

Name : _____

Class : 12 MT ____

KW
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AW

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2001 AP4

YEAR 12 TRIAL HSC

MATHEMATICS

[2/3 UNIT]

*Time allowed - 3 hours
(plus 5 minutes reading time)*

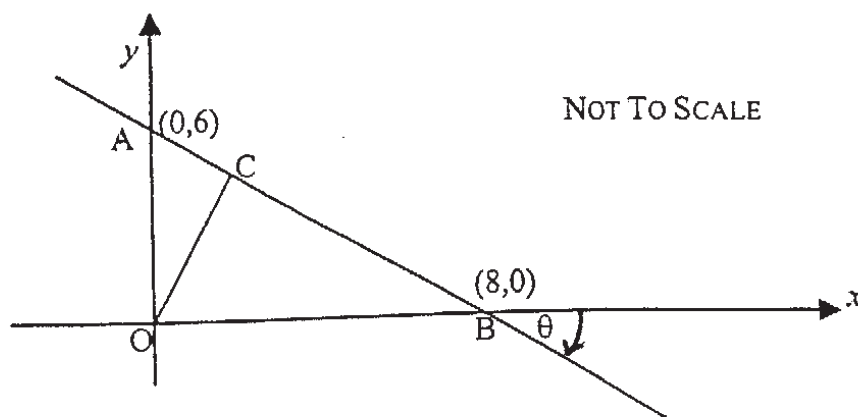
DIRECTIONS TO CANDIDATES:

- Attempt ALL questions.
- All questions are of equal value
- Standard Integrals are provided.
- Approved calculators may be used.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Each question attempted is to be returned on a new page clearly marked Question 1, Question 2, etc on the top of the page.
- Each page must show your class and your name.

Students are advised that this is a school based Trial Examination *only* and cannot in any way guarantee the complete content nor format of the Higher School Certificate Examination.

QUESTION 1**Use a new page****Marks**

- (a) Factorise $3x^2 - 2x - 1$. 2
- (b) Solve and graph the solution of $|2x + 1| < 2$ on a number line 2
- (c) Find the value of $8^{\frac{1}{2}}$ correct to 3 decimal places. 2
- (d) Find the primitive function for $x^{-2} + 6$. 2
- (e) Find the exact value of $\tan 60^\circ + \tan 150^\circ$. 2
- (f) Solve $\tan \alpha = \sqrt{3}$ for $0^\circ \leq \alpha \leq 360^\circ$ 2

QUESTION 2**Start a new page****Marks**

- (a) Find the gradient of the line AB 1
- (b) Show that the equation of AB is $3x + 4y - 24 = 0$ 2
- (c) Calculate the angle θ to the nearest degree. 2
- (d) Given that OC meets AB at right angles, calculate the distance OC. 2
- (e) (i) Show OC has the equation $4x - 3y = 0$ 2
- (ii) Find the distance of BC. 2
- (iii) Show that $\frac{OC}{BC} = \frac{OA}{OB}$. 1

QUESTION 3

Use a new page

Marks(a) Obtain all solutions to $9^x - 28 \times 3^x + 27 = 0$.**2**

(b) Find the indefinite integrals for:

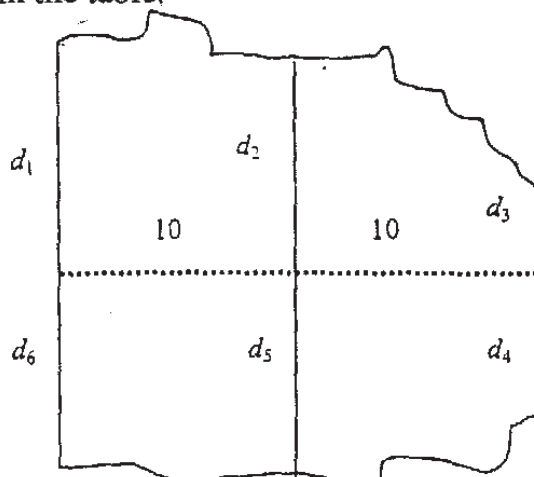
(i)
$$\int \frac{4x}{x^2 - 7} dx$$

1

(ii)
$$\int \frac{3x^2 - 7x + 2}{x^2} dx$$

2(c) Evaluate $\int_0^1 2xe^{(3x^2-5)} dx$ Give your answer in scientific notation to 3 significant figures. **3**

(d) The diagram shows the face of a vertical cliff.

The distances d_1 to d_6 are given in the table.

d_1	d_2	d_3	d_4	d_5	d_6
15	14	5.4	8.8	15	14.4

(i) Find an estimate for the area of the cliff face using the trapezoidal rule. Give your answer to the nearest square metre. **2**(ii) Is the area greater than or less than the actual area of the cliff? Justify your answer. **2**

QUESTION 4

Start a new page

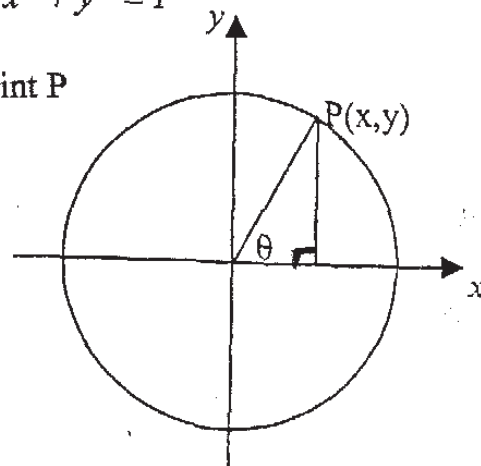
Marks

(a) For the quadratic function $f(x) = Ax^2 - 7x + 3$, $f(2) = -3$.

- (i) Find the value of A . 1
- (ii) If the two roots of the equation $f(x) = 0$ are α and β ,
Find the value of $\alpha^2 + \beta^2$ 2

(b) The unit circle shown has the equation $x^2 + y^2 = 1$

- (i) Write the co-ordinates of the point P
In terms of the angle θ . 1
- (ii) Explain why $\sin^2 \theta + \cos^2 \theta = 1$ 1
- (iii) If $\sin \theta = \frac{8}{17}$ find 2 possible
values for $\cos \theta$ 2

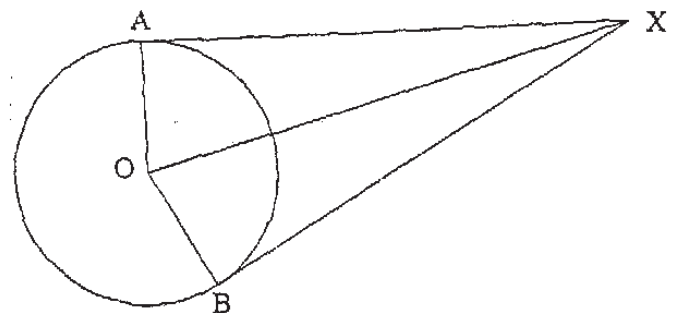


(c) The figure shows a circle, centre O.

AX and BX are tangents to the circle from the external point X.

OA and OB are the radii at the points of contact of the tangents.

$AX \perp OA$ and $BX \perp OB$



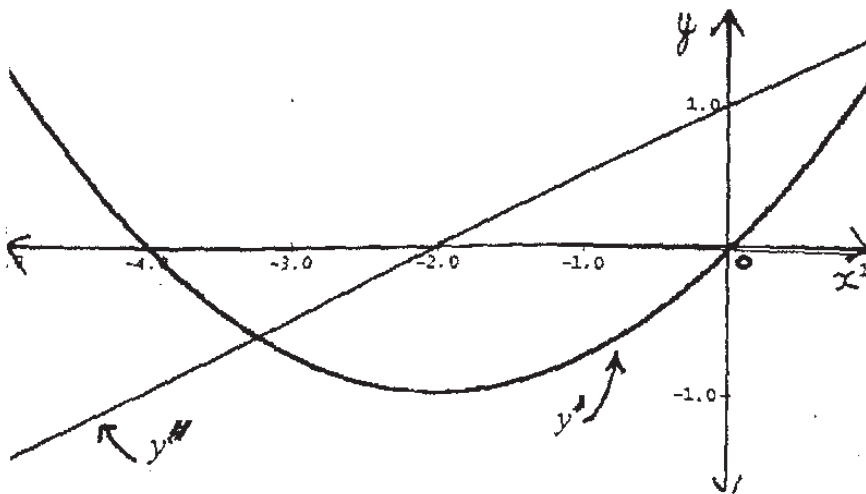
- (i) By considering the triangles AOX and BOX prove that $AX = BX$. 3
- (ii) If $AO = r$ and $\angle AOX = \theta$,
show that the area of OAXB = $r^2 \tan \theta$. 2

QUESTION 5

Start a new page

Marks

- (a) In a geometric sequence $T_1 = 27$ and $T_4 = 1$
- (i) Find the common ratio, r . 1
- (ii) Find the limiting sum. 2
- (b) Consider the series $97 + 91 + 85 + 79 + \dots$
- (i) Find the common difference, d . 1
- (ii) Find the largest n such that $S_n < 0$. 2
- (c) The point P moves such that its distance from the point $(0,2)$ is the same as the distance from the line $y = -2$.
What is the equation of the ~~line~~
locus? 3
- (d) The graph shows y' and y'' for the function $y = f(x)$.



Sketch the graph of $y = f(x)$ clearly showing the x values of any turning points and points of inflexion.

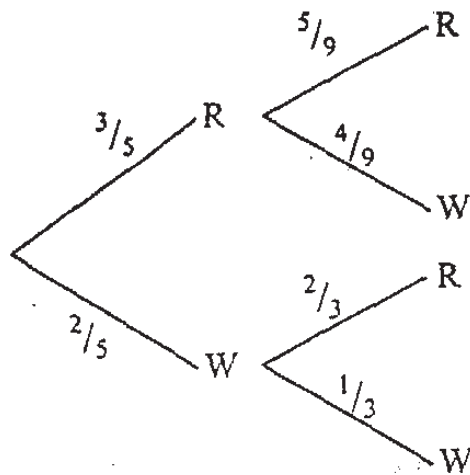
3

QUESTION 6

Start a new page

Marks

(a)



Some red and white balls are placed in a bag.

The tree diagram shows the probabilities relating to the situation of two balls from the bag, without replacement.

- Find
- | | | |
|-------|---|---|
| (i) | the probability that the two balls are different colours. | 2 |
| (ii) | the probability that the two balls are the same colour.. | 1 |
| (iii) | the number of red balls and white balls in the bag. | 1 |
- (b) For the function $f(x) = 2x.e^{0.5x}$
- | | | |
|------|--|---|
| (i) | Show it has a minimum at $x = -2$ and state the minimum value at this point. | 6 |
| (ii) | State the region(s) for where the curve is increasing. | 2 |

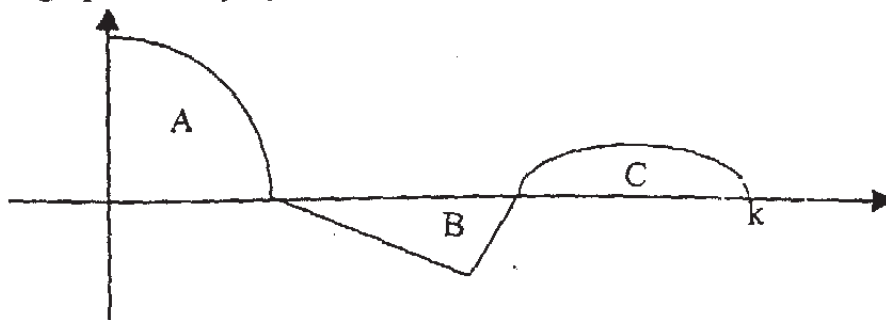
QUESTION 7

Start a new page

Marks

- (a) The graph shows
- $y = f(x)$
- for
- $0 \leq x \leq k$

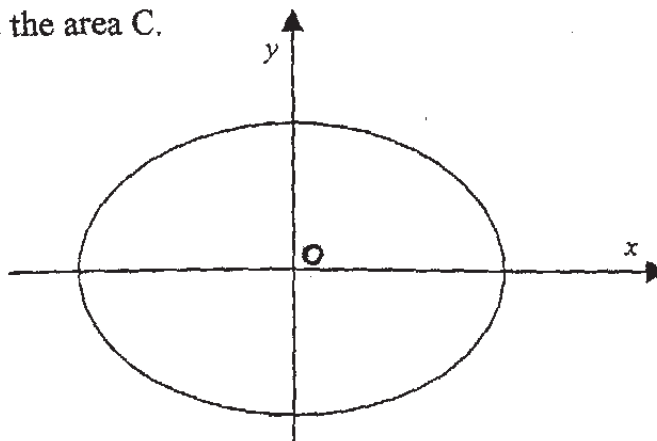
2



The value of $\int_0^k f(x) dx$ is known to be 3.5 units

If $A = 5$ and $B = 4$ find the area C .

- (b)



The curve represented on the graph is an ellipse which has the equation $4x^2 + 9y^2 = 36$

- (i) Show that the curve crosses the
- x
- axis at
- $(3,0)$
- and
- $(-3,0)$
- 1

- (ii) Obtain the volume generated when the curve is rotated around the
- x
- axis. 3

- (c) Michael has decided to invest in a superannuation fund. He calculates that he will need \$1 000 000 if he is to retire in 20 years time and maintain his present lifestyle. The superannuation fund pays 12% per annum interest on his investments.

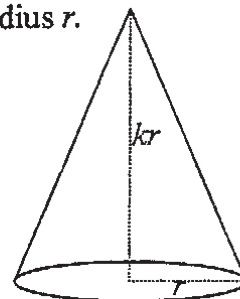
- (i) Michael invests \$
- P
- at the beginning of each year. Show that at the end of the first year his investment is worth \$
- $P(1.12)$
- . 1

- (ii) Show that at the end of the third year his investment is given by the expression \$
- $P(1.12)(1.12^2 + 1.12 + 1)$
- . 2

- (iii) Find a similar expression for his investment after 20 years and hence find the value of
- P
- needed to realise the total of \$1 000 000 required for his retirement. 3

QUESTION 8	Start a new page	Marks
(a) (i)	Show that the discriminant for the quadratic equation $kx^2 + (k+3)x - 1 = 0$ is given by $k^2 + 10k + 9$. Hence find for what value of k does the equation have real roots.	3
(ii)	For what value of k is the quadratic expression $kx^2 + (k+3)x - 1 = 0$ positive definite?	2
(b) (i)	Show that $\frac{d}{dx}(x \ln x - x) = \ln x$	2
(ii)	Hence evaluate $\int_1^{e^2} \ln x \cdot dx$. Leave your answer in exact form.	2
(c)	Find the equation of the tangent to the curve $y = \ln(\sqrt{x})$ when $x = e$.	3

QUESTION 9	Start a new page	Marks
(a)	For the parabola $8x = y^2$ find	
(i)	The Vertex	1
(ii)	The Focus	1
(iii)	The Directrix	1
(b)	If $\log_x a = 3.6$ and $\log_x b = 2$ find:	
(i)	$\log_x \sqrt[3]{a}$	1
(ii)	$\log_x ab$	1
(iii)	$\log_x \frac{a}{b}$	1
(c)	The diagram represents a right conical container, with radius r . The height of the container = kr . Also the sum of the radius and the height = 1m.	



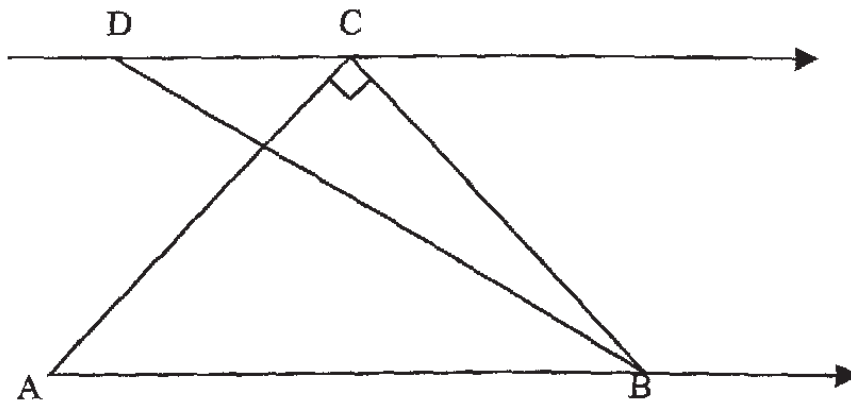
- (i) Show that the volume of the cone is given by $V = \frac{\pi}{3} \cdot \frac{k}{(1+k)^3}$ 2
- (ii) Find the value of k which maximises the volume of the cone. 3
- (iii) Calculate the maximum volume. 1

QUESTION 10

Start a new page

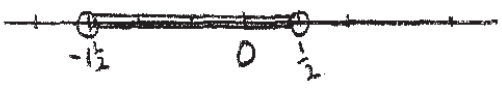

Marks

- (a) Sketch $y = 2 \sin x - 1$ for $0^\circ \leq x \leq 180^\circ$ 3
- (b) To comply with regulations, a factory must make hourly measurements of the quality of fumes produced by its furnaces. The measured quantity of fumes, L litres, that has been produced by each of its furnaces t hours after the furnace has been lit is given by the expression $L = t + 1.2^t$.
- (i) A furnace is lit at 6 a.m. What is the measured quantity of fumes from the furnace after one hour? 1
- (ii) A second furnace is lit at 7 a.m. Show that the total measured quantity of fumes from the two furnaces by 8.45 a.m. is 5.64 litres. 1
- (iii) At the beginning of each hour of the day, an additional furnace is lit. Write an expression to find the total measured quantity of fumes from the furnaces after n hours, where n is a positive whole number. 2
- (iv) On a given day, the first furnace is lit at 6 a.m. and an additional furnace is lit every hour, until the last furnace is lit at 4 p.m. Using the formulas for the sum of an arithmetic series and the sum of a geometric series, calculate the total measured quantity of fumes produced by 5 p.m. 2
- (c) A, B and C are the vertices of an isosceles triangle with a right angle at C. D is a point such that $DB = AB$ and angle DBA is acute $DC \parallel AB$. 3



Find the size of $\angle DBC$

Question

<p>a) $(3x + 1)(x - 1)$</p>	<p>2</p>
<p>b) $2 \left x - \frac{1}{2} \right < 2$ $\left x - \frac{1}{2} \right < 1$</p> 	<p>or $2x + 1 < 2$ $-2x - 1 < 2$ or * $-2 < 2x + 1 < 2$ $2x < 1$ $-2x < 3$ $-3 < 2x < 1$ $x < \frac{1}{2}$ $x > -\frac{1}{2}$ $-\frac{1}{2} < x < \frac{1}{2}$ *</p> <p>2</p>
<p>c) $8^{\frac{1}{2}} = \sqrt{8} \doteq 2.828$</p>	<p>1 for answer 1 for correct to 3 dp.</p>
<p>d) $\int x^{-2} + b \, dx = -1x^{-1} + bx + c$ $= \frac{-1}{x} + bx + c$</p>	<p>2</p>
<p>e) $\tan 60 = \sqrt{3}$ $\tan 150 = \frac{-1}{\sqrt{3}}$ $\sqrt{3} - \frac{1}{\sqrt{3}}$ $\frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$</p>	<p>1 for exact ratio 1 for simplifying.</p> <p>2</p>
<p>f) $\tan \alpha = \sqrt{3}$  $\tan 60 = \sqrt{3}$ $\alpha = 60^\circ$ or $180^\circ + 60^\circ$ $\therefore \alpha = 60^\circ \text{ \& } 240^\circ$</p>	<p>1 for 60° 1 for 240° $\frac{1}{2}$</p>

$$\begin{aligned} \text{Q 2(a)} \quad m &= \frac{6-0}{0-8} \\ &= -\frac{3}{4} \end{aligned}$$

1.

$$\begin{aligned} \text{(b)} \quad y \text{ intercept} &= 6 & \text{OR} \\ y &= -\frac{3}{4}x + 6 & y - 0 &= \frac{3}{4}(x - 8) \\ 4y &= -3x + 24 & y &= \frac{3}{4}x - 6 \\ 3x + 4y - 24 &= 0 & 3x + 4y - 24 &= 0 \end{aligned}$$

2.

$$\begin{aligned} \text{(c)} \quad \theta &= \angle ABO & \text{OR} \quad \text{inclination} &= \tan^{-1}\left(-\frac{3}{4}\right) \\ \therefore \tan \theta &= \frac{6}{8} & &= 143^\circ \\ \theta &= 37^\circ & \theta &= 180^\circ - 143^\circ \\ & & &= 37^\circ \end{aligned}$$

2

$$\begin{aligned} \text{(d)} \quad d &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|0 + 0 - 24|}{\sqrt{3^2 + 4^2}} \\ &= \left| \frac{-24}{5} \right| = 4.8 \end{aligned}$$

2.

$$\begin{aligned} \text{e) (i)} \quad OC \perp AB &\therefore \text{gradient} = \frac{4}{3} \\ y \text{ intercept is } 0 & \\ \therefore y &= \frac{4}{3}x \\ 4x - 3y &= 0 \end{aligned}$$

2

Solutions

Marks/Comments

$$2(e)(ii) \quad 4x - 3y = 0 \quad \text{OR.} \quad OB = 8$$

$$y = \frac{4}{3}x \quad OC = 4.8$$

$$\text{sub into } 3x + 4y - 24 = 0 \quad BC^2 = OB^2 - OC^2$$

$$3x + 4\left(\frac{4}{3}x\right) - 24 = 0 \quad = 8^2 - 4.8^2$$

$$\frac{25x}{3} = 24 \quad BC^2 = 40.96$$

$$x = \frac{72}{25} \quad BC = 6.4$$

$$y = \frac{96}{25}$$

$$d = \sqrt{\left(8 - \frac{72}{25}\right)^2 + \left(0 - \frac{96}{25}\right)^2}$$

$$BC = 6.4$$

$$(iii) \quad \frac{OC}{BC} = \frac{4.8}{6.4} = \frac{3}{4}$$

$$\frac{OA}{OB} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \frac{OC}{BC} = \frac{OA}{OB}$$

2.

|

Solutions	Marks/Comments
<p>Q3 a) let $u = 3^x$ $u^2 - 28u + 27 = (u-27)(u-1) = 0$ when $3^x = 27$ $x=3$ OR $3^x = 1$ $x=0$</p>	<p>1 1 for both solutions in x.</p>
<p>b) i) $\int \frac{4x}{x^2-7} dx = 2 \ln(x^2-7) + c$ ii) $\int \frac{3x^2-7x+2}{x^2} dx = \left(\left(3 - \frac{7}{x} + \frac{2}{x^2} \right) dx \right)$ $= 3xc - 7hx - \frac{2}{x} + c$</p>	<p>1 1 1</p>
<p>c) $\int_0^1 2x \cdot e^{(3x^2-5)} dx = \left[\frac{1}{3} e^{3x^2-5} \right]_0^1$ $= \frac{1}{3} e^{-2} - \frac{1}{3} e^{-5}$ $= 0.0428657$ $\approx 4.29 \times 10^{-2}$</p>	<p>1 1 1</p>
<p>$A = \frac{10}{2} [d_1 + d_2] + \frac{10}{2} [d_2 + d_3] + \frac{10}{2} [d_3 + d_4] + \frac{10}{2} [d_4 + d_5]$</p>	
<p>d) i) $A \approx 10 \left(\frac{d_1}{2} + d_2 + \frac{d_3}{2} + \frac{d_4}{2} + \frac{d_5 + d_6}{2} \right)$ $= 508 \text{ m}^2$</p>	<p>← or equivalent expression. 1</p>
<p>ii) <u>less than actual area.</u> If we connect the ends of d_1, d_2, d_2, d_3, d_3, d_4, d_4, d_5, d_5, d_6 areas of the cliff are not included.</p>	<p>1 1</p>

Solutions

Marks/Comments

Q4(a)(i) $A(2)^2 - 7(2) + 3 = -3$

$4A - 11 = -3$

$4A = 8$

$A = 2$

1

(ii) $2x^2 - 7x + 3 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{-b}{a}\right)^2 - \frac{c}{a} \times 2$

$= \left(\frac{7}{2}\right)^2 - \frac{3}{2} \times 2$

$= \frac{49}{4} - 3$

$= \frac{43}{4} = \frac{37}{4} = 9\frac{1}{4}$

1

1

If $\alpha^2 + \beta^2$ is calculated correctly using the wrong value of A from (i) give full marks.

2.

OR BY FINDING ROOTS (3 and $\frac{1}{2}$) and squaring.

(b) (i) $\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$

$x = r \cos \theta$ $y = r \sin \theta$

$P(\cos \theta, \sin \theta)$

1

(ii) $x^2 + y^2 = 1$ for all points on circle.

$P(\cos \theta, \sin \theta)$ lies on circle.

$\therefore \cos^2 \theta + \sin^2 \theta = 1$

$\sin^2 \theta + \cos^2 \theta = 1$

OR USING PYTHAGORAS THM.

(iii) $\cos^2 \theta + \left(\frac{8}{17}\right)^2 = 1$

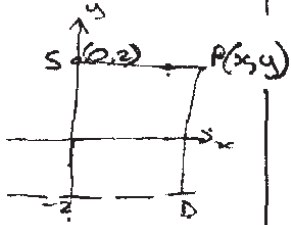
$\cos^2 \theta + \frac{64}{289} = 1$

$\cos^2 \theta = 1 - \frac{64}{289} = \frac{225}{289}$

$\cos \theta = \pm \frac{15}{17}$

1 for working
1 for answer

2

Solutions	Marks/Comments
<p>Q5a) i) $T_1 = a = 27$, $T_4 = ar^3 = 1$ $r^3 = \frac{1}{27}$ $r = \frac{1}{3}$</p>	<p>1</p>
<p>ii) $a = 27$ $r = \frac{1}{3}$ $\frac{a}{1-r} = S_{\infty}$ $S_{\infty} = \frac{27}{\frac{2}{3}}$ $= 40\frac{1}{2}$</p>	<p>1 1</p>
<p>b) i) $d = -6$</p>	<p>1</p>
<p>ii) $S_n = \frac{n}{2} (2a + (n-1)d)$ $0 = \frac{n}{2} (194 - 6n - 6)$ $0 = 94n - 3n^2$ gives $n = 31\frac{1}{3}$ $n(94 - 3n)$ whence $n = 31$ is largest n for which $S_n > 0$</p>	<p>1</p>
<p>c) Let $D = (x, -2)$ $S = (0, 2)$ and $PS = PD$ $\therefore PS^2 = PD^2$ $(x-0)^2 + (y-2)^2 = (x-x)^2 + (y-(-2))^2$ $x^2 + y^2 - 4y + 4 = 0 + y^2 + 4y + 4$ $x^2 = 8y$ \therefore locus of P is $y = \frac{x^2}{8}$</p> 	<p>1 for distance statement. 1 for Algebra manipulation 1 for Equation.</p>

Solutions

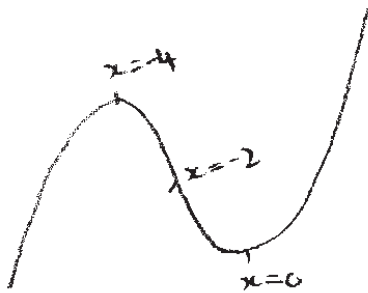
Marks/Comments

5 d) $x = -4$ is a max $\because y'' < 0$ $y' = 0$

$x = -2$ is point of inflexion

$$y'' = 0 \quad y' < 0$$

$x = 0$ is min $\because y' = 0$ $y'' > 0$



12

Solutions	Marks/Comments
$b) i) P(RW) = \frac{3}{5} \times \frac{4}{9} = \frac{12}{45}$ $P(WR) = \frac{2}{5} \times \frac{2}{3} = \frac{4}{15} = \frac{12}{45}$ $P(2 \text{ different}) = \frac{24}{45} = \frac{8}{15}$	<p>1</p> <p>1</p>

<p>(iii) $R=6$ $W=4$</p> <p>(ii) $P(2 \text{ same}) = 1 - P(\text{diff}) = 1 - \frac{24}{45} = \frac{21}{45} = \frac{7}{15}$</p>	<p>①</p> <p>①</p>
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<p>(b) $f(x) = 2x e^{0.5x}$</p> <p>(i) $f'(x) = 2 \cdot e^{0.5x} + 2x \times \frac{1}{2} e^{0.5x}$ $= 2e^{0.5x} + x e^{0.5x}$</p> <p>For stationary points $f'(x) = 0$ $e^{0.5x} (2+x) = 0$ $2+x = 0$ $x = -2$</p> <p>TEST NATURE at $x = -2$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">-2.1</td> <td style="text-align: center;">-2</td> <td style="text-align: center;">-1.9</td> </tr> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">-2^-</td> <td style="text-align: center;">-2</td> <td style="text-align: center;">-2^+</td> </tr> <tr> <td style="text-align: center;">$f'(x)$</td> <td style="text-align: center;">-0.035</td> <td style="text-align: center;">0</td> <td style="text-align: center;">$+0.039$</td> </tr> </table> <p style="text-align: center;">\ - /</p> <p>\therefore a rel min T.pt at $x = -2$</p> <p>at $x = -2$ $f(-2) = 2(-2) e^{0.5(-2)}$ $= -4e^{-1}$</p> <p>\therefore min value = $-4e^{-1}$</p>		-2.1	-2	-1.9	x	-2^-	-2	-2^+	$f'(x)$	-0.035	0	$+0.039$	<p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>1</p>
	-2.1	-2	-1.9										
x	-2^-	-2	-2^+										
$f'(x)$	-0.035	0	$+0.039$										

<p>(ii) Increasing function $f'(x) > 0$ $e^{0.5x} (2+x) > 0$ $e^{0.5x} > 0 \quad \forall x$ $\therefore 2+x > 0$ $x > -2$</p>	<p>1</p> <p>1</p>
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Solutions	Marks/Comments
<p>7 a) $3 \cdot 5 = 5 - 4 + C$ $2 \cdot 5 = C$</p>	<p>1 1</p>
<p>b) i) $4x^2 + 9y^2 = 36$ if $y = 0$ $4x^2 = 36$ $x^2 = 9$ $x = \pm 3$ as req'd</p>	<p>1 or straight substitution of points</p>
<p>ii) $V = \pi \int_{-3}^3 y^2 dx$ $= \pi \int_{-3}^3 4 - \frac{4}{9}x^2 dx$ $= \pi \left[4x - \frac{4}{27}x^3 \right]_{-3}^3$ $= \pi [(12 - 4) - (-12 + 4)]$ $= 16\pi$ cubic units</p>	<p>1 for manipulation of y^2 and sub into correct formula 1 for integral. 1 for answer.</p>
<p>c) i) $A_1 = P + (P \cdot 0.12) = P + 0.12P$ $= P(1 + 0.12)$ $= P(1.12)$ ii) $A_2 = (A_1 + P) \cdot 1.12$ $A_3 = (A_2 + P) \cdot 1.12$ $A_3 = P(1.12)^3 + P(1.12)^2 + P(1.12)$ $A_3 = P(1.12)(1.12^2 + 1.12 + 1)$ iii) $A_{20} = P(1.12)(1.12^{19} + 1.12^{18} + \dots + 1.12 + 1)$ $1000000 = P(1.12) \left(\frac{1(1.12^{20} - 1)}{1.12 - 1} \right)$ $P = \frac{1000000}{(1.12) \left(\frac{1.12^{20} - 1}{0.12} \right)}$ $= \\$12391.77$</p>	<p>① or equiv. eg: $A_1 = P \left(1 + \frac{12}{100} \right)^1$ ① ① ① ① GP $a=1$ $r=1.12$ $n=20$ ①</p>

Q8(a) (i) $\Delta = b^2 - 4ac$
 $= (k+3)^2 - 4 \times k \times (-1)$
 $= k^2 + 6k + 9 + 4k$
 $\Delta = k^2 + 10k + 9$

For real root $\Delta \geq 0$
 $\Rightarrow k^2 + 10k + 9 \geq 0$
 $(k+9)(k+1) \geq 0$



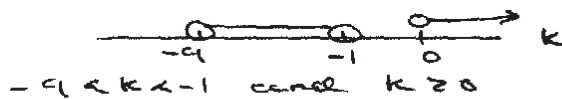
$\therefore k \leq -9$ or $k \geq -1$

1

(1)

1 (2)

(ii) PDQF if $\Delta < 0$ and $a > 0$
 $\Rightarrow (k+9)(k+1) < 0$ and $k > 0$



\therefore no solution possible for k
 \therefore never a PDQF

1

1

(b) (i) $\frac{d}{dx} [x \ln x - x] = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$
 $= \ln x + 1 - 1$
 $= \ln x$ QED.

1

1

(ii) $\int_1^{e^2} \ln x \, dx = [x \ln x - x]_1^{e^2}$
 $= [e^2 \ln e^2 - e^2] - [1 \ln 1 - 1]$
 $= e^2 \times 2 - e^2 - (0 - 1)$
 $= 2e^2 - e^2 - (0 - 1)$
 $= e^2 + 1$

1 for subst.

1 for evaluation

(c) $y = \ln \sqrt{x}$
 $\frac{dy}{dx} = \frac{1}{2x}$

Gravel. of Tang at $x=e$
 $m_T = \frac{1}{2e}$
 at $x=e$ $y = \ln \sqrt{e} = \frac{1}{2}$

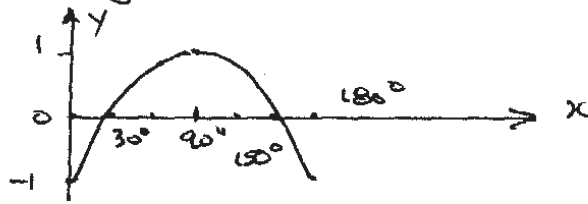
1 for differe
 (for $(e, \frac{1}{2})$ and
 $m_T = \frac{1}{2e}$

11.
 Equ. of Tangent
 $y - \frac{1}{2} = \frac{1}{2e} (x - e)$
 $\Rightarrow y = \frac{1}{2e} x$

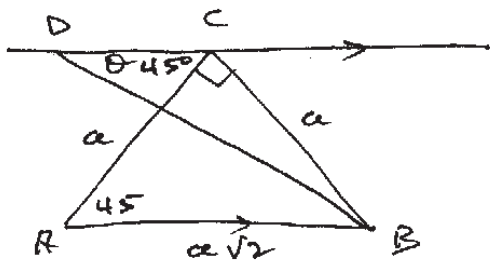
1 for "equ"

ying

Solutions	Marks/Comments								
Q9(a) (i) Vertex = (0, 0)	1								
(ii) $y^2 = 4Ax$, $A = 2$ \therefore focus $S = (2, 0)$	1								
(iii) Directrix $x = -2$	1								
(b) (i) $\log_x \sqrt[3]{a} = \frac{1}{3} \log_x a = \frac{3.6}{3} = 1.2$	1								
(ii) $\log_x ab = \log_x a + \log_x b = 3.6 + 2 = 5.6$	1								
(iii) $\log_x \frac{a}{b} = \log_x a - \log_x b = 3.6 - 2 = 1.6$	1								
(c) (i) $h = kr$ $r + kr = 1 \Rightarrow r = \frac{1}{1+k}$ $V = \frac{\pi}{3} r^2 h = \frac{\pi}{3} \cdot \frac{1}{(1+k)^2} \times \frac{k}{1+k}$ $= \frac{\pi}{3} \cdot \frac{k}{(1+k)^3}$ q. ed.	1 1								
(ii) $\frac{dV}{dk} = \frac{\pi}{3} \left[\frac{1 \cdot (1+k)^3 - k \cdot 3(1+k)^2}{(1+k)^6} \right]$ $= \frac{\pi}{3} \times (1+k) \frac{[1+k-3k]}{(1+k)^6}$ $= \frac{\pi}{3} \times \frac{1-2k}{(1+k)^4}$ When $\frac{dV}{dk} = 0$ when $k = \frac{1}{2}$ Test nature at $k = \frac{1}{2}$ <table border="1" data-bbox="207 1814 590 1971"> <tr> <td>k</td> <td>$\frac{1}{2}^-$</td> <td>$\frac{1}{2}$</td> <td>$\frac{1}{2}^+$</td> </tr> <tr> <td>$\frac{dV}{dk}$</td> <td>+</td> <td>0</td> <td>-</td> </tr> </table> \therefore Rel max T. pt at $k = \frac{1}{2}$	k	$\frac{1}{2}^-$	$\frac{1}{2}$	$\frac{1}{2}^+$	$\frac{dV}{dk}$	+	0	-	1 ? 1 for justifying at $k = \frac{1}{2}$ V is max.
k	$\frac{1}{2}^-$	$\frac{1}{2}$	$\frac{1}{2}^+$						
$\frac{dV}{dk}$	+	0	-						
(iii) $V_{max} = \frac{\pi}{3} \cdot \frac{\frac{1}{2}}{(1+\frac{1}{2})^3} = \frac{4\pi}{81}$	1								

Solutions	Marks/Comments
<p>Q10(a) $y = 2 \sin x - 1$</p> 	<p>1 for $(0, -1) + (90°, 1)$ 1 for concavity 1 for $30°$ and $150°$</p>
<p>(b) (i) $F_1 = t + 1.2^t$ when $t=1$ $F_1 = 1 + 1.2^1 = \underline{2.2}$</p>	<p>1</p>
<p>(ii) $L_2 = F_1 + F_2$ $= 1 + 1.2 + 2 + 1.2^2$ $= 3 + 1.2 + 1.2^2$ $= \underline{5.64}$ <i>quad</i></p>	<p>1</p> <div style="text-align: right; margin-right: 20px;"> $\begin{array}{r} 3 \\ 1.2 \\ 1.44 \\ \hline 5.64 \end{array}$ </div>
<p>(iii) $L_n = F_1 + F_2 + F_3 + \dots + F_n$ $= [1 + 1.2] + [2 + 1.2^2] + [3 + 1.2^3] + \dots + [n + 1.2^n]$ $= [1 + 2 + 3 + \dots + n] + [1.2 + 1.2^2 + 1.2^3 + \dots + 1.2^n]$ $= \frac{n}{2}(1+n) + 1.2 \frac{(1.2^n - 1)}{1.2 - 1}$</p>	<p>1 1 G. Series</p>
<p>(iv) from 6am to 4pm $n=11$ so $L_{11} = \frac{11}{2}(1+11) + 1.2 \frac{(1.2^{11} - 1)}{0.2}$ $= 11 \times 6 + 6(1.2^{11} - 1)$ $= 104.6$ litres (1dp)</p>	<p>1 1</p>
<p>(c) Let AC = 1 unit 1. $\therefore DB = \sqrt{2}$ 2. as $\angle CAB = 45^\circ (= \angle CBA)$ so $\angle DCA = 45^\circ$ (Alt. Angs equal as $DC \parallel AB$) so $\angle DCB = 135^\circ$ 3. $\frac{\sin \theta}{1} = \frac{\sin 135^\circ}{\sqrt{2}} = \frac{1}{2}$ $\angle \theta = 30^\circ \equiv \angle BDC$ so $\angle DBC = 45^\circ - 30^\circ = \underline{15^\circ}$</p>	<p>1 for get to $\sqrt{2}$ 1 getting to 135° 1</p>

Q 10(c)



Let $AC = a$

1. $\therefore CB = a$
 $\therefore AB = a\sqrt{2}$
 $\therefore DB = a\sqrt{2}$ (1)

2. $\angle CAB (= \angle CBA) = 45^\circ$
 $\therefore \angle DCA = 45^\circ$ (Alt \angle s equal as $DC \parallel AB$) (1)

3. $\frac{\sin \theta}{BC} = \frac{\sin 135^\circ}{DB}$
 $\frac{\sin \theta}{a} = \frac{\sin 135^\circ}{a\sqrt{2}} = \frac{\frac{1}{\sqrt{2}}}{a\sqrt{2}} = \frac{1}{2a}$

$\therefore \sin \theta = \frac{1}{2}$
 $\angle \theta = 30^\circ$ (1)
 $\angle BDC = 30^\circ$
 $\angle DBC = 45^\circ - 30^\circ = 15^\circ$ (1)