RP

CHELTENHAM GIRLS' HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 1994

MATHEMATICS

2/3 UNIT (COMMON)

Time allowed - Three hours
(Plus 5 minutes reading time)

Directions to Candidates:

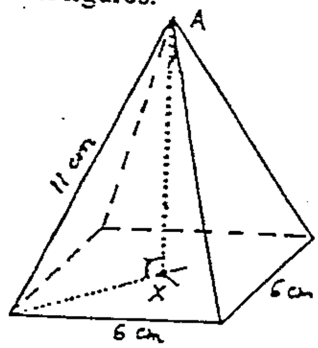
- Attempt all questions.
- All questions are of equal value.
- Show all necessary working as marks may be deducted for careless or badly arranged work.
- Approved calculations may be used.

THIS IS A SCHOOL PAPER AND DOES NOT NECESSARILY REFLECT THE HSC EXAMINATION

THIS ASSESSMENT TASK IS WORTH 40%

QUESTION 1

a) Evaluate $\dfrac{4.1 \times 10^5}{3.7 \times 10^{11}}$ in scientific notation to three significant figures.



b) Factorise $a^2 - 15a - 16$.

c) In the square pyramid shown, find the height AX.

d) The roots of $ax^2 - bx - 10 = 0$ are -1 and 5.
Find the values of a and b.

e) Sketch the solution set of $4 - 3a < 2a + 11$.

QUESTION 2

a) Sketch (not on grid paper) the region defined by $y < \sqrt{81 - x^2}$.

b) Show that the line $5x - 12y - 26 = 0$ is a tangent to the circle $x^2 + y^2 = 4$

c) A function is defined: $f(x) = \begin{cases} -x - 1 & \text{for } x < 0 \\ -1 & \text{for } x = 0 \\ x - 1 & \text{for } x > 0 \end{cases}$

i) Evaluate $f(-2) - f(3)$.
ii) Draw a neat sketch of the function $y = f(x)$.
iii) Find a single equation that could replace the equation of function given.

d) M (5,7) is the mid point of the interval AB, where A is the point (−1, 4).

Find the equation of HB, a line perpendicular to AB.

QUESTION 3

a) Differentiate:

i) $\dfrac{x^2 - 5x + 3}{x}$

ii) $\dfrac{5}{\sqrt{2x - 1}}$

iii) $x^2 \cos x$

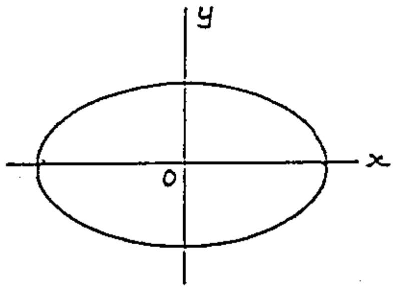
b) Find:

i) $\displaystyle\int \frac{\sin x}{\cos x}\,dx$

ii) $\displaystyle\int_1^2 (x + 2)^2\,dx$

Question 3 (continued)

c) Mr Radford was asked to design a soccer ball for the world cup. He came up with the design shown in the diagram which is centred at the origin and has the equation $9x^2 + 16y^2 = 144$ which shows his lack of soccer knowledge.



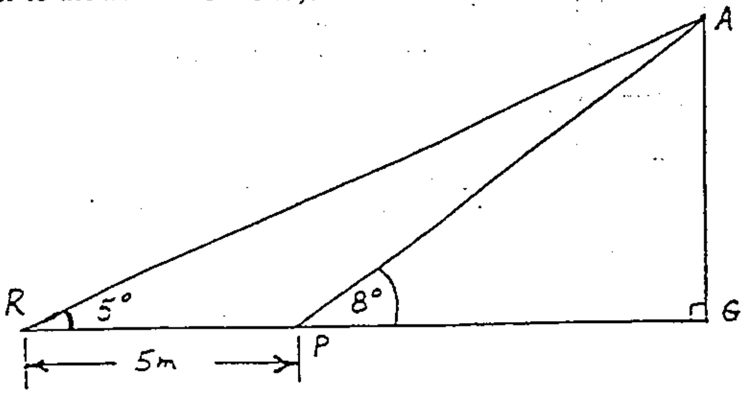
i) Verify that this ellipse (oval) has x intercepts ±4 and y intercepts ±3.

ii) Find, correct to 2 decimal places, the volume of this ball formed when this ellipse is rotated about the x axis.

QUESTION 4

a) If it takes three depressed Italians 12 minutes to drink one (1) litre of Chianti, how long will it take seven depressed Italians to drink the same amount at the same rate? (Answer to the nearest second).

b)

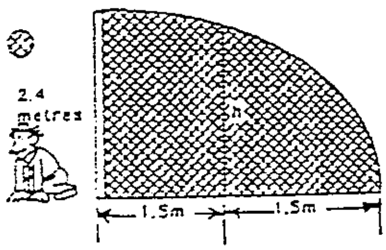


Roberto, in the World Cup, knows that from his position directly in front of the goalkeeper, he must kick the ball through the point A to score a goal. If he runs the ball forward an extra five metres, his kicking angle increases from 5° to 8°.

i) Calculate the kicking distance to A, from the 8° mark (2 decimal places).

ii) Find how far Roberto was from the goalkeeper at the 5° mark.

c) The side of the goal that Roberto was shooting at is shown in the diagram. The visible area of net is 4.6 square metres. Use Simpson's Rule to find the height of the net "h" to one decimal place.



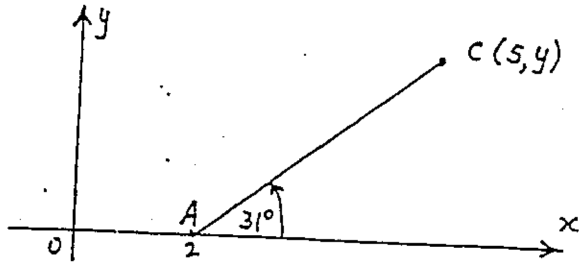
d) If there are 11 players on a soccer team, in how many different ways can a captain and vice-captain be selected?

QUESTION 5

a) Find the (i) period, (ii) amplitude of the function $y = 1 + 3\cos 2x$.

b) i) The line AC is inclined at an angle of 31° to the x axis. Use trigonometry to find the y coordinate of C (to 1 decimal place).

ii) Find the equation of AC.



b) For this year's World Cup, the Brazilians had 80 000 players to choose from. The rate of increase of eligible players is exponential and is given by $\frac{dP}{dt} = kP$.

It is expected that in two year's time Brazil will have 120 000 players to choose from. The next World Cup is in four year's time (1998).

i) Find the number of players that Brazil will have available in 1998.

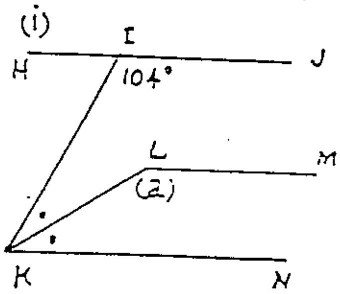
ii) Find the rate at which the number of players is increasing in the year 2000 (six year's time).

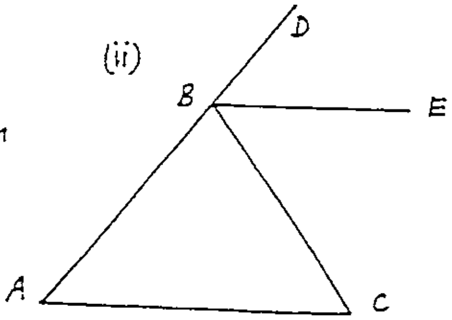
QUESTION 6

a) Solve the equation $4^x - 12(2^x) + 32 = 0$.

b) i) Find the value of (a) giving reasons.

ii) Given AB = BC and BE // AC, prove that BE bisects the angle CBD.





c) For the curve $y = x^3 - 3x^2 + 1$ in the domain $-2 \le x \le 4$:-

i) Determine the nature of any stationary points.

ii) Find any point of inflexion.

iii) Sketch the curve for the given domain.

iv) Find the gradient of the normal to the curve at $x = 3$.

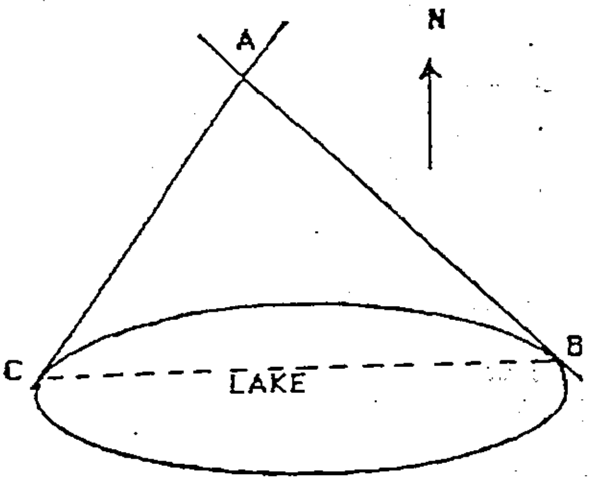
QUESTION 7

a) An arithmetic series is given by: 10 + ... + 60 with a sum of 3535.

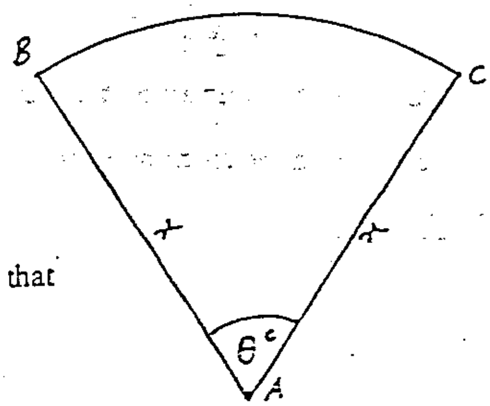
i) Find the number of terms in this series.

ii) Find the common difference.

b) From A, the intersection of two straight roads AB and AC, the bearing of B is 120°T and C is South West of A. AB = 12.3 km, AC = 15.2 km.
Calculate BC, the width of the lake (1 decimal place).



NOT TO SCALE



c) ABC is the sector of a circle.

i) Write an expression for the arc BC.

ii) Write an expression for the area of ABC.

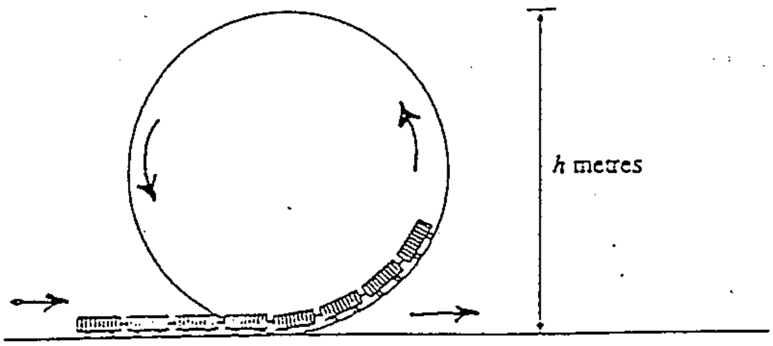
iii) If ABC has a perimeter of 12 metres, show that

Area of ABC = $\dfrac{72\theta}{(\theta + 2)^2}$.

iv) Hence or otherwise show that the the maximum area is 9 units^2.

QUESTION 8

a)



For a roller coaster to complete a loop safely, it must enter the loop with a speed of at least $v\ ms^{-1}$ where $v^2 = 20h$.

i) Find the height of a loop that can be completed at an entry speed of $18\ ms^{-1}$.

ii) If a loop is 14 m high and the roller coaster enters with a speed of $19\ ms^{-1}$, explain why it will complete the loop safely.

b) A parabola has the equation $y = Ax^2$ and the line $y = 20x + 20$ is one of its tangents.

i) Find the value of A.

ii) Sketch the parabola and the line showing the point of contact.

iii) Find the coordinates of the focus and the equation of the directrix of the parabola.

QUESTION 9

a) A soccer ball is being inflated so that after t seconds, the rate of increase R, of its volume is given by $R = 4\pi kt^2$, where k is a constant.
Before inflation, the ball contained 2000 cm^3 of air. If $k = 0.03$, find the volume of air in the ball after 5 seconds.

b) A fast soccer linesman runs only along the side line. When he (she?) was first observed he was on the half way line ($x = 0$) travelling with a velocity of $2\pi\ ms^{-1}$. His acceleration (a) is ms^{-2} at a time t seconds ($t \geq 0$) is:

$$a = 4\pi^2 \cos \pi t.$$

b) i) Find the velocity function.
ii) Find the displacement function (x) in terms of t.
iii) Find when the linesman was stationary in the first four seconds.
iv) Show that the furthest distance he attained from the halfway line in the first four seconds was $2(2 + \sqrt{3}) + \frac{19\pi}{3}$ metres.

QUESTION 10

a) The pendulum of a clock is 15cm long and travels through an angle of 28° on each swing. Find (to two decimal places) the distance that the end of the pendulum moves on each swing.

b) In the quadratic equation $x^2 + hx + m = 0$, one root is triple the other. Prove that $3h^2 = 16m$.

c) Sketch A (1,0), B (4,0) and C (0,2).

  i) Show that CB = 2(CA).

  ii) If P(x,y) moves so that PB = 2PA:
  (*a*) find the equation of the locus of P;
  (*b*) describe and sketch the locus of P giving its main features.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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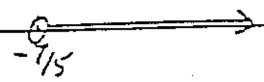
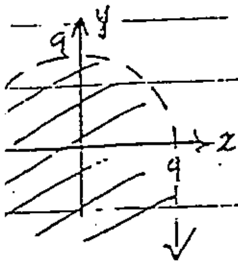
) $1 \cdot 11 \times 10^{-6}$ (b) $(a-16)(a+1)$ (c) $\sqrt{103}$ cm

) $\frac{b}{a} = 4$, $\frac{-10}{a} = -5$ ∴ $a = 2$, $b = 8$ (e) 

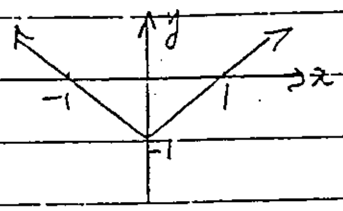


(b) Dist of $(0,0)$ from $5x - 12y - 26 = 0$ is $\left|\frac{26}{\sqrt{5^2+12^2}}\right| = 2$

Since radius is 2 ∴ line is tangent.

)(i) $f(-2) - f(3) = 1 - 2 = -1$

(iii) $y = |x - 1|$



) B is $(11, 10)$, HB: $2x + y - 32 = 0$

(i) $1 - \frac{3}{x^2}$ (ii) $\frac{-5}{\sqrt{(2x-1)^3}}$ (iii) $2x\cos x - x^2 \sin x$

)(i) $-\ln\cos x + C = \ln\sec x + C$ (ii) $\left[\frac{1}{3}(x+2)^3\right]_1^2 = 12\frac{1}{3}$

(i) when $y = 0$ $x = \pm 4$; when $x = 0$ $y = \pm 3$

(ii) $V = 2\pi \int_0^4 \left(\frac{144 - 9x^2}{16}\right) dx = 48\pi\ u^3$

) $5\frac{1}{7}$ mins (b)(i) $\frac{PA}{\sin 5^\circ} = \frac{5}{\sin 3^\circ}$ ∴ $PA = 8.33$ m (ii) $\frac{RA}{\sin 172^\circ} = \frac{5}{\sin 3^\circ}$ ∴ $RA = 13.3$ m

) $4 \cdot 6 = \frac{1}{6} \times 3 \{2.4 + 4h + 0\} \Rightarrow h = 1.7$ (d) $11 \times 10 = 110$ ways

)(i) $\frac{2\pi}{2} = \pi$ (ii) 3 (b) $\tan 31^\circ = \frac{y}{3} \Rightarrow y = 1.8$; AC: $3x - 5y - 6 = 0$

)(i) $P = P_0 e^{kt}$ where $P_0 = 80000$ and when $t = 2$, $P = 120000$

∴ $P = 80000\, e^{(\frac{1}{2}\ln\frac{3}{2})t}$ & when $t = 4$ $P = 180000$

(ii) when $t = 6$, $P = 270000$ and $\frac{dP}{dt} = 54737.8$

(a) $x = 2, 3$ (Let $y = 2^x$)

(b)(i) $I\hat{K}N = 76^\circ$ (Coint ∠'s supple; HJ ∥ KN)

$L\hat{K}N = 38^\circ$ (given angle bisected)

$a = 142$ (coint ∠'s supple; LM ∥ KN)

(b)(ii) $B\hat{A}C = B\hat{C}A$ (base angles of isos △ABC; AB = AC)

$D\hat{B}E = B\hat{A}C$ (corresp; AC ∥ BE)

$E\hat{B}C = B\hat{C}A$ (alt; AC ∥ BE)

∴ $D\hat{B}E = E\hat{B}C$

∴ BE bisects $C\hat{B}D$

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6. (c) $y = x^3 - 3x^2 + 1$

$\frac{dy}{dx} = 3x^2 - 6x$

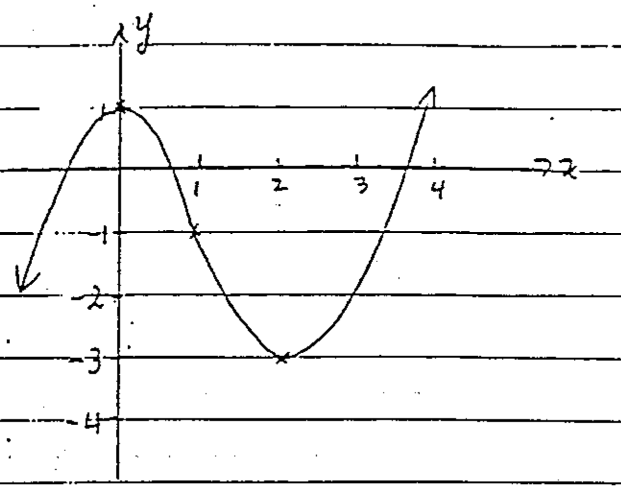
$\frac{d^2y}{dx^2} = 6x - 6$

(i) max turning pt at $(0, 1)$

min turning pt at $(2, -3)$

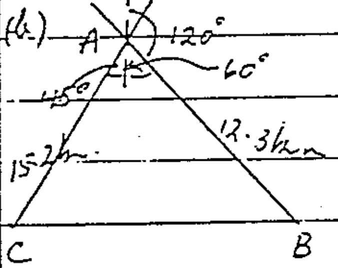
(ii) pt of inflexion at $(1, -1)$

(iv) grad of normal at $x = 3$ is $-\frac{1}{9}$



7. (a)(i) $\frac{n}{2}(10 + 60) = 3535 \Rightarrow n = 101$ (ii) $d = \frac{1}{2}$ [ie $60 = 10 + 100d$]

(b)



$BC^2 = 12.3^2 + 15.2^2 - 2 \times 12.3 \times 15.2 \cos 105°$

$BC = 21.88 = 21.9$ km (to 1 dec pl)

(c)(i) $l = r\theta$ (ii) $A = \frac{1}{2}r^2\theta$ (iii) $r\theta + 2r = 12 \Rightarrow r = \frac{12}{2+\theta}$

$\therefore A = \frac{1}{2}\left(\frac{12}{2+\theta}\right)^2 \cdot \theta = \frac{72\theta}{(2+\theta)^2}$

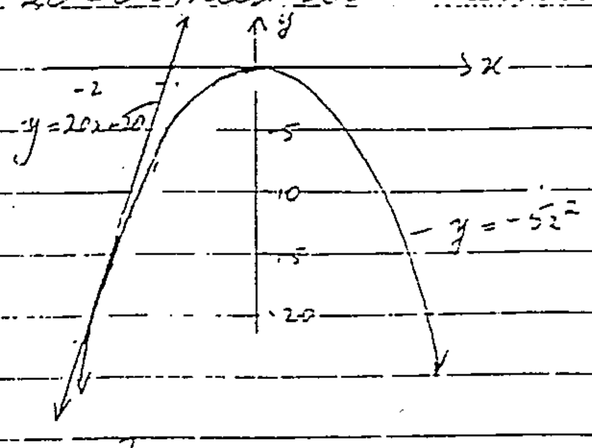
(iv) For max area $\frac{dA}{d\theta} = 0$ and $\frac{d^2A}{d\theta^2} < 0$; $\frac{dA}{d\theta} = \frac{144 - 72\theta}{(2+\theta)^3} \Rightarrow \theta = 2$

8. (a) (i) 16.2m (ii) when $h = 14$ $v^2 = 280 \Rightarrow v = 16.7$ and since speed of $19\ ms^{-1} > 16.7$ it will complete loop safely.

(b)(i) For it to be tangent $Ax^2 - 20x - 20 = 0$ must have one solution

$\therefore \Delta = 0$ $\therefore A = -5$

(ii) $(0, -\frac{1}{5})$ and $y = \frac{1}{5}$



9. (a) $R = 4\pi k t^2$

$\therefore \frac{dV}{dt} = 4\pi \times 0.03t^2$

$V = 4\pi \times 0.01t^3 + C$

when $t = 0$, $V = 2000$ $\therefore C = 2000$

$\therefore V = 0.04\pi t^3 + 2000$

when $t = 5$

$V = 2015.7\ cm^3$ (to 1 dec pl)

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9 (b) $a = 4\pi^2 \cos \pi t$

(i) $v = 4\pi \sin \pi t + 2\pi$

(ii) $x = 2\pi t - 4\cos \pi t + 4$

(iii) when $v = 0$, $t = 1\frac{1}{6}, 1\frac{5}{6}, 3\frac{1}{6}, 3\frac{5}{6}$

(iv) when $t = 19/6$ *

$x = 2\pi\left(\frac{19}{6}\right) - 4\cos\frac{19\pi}{6} + 4$

$= \frac{38\pi}{6} + 2\sqrt{3} + 4$

Max dist is $\left[2(2+\sqrt{3}) + \frac{19\pi}{3}\right]$

[*NB max since it is a neg cos graph]

Q10 (a) $l = r\theta$

$= 15 \times \frac{28}{180} \times \pi$

Distance $= 7.33$ cm (to 2 dec pl)

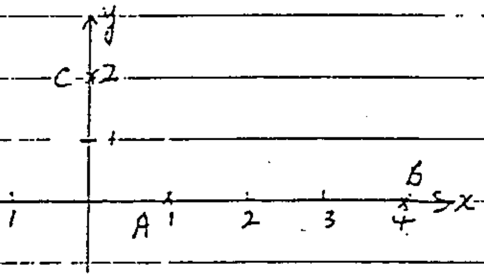
(b) $x^2 + hx + m = 0$; roots $\alpha, 3\alpha$

$\therefore 4\alpha = -h$ and $3\alpha^2 = m$

$\therefore 3\left(\frac{-h}{4}\right)^2 = m$

$3h^2 = 16m$

(c)



(i) $CB = \sqrt{(-4)^2 + 2^2} = 2\sqrt{5}$

$CA = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$

$\therefore CB = 2CA$

(ii) (a) $\sqrt{(x-4)^2 + y^2} = 2\sqrt{(x-1)^2 + y^2}$

$(x-4)^2 + y^2 = 4(x-1)^2 + 4y^2$

$x^2 + y^2 = 4$

(b) Locus of P is a circle centre (0,0) radius 2 units