AT
KW
LB
PV
FH
AW

Name: __________
Class: 12MT2_ or 12MTX_
Teacher: __________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2005

YEAR 12

AP4 EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **
- Each question is to be returned in a separate bundle.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

*****Each page must show your name and your class. *****

QUESTION ONE. **(12 MARKS.)** **MARKS**

(a) Evaluate $\sqrt{\dfrac{e^3 - 2.3}{3\pi}}$ correct to 3 significant figures. 2

(b) Completely factorise $x^3 + 2x^2 - 8x$. 2

(c) Differentiate with respect to x:

(i) $3x^2(4x-1)$ 2

(ii) $\dfrac{\log_e x}{x^3}$ 3

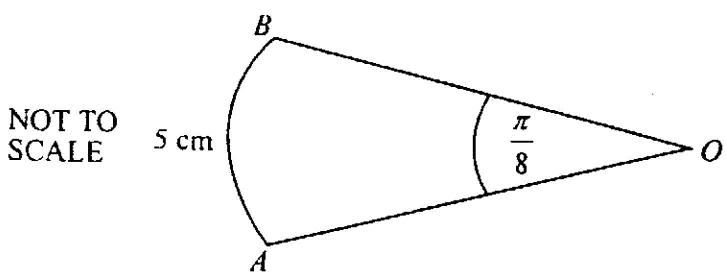
(d) Solve the pair of simultaneous equations: 2

$$x - 2y = 9$$
$$2x + y = 8$$

(e) Find the primitive of $\dfrac{1}{x-1}$. 1

QUESTION TWO. **START A NEW PAGE.** **(12 MARKS)**

(a) In the diagram, AB is an arc of a circle with centre O. The length of the arc AB is 5cm and angle AOB is $\frac{\pi}{8}$ radians. Find the length of AO, to the nearest cm. 2



(b) Solve $2^{2x} - 9 \ . \ 2^x + 8 = 0$ 2

(c) D(0,-2), E(4,0) and F(2,4) are three points on the number plane.

(i) Draw a diagram to represent this information. 1

(ii) Calculate the length of the interval DF. 1

QUESTION TWO CONTINUED.

MARKS

(iii) Calculate the gradient of DF. 1

(iv) Write the equation of the line DF in general form. 1

(v) Calculate the perpendicular distance from E to the line DF, giving your answer with a rational denominator. 2

(vi) Calculate the area of the Δ DEF. 2

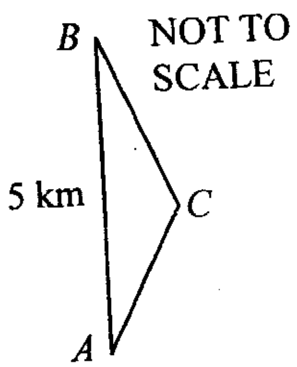
QUESTION THREE. **START A NEW PAGE.** **(12 MARKS)**

(a) Differentiate $\dfrac{\tan 3x}{3x+2}$. 2

(b) Find $\displaystyle\int \frac{3}{\sqrt{e^x}}\,dx$. 2

(c) Evaluate $\displaystyle\int_2^4 \frac{3x}{x^2-1}\,dx$ leaving your answer exact and in its simplest form. 3

(d)



The buoy B is directly north of the buoy A. Because of the direction of the wind, to sail from A to B a boat must first sail from A to C on a bearing 025° and then turn and sail from C to B on a bearing of 335°.

If A and B are 5km apart,

(i) find the size of $\angle$ABC. 1

(ii) Hence, find the distance from A to C, 2

(iii) Calculate the total distance sailed in going from A to B via C, to the nearest km. 2

QUESTION FOUR. **START A NEW PAGE.** **(12 MARKS)** **MARKS**

(a) Solve $4\cos x = 1$ for $0 \le x \le 2\pi$. Express your answer in radians, correct to 2 decimal places. 2

(b) The quadratic equation $2x^2 + 9x - 4 = 0$ has roots of α and β. 2
Hence evaluate $\alpha^2 + \beta^2$

(c) Shopping trolleys are 80cm long. When two trolleys are pushed together the resulting pair is 100cm long. When three are pushed together the resulting triple is 120 cm long.

(i) How long will a set of 10 trolleys pushed together be? 2

(ii) For safety reasons trolley collectors are not allowed to push a set of trolleys more than 4.5 metres long. 2
What is the greatest number of trolleys that can be pushed?

(d) For the parabola $4x = 8y - y^2$,

(i) Find the coordinates of the vertex. 2

(ii) Find the coordinates of the focus. 1

(ii) Sketch the curve clearly labeling the vertex and focus. 1

QUESTION FIVE. **START A NEW PAGE.** **(12 MARKS)**

(a) Consider the function defined by $f(x) = 2x^3 - 3x^2 - 36x + 26$

(i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. 3

(ii) Find the coordinates of any point of inflection. 2

(iii) Sketch the graph of $f(x) = 2x^3 - 3x^2 - 36x + 26$ by showing the above information. 2

(iv) For what values of x is the curve concave down and decreasing ? 2

QUESTION FIVE CONTINUED. **MARKS**

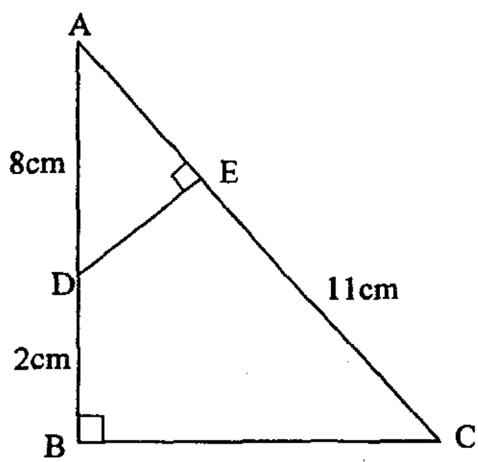
(b) Two-digit numbers are formed from the digits 2, 3, 4, 6 with no repetition of digits allowed.

(i) Write down the set S of all possible outcomes. 2

(ii) A two-digit number is then selected at random from the set S. Find the probability that the number is prime. 1

QUESTION SIX. **START A NEW PAGE.** **(12 MARKS)**

(a) ABC is a right- angled triangle in which, $\angle ABC = 90^0$. Points D and E lie on AB and AC respectively such that AC is perpendicular to DE. AD =8cm, EC=11cm, and DB=2cm.



Not to Scale

(i) Prove that ΔABC is similar to ΔAED. 3

(ii) Find the length of AE. 1

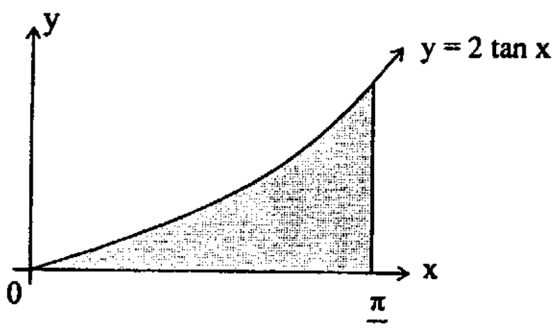
QUESTION SIX CONTINUED. **MARKS**

(b) Consider the geometric series : $1 + (5 - \sqrt{p}) + (5 - \sqrt{p})^2 + (5 - \sqrt{p})^3 + \ldots\ldots$

(i) Find the values of p for which the geometric series has a limiting sum? 2

(ii) Find the limiting sum of the series given that p is 20. write your answer with a rational denominator. 2

(c) The shaded area between the curve $y = 2\tan x$, the x-axis and the line $x = \dfrac{\pi}{3}$ is rotated about the x-axis.

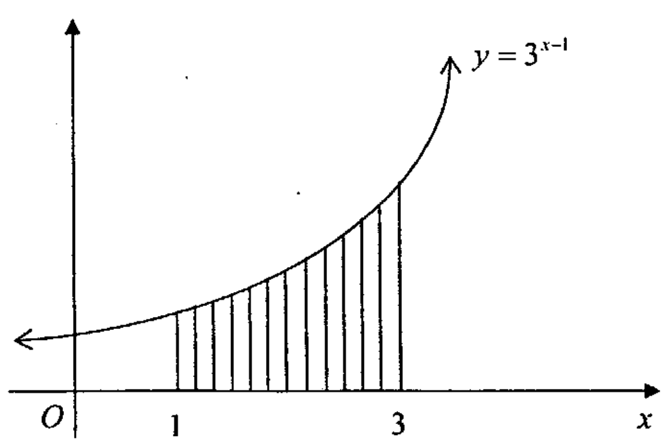


NOT TO SCALE

Calculate the volume of the solid formed. 4

QUESTION SEVEN. **START A NEW PAGE.** **(12 MARKS)**

(a) The diagram below shows the shading of a region bounded by the graph $y = 3^{x-1}$ and the lines $x = 1$ and $x = 3$.



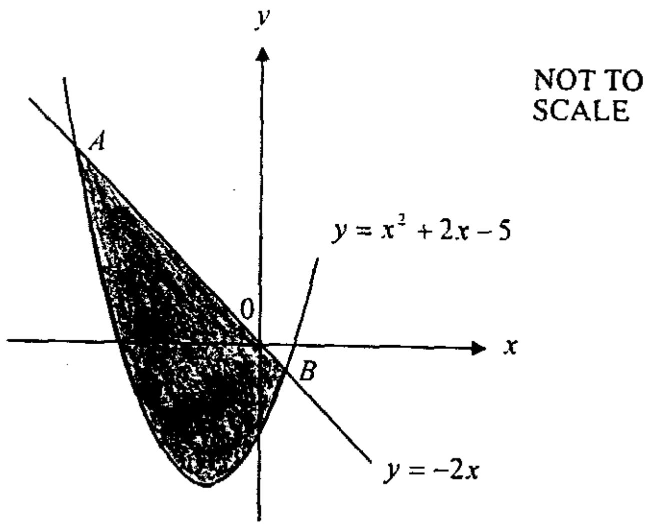
QUESTION SEVEN CONTINUED. **MARKS**

(i) Copy and complete the following table giving your answer correct to three decimal places: 1

x	1	1.5	2	2.5	3
$y = 3^{x-1}$	1	1.732			

(ii) Use Simpson's Rule with five function values to approximate the shaded area to three decimal places. 2

(b)



The diagram shows the graphs of $y = x^2 + 2x - 5$ and $y = -2x$. These two graphs intersect at the point A and B.

(i) Find the x values of the points of intersection A and B 2

(ii) Calculate the area of the shaded region. 3

(c) Write down the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. 2

(d) Evaluate $\sum_{n=2}^{4} -2^n$ 2

QUESTION EIGHT. **START A NEW PAGE.** **(12 MARKS)** **MARKS**

(a) The probability of winning a particular game of chance is 20%.

(i) What is the probability of winning all three games? 1

(ii) If she plays three games, what is the probability she will win at least one? 2

(iii) What is the least number of games she must play before the probability of her winning at least one game is 99% 3

(b) When Penny began her first job at Hornsby, she decided to open an account and deposit $50 per month into the account. The bank guaranteed an interest rate of 8%p.a. compounding every 6 months if she agreed not to make any withdrawals.

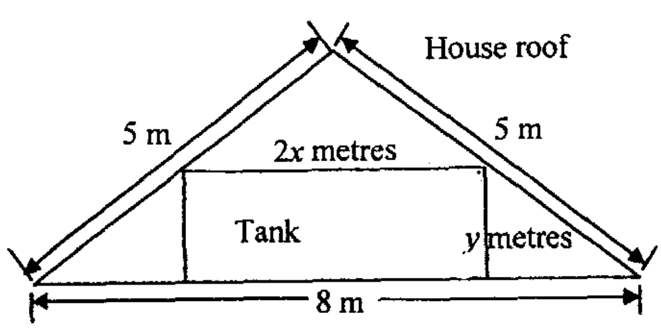
(i) How much will be in this account at the end of 12 months? 1

(ii) How much will be in this account at the end of 30 years? 2

(iii) How much will be in this account at the end of 30 years if Penny doubles the amount of her monthly deposit to $100 per month at the beginning of the 11^{th} year, and doubles again to $200 per month at the beginning of the 21^{st} year? 3

QUESTION NINE. **START A NEW PAGE.** **(12 MARKS)**

(a) The diagram below shows the cross section of a cylindrical hot water tank, with diameter $2x$ metres and height y metres that fits exactly into the roof of a house. The cross section of the roof is an isosceles triangle with the base 8 metres and equal sides 5 metres in length.



NOT TO SCALE

QUESTION NINE CONTINUED. **MARKS**

(i) Explain why the roof is 3 metres high? 1

(ii) By using similar triangles, show that $y = \frac{3}{4}(4 - x)$. 2

(iii) Show that the volume of the tank, V metres^3, is given by 1

$$V = \frac{3\pi}{4}(4x^2 - x^3)$$

(iv) Use calculus to find the radius of the tank that gives it a maximum volume. 3

(v) Calculate the maximum volume. 1

(b) (i) If the line $y = x + m$, cuts the circle $x^2 + y^2 = 4$, show that the x coordinates of the points of intersection can be found by solving $2x^2 + 2xm + m^2 - 4 = 0$ 2

(ii) For what values of m will the line $y = x + m$, be a tangent to the circle. 2

QUESTION TEN. **START A NEW PAGE.** **(12 MARKS)**

(a) (i) Simplify $\log_e e^{2ax}$ 1

(ii) Hence evaluate $\int_a^b \log_e e^{2ax}\,dx$ 2

(b) Express $\frac{\cos^3\theta}{\sin\theta} + \sin\theta\cos\theta$ as a single trigonometric ratio. 2

Hence, solve $\frac{\cos^3\theta}{\sin\theta} + \sin\theta\cos\theta = 1$ for $0 \le \theta \le 2\pi$.

MARKS

(c) (i) Sketch , on the same set of axes, the graphs of $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le x \le \pi$. 2

(ii) Write down the values of x for which $\sin x = 1 - \cos x$ in the domain $0 \le x \le \pi$. 1

(iii) Evaluate the integral $\int_0^{\pi} (1 - \cos x - \sin x)\,dx$. 2

(iv) Calculate the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le x \le \pi$. 2

END OF TEST.

TRIAL MATHEMATICS

SOLUTIONS: 2005

Question 1:

(a) 1.3737 ← ①

1.37 ← ① for sig. figs

(target question for rounding off.)

(b) x^3+2x^2-8x

$= x(x^2+2x-8)$ ← ①

$= x(x+4)(x-2)$ ← ①

(c)(i) $3x^2(4x-1)$

$= 12x^3-3x^2$ ← ①

So $\frac{d\,(12x^3-3x^2)}{dx}$

$= 36x^2-6x$ ← ①

OR using the product rule ①

$y' = 3x^2 \cdot 4 + (4x-1)6x$

$= 12x^2+24x^2-6x$

$= 36x^2-6x.$

(ii) $\frac{d\log_e x}{dx} = \frac{vu'-uv'}{v^2}$ ①

So $\frac{dy}{dx} = \frac{x^3 \cdot \frac{1}{x} - \log_e x \cdot 3x^2}{(x^3)^2}$

① → $= \frac{x^2-3x^2\log_e x}{x^6}$

$= \frac{1-3\log_e x}{x^4}$ ← ①

e) $\int \frac{1}{x-1}\,dx = \ln(x-1)+C$ ①

Question 2:

(a) $l = 5\text{cm}$, $\angle AOB = \pi/8$

$l = r\theta$

$5 = r \cdot \pi/8$ ← ①

$r = 40/\pi$

$r = 13\text{cm}$ ← ①

(b) Let $u = 2^x$

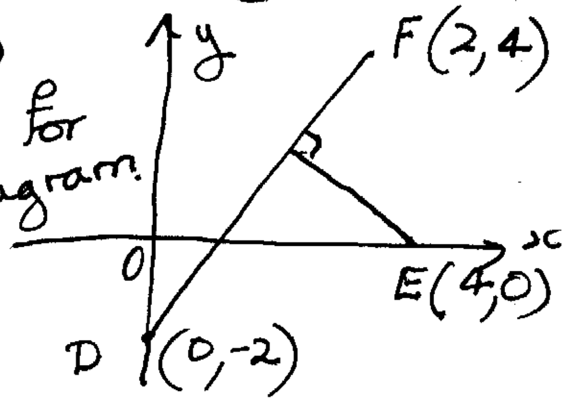
then $u^2-9u+8=0$

① → $(u-1)(u-8)=0$

$u = 2^x = 1$, $u = 2^x = 8$

$x = 0$, $x = 3$ ①

(c)(i) ① for diagram



(ii) $DF = \sqrt{(2-0)^2+(4-2)^2}$

$= \sqrt{4+36}$

$= \sqrt{40}$ ← ①

OR $2\sqrt{10}$

(iii) $m = \frac{4--2}{2-0} = \frac{6}{2} = 3$ ①

(2)

(iv) $y - 4 = 3(x-2)$

$y - 4 = 3x - 6$

$y = 3x - 2$

$3x - y - 2 = 0 \leftarrow$ (1)

(v) $d = \left| \dfrac{3.4 - 0 - 2}{9+1} \right| \leftarrow$ (1)

$= \dfrac{10}{\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}}$

$= \sqrt{10}\ u^2 \leftarrow$ (1)

(vi) $A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \sqrt{40} \times \sqrt{10} \leftarrow$ (1)

$= \frac{1}{2}\sqrt{400}$

$= \frac{1}{2} \times 20$

$= 10\ u^2 \leftarrow$ (1)

Question 3:

(a) $\dfrac{d\left(\dfrac{\tan 3x}{3x+2}\right)}{dx}$ (1) (1)

$= \dfrac{(3x+2)3\sec^2 3x - \tan 3x \cdot 3}{(3x+2)^2}$

OR

$= 3\left[\dfrac{(3x+2)\sec^2 3x - \tan 3x}{(3x+2)^2}\right]$

(b) $\int 3e^{-x/2}\,dx \leftarrow$ (1)

$= 3\int e^{-\frac{1}{2}x}\,dx$

$= 3 \times -2e^{-\frac{1}{2}x} + C$

$= -6e^{-\frac{1}{2}x} + C. \leftarrow$ (1)

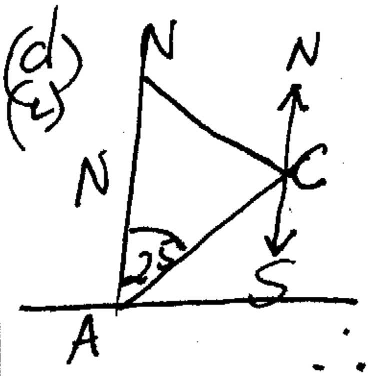
(c) $\displaystyle\int_2^4 \frac{3x}{x^2-1}\,dx$

$= \left[\frac{3}{2}\ln(x^2-1)\right]_2^4 \leftarrow$ (1)

$= \frac{3}{2}\left[\ln 15 - \ln 3\right] \leftarrow$ (1)

$= \frac{3}{2}\ln 5 \leftarrow$ (1)

(d) (i)



$\angle ACS = 25^\circ$ (alt. ∠s on ∥ lines are =)

$\therefore \angle BCA = 335 - 205 = 130^\circ$ ← (1)

(ii) $\dfrac{d}{\sin 25^\circ} = \dfrac{5}{\sin 130^\circ} \leftarrow$ (1)

$d = \dfrac{5 \times \sin 25}{\sin 130}$

$d = 2.8 \leftarrow$ (1)

(iii) $\triangle ABC$ is an isos. $\triangle$

$(130 + 25 + \angle ABC = 180^\circ)$

$\angle ABC = 25 \leftarrow$ (1)

(∠ sum of △ from ∠)

So total distance is (1)

$2.8 \times 2 = 5.6$ km

$= 6$ km

Question 4:

(a) $4\cos x = 1$

$\cos x = \frac{1}{4}$

$x = 1.32 \leftarrow$ (1)

$x = 2\pi - 1.318 \leftarrow$ (1)

$\therefore x = 4.97 \leftarrow$ (1)

(b) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

(1) $\rightarrow = \left(\frac{-9}{2}\right)^2 - 2\times -2$

$= 24\frac{1}{4} \leftarrow$ (1)

(c) 80, 100, 120 . . .

(i) $T_{10} = ?$

an AP where $a = 80$, $d = 20$ $\leftarrow$ (1)

So $T_{10} = 80 + (9)(20)$

$= 260$ cm $\leftarrow$ (1)

(ii) (1) $\rightarrow$ $450 \geq 80 + 20(n-1)$

$37 \geq 2n - 2$

$19\frac{1}{2} \geq n$

$\therefore$ trolley $= 19$ (integer)

(1)

(d) (i) $4x = 8y - y^2$ (1)

$-(4\times 1)\,x = y^2 - 8y$

$-4x + 16 = y^2 - 8y + 16$

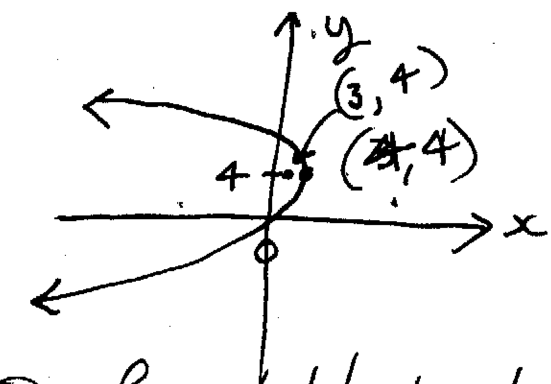
$-4(x-4) = (y-4)^2$

$\therefore$ Vertex $(4, 4)$

(1)

(ii) Focus is $(3, 4)$ since $a = 1$ and parabola opens to the left. $\leftarrow$ (1)

(iii)



(1) for sketch & pts.

Question 5:

a(i) $f(x) = 2x^3 - 3x^2 - 36x + 26$

$f'(x) = 6x^2 - 6x - 36$

Stat. pts when $f'(x) = 0$

ie $0 = 6(x^2 - x - 6)$

$0 = (x-3)(x+2)$

$x = 3, -2 \leftarrow$ (1)

$f(3) = 2\times 3^3 - 3\times(3)^2 - 36\times 3 + 26$

$= -55$

$f(-2) = 2\times(-2)^3 - 3\times(-2)^2 - 36\times -2 + 26$

$= 70$

$\therefore$ Stat. pts at $(3, -55)$ & $(-2, 70)$

(1) for the y-values.

(4)

To find nature of the points:

$y'' = 12x - 6$

(1) At $x = 3$, $y'' = 30 > 0$ ∴ concave up so relative minimum

At $x = -2$, $y'' = -30 < 0$ ∴ concave down so rel. max.

(ii) Possible pt. of inflexion at $y'' = 0$

ie $12x - 6 = 0$

$x = 1/2$

So $y = 2\times\left(\frac{1}{2}\right)^3 - 3\times\left(\frac{1}{2}\right)^2 - 36\times\left(\frac{1}{2}\right) + 26$

$y = 7\frac{1}{2}$ (1)

∴ Poss. pt. of inflexion: $\left(\frac{1}{2}, 7\frac{1}{2}\right)$

Check:

x	0	$\frac{1}{2}$	1
y''	−6	0	6

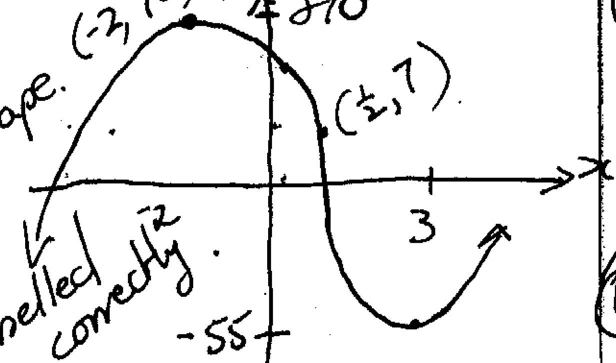
(1)

−ve ╲_╱ +ve

change in concavity

∴ pt. of inflexion.

(iii) (1) for shape. (1) pts labelled correctly.



(iv) Concave down when $f''(x) < 0$

ie $f''(x) < \frac{1}{2}$

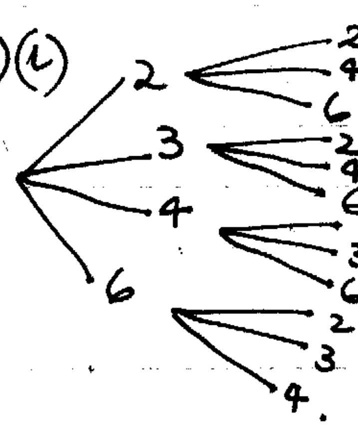
decreasing when $f'(x) < 0$

ie $-2 < x < 3$ (1)

So to satisfy both conditions

$-2 < x < \frac{1}{2}$ ← (1)

(b)(i)



$S = 23, 24, 26, 32, 34, 36, 42, 43, 46, 62, 63, 64.$

(2) (−1 for each number missing)

(ii) $P(\text{prime}) = \frac{2}{12} = 1/6$ ← (1)

Question 6:

(a) To prove: $\triangle ABC \parallel\!\mid \triangle AED$

Proof: $\angle DAE = \angle BAC$ ← (1) (common)

$\angle AED = \angle ABC = 90°$ (Given)

∴ $\triangle ABC \parallel\!\mid \triangle AED$ (1)

(AAA - equiangular) (1)

(ii) $\frac{AD}{AC} = \frac{AE}{AB}$

$\frac{8}{11 + AE} = \frac{AE}{10}$

$80 = AE^2 + 11AE$

$AE^2 + 11AE - 80 = 0$

let $AE = x$

So $(x+16)(x-5)=0$

$x = 5$ or $x \neq -16$

ie $x = 5$ ← (1)

(b) For limiting sum

$|r| < 1$ —— (1)

ie $|5 - \sqrt{p}| < 1$

OR

$-1 < 5 - \sqrt{p} < 1$

OR $-6 < -\sqrt{p} < -4$ } (1)

OR $16 < p < 36$ except $p \neq 25$.

(as if $p = 25$ it will not be a series.)

(ii) If $r = 5 - \sqrt{p}$

$S_\infty = \frac{1}{1 - (5 - \sqrt{20})}$ ← (1)

(1) $= \frac{1}{2\sqrt{5} - 4} \times \frac{2\sqrt{5} + 4}{2\sqrt{5} + 4}$

$= \frac{2\sqrt{5} + 4}{20 - 16}$

$= \frac{2\sqrt{5} + 4}{4}$ OR $\frac{\sqrt{5} + 2}{2}$

(c) $y = 2\tan x$

$V = \pi \int_0^{\pi/3} (2\tan x)^2 \, dx$ (1)

$= 4\pi \int_0^{\pi/3} (\sec^2 x - 1) \, dx$ ← (1)

$= 4\pi \left[\tan x - x \right]_0^{\pi/3}$ ← (1)

$= 4\pi \left[\tan\frac{\pi}{3} - \frac{\pi}{3} \right] - \left[\tan 0 - 0 \right]$ ← (1)

$= 4\pi \left(\sqrt{3} - \frac{\pi}{3} \right) u^3$ ← (1)

Question 7:

(a)(i)

x	1	1.5	2	2.5	3
y	1	1.732	3.000	5.196	9.000

(1) (-1 for any number incorrect - don't worry about sig. figs as question is targetted in question (1a).

(ii) $A = \frac{h}{3} \left[f(1) + f(3) + 4\left[f(1.5) + f(2.5) \right] + 2f(2) \right]$ (1)

$A = \frac{\frac{1}{2}}{3} \left[(1 + 9) + 4(1.732 + 5.196) + 2(3) \right]$

$= 7.285$ ← (1)

(Q7b)

(i) pts. of intersection

$-2x = x^2 + 2x - 5$

$0 = x^2 + 2x - 5$ ← (1)

$0 = x^2 + 4x - 5$

$= (x+5)(x-1)$

$x = -5, 1$ ← (1)

(ii) $A = \int_{-5}^{1} (-2x - (x^2 + 2x - 5)) dx$

(1)

$= \int_{-5}^{1} (-x^2 - 4x + 5) dx$

(1)

$= \left(-\frac{1}{3} - 2 + 5\right) - \left(\frac{125}{3} - 50 - 25\right)$

$= \frac{8}{3} + \frac{100}{3}$

$= 36u^2$ ← (1)

(c) $y = e^{2x}$

$y' = 2e^{2x}$

at $x = 1$ $y = e^2$ (1)

$\therefore m = 2e^2$

So eqn:

$y - e^2 = 2e^2(x-1)$

$y - e^2 = 2e^2x - 2e^2$ (1)

$y = 2e^2x - e^2$ ←

(d) $\sum_{n=2}^{4} -2^n$ (1)

$= 2^2 + -2^3 + -2^4$ ←

$= -4 - 8 - 16$ (1)

$= -28$ ←

Question 8

(a) $P(WWW) = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$

$= \frac{1}{125}$ ← (1)

(b) $P(WLL) + P(LWL) + P(LLW) + P(WWL) + P(WL\ldots$

$+ P(LWW) + P(WWW)$

$= 1 - P(LLL)$ ← (1)

$= 1 - (.8 \times .8 \times .8)$

$= 0.488$

OR 48.8% ← (1)

(iii) P(win at least 1 game in 'n' games)

$= 1 - (.8)^n$

$\therefore 1 - (.8)^n > .99$ ← (1)

$-(.8)^n > -.01$

$(.8)^n < .01$ (1)

So $n(\log .8) < \log(.01)$

$\therefore n > \frac{\log(.01)}{\log(.8)}$

$n > 20.63$

∴ Elise needs to play 21 games. ← (1)

(Q8b)

(i) 6 months - \$300, $r = 8/2 = 4\%$

$u_1 = 300(1.04) + 300(1.04)^2$

$= \$636.48$ ←

(ii) 30 yrs $60 = n$

Total $= 300(1.04 + 1.04^2 + .. 1.04^{60})$

this is a G.P.

with $a = 1.04$,
$r = 1.04$
$n = 60$ (1)

$= 300 \times \dfrac{1.04(1.04^{60} - 1)}{1.04 - 1}$

$= \$74253.09$ ← (1)

(iii) $A_{10} = 300(1.04^{60} + 1.04^{59} + \ldots 1.04^{41})$

next 10 yrs:

A_{11} to $A_{20} = 600(1.04^{40} + 1.04^{39} + \ldots)$ ← (1)

last 10 yrs

A_{21} to $A_{30} = 1200(1.04^{20} + 1.04^{19} + \ldots 1.04)$

∴ Total: $300 \times 1.04^{41}\left(\dfrac{1.04^{20} - 1}{.04}\right)$

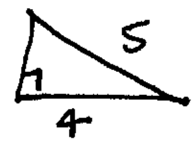
$+ 600 \times 1.04^{21}\left(\dfrac{1.04^{20} - 1}{.04}\right) +$ (1)

$1200 \times 1.04\left(\dfrac{1.04^{20} - 1}{.04}\right)$

$= \$122482.57$ ← (1)

QUESTION NINE:

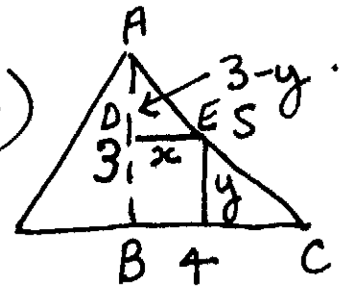
(a)(i) Using pythagoras: and the fact that the roof is isosceles (1)



$5^2 - 4^2 = 3^2$

∴ roof 3m.

(ii)



$\dfrac{3}{4} = \dfrac{3-y}{x}$ (1)

$12 - 4y = 3x$

$4y = 12 - 3x$ (1)

So $y = \frac{3}{4}(4 - x)$

(iii) $V = \pi r^2 h$
where $x = r$ &
$h = y = \frac{3}{4}(4 - x)$

$V = \pi x^2 y$ (1)

$V = \pi x^2 \cdot \frac{3}{4}(4 - x)$

$= \dfrac{3\pi}{4}(4x^2 - x^3)$

9(a)(iv) $V = \frac{3\pi}{4}(4x^2 - x^3)$

$\frac{dV}{dx} = \frac{3\pi}{4}(8x - 3x^2)$

$= \frac{3\pi x}{4}(8 - 3x)$ (1)

$\frac{dV}{dx} = 0$ for max.

$0 = \frac{3\pi x}{4}(8 - 3x)$

$x = 0, \ 8/3$ (1)

$x = 0$ cannot exist.

to check if max

$\frac{d^2V}{dx^2} = \frac{3\pi}{4}(8 - 6x)$ (1)

at $x = 8/3$

$\frac{d^2V}{dx^2} = -18{\cdot}8 < 0$

∴ concave down

rel. max.

∴ Radius for max volume is $2\frac{2}{3}$ m.

(v) Max vol $= \frac{3\pi}{4} \times \frac{64}{9} \times \frac{4}{3}$

(1) $= \frac{64\pi}{9}$ m^3 OR 22.34 m^3

(b)(i) $y = x + m$

$x^2 + y^2 = 4$

$x^2 + (x+m)^2 = 4$ (1)

$x^2 + x^2 + 2xm + m^2 = 4$ (1)

$2x^2 + 2xm + m^2 - 4 = 0$

(ii) $\Delta = 0$

$2m^2 - 4.2(m^2 - 4) = 0$ (1)

$4m^2 - 8m^2 + 32 = 0$

$-4m^2 + 32 = 0$ (1)

$-4m^2 = -32$

$m = \pm\sqrt{8}$

OR $m = 2\sqrt{2}$ and $-2\sqrt{2}$

Question 10:

(i) $\log_e e^{2ax} = 2ax$ (1)

OR let $\log_e e^{2ax} = y$ (1)

$\therefore e^y = e^{2ax}$

$\therefore y = 2ax$.

(ii) $\int_a^b \log_e e^{2ax} dx = \int_a^b 2ax\,dx$ (1)

$= \left[ax^2\right]_a^b$ (1)

$= ab^2 - a a^2 = ab^2 - a^3$

(b)

$$\frac{\cos^3\theta}{\sin\theta} + \sin\theta\cos\theta =$$

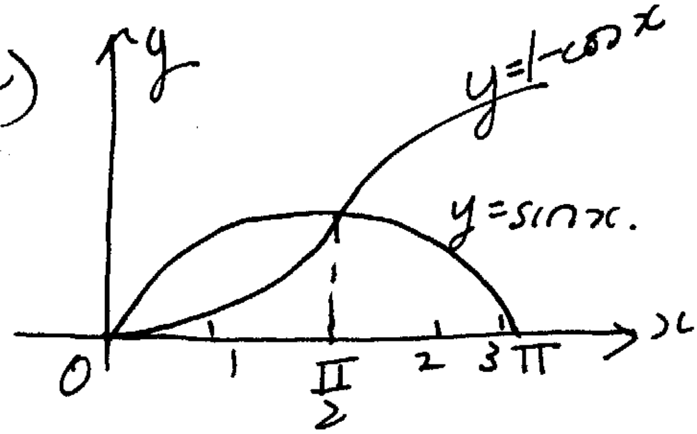
$$\frac{\cos\theta}{\sin\theta}(\cos^2\theta + \sin^2\theta) \quad \leftarrow (1)$$

$$= \cot\theta$$

So $\cot\theta = 1$ — (1)

$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

(c)



(1) for each graph.

(ii) $x = \frac{\pi}{2}$ & 0 ↙ (1) (from graph)

(iii) $\int_0^{\pi} (1-\cos x - \sin x)\,dx$

$$= \left[x - \sin x + \cos x\right]_0^{\pi} \leftarrow (1)$$

$$= \pi - 0 - 1 - (0 - 0 + 1)$$

$$= \pi - 2. \leftarrow (1)$$

(iv) $A = \int_0^{\frac{\pi}{2}} (\sin x - (1-\cos x))\,dx + \int_{\frac{\pi}{2}}^{\pi} (1 - \cos x - \sin x)\,dx$

$$= \left[-\cos x - x + \sin x\right]_0^{\pi/2} + \left[x - \sin x + \cos x\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \left(0 - \pi/2 + 1 - (-1)\right) + \left(\pi - 0 - 1 - \left(\frac{\pi}{2} - 1\right)\right) \quad (1)$$

$$= 2 - \pi/2 + \pi/2$$

$$= 2 \quad \leftarrow (1)$$

END OF SOLUTIONS.

(thank goodness)