GF
AW
AT
NB
YR
JG
LB
VL

Name: ____________________

Class: 12MT2__ or 12MTX__

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2006 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 10.

Each page must show your name and your class. *

Question 1 (12 marks) **Marks**

(a) The base length x, of a square pyramid of volume, V and perpendicular height h, is given by

$$x=\sqrt{\frac{3V}{h}}$$

Find x, correct to two decimal places, if $V=750$ and $h=8{\cdot}45$. **2**

(b) Solve the equation $\frac{2x-1}{2}-\frac{x+1}{3}=\frac{1}{2}$. **2**

(c) Express $4\sqrt{27}-2\sqrt{12}$ in its simplest surd form. **2**

(d) Factorise completely

$2y-18y^3$ **2**

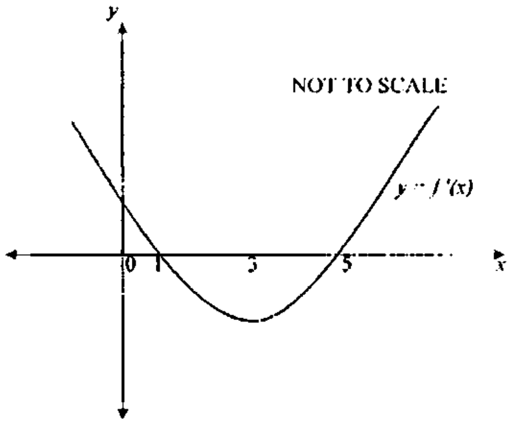
(e) Solve $\tan\theta=-1$ for $0<\theta\le 2\pi$ **2**

(f) Solve $|2x-3|\le 4$ **2**

End of Question 1

Question 2 (12 marks) **START A NEW PAGE** **Marks**

(a)



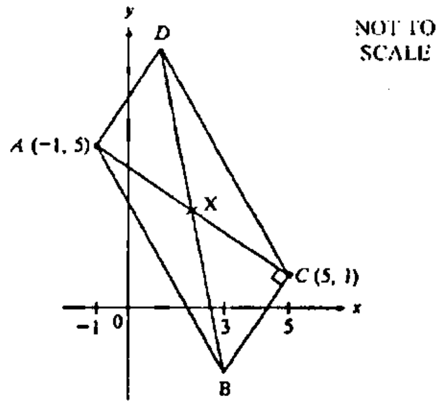
Consider the above graph of $y = f'(x)$

(i) State the value of x for which there is a minimum turning point for the graph $y = f(x)$ **1**

(ii) For what values of x in the domain is the curve $y = f(x)$ increasing. **1**

Question 2 cont'd next page

(b)



In the diagram above, $ABCD$ is a parallelogram. A and C are the points $(-1,5)$ and $(5, 1)$ respectively.
AC is perpendicular to BC. Their diagonals bisect each other at X.

(i) Find the midpoint of AC. **1**

(ii) Show that the midpoint of AC lies on the line BD whose equation is $5x + y - 13 = 0$ **1**

(iii) Show that the equation of BC is $3x - 2y - 13 = 0$ **2**

(iv) By solving simultaneously, show that the coordinates of B are $(3,-2)$. **2**

(v) Find the coordinates of D. **1**

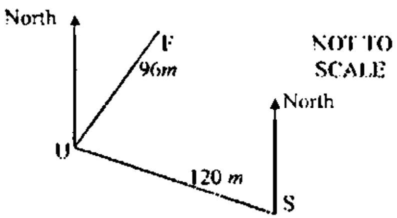
(vi) Show that the perpendicular distance from A to BC is $2\sqrt{13}$. **1**

(vii) Hence calculate the area of the parallelogram $ABCD$. **2**

End of Question 2

Question 3 (12 marks) **START A NEW PAGE** **Marks**

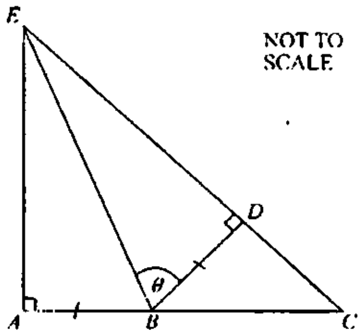
(a) A woman walks 120 metres from a point S to a point U on a bearing 312°, then turns and walks another 96 metres on a bearing of 056° to F.



(i) How far is the woman from her starting point, correct to three decimal places? **2**

(ii) Hence find the bearing of the woman from her starting point, correct to the nearest degree. **2**

(b)



In the diagram, ACE is a right-angled triangle. The point B lies on AC and the point D lies on CE. Also $\angle BDE = 90^\circ$, $AB = BD$ and $\angle DBE = \theta$.

(i) Show that $\Delta ABE \equiv \Delta DBE$. **2**

(ii) Show that $\angle ACE = 2\theta - 90^\circ$. **1**

(iii) Show that $\Delta ACE ||| \Delta DCB$. **2**

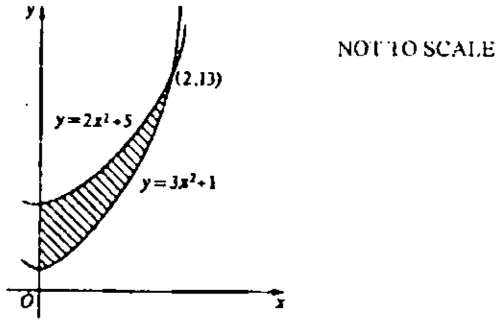
(iv) Hence show that $EA : AB = CE : CB$. **1**

(c) Find the values of k for which the equation $kx^2 - (k+1)x = -1$ has two distinct roots. **2**

End of Question 3

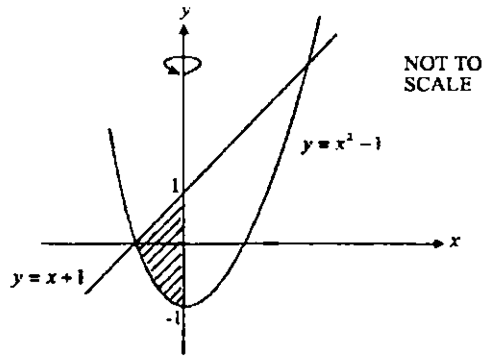
Question 4 (12 marks) **START A NEW PAGE** **Marks**

(a) The diagram shows parts of the curves $y = 2x^2 + 5$ and $y = 3x^2 + 1$, intersecting at (2, 13).



Find the area of the shaded region. **3**

(b)



The graph of the line $y = x + 1$ and the curve $y = x^2 - 1$ are shown on the diagram above.

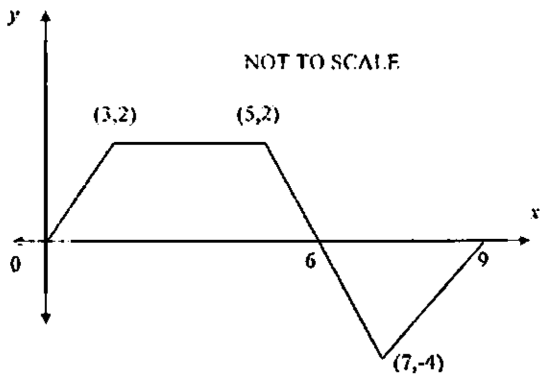
(i) Find the coordinates of the points of intersection of the two graphs. **1**

(ii) The shaded region shown on the graph is rotated about the y axis. By considering two integrals or otherwise find the volume of the solid formed. **3**

Question 4 cont'd next page

Question 4 continued **Marks**

(c)



The graph of $y = f(x)$ is illustrated.

Evaluate $\int_0^9 f(x)\,dx$. **2**

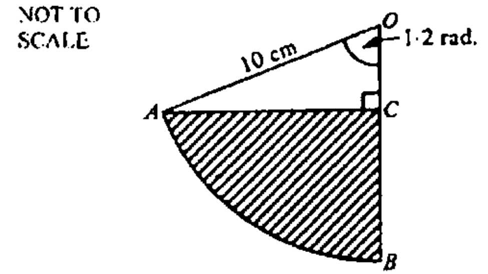
(d) Show that $\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} (\tan^2 x + 1)\,dx = \frac{4\sqrt{3}}{3}$. **3**

End of Question 4

Question 5 (12 marks) **START A NEW PAGE** **Marks**

(a) Express $\frac{3\pi}{8}$ radians in degrees and minutes. **1**

(b)



OAB is a sector of the circle, centre O, radius $10cm$, and $\angle AOB = 1{\cdot}2$ radians. Given that AC is perpendicular to OB,

(i) show that $BC = 6.38cm$, correct to two decimal places **2**

(ii) calculate the perimeter of the shaded region, correct to one decimal place. **2**

(iii) calculate the area of the shaded region, correct to one decimal place. **2**

(c) For $y = 2\cos x - 1$ and $y = \sin 2x$ in the domain $0 \le x \le 2\pi$

(i) state the period of $y = \sin 2x$ **1**

(ii) state the amplitude of $y = 2\cos x - 1$ **1**

(iii) on the same number plane, sketch the graphs for both equations in the given domain. **2**

(iv) Hence state the number of solutions for the equation $2\cos x - 1 = \sin 2x$ **1**

End of Question 5

Question 6 (12 marks) **START A NEW PAGE** **Marks**

(a) Differentiate with respect to

(i) $\log_e(2x-1)$ **1**

(ii) $\cos^2 3x$ **2**

(iii) $\dfrac{2\tan x}{\sqrt[3]{e^x}}$ **3**

(b) Find

(i) $\displaystyle\int_0^1 e^{\frac{x}{2}}\,dx$ **2**

(ii) $\displaystyle\int \frac{\cos 2x}{3+\sin 2x}\,dx$ **2**

(c) Using two applications of the trapezoidal rule, evaluate correct to 4 significant figures,

$$\int_{-1}^{1}\sqrt{\ln(2+x)}\,dx$$ **2**

End of Question 6

Question 7 (12 marks) **START A NEW PAGE** **Marks**

(a) Joan invested \$8000 at 15% pa for two years compounded monthly. What will her investment be worth at the end of the two years? **2**

(b) The equation of a parabola is given by $x^2-4x-2y+8=0$
Find the

(i) Vertex **2**
(ii) Equation of the normal to the parabola at the point (0,4). **2**

(c) The function $f(x)=e^x+e^{-x}$ is defined for all real values of x.

(i) Show $f(x)$ is an even function. **1**
(ii) Find the stationary point and its nature.
Hence sketch the curve of $y=f(x)$. **3**
(iii) Show that the equation of the tangent at $x=1$ is

$$y=\left(e-\frac{1}{e}\right)x+\frac{2}{e}$$ **2**

End of Question 7

Question 8 (12 marks) **START A NEW PAGE** **Mark**

(a) Solve $5^{2x}=5+4(5^x)$ **3**

(b) A woman buys a ticket in a raffle in which there are three prizes and 50 tickets sold. What is the probability that she

(i) does not win a prize **2**

(ii) wins the third prize? **2**

(c) A funding body gives a grant to a sports organisation each year from 2000. The amount of the grant in 2000 is \$20000, and thereafter it is 90% of the amount of the previous year's grant.

(i) Show that the year in which the value of the grant first falls below \$4000 is 2016. **3**

(ii) Calculate the total amount paid in grants to the sports organisation during the years from 2000 to 2009 inclusive. **2**

End of Question 8

Question 9 (12 marks) **START A NEW PAGE** **Marks**

(a) Consider the geometric series

$$2+2\sin^2 x+2\sin^4 x+\ldots,\ \text{for } 0<x<\frac{\pi}{4}$$

(i) Show that the limiting sum exists. **2**

(ii) Find the limiting sum as $x\to\dfrac{\pi}{4}$. **2**

(b) Consider the arithmetic series $\log_2 k+2\log_2 k+3\log_2 k+\ldots$.
Find the value of k given the sum of the first ten terms of the series is 165. **2**

Question 9 cont'd next page

Question 9 continued

Marks

(c) Tara's grandparents deposit $\$P$ into a trust fund account to provide for the cost of her education at university. The balance of the money in the trust account during each year will earn 11% p.a. interest compounded annually. \$10000 will be withdrawn at the end of each year, immediately after interest is paid into the account.

(i) Write an expression for the amount $\$A_1$, in the account just after the first withdrawal in terms of P. **1**

(ii) Derive expressions for $\$A_2$ and $\$A_3$.
Hence show that just after the sixth withdrawal, the amount $\$A_6$, is given by

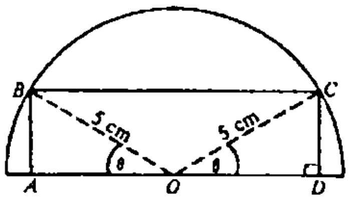
$$P(1\cdot 11)^6 - \frac{10000(1\cdot 11^6 - 1)}{0\cdot 11}$$ **3**

(iii) Find the amount $\$P$ if the account is to be empty after the sixth withdrawal. **2**

End of Question 9

Question 10 (12 marks) **START A NEW PAGE** **Marks**

(a)



NOT TO SCALE

The diagram shows a rectangle $ABCD$ inside a semi-circle centre O and radius $5cm$ such that the acute angle, $\angle BOA = \angle COD = \theta$ radians.

(i) Show that $OD = 5\cos\theta$. **1**

(ii) Hence show that the perimeter, P cm, of the rectangle is given by $P = 20\cos\theta + 10\sin\theta$. **1**

(iii) Determine the value of θ, correct to two decimal places, for which P is maximum **3**

Question 10 cont'd next page

Question 10 continued **Marks**

(b) An underground wine cellar is in the shape of a rectangular prism with a floor area of $12\,m^2$ and a ceiling height of 2 m. At 2pm one Saturday, water begins to enter the cellar. The rate at which the volume, V, of water in the cellar changes over time t hours, is given by

$$\frac{dV}{dt} = \frac{24t}{t^2+15}$$

where $t = 0$ represents 2pm on Saturday and where V is measured in cubic metres. The cellar is initially dry.

(i) Show that the volume of water in the cellar at time t is given by **2**

$$V = 12\ln\left(\frac{t^2+15}{15}\right),\ t \ge 0$$

(ii) Find the time when the cellar will be completely filled with water if the water continues to enter the cellar at the given rate. Express your answer to the nearest minute. **2**

(iii) The owners return to the house and manage to simultaneously stop the water entering the cellar and start the pump in the cellar. This occurs at 6pm on Saturday. Find the volume of water in the cellar at this time as an exact value. **1**

(iv) The rate at which the water is pumped out of the cellar is given by **2**

$$\frac{dV}{dt} = \frac{t^2}{k} \text{ where } k \text{ is a constant.}$$

At exactly 8pm the cellar is emptied of water.

Find the value of k. Express your answer correct to 4 significant figures.

End of Exam

Mathematics APA 2006 Trial

Question 1

(a) $x = \sqrt{\frac{3 \times 750}{8.45}}$ ✓

$= 16.3178487966$

$= 16.32$ (2 dp) ✓

(b) $\frac{2x-1}{2} - \frac{x+1}{3} = \frac{1}{2}$

$\frac{3(2x-1)-2(x+1)}{6} = \frac{1}{2}$

$6x - 3 - 2x - 2 = 3$ ✓

$4x - 5 = 3$

$4x = 8$

$x = 2$ ✓

cfor 1 error only

c) $4\sqrt{27} - 2\sqrt{12}$

$4\sqrt{9\times3} - 2\sqrt{4\times3}$

$12\sqrt{3} - 4\sqrt{3}$ ✓ (1st mark either one correct with an attempt to simplify surd on other one)

$= 8\sqrt{3}$ ✓

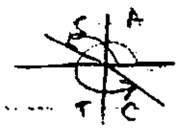
d) $2y - 18y^3$

$2y(1-9y^2)$ ✓

$2y(1-3y)(1+3y)$ ✓

e) $\tan\theta = -1$

$\theta = \frac{3}{4}\pi, \frac{7}{4}\pi$ ✓ ✓



f) $|2x-3| \le 4$

$-4 \le 2x - 3 \le 4$ ✓

$-1 \le 2x \le 7$

$-\frac{1}{2} \le x \le \frac{7}{2}$ ✓

do not accept for 2
Award 1
$x \geq -\frac{1}{2}$ or $x \le \frac{7}{2}$

but

$x \geq -\frac{1}{2}$ and $x \le \frac{7}{2}$

IS correct

via

$2x-3 \le 4$ and $2x-3 \geq -4$ ✓

$2x \le 7$ and $2x \geq -1$

$x \le 3\frac{1}{2}$ and $x \geq -\frac{1}{2}$

(2)

Question 2

(a) (i) $x = 5$ ✓

(ii) $x < 1$ or $x > 5$ ✓

(b) (i) $\left(\frac{-1+5}{2}, \frac{5+1}{2}\right)$

$= X(2, 3)$ ✓

(ii) $5\times2 + 3 - 13 = 0$

$13 - 13 = 0$ ✓

$\therefore$ (2,3) lie on BD

(iii) Grad AC $= \frac{1-5}{5+1}$

$= \frac{-4}{6}$

$= \frac{-2}{3}$

$\therefore$ grad of BC $= \frac{3}{2}$ ✓

$\therefore$ Eqn of BC

$y - 1 = \frac{3}{2}(x-5)$

$2y - 2 = 3x - 15$ ✓

$3x - 2y - 13 = 0$

(iv) $5x + y - 13 = 0$ — (A)

$3x - 2y - 13 = 0$ — (B)

from (A) $y = 13 - 5x$

sub into B

$3x - 2(13-5x) - 13 = 0$

$3x - 26 + 10x - 13 = 0$ ✓ for method of method of elimination

$13x - 39 = 0$

$x = 3$

$y = 13 - 5x$

$= 13 - 5\times3$

$= 13 - 15$ ✓ cfm on their x

$= -2$

$\therefore B(3, -2)$

(v) Let D be (h, k)

X is midpt of DB

$\frac{h+3}{2} = 2$

$h = 1$

$\frac{k-2}{2} = 3$

$k = 8$

$\therefore D(1, 8)$ ✓

or use diagram using gradients

(vi) $A(-1, 5)$

BC $3x - 2y - 13 = 0$

$d_\perp = \frac{|3\times-1 - 2\times5 - 13|}{\sqrt{9+4}}$

$= \frac{|-3-10-13|}{\sqrt{13}}$

$= \frac{|-26|}{\sqrt{13}}$ ✓

$= \frac{26}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$

$= \frac{26\sqrt{13}}{13} = 2\sqrt{13}$

(or via distance formula

$AC = \sqrt{(5-1)^2 + (-1-5)^2}$)

B(3, -2) C(5, 1)

(vii) $d_{BC} = \sqrt{(3-5)^2 + (-2-1)^2}$

$= \sqrt{4+9}$ ✓

$= \sqrt{13}$

$\therefore$ Area of ABCD

$= 2\sqrt{13} \times \sqrt{13}$ ✓

$= 26 \text{ units}^2$

or Area of $\triangle ABC = \frac{1}{2} BC \times AC$

$AC = \sqrt{52}$

$= \frac{1}{2} \times \sqrt{13} \times \sqrt{52}$

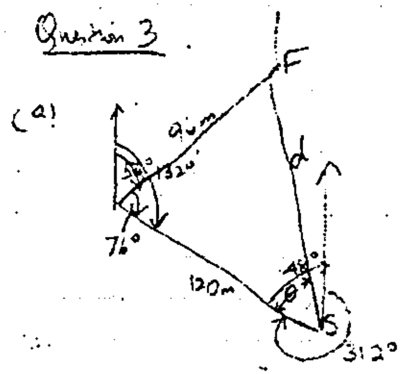
$= 13$

$\therefore$ Area of ABCD $= 2 \times 13$

$= 26 \text{ units}^2$

Question 3

(a)



i) $d^2 = 96^2 + 120^2 - 2 \times 96 \times 120 \times \cos 76°$ ✓

$d = 134.32095713 4$ ✓

$= 134.321 \text{ m}$

ii) $\frac{\sin \theta}{96} = \frac{\sin 76°}{134.321}$ ✓

$\sin \theta = \frac{96 \sin 76°}{134.321}$

$\theta = 43.905898 1325$

$= 44°$

Bearing $= 312 + 44$

$= 356°$ ✓

(nearest degree) or N 4° W

Question 3

(b) (i) $\triangle$'s ABE and DBE are both right angled triangles

AB = DB (given)

$\angle EAB = \angle EDB = 90°$ (given) } ✓

EB in common

$\therefore \triangle ABE \equiv \triangle DBE$ (RHS) } ✓

(ii) $\angle ABE = \theta$ (corresponding $\angle$'s of congruent Δ's)

$\angle BDC + \angle BCD = \angle DBA$ (sum of interior angles of $\triangle BDC$ equal opposite exterior angle) ✓

$\angle BCD = \angle ACE$ (same angle)

$\angle BDC = 90°$

$\therefore 90° + \angle ACE = 2\theta$

$\therefore \angle ACE = 2\theta - 90°$

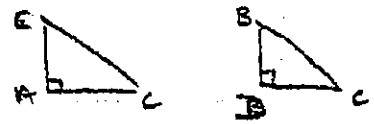
but must continue to retain mark

(iii) $\angle CAE = \angle BDC = 90°$ (given) } ✓

$\angle ACE = \angle BCD$ (common angle) ✓

$\therefore \triangle ACE \ ||| \ \triangle DCB$ (equiangular)

(iv)



Since $\triangle ACE \ ||| \ \triangle DCB$

EA : BD = CE : CB

Given BD = AB

$\therefore$ EA : AB = CE : CB

✓ for writing ratio then linking BD=AB (given)

Question 3

(i) $kx^2-(k+1)x+1=0$

$\Delta = (k+1)^2 - 4 \times k \times 1$

$= k^2+2k+1-4k$

$= k^2-2k+1$

$= (k-1)^2$

For 2 distinct roots

$(k-1)^2 > 0$ ✓

$\therefore$ k is all reals $k \neq 1$. ✓

Question 2

(a) $A = \int_0^2 (2x^2+5)\,dx - \int_0^2 (3x^2-1)\,dx$ ✓

$= \left[\frac{2x^3}{3}+5x\right]_0^2 - \left[\frac{3x^3}{3}-x\right]_0^2$ ✓

$= \left(\frac{2\times 8}{3}+10\right) - (8-2)$

$= 5\frac{1}{3}$ units^2 ✓

(b) (i) $y = x+1$

$y = x^2-1$

$x^2-1 = x+1$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$, or $x=-1$

Points of intersection are $(2,3)$ and $(-1,0)$ ✓

(ii) $V = \pi\int_0^1 (y-1)^2\,dy + \pi\int_{-1}^0 (y+1)\,dy$. ✓

$= \pi\int_0^1 (y^2-2y+1)\,dy + \pi\left[\frac{y^2}{2}+y\right]_{-1}^0$

$= \pi\left[\frac{y^3}{3}-\frac{2y^2}{2}+y\right]_0^1 + \pi\left[\frac{y^2}{2}+y\right]_{-1}^0$ ✓

$= \pi\left[\left(\frac{1}{3}-1+1\right)-0\right] + \pi\left[0-\left(\frac{1}{2}-1\right)\right]$

$= \frac{\pi}{3}+\frac{\pi}{2}$

$= \frac{5\pi}{6}$ u^3 ✓

(accept cal decimal ans.)

or $(= 2.61799\ldots$ or $= 2.6(\ldots)$ correct to ... dec pl) ✓

(c) $\int_0^4 f(x)\,dx = \left[\frac{2}{2}(2+6)\right] - \left(\frac{1}{2}\times 3\times 4\right)$

$= 2$ ✓

d) $\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \sec^2 x\,dx$ ✓

$= \left[\tan x\right]_{\frac{\pi}{6}}^{\frac{2\pi}{3}}$

$= \tan\frac{2\pi}{3} - \tan\frac{\pi}{6}$ ✓

$= -\sqrt{3} - \frac{1}{\sqrt{3}}$

$= \frac{-3-1}{\sqrt{3}}$

$= -\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ✓

$= -\frac{4\sqrt{3}}{3}$

Question 5

(a) $\frac{3\pi}{8} \times \frac{180}{\pi}$

$= 67°30'$ ✓ accept $67.5°$

(b)(i) $\cos \angle AOC = \frac{OC}{OA}$

$\cos 1.2 = \frac{OC}{10}$

$OC = 10 \cos 1.2$

$= 3.62357$ ✓

$BC = 10 - 3.62357$ ✓ (cfm)

$\doteqdot 6.3764$

$= 6.38$ (2 dec pl)

(ii) arc $AB = 10 \times 1.2$

$= 12$

$AC = 10 \sin 1.2$ ✓

$= 9.32039$

$P = 12 + 9.320 + 6.376$

$= 27.696$

$= 27.7$ ✓

(iii) Area of sector $= \frac{1}{2} \times 10^2 \times 1.2$

$= 60 \text{ cm}^2$ ✓

area of $\Delta AOC = \frac{1}{2} \times 10 \times 3.62357 \times \sin 1.2$

$= 16.867$

Shaded area $= 60 - 16.867$

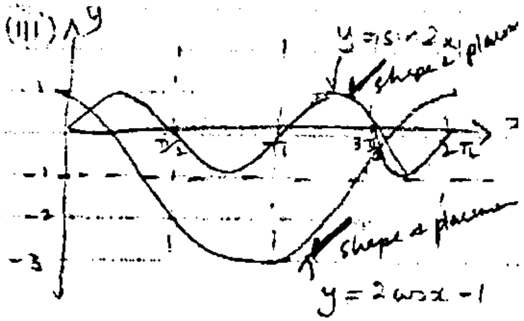
$= 43.113$

$= 43.1 \text{ cm}^2$ ✓

(c)(i) period $= \pi$ ✓

(ii) amplitude $= 2$ ✓

(iii)



(iv) 2 solutions ✓

Question 6

(a) (i) $\frac{2}{2x+1}$ ✓

if no chain rule → zero

(ii) $y = (\cos 3x)^2$

$\frac{dy}{dx} = 2(\cos 3x)^1 \cdot -3\sin 3x$ ✓

$= -6\cos 3x \sin 3x$ ✓ either

(Ext 1 might simplify to $-3\sin 6x$ correctly *)

(iii) $u = 2\tan x \quad v = e^{x/2}$

$\frac{du}{dx} = 2\sec^2 x \quad \frac{dv}{dx} = \frac{1}{2}e^{x/2}$ ✓

$\frac{dy}{dx} = \frac{2e^{x/2}\sec^2 x - \frac{2\tan x}{2} \cdot e^{x/2}}{e^x}$ ✓ correct use of quotient rule

$= \frac{e^{x/2}(2\sec^2 x - \tan x)}{e^x}$

$= \frac{2\sec^2 x - \tan x}{e^{x/2}}$

not required.

Students might not set up the 2 derivatives and go straight to "rule". Award 3 if correct (ie look for derivatives applied correctly)

(b) (i) $\int_0^1 e^{x/2}\,dx = 2\left[e^{x/2}\right]_0^1$ ✓

$= 2(e^{1/2} - e^0)$

$= 2(e^{1/2} - 1)$ ✓

(ii) $\int \frac{\cos 2x}{3+\sin 2x}\,dx = \frac{1}{2}\int \frac{2\cos 2x}{3+\sin 2x}\,dx$ ✓

$= \frac{1}{2}\ln(3+\sin 2x) + C$ ✓

d)

x	-1	0	1
$f(x)$	$\sqrt{\ln 1}$	$\sqrt{\ln 2}$	$\sqrt{\ln 3}$

$\int_{-1}^{1} \sqrt{\ln(2+x)}\,dx = \frac{1}{2}\left[\sqrt{\ln 1} + 2\sqrt{\ln 2} + \sqrt{\ln 3}\right]$ ✓

$= 1.35662814814$ Zeither

1.357 (4 sig figs) ✓

if use 5 values award (1)

x	-1	-0.5	0	0.5	1
$f(x)$	$\sqrt{\ln 1}$	$\sqrt{\ln 1.5}$	$\sqrt{\ln 2}$	$\sqrt{\ln 2.5}$	$\sqrt{\ln 3}$

$1.4753\ldots \doteqdot 1.475$ (4 sig figs)

Q7

(a) $r = \frac{15}{12}\%$ per month

$= 0.0125$

$A = 8000(1+0.0125)^{24}$ ✓

$= 10778.8084033$

$= \$10778.81$ ✓

Investment worth $10778.81 at end of 2 years

(b)(i) $x^2 - 4x - 2y + 8 = 0$

$x^2 - 4x (+2^2) = 2y - 8 (+2^2)$

$(x-2)^2 = 2y - 4$ ✓

$(x-2)^2 = 2(y-2)$

$\therefore$ Vertex $(2, 2)$ ✓

other method

$x = -\frac{b}{2a}$ (1)

$x = -\frac{-4}{2}$

$= 2$ (2)

$f(2) = \frac{1}{2}(2^2 - 4\times 2 + 8)$ (3)

$= 2$

vertex $(2,2)$

(iii) $2y = x^2 - 4x + 8$

$y = \frac{1}{2}x^2 - 2x + 4$

$\frac{dy}{dx} = x - 2$

at $(0,4)$ $\frac{dy}{dx} = -2$

$\therefore$ grad of normal $= \frac{1}{2}$ ✓

Eqn of normal

$y - 4 = \frac{1}{2}(x - 0)$

$2y - 8 = x$

$x - 2y + 8 = 0$ ✓

(c)(i) $f(x) = e^x + e^{-x}$

$f(-x) = e^{-x} + e^{x}$

$= f(x)$ ✓

$\therefore f(x)$ is even

(ii) $f'(x) = e^x - e^{-x}$

For st pts $f'(x) = 0$

$e^x - e^{-x} = 0$

$e^x - \frac{1}{e^x} = 0$

$e^{2x} = 1$

$2x = \ln 1$

$2x = 0$

$\therefore x = 0$ ✓

$f(0) = e^0 + e^0$

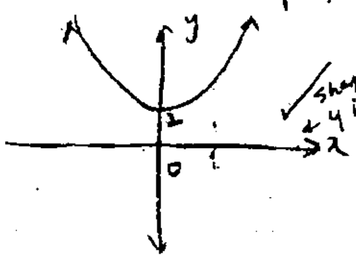
$= 2$

Stat pt at $(0, 2)$

Nature

x	-1	0	1
$f'(x)$	$-$	0	$+$

$\therefore (0,2)$ minimum stat. pt. ✓



✓ shape + y inter

(iii) At $x = 1$, $f(1) = e + e^{-1}$

$= e + \frac{1}{e}$

$f'(1) = e - e^{-1}$

$= e - \frac{1}{e}$ ✓

Eqn of tangent at $(1, e + \frac{1}{e})$

is

$y - (e + \frac{1}{e}) = (e - \frac{1}{e})(x - 1)$

$y \cancel{- e} - \frac{1}{e} = (e - \frac{1}{e})x \cancel{- e} + \frac{1}{e}$ ✓

$y = (e - \frac{1}{e})x + \frac{2}{e}$

Q8

a) Let $a = 5^x$

$a^2 = 5 + 4a$

$a^2 - 4a - 5 = 0$

$(a+1)(a-5) = 0$

$\therefore a = -1,\ 5$

$\therefore 5^x = -1$ no solution

$5^x = 5 \quad \therefore x = 1$

b) (i) $\frac{49}{50} \times \frac{48}{49} \times \frac{47}{48}$

$= \frac{47}{50}$

(ii) $\frac{49}{50} \times \frac{48}{49} \times \frac{1}{48}$

$= \frac{1}{50}$

(c) i) $r = 0.9,\ a = 20000$

$T_n < 4000$

$20000(0.9)^{n-1} < 4000$

$(0.9)^{n-1} < 0.2$

$(n-1)\log 0.9 < \log 0.2$

$n-1 > \frac{\log 0.2}{\log 0.9}$

$n > 1 + \frac{\log 0.2}{\log 0.9}$

$n > 16.3$

$\therefore 17^{th}$ year from 2000

ie 2016.

(ii) $S_{10} = \frac{20000(1-0.9^{10})}{1-0.9}$

$= \$130304.31$

Q9

(a) (i) $r = \frac{2\sin^2 x}{2}$

$= \sin^2 x$

particular method

For $0 < x < \frac{\pi}{4}$

$\sin^2 0 < r < \sin^2 \frac{\pi}{4}$

$0 < r < \frac{1}{2}$

OR general method

$-1 \leq \sin x \leq 1$ diagram $\sin x$

but $0 < \sin^2 x < 1$

$\therefore 0 < r < 1$

$\therefore$ limiting sum exists as $-1 < r < 1$

Since $-1 < r < 1$, limiting sum exists

(ii) $S_\infty = \frac{2}{1 - \sin^2 x}$

$= \frac{2}{\cos^2 x}$

as $x \to \frac{\pi}{4}$, $S_\infty \to \frac{2}{\cos^2 \frac{\pi}{4}}$

$\therefore S_\infty \to 4$

b) $a = \log_2 k,\ d = \log_2 k$

$S_{10} = \frac{10}{2}\left[2\log_2 k + 9\log_2 k\right] = 165$

$11 \log_2 k = \frac{165}{5} = 33$

$\log_2 k = 3$

$k = 2^3$

$= 8$

c) (i) $A_1 = P(1\cdot11) - 10000$ ✓

(ii) $A_2 = A_1(1\cdot11) - 10000$

$= [P(1\cdot11) - 10000](1\cdot11) - 10000$

$= P(1\cdot11)^2 - 10000(1\cdot11) - 10000$

$= P(1\cdot11)^2 - 10000(1\cdot11 + 1)$ ✓

$A_3 = P(1\cdot11)^3 - 10000(1\cdot11^2 + 1\cdot11 + 1)$ ✓

$A_6 = P(1\cdot11)^6 - 10000(1\cdot11^5 + 1\cdot11^4 + \dots + 1)$

$= P(1\cdot11)^6 - 10000\dfrac{(1\cdot11^6 - 1)}{1\cdot11 - 1}$ ✓

$= P(1\cdot11)^6 - 10000\dfrac{(1\cdot11^6 - 1)}{0\cdot11}$ ← given in question

(iii) $P(1\cdot11)^6 = 10000\dfrac{(1\cdot11^6 - 1)}{0\cdot11}$

$P = \dfrac{10000(1\cdot11^6 - 1)}{(1\cdot11)^6(0\cdot11)}$ ✓

$= \$42305\cdot3785274$ ✓

$= \$42305\cdot38$

Q10

a) (i) $\cos\theta = \dfrac{CD}{5}$ ✓

$\therefore CD = 5\cos\theta$

(ii) $\sin\theta = \dfrac{DC}{5}$

$\therefore DC = 5\sin\theta$, $AD = 2CD = 10\cos\theta$

Perimeter $= 4CD + 2CD$ ✓ (must show sin ratio working)

$= 4 \times 5\cos\theta + 2 \times 5\sin\theta$

$= 20\cos\theta + 10\sin\theta$

$P = 2AD + 2CD$

$= 2\times10\cos\theta + 2\times5\sin\theta$

$= 20\cos\theta + 10\sin\theta$

(iii) $\dfrac{dP}{d\theta} = -20\sin\theta + 10\cos\theta$ ✓

for stat pts $\dfrac{dP}{d\theta} = 0$

$20\sin\theta = 10\cos\theta$

$\tan\theta = \frac{1}{2}$

$\theta = 0\cdot463647609$ ✓ $\theta = 26° 34'$ (lose 1 mark without rad angle).

θ must be acute $\therefore \theta = \cdot46\ldots$ must be in radians

$\dfrac{d^2P}{d\theta^2} = -20\cos\theta - 10\sin\theta$

for θ acute $0 < \cos 0\cdot46 < 1$

$0 < \sin 0\cdot46 < 1$

$\therefore \dfrac{d^2P}{d\theta^2} < 0$ for $\theta = 0\cdot46$ ✓ test with sensible θ. (if test degrees ÷ give ie $\frac{2}{3}$)

$\therefore \theta = 0\cdot46$ gives max perimeter

$\therefore \theta = 0\cdot46$

Question 10

(b) (i) $\frac{dV}{dt} = \frac{24t}{t^2+15}$

$V = 12\int \frac{2t}{t^2+15}\, dt$

$V = 12 \ln(t^2+15) (+c)$ ✓

$t=0, \quad V=0$ no marks without brackets.

$0 = 12 \ln(15) + c$

$c = -12 \ln 15$ ✓

$V = 12 \ln\left(\frac{t^2+15}{15}\right), \; t>0$ (given in quest)

(ii) $V = 12 \times 2 = 24 m^3$

$24 = 12 \ln\left(\frac{t^2+15}{15}\right)$

$2 = \ln\left(\frac{t^2+15}{15}\right)$

$e^2 = \frac{t^2+15}{15}$ → ✓

$15e^2 = t^2+15$

$t = \sqrt{15e^2-15}, \quad t>0$ 1 answer

$= 9.789578208$ ∴ ie 9hr 47 mins after 2pm

The cellar will be filled with water at 11.47pm ✓ on Sat. (to nearest minute)

(must give time)

(iii) 6pm Sat ie $t=4$.

$V = 12 \ln\left(\frac{t^2+15}{15}\right)$

$= 12 \ln\left(\frac{16+15}{15}\right)$

$= 12 \ln\left(\frac{31}{15}\right)$ ✓ Except any correct rounding off

(iv) $\frac{dV}{dt} = \frac{t^2}{k}$

$V = \frac{1}{k}\int t^2\, dt$

$= \frac{t^3}{3k} + c$

when $t=0$, $V = 8.711244$ $\left(V = 12 \ln\frac{31}{15}\right)$

$8.711244 \left(12 \ln \frac{31}{15}\right) = 0 + c$

∴ $V = \frac{t^3}{3k} + 8.711244 \; \left(12 \ln\frac{31}{15}\right)$ → (1)

at 8pm ie $t=2$, $V=0$

$0 = \frac{8}{3k} + 8.711244 \; \left(12 \ln\frac{31}{15}\right)$

$k = \frac{8}{3 \times -8.711244}$

$= -0.3061$ (4 sig figs)