
Find the value of x.

(f) Find $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$ **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) A cylindrical block of height h cm base radius r cm is cut from a solid sphere of radius 15 cm. O is the centre of the sphere.



(i) Show that the volume of the cylinder is given by $v = 225\pi h - \frac{\pi}{4}h^3$ **3**

(ii) Find the height of the cylinder of maximum volume which can be cut from the sphere. **3**

(b) If α and β are the two roots of $2x^2 + 3x - 4 = 0$, find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha^3 + \beta^3$ **2**

(c) A family borrows \$120 000 to extend their house. The bank loans them the money for 10 years at an interest rate of 6% p.a. on the balance owing at the end of each month. The loan is to be repaid in equal monthly instalments of \$$M$.
Let A_n be the amount owing after n months.

(i) How much do they owe at the end of the first month ***before*** the first payment is made? **1**

(ii) Show that at the end of the second month immediately ***after*** the second payment has been made the amount owing is given by **2**

$$A_2 = (121203 - 2.005M)$$

(iii) Show that at the end of n months after n repayments have been made the amount owing will be given by **2**

$$A_n = 120\,000 \times 1.005^n - 200M(1.005^n - 1)$$

(iv) Hence find the value of each monthly instalment \$$M$. **1**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

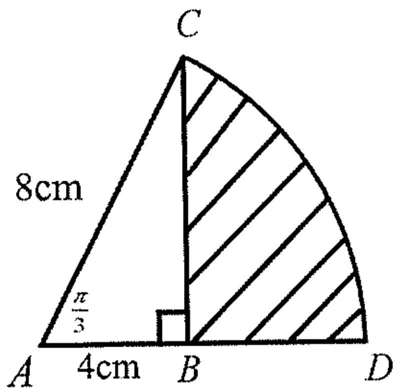
(a) Water is draining from a storage tank at a rate that is proportional to the volume of water that is left contained in the tank. Let V be the amount of water in the tank.

(i) Show that $V = V_0 e^{-kt}$ is a solution to the differential equation $\dfrac{dV}{dt} = -kV$. **1**

(ii) When full, the water tank holds 1000 L of water. After 40 minutes, the tank has 800 L of water. How much water will be left in the tank after 1 hour to the nearest litre? **2**

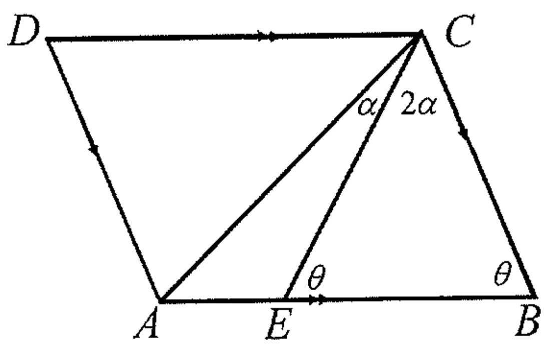
(iii) How long in hours and minutes will it take for there to be 10 L of water left in the tank? **2**

(b) A right-angled triangle ACB is placed in a sector of a circle with radius 8 cm. $AB = 4$ cm and $\angle BAC = \dfrac{\pi}{3}$ radians. **2**



Calculate the exact area of the shaded region BCD.

(c) $ABCD$ is a rhombus. The angles involving α and θ are marked on the diagram. **3**



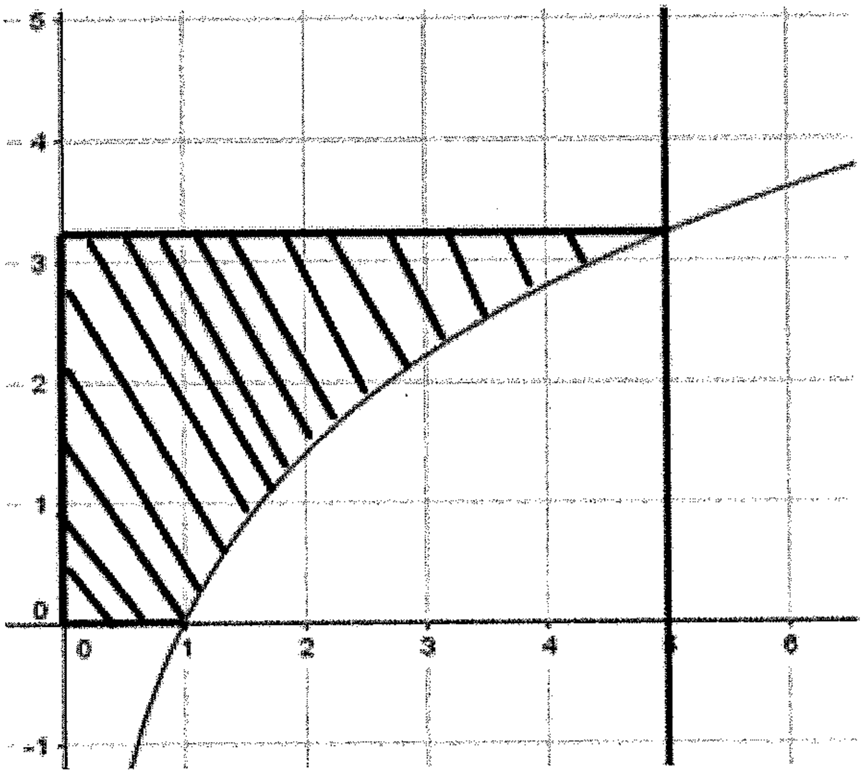
Determine the size of α and θ giving clearly worded reasons.
(Diagram is not drawn to scale.)

Question 16 continues on page 13

(d) **(i)** Show that $\frac{d}{dx}(x\log_e x - x) = \log_e x$ **1**

(ii) Hence, or otherwise, find $\int \log_e(x^2)dx$ **1**

(iii) The graph shows the curve $y = \log_e(x^2)$ which meets the line $x = 5$ at Q. **3**



Using your answers from (i) and (ii), or otherwise, find the exact area of the shaded section.

End of Paper

Caringbah High School

Examination: 2018 2unit Trial CHS

Student Number

1. $a=2$ $T=\frac{2\pi}{n}=\frac{2\pi}{3}$ so $n=3$ $y=2\sin 3x$ [B]

2. $\int \frac{dx}{3x+1}=\frac{1}{3}\ln(3x+1)+C$ [A]

3. + o − max P = [A]

4. $(x+1)^2=8(y-3)$ vertex $(-1,3)$ $a=2$ focus: $(-1,5)$ [B]

5. $T_4=ar^3=192$ $r^3=\frac{-24}{192}=\frac{-1}{8}$ so $r=\frac{-1}{2}$

$T_7=ar^6=-24$ $S=\frac{a}{1-r}$ $a\times -\frac{1}{8}=192$ so $a=-1536$

$S=\frac{-1536}{1+\frac{1}{2}}=\frac{-3072}{3}=-1024$ [D]

6. $\int_2^5 1-f(x)\,dx = x\Big]_2^5-7=3-7=-4$ [B]

7. $y=\log_e(2x-1)$ D: $2x-1>0$ $x>\frac{1}{2}$. [C]

8. $\frac{dA}{dt}=\frac{\pi t}{32}$ $\therefore A=\frac{\pi t^2}{64}+C$ $t=0$ $A=36\pi$ [D]

9. $100\times 201-33\times 201=67\times 201=13467$ [D]

10. $N(0)=\frac{A}{1+1}=2\times 10^5$

$A=4\times 10^5$ [C]

Q11 a) $\frac{2}{\sqrt{7}-1}\times\frac{\sqrt{7}+1}{\sqrt{7}+1}=\frac{2\sqrt{7}+2}{6}=\frac{\sqrt{7}+1}{3}$

b) $\log_3 9+\log_3 27=2\log_3 3+3\log_3 3$

$=5\times 1=5$

c) $\int_0^3 e^{2x+3}\,dx=\frac{1}{2}e^{2x+3}\Big]_0^3=\frac{1}{2}(e^9-e^3)$

d) i) $\frac{d\tan(x^2)}{dx}=2x\sec^2(x^2)$

ii) $\frac{d\,x^2e^x}{dx}$ $u=x^2$ $v=e^x$ $u'=2x$ $v'=e^x$

$=x^2e^x+2xe^x$

$=xe^x(x+2)$

iii) $\frac{d\frac{1}{3}x^{-6}}{dx}=\frac{-6}{3}x^{-7}$

$=\frac{-2}{x^7}$

e) $\sqrt{2}+\sqrt{8}+\sqrt{18}=\sqrt{2}+2\sqrt{2}+3\sqrt{3}$

$a=\sqrt{2}$ $d=\sqrt{2}$

$S_{50}=\frac{50}{2}(2\sqrt{2}+49\sqrt{2})$

$=25\times 51\sqrt{2}$

$=1275\sqrt{2}$

f) $\frac{\sin^3\theta}{\cos\theta}+\sin\theta\cos\theta=\frac{\sin^3\theta+\sin\theta\cos^2\theta}{\cos\theta}$

$=\frac{\sin\theta}{\cos\theta}(\sin^2\theta+\cos^2\theta)$ $\sin^2\theta+\cos^2\theta=1$

$=\frac{\sin\theta}{\cos\theta}=\tan\theta$

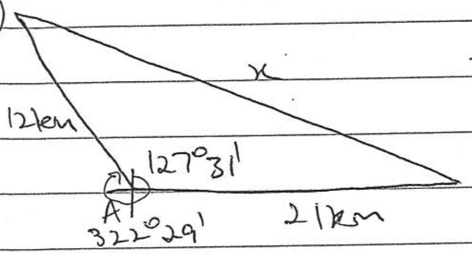
Start here for Question Number: 12

(a) $h=9$, $A=\frac{h}{3}\left[(y_0+y_4)+4(\text{odds})+2(\text{evens})\right]$

$=\frac{9}{3}\left[(12+18)+4(16+14)+2(14)\right]$

$=534\,m^2$

(b)



$x^2=12^2+21^2-2(12)(21)\cos 127°31'$

$x \doteqdot 30km$

(c) $y=x^3-6x^2-9x+4$

$y'=3x^2-12x-9$

$=3(x^2-4x-3)$

$y'=0$, stat. pt.

$x^2-4x-3=0$

$x=\frac{4\pm\sqrt{(-4)^2-4(1)(-3)}}{2}$

$=\frac{4\pm\sqrt{16+12}}{2}$

$=\frac{4\pm\sqrt{28}}{2}$

$=2\pm\sqrt{7}$

$x=4.65$, $y=-67.04$ (min)

$x=-0.65$, $y=7.04$ (max)

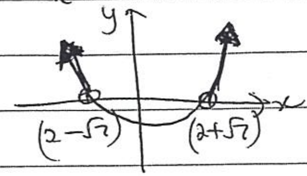
$y''=6x-12$

when $x=4.65$, $y''=15.87>0$, min.

when $x=-0.65$, $y''=-15.87<0$, max

(iii) Increasing $y'>0$, concave up $y''>0$.

$x^2-4x-3>0$.



$6x-12>0$.

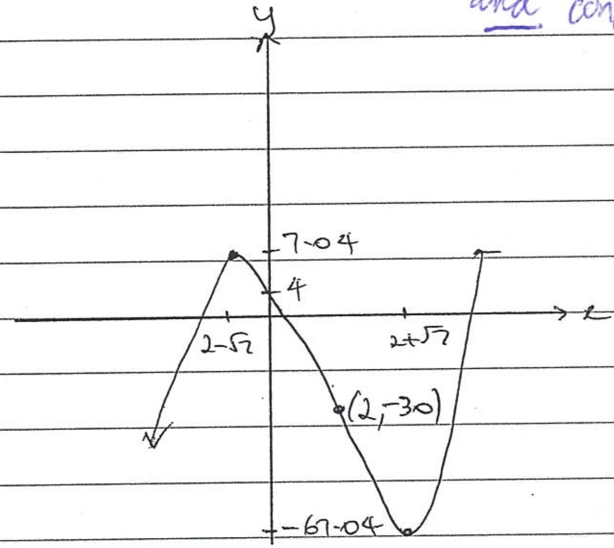
$6x>12$

$x>2$.

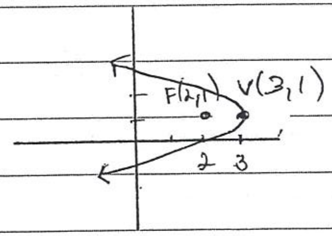
$\therefore x<2-\sqrt{7}$, $x>2+\sqrt{7}$

$\therefore x>2+\sqrt{7}$ for increasing and concave up.

(iv)



(d) $F(2,1)$ $V(3,1)$



(i) $a=1$

(ii) $x=4$

(iii) $(y-k)^2=-4(x-h)$

$(y-1)^2=-4(x-3)$

$y^2-2y+1=-4x+12$

$4x+y^2-2y-11=0$.

Additional writing space on back page.

Caringbah High School

Examination:..............................

Student Number

Q13 a) $V = \pi \int_0^3 (10-3x)^2 \, dx$ or $\pi \int (100 - 60x + 9x^2) \, dx$

$= \pi \left[\frac{(10-3x)^3}{9} \right]_0^3$ $= \pi \left[100x - 30x^2 + 3x^3 \right]_0^3$

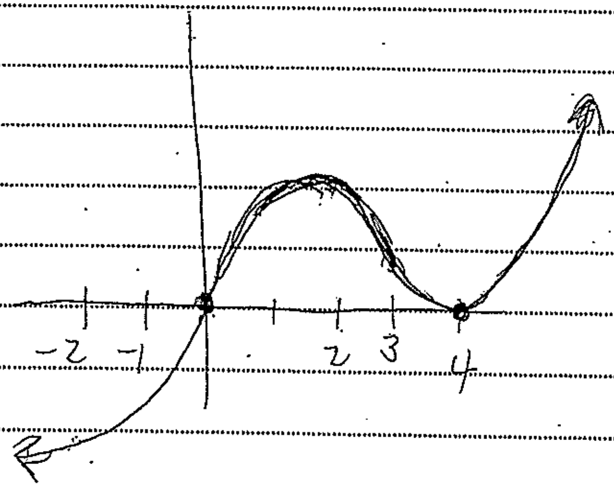
$= \pi \times 300 - 270 + 81$

$= \pi \left(\frac{-1}{9} - \left(\frac{-1000}{9} \right) \right)$ $= 111\pi u^3$

$= \frac{999\pi}{9} u^3$

$= 111\pi u^3$

b)



c) i) $\int (x+6)^{-\frac{1}{2}} dx = 2(x+6)^{\frac{1}{2}} + C$

ii) $\frac{1}{2} \int_0^2 \frac{2x \, dx}{x^2+4} = \frac{1}{2} \ln(x^2+4) \Big]_0^2 = \frac{1}{2}(\ln 8 - \ln 4)$

$\frac{1}{2} \ln 2$

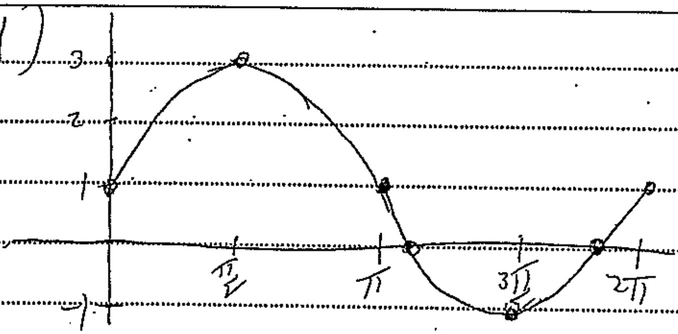
11std

Caringbah High School

Examination:..............................

Student Number

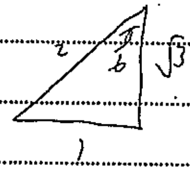
d)



ii) $1 + 2\sin x = 0$

$\sin x = \frac{-1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$



e.) $16x^2 - (16a^2 - 24a + 9)$

$16x^2 - (4a-3)^2$

$(4x - 4a + 3)(4x + 4a - 3)$

11std

Q1.4 $\sqrt{3}x - y + \sqrt{3} = 0$

i) $A: y = 0 \quad \sqrt{3}x + \sqrt{3} = 0$

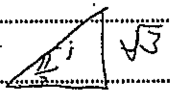
$x = -1 \quad (-1, 0)$

$B: x = 0 \quad -y + \sqrt{3} = 0$

$y = \sqrt{3} \quad (0, \sqrt{3})$

ii) $m_{AB} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

iii) $\angle BAO \quad \tan\theta = \sqrt{3}$



$\theta = \frac{\pi}{3}$ or $60°$

IV) $AC: \frac{1}{2} AC \times AO \times \sin\frac{\pi}{3} = 3\sqrt{3}$

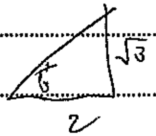
$\frac{1}{2} \times AC \times 1 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

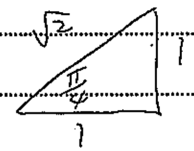
$\frac{\sqrt{3}}{4} AC = 3\sqrt{3}$

$\therefore AC = 12$.

b) $\tan\frac{\pi}{3} + \operatorname{cosec}\frac{\pi}{4}$

$\sqrt{3} + \sqrt{2}$





=

c) $\log_e x = 2\log_e 2x$

$x = 4x^2$

$4x^2 - x = 0$

$x(4x - 1) = 0$

$x = 0, \frac{1}{4}$

but $x > 0$ for $\log_e x$

so $x = \frac{1}{4}$ is the only solution

11std

d) ~~4k - k~~ $x^2 - (k+2)x + 4 = 0$

$\Delta = (k+2)^2 - 16 \geq 0$

$k^2 + 4k - 12 \geq 0$

$(k+6)(k-2) \geq 0$

$k \leq -6, \ k \geq 2$

e) $\frac{12 + x}{24} = \frac{40}{12}$

$144 + 12x = 960$

$12x = 816$

$x = 68$.

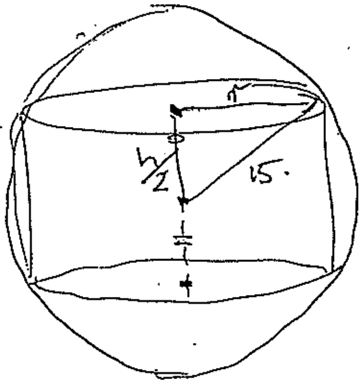
f) $\lim_{x \to 3} \frac{(x-3)(x+2)}{x-3} = \lim_{x \to 3} x + 2$

$= 5$

11std

Q15

(a)



i) $15^2 = r^2 + \left(\frac{h}{2}\right)^2$ 1

$225 = r^2 + \frac{h^2}{4}$

$225 - \frac{h^2}{4} = r^2$ 1

$V = \pi r^2 h$

$V = \pi\left(225 - \frac{h^2}{4}\right)h$ 1

$V = 225\pi h - \frac{\pi h^3}{4}$

(ii) $V = 225\pi h - \frac{\pi h^3}{4}$

$\frac{dv}{dh} = 225\pi - \frac{3\pi h^2}{4}$ ✓ 1

when $\frac{dv}{dh} = 0$

$\frac{3\pi h^2}{4} = 225\pi$

$3h^2 = 900$

$h^2 = 300$

$h = 10\sqrt{3}$. ✓ 1

$\frac{d^2v}{dh^2} = \frac{-6\pi h}{4}$ ✓ 1

∴ Max value when $h = 10\sqrt{3}$.

Caringbah High School

Examination: ....

Student Number

b) $2x^2 + 3x - 4 = 0$

i) $\alpha + \alpha = \frac{-3}{2}$ ✓ 1

ii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)\left((\alpha+\beta)^2 - 3\alpha\beta\right)$ ✓ 1 $\quad \alpha\beta = \frac{-4}{2} = -2$

$= \frac{-3}{2}\left(\frac{9}{4} - 3\times(-2)\right)$

$= \frac{-3}{2}\left(\frac{9}{4} + 6\right)$

$= \frac{-3}{2} \times \frac{33}{4}$

$= \frac{-99}{8}$ ✓ 1

c) $A_0 = 120000 \quad n = 120 \quad r = 0.005$

$A_0 = 120000 \times 1.005$

$= 120,600$ 1

ii) $A_1 = 120600 - M$ 1

~~$A_2 = 120600 \times 1.005 - M \times 1.005 - M$~~

$= 121203 - 2.005M$ 1

iii) $A_3 = 120000 \times 1.005^3 - m(1 + 1.005 + 1.005^2)$

$\vdots$

$A_n = 120000 \times 1.005^n - m(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})$

$\left(\frac{1(1.005^n - 1)}{1.005 - 1} \right)$

$= 120000 \times 1.005^n - 200m(1.005^n - 1)$

iv) $m = \frac{120000 \times 1.005^{120}}{200(1.005^{120} - 1)}$

$= \$1332.25$

Caringbah High School

Examination:....................................

Student Number

Q16 i) $V = V_0 e^{-kt}$

$\frac{dV}{dt} = -k(V_0 e^{-kt})$

$= -kV$

ii) when $t = 0$ $V = 1000$

$\therefore V = 1000 e^{-kt}$

when $t = 40$ $V = 800$

$800 = 1000 e^{-40k}$

$e^{-40k} = \frac{4}{5}$

$-40k = \ln \frac{4}{5}$

$k = \frac{-\ln \frac{4}{5}}{40}$

$V = 1000 e^{-60\left(\frac{-\ln \frac{4}{5}}{40}\right)}$

$= 716L$

iii) $10 = 1000 e^{\left(\frac{\ln(\frac{4}{5})t}{40}\right)}$

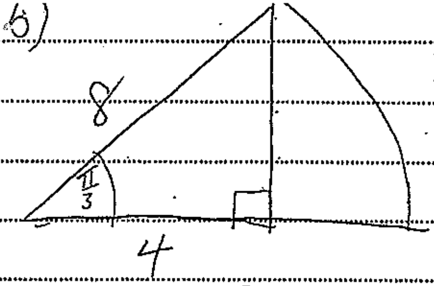
$0.01 = e^{\frac{\ln(\frac{4}{5})t}{40}}$

$\frac{\ln \frac{4}{5}}{40} t = \ln 0.01$

$t = \frac{40 \ln 0.01}{\ln(\frac{4}{5})} = 826 min$

$= 13 hrs\ 46 min.$

b)

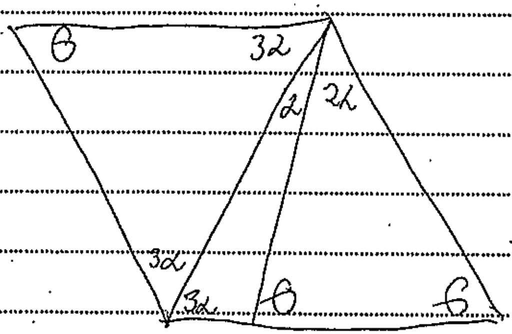


$\frac{1}{2} \times 4 \times 8 \times \sin\frac{\pi}{3}$ $\qquad$ $\frac{1}{2} \times 8^2 \times \frac{\pi}{3}$

$= \frac{16\sqrt{3}}{2} = 8\sqrt{3}$ $\qquad$ $= \frac{32\pi}{3}$

Area $= \frac{32\pi}{3} - 8\sqrt{3}$

c)



$12\alpha + 2\theta = 360$

$2\alpha + 2\theta = 180$

$10\alpha = 180$

$\alpha = 18°$

$36 + 2\theta = 180$

$\theta = 72°$

d) i) $\frac{d\,(x \log_e x - x)}{dx}$

$u = x \qquad v = \log_e x$

$u' = 1 \qquad v' = \frac{1}{x}$

$= x \times \frac{1}{x} + \log_e x - 1$

$= 1 + \log_e x - 1$

$= \log_e x$

$\int \log_e(x^2)\,dx = 2\int \log_e x\,dx$

$= 2(x \log_e x - x) + C$

ii) Area $= (2\log_e 5) \times 5 - 2\int_1^5 \log_e(x^2)\,dx$

$= 10\log_e 5 - 2\left[x \log_e x - x\right]_1^5$

$= 10\log_e 5 - 2\left[5\log_e 5 - 5 - (1\log_e 1 - 1)\right]$

$10\log_e 5 - (10\log_e 5 - 4)$

$= 4$

Q 16 d ii) Alternative method:

$$\int_0^{\ln 25} \log_e(x^2)\, dy$$

$y = \log_e x^2$

$y = 2\log_e x$

$\log_e x = \frac{y}{2}$

$x = e^{\frac{y}{2}}$

$$= \int_0^{\ln 25} e^{\frac{y}{2}}\, dy$$

$$= 2e^{\frac{y}{2}}\Big]_0^{\ln 25}$$

$$= 2\left[e^{\frac{1}{2}\ln 25} - e^{0}\right]$$

$$= 2\left[e^{\ln 5} - 1\right]$$

$$= 2\left[4\right]$$

$$= 8.$$