

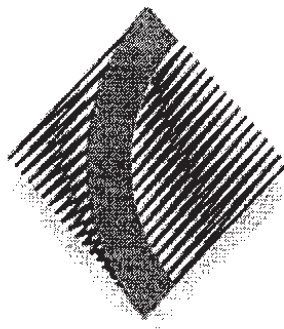
HB.
KW
AW
GF
NB
FH
LB

Name: _____

Class: 12MT2__ or 12MTX__

Teacher: _____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2003 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

*Time allowed - 3 HOURS
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- Each question is to be returned in a separate bundle.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- A Table of Standard Integrals is provided.

****Each page must show your name and your class. ****

Question 1

Marks

- (a) If $\log_8 x = \frac{7}{3}$ find the value of x 1
- (b) Evaluate $\pi(29 \cdot 3)^2$ correct to 4 significant figures 2
- (c) Simplify $\frac{\sqrt{3}+1}{2\sqrt{3}-1}$ by rationalising the denominator 2
- (d) Solve $x^3 = 4x^2$ 2
- (e) Find the x and y intercepts of the line $4x - 3y + 5 = 0$ 2
- (f) \$1 was invested for 10 years at 10% pa. How much was this investment worth at the end of the investment period if the interest is
- (i) compounded annually 1
- (ii) compounded monthly 2

Question 2

(Please start a new page)

- (a) Differentiate with respect to x
- (i) $\frac{x}{x+1}$ 2
- (ii) $e^{2x} \sin x$ (factorise your answer) 2
- (b) (i) Find $\int \sin 2x dx$ 1
- (ii) Evaluate $\int_0^3 e^{2x} dx$ 2
- (c) The coordinates of three points are $A(-1,3)$, $B(2,8)$ and $C(9,11)$
- (i) Show that the distance BC is $\sqrt{58}$ units 1
- (ii) Find the equation of the line BC 2
- (iii) Find the perpendicular distance from the point A to the line BC .
(Leave your answer in simplified surd form) 2

Question 3

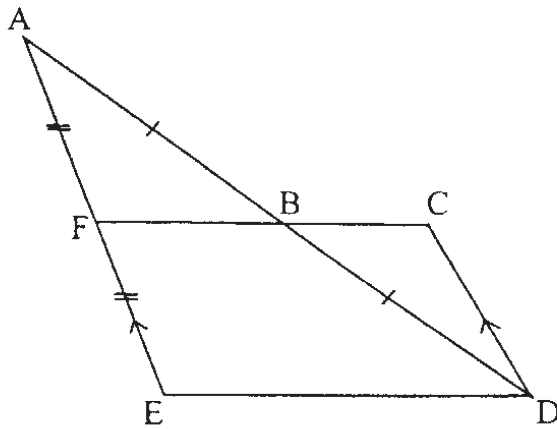
(Please start a new page)

Marks

- (a) Sketch the graph of $y = \frac{1}{x+1}$ and state the domain and range of the function

3

(b)



NOT TO SCALE

In the diagram, the line FC bisects AE at F and AD at B . The line AE is parallel to CD . Copy the diagram onto your working page.

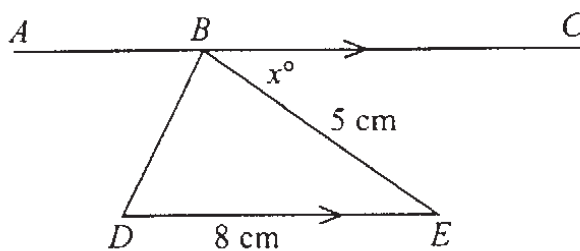
- (i) Prove that $\triangle ABF$ is similar to $\triangle ADE$

2

- (ii) Hence explain why $ED = 2BF$

1

(c)



NOT TO SCALE

In the diagram, AC is parallel to DE , $BE = 5 \text{ cm}$, $DE = 8 \text{ cm}$ and $\angle CBE = x^\circ$. The area of $\triangle DBE$ is 10 cm^2 . Find the value of x , giving reasons.

3

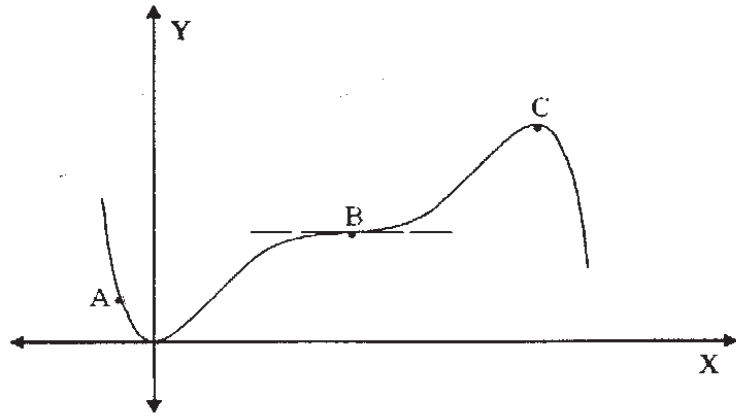
(Question 3 continued next page)

Question 3

Marks

(d)

	$f'(x)$	$f''(x)$
A		
B		
C		



Copy the table onto your working page. Complete the table by indicating whether the derivatives are positive (+), negative (-) or equal to zero (0) at the points A, B and C. B is a horizontal point of inflexion.

3

Question 4

(Please start a new page)

Consider the function defined by $y = x^3 - 12x^2 + 36x + 10$

- (a) Find $\frac{dy}{dx}$ 1
- (b) Find the coordinates of the two stationary points 2
- (c) Determine the nature of the stationary points 2
- (d) Find the coordinates of the point of inflexion 2
- (e) Sketch the curve $y = f(x)$ for $-1 \leq x \leq 9$ showing stationary points and the point of inflexion 2
- (f) Apply Simpson's Rule, with five function values to find an approximation for 3

$$\int_0^8 (x^3 - 12x^2 + 36x + 10) dx$$

Question 5

(Please start a new page)

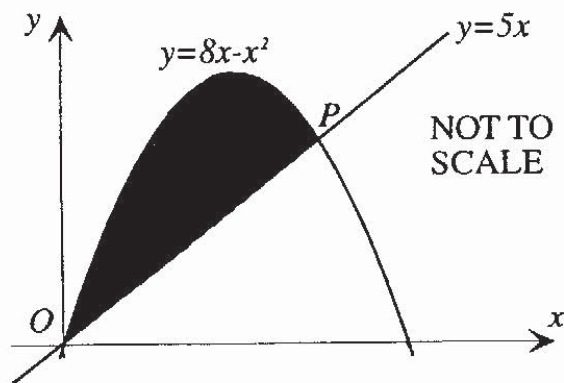
Marks

- (a) Show that $\frac{\tan \theta}{\sec \theta - 1} - \frac{\tan \theta}{\sec \theta + 1} = 2 \cot \theta$ 2
- (b) (i) If $y = x \log_e x$ show that $\frac{dy}{dx} = 1 + \log_e x$ 1
- (ii) Hence find $\int \log_e x dx$ 2
- (c) A , B and C are three towns. B is 10 kilometres from A in the direction $N40^\circ E$ and C is 16 kilometres from A in the direction $N70^\circ W$
- (i) Draw a neat sketch showing the above information 1
- (ii) Calculate the distance from B to C , correct to 3 significant figures 3
- (d) A bag contains three red and four black discs. One disc is drawn at random from the bag and replaced only if it is black. Another disc is then drawn. Find the probability of selecting
- (i) a red then a black disc 1
- (ii) at least one red disc 2

Question 6

(Please start a new page)

(a)



The graphs of $y = 5x$ and $y = 8x - x^2$ intersect at the origin and point P

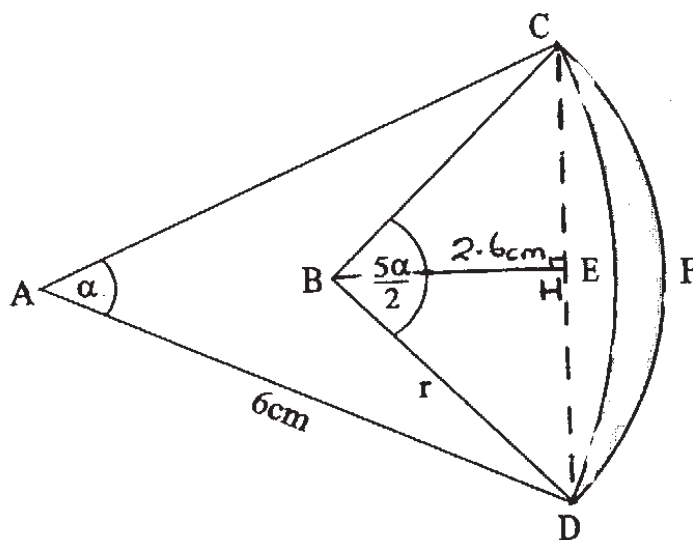
- (i) Show that the coordinates of P are $(3, 15)$ 1
- (ii) Find the area of the shaded region 2
- (iii) This region is rotated about the x - axis. Find the volume of the solid obtained 3
- (Question 6 continued next page)

- (b) Find the equation of the parabola which has its vertex at $(2,0)$ and its Directrix given by $x = 5$ 1
- (c) (i) In the following series, for what values of x will a sum to infinity exist? 2
- $$(1-x)^{\frac{1}{2}} + (1-x) + (1-x)^{\frac{3}{2}} + \dots$$
- (ii) Find the value of x if the series has a limiting sum of $2\sqrt{3} + 3$ 3

Question 7

(Please start a new page)

- (a) In a certain strain of plant, the probability that a seed will produce a pink flower is 0.3
- (i) If two seeds were planted, what is the probability that at least one flower is pink 2
- (ii) Find the least number of seeds that must be planted to be sure that the probability of obtaining at least one pink flower exceeds 0.99 2
- (b) The point A is the centre of the sector $ACED$. This sector has a radius of 6 cm . The point B is the centre of the sector $BCFD$, with radius $r\text{ cm}$.
 $\angle CAD = \alpha$, $\angle CBD = \frac{5\alpha}{2}$ and the altitude $BH = 2.6\text{ cm}$.
 The area of the sector $BCFD$ is 90% of the area of the sector $ACED$



- (i) Show that the radius r of the sector centred at B is 3.6 cm 3
- (ii) If the angles are measured in radians, find α correct to two decimal places 2
- (c) Solve $3\sec^2 x = 4$ for $0 \leq x \leq 2\pi$ 3

Question 8

(Please start a new page)

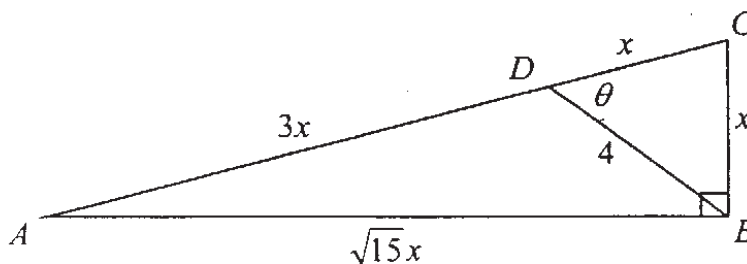
Marks

- (a) Graph the solution of $3x \leq 16 \leq -7x$ on a number line 2
- (b) Consider the equation $x^2 + (k+2)x + 4 = 0$. For what values of k does the equation have
- (i) equal roots 2
- (ii) distinct real roots 2
- (c) (i) Given the function $y = 3 \sin x$. State the highest and lowest possible values for y 2
- (ii) Draw the graphs of $y = 3 \sin x$ and $y = x - 1$ on the same set of axes for $-2\pi \leq x \leq 2\pi$ 3
- (iii) How many solutions does the equation $3 \sin x = x - 1$ have? 1

Question 9

(Please start a new page)

(a)



In the diagram, ABC is a right angled triangle where $AB = \sqrt{15}x$ cm and $BC = x$ cm.

The point D lies on AC and $CD = BC = x$ cm, $AD = 3x$ cm and $BD = 4$ cm. Let $\angle BDC = \theta$

- (i) Use the cosine rule to show that $\cos \theta = \frac{2}{x}$ 1
- (ii) Write an exact ratio for $\sin \angle ACB$ 1
- (iii) Use the sine rule to show that $\sin \theta = \frac{\sqrt{15}x}{16}$ 2
- (iv) Hence show that $15x^4 - 256x^2 + 1024 = 0$ 2

(Question 9 continued next page)

- (b) Peter and John start work on the same day. Peter starts on a salary of \$32 000. His salary increases by \$1 300 each year. John starts on a salary of \$30 000 and increases by \$1550 each year. Calculate
- (i) Peter's salary in his eleventh year of employment 2
- (ii) The total amount John has earned in his eleven years of employment 2
- (iii) How many years would Peter and John have to work before their total earnings were equal? 2

Question 10

(Please start a new page)

- (a) A length of wire 120 *cm* long is to be cut into two pieces. One of the pieces is bent into the shape of a circle and the other into an equilateral triangle.
- (i) If x is the circumference of the circle, write an expression for the perimeter P of the equilateral triangle in terms of x 1
- (ii) Show that the sum of the areas A formed by the two pieces of wire is given by 2

$$A = \frac{x^2}{4\pi} + \frac{\sqrt{3}}{36}(120 - x)^2$$

- (iii) Find the value of x such that the sum of the areas is a minimum 3
- (b) Gayle invests in a superannuation fund which pays 5% pa interest compounded annually. She pays \$12 000 into the fund on 1st of July each year.
- (i) What is the value of Gayle's investment on 30th June, one year after she makes her first payment? 1
- (ii) What is the value of the investment on the 30th June, ten years after she made her first payment? 2
- (iii) After making her tenth payment, Gayle considers increasing her annual payment to M dollars each year. Show that if Gayle does do this, the value of her investment twenty years after her first payment of \$12 000 was made, would be approximately equal to $13 \cdot 2068(12000 \times 1 \cdot 05^{10} + M)$ 3

END OF TEST

3c) $LBSO = \pi$
 (alt. L's, 11 lines)
 $Area = \frac{1}{2} \times 5 \times 8 \times \sin \pi$
 $= 10$

$\sin \pi = \frac{1}{2}$
 $\pi = 30^\circ$

Acc. * d)

	$f'(x)$	$f''(x)$
A	-	+
B	0	0
C	0	-

QUESTION 4

a) $\frac{dy}{dx} = 3x^2 - 24x + 36$

b) $3x^2 - 24x + 36 = 0$
 $x = 6, 2$

$\therefore (6, 10), (2, 42)$

c) $\frac{d^2y}{dx^2} = 6x - 24$

when $x = 2, \frac{d^2y}{dx^2} = 12 > 0$

\therefore min. at $(2, 42)$

when $x = 6, \frac{d^2y}{dx^2} = 12 < 0$

\therefore maximum at $(6, 10)$

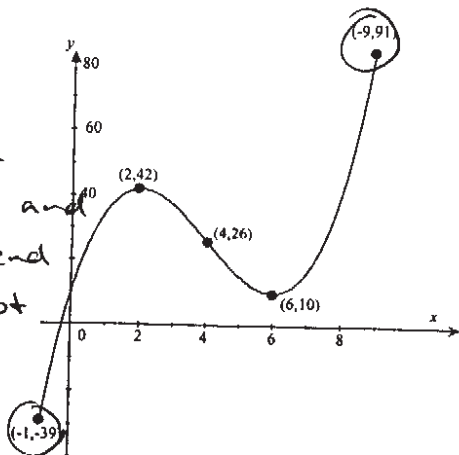
d) $6x - 24 = 0 \Rightarrow x = 4$

when $x = 3, \frac{d^2y}{dx^2} = -6$
 when $x = 5, \frac{d^2y}{dx^2} = 6$

change of sign

\therefore pt. of inflexion at $(4, 26)$

e)



Graph must stop at $x = -1$ and $x = 9$, but end ordinates not required

f) See after Qn. 5 soln.

QUESTION 5

a) LHS =
 $\frac{\tan \theta (\sec \theta + 1) - \tan \theta (\sec \theta - 1)}{\sec^2 \theta - 1}$
 $= \frac{\tan \theta \sec \theta + \tan \theta - \tan \theta \sec \theta + \tan \theta}{\sec^2 \theta - 1}$
 $= \frac{2 \tan \theta}{\sec^2 \theta - 1}$
 $= \frac{2}{\tan \theta}$
 $= 2 \cot \theta$

(OR ANY OTHER REASONABLE METHOD)

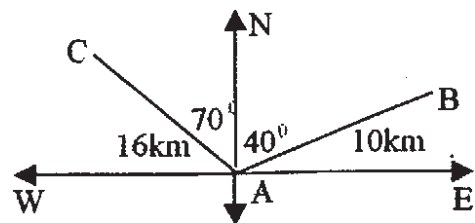
b) (i) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \log_e x$
 $= 1 + \log_e x$

(ii) $\int 1 + \log_e x \, dx =$
 $x \log_e x + c$

$\therefore x + \int \log_e x \, dx =$
 $x \log_e x + c$

$\therefore \int \log_e x \, dx =$
 $x \log_e x - x + c$

c) (i)



(ii) $a^2 = 16^2 + 10^2 - 2 \times 16 \times 10 \times \cos 110^\circ$
 $a^2 = 465.446 \dots$
 $a = 21.6$ (3 sf)

(-1 for incorrect rounding)

for graph for labelling (permanently)

$$5(d) (i) \frac{\sqrt{3}}{2} \times \frac{4}{6} = \frac{2}{3}$$

$$(ii) \frac{\sqrt{3}}{2} + \frac{4}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{4}{6}$$

$$= \frac{33}{49}$$

OR

$$1 - \frac{4}{2} \times \frac{4}{2} = \frac{33}{49}$$

$$4 (f) h = \frac{8 \pm 0}{4} = 2$$

$$\int \equiv \frac{\sqrt{3}}{3} \times \left[10 + 42 + 2 \times 26 + \right]$$

$$4 \times (42 + 10)$$

$$\equiv 208$$

QUESTION 6

$$a) (i) 5x = 8x - x^2$$

$$x^2 - 3x = 0$$

$$x = 0, 3$$

when $x = 3, y = 15$

$$\therefore P_3 (3, 15)$$

$$(ii) A = \int_0^3 (8x - x^2 - 5x) dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3$$

$$= \frac{27}{2} - 9$$

$$= 4\frac{1}{2} \text{ units}^2$$

$$(iii) V = \pi \int_0^3 (8x - x^2)^2 - (5x)^2 dx$$

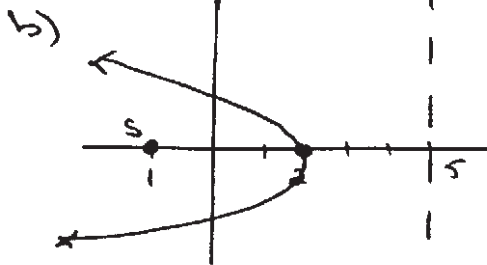
$$= \pi \int_0^3 (64x^2 - 16x^3 + x^4 - 25x^2) dx$$

$$= \pi \left[\frac{x^5}{5} - 4x^4 + 13x^3 \right]_0^3$$

$$= \pi \times \left(\frac{243}{5} + 324 + 351 \right)$$

$$= 75.6\pi \text{ or } \frac{378\pi}{5}$$

(or 237.05) $\frac{5}{5}$ units³



OR
(1)

In form $(y-k)^2 = -4a(x-h)$

$$y^2 = -4 \times 3(x-2)$$

$$y^2 = -12(x-2)$$

c) (i) $r = \sqrt{1-x}$

Limiting sum exists when $|\sqrt{1-x}| < 1$

$\sqrt{1-x} < 1$
 $1-x < 1$
 $x > 0$

Not all required - investigation good enough

But the domain of $\sqrt{1-x}$ is $x \leq 1$

\therefore Limiting sum exists for $0 < x < 1$ (acc. $0 < x \leq 1$)

$$(ii) S_{\infty} = \frac{a}{1-r}$$

$$2\sqrt{3} + 3 = \frac{a}{1-\sqrt{1-x}}$$

$$(2\sqrt{3} + 3)(1 - \sqrt{1-x}) = \sqrt{1-x}$$

$$2\sqrt{3} - 2\sqrt{3}\sqrt{1-x} + 3 - 3\sqrt{1-x} = \sqrt{1-x}$$

$$2\sqrt{3} + 3 = \sqrt{1-x} + 2\sqrt{3}\sqrt{1-x} + 3\sqrt{1-x}$$

$$2\sqrt{3} + 3 = (2\sqrt{3} + 4)\sqrt{1-x}$$

$$\therefore \sqrt{1-x} = \frac{2\sqrt{3} + 3}{2\sqrt{3} + 4}$$

$$\therefore \sqrt{1-x} = \frac{\sqrt{3}}{2}$$

$$\therefore 1-x = \frac{3}{4}$$

$$\therefore x = \frac{1}{4}$$

QUESTION 7

a) (i) $P(\text{Pink, not Pink}) + P(\text{not Pink, Pink}) + P(\text{Pink, Pink})$
 $= (0.3)(0.7) + (0.7)(0.3) + (0.3)^2$
 $= 0.51$

or

$1 - P(\text{not Pink, not Pink})$
 $= 1 - (0.7)^2$
 $= 0.51$

(ii) $1 - (0.7)^n \geq 0.99$

$\therefore (0.7)^n \leq 0.01$

$n \log(0.7) \leq \log(0.01)$
 $n > \frac{\log(0.01)}{\log(0.7)}$

$n > 12.91$

\therefore 13 seeds

(on trunk + rone)

b) (i) Area of Aced

$= \frac{1}{2} \times 6^2 \times \alpha = 18\alpha$

Area of BCFD = $\frac{1}{2} \times r^2 \times \frac{3\alpha}{2}$
 $= \frac{5r^2\alpha}{4}$

Area of BCFD = 0.9 x Area of Aced

$\frac{5r^2\alpha}{4} = 0.9 \times 18\alpha$

$\therefore r^2 = 12.96$

$r = 3.6 \text{ cm}$

(ii) $\triangle BCD$ is isosceles

$\therefore \angle DBH = \frac{1}{2} \times \frac{5\alpha}{2} = \frac{5\alpha}{4}$

$\therefore \cos \frac{5\alpha}{4} = \frac{2.6}{3.6}$

$\therefore \alpha = 0.61$

c) $\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}}$ ← incorrect for no +

$\therefore \cos x = \pm \frac{\sqrt{3}}{2}$ cfm

$\frac{1}{2}$ for degrees, not radians. cfm

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

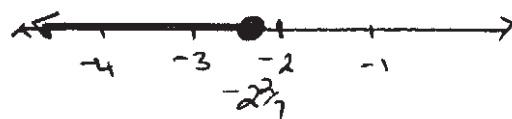
No, changed my mind.

QUESTION 8

a) $3x \leq 16$ and $-7x \geq 16$ (simult.)

$\therefore x \leq 5\frac{1}{3}$ and $x \leq -2\frac{2}{7}$

$\therefore x \leq -2\frac{2}{7}$



b) (i) $\Delta = 0$ -1 for wrong Δ .

$(k+2)^2 - 4 \cdot 1 \cdot 4 = 0$ -1 for wrong sign.

$\therefore (k+6)(k-2) = 0$

$\therefore k = -6, 2$

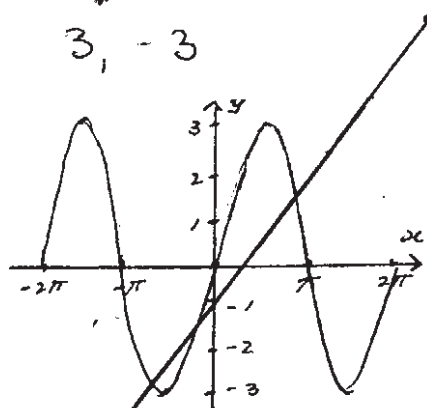
(ii) $\Delta > 0$

$(k+6)(k-2) > 0$

$\therefore k < -6$ or $k > 2$

c) (i) 3, -3

(ii)



(iii)

3

2
 $\frac{3}{2} \sin x$
 $\frac{3}{2} = x - 1$
 correct domain.

QUESTION 9

a) (i) $\cos \theta = \frac{x^2 + 4^2 - x^2}{2 \times x \times 4}$
 $= \frac{2}{x}$

(ii) $\sin \angle ACB = \frac{\sqrt{15} x}{4x}$
 $= \frac{\sqrt{15}}{4}$

(iii) $\frac{\sin \theta}{x} = \frac{\sin \angle ACB}{4}$
 $\sin \theta = \frac{x \cdot \frac{\sqrt{15}}{4}}{4}$

$= \frac{\sqrt{15} x}{16}$

(iv) $\sin^2 \theta + \cos^2 \theta = 1$
 $\left(\frac{\sqrt{15} x}{16}\right)^2 + \left(\frac{2}{x}\right)^2 = 1$

$\frac{15x^2}{256} + \frac{4}{x^2} = 1$

$15x^4 + 1024 = 256x^2$
 $\therefore 15x^4 - 256x^2 + 1024 = 0$

b) (i) $T_{11} = 32000 + 10 \times 300$
 $= \$45000$

(ii) $S_{11} = \frac{11}{2} \times (2 \times 30000 + 10 \times 1550)$
 $= \$415250$

(iii) Peter's total =
 $\frac{n}{2} (2 \times 32000 + 1300(n-1))$

John's total =
 $\frac{n}{2} (2 \times 30000 + 1550(n-1))$

$\therefore \frac{n}{2} (64000 + 1300(n-1)) = \frac{n}{2} (60000 + 1550(n-1))$

$250n = 4250$

$\therefore n = 17$

$\therefore 17$ years.

QUESTION 10

a) (i) $P = 120 - x$ cm

(ii) $2\pi r = x$

$\therefore r = \frac{x}{2\pi}$

\therefore Area of circle = πr^2

$= \pi \times \frac{x^2}{4\pi^2}$

$= \frac{x^2}{4\pi}$

Area of $\Delta = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} \left(\frac{120-x}{3}\right)^2 \times \sin 60^\circ$
 $= \frac{\sqrt{3}}{36} (120-x)^2$

$\therefore A = \frac{x^2}{4\pi} + \frac{\sqrt{3}}{36} (120-x)^2$

(vi) $\frac{dA}{dx} = \frac{2x}{4\pi} + 2 \cdot \frac{\sqrt{3}}{36} (120-x)$

$= \frac{x}{2\pi} + \frac{\sqrt{3}}{18} (120-x)$

when $\frac{dA}{dx} = 0$

$x \left(\frac{1}{2\pi} + \frac{\sqrt{3}}{18} \right) - \frac{20\sqrt{3}}{3} = 0$

$\therefore x = \frac{20\sqrt{3}}{3}$

$\frac{1}{2\pi} + \frac{\sqrt{3}}{18}$

$= \frac{120\pi\sqrt{3}}{9 + \pi\sqrt{3}}$

$\frac{d^2A}{dx^2} = \frac{1}{2\pi} + \frac{\sqrt{3}}{18} > 0 \therefore$

minimum Area for this value of x .

$$10 \text{ b) (i) } 12000 \times 1.05 \\ = \$12600$$

$$(ii) \text{ Value after 2 years} = \\ 12000 (1.05)^2 + 12000 (1.05)$$

$$\text{After 3 years} = \\ 12000 (1.05)^3 + \dots + 12000 (1.05)$$

$$\therefore \text{After 10 years} =$$

$$12000 (1.05)^{10} + 12000 (1.05)^9 + \\ \dots + 12000 (1.05)$$

$$= 12000 (1.05 + \dots + (1.05)^{10}) \\ = 12000 \times 1.05 \times \left(\frac{1.05^{10} - 1}{1.05 - 1} \right)$$

$$= \$158,481.45$$

$$(iii) A_{20} = 12000 (1.05)^{20} + \\ \dots + 12000 (1.05)^n +$$

$$M (1.05)^{10} + \dots + M (1.05)^9$$

$$= 12000 \times (1.05)^n (1 + 1.05 + \dots \\ + (1.05)^9)$$

$$+ M \times 1.05 (1 + 1.05 + \dots \\ + (1.05)^9)$$

$$= 1 \left(\frac{1.05^{10} - 1}{1.05 - 1} \right) (12000 + 1.05^n + 1.05^9)$$

$$= 12057789 \times 1.05 (12000 + 1.05^{10} + M)$$

$$= 13.2068 (12000 \times 1.05^{10} + M)$$

2003 AP 4 MARKS COMMENTS.

QUESTION 1/ f(i) \$2.59 (not \$1.10)

QUESTION 2/ b) Accept $201 \cdot 2143 \dots$ as well as $\frac{1}{2}(e^b - 1)$
c) Give 1 mark for substitution and $\frac{1}{2}e^b - \frac{1}{2}$
into perpendicular distance formula.

c) (ii) 1 mark for correct eqn. in any form.

Denominator must be rationalized to gain full marks. (-1 for $\frac{26}{\sqrt{58}}$)
b) (i) Accept $-\frac{1}{2} \cos 2x$ for 1 mark.

QUESTION 3/ b) (i) Must be SAS Similarity.

Too many students assumed $FC \parallel ED$ + used corresponding angles to prove similarity by equiangular. (No marks for this though)

(ii) Must state, or show, corresponding sides in proportion to gain mark.

d) Graph not labelled $y = f(x)$
 \therefore accept answers for a graph of $y = f'(x)$
ie

A	+	-
B	+	0
C	+	0

QUESTION 4/ e) End points must be shown + labelled $(-1, -39)$, $(9, 91)$ to gain second mark.

QUESTION 6/ a) (iii) Lots of students subtracted then squared. Lose 1 mark for each mistake made.

QUESTION 7/ c) Lose 1 mark for \pm not included earlier. CFM's from then on.

Lose $\frac{1}{2}$ mark for giving solutions in degrees, not radians.

QUESTION 8, b)(i) Give any CFM's please.

b)(ii) Correct answer is $k < -6$ or $k > 2$
(remove equal signs from solutions)

QUESTION 10, a)(ii) Only worth 2 marks.

No third mark for giving $A = \frac{x^2}{4\pi} + \frac{\sqrt{3}(120-x^2)}{36}$
as it was given to be shown.