JG
AW
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GB
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Name: ________

Class: 12MT2__ or 12MTX__

Teacher: ____________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2008 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 10.

***Each page must show your name and your class. ***

Question 1 **Marks**

(a) Evaluate $2\pi^2 + 3e$, correct to 3 significant figures. **2**

(b) Simplify $2\sqrt{75} - 3\sqrt{3}$. **1**

(c) Factorise $x^3 + 27y^3$. **1**

(d) Solve for x, $3^x \times 2^x = 1$. **1**

(e) Find the exact value of $\tan(-\frac{2\pi}{3})$. **1**

(f) Find the primitive function of $\frac{1}{2x^3}$. **1**

(g) Solve for x

(i) $(4-x)(2x+3) < 0$ **1**

(ii) $\frac{2x+1}{5} - \frac{x-1}{3} = 1$ **2**

(h) Given $\log_a 7 = x$ and $\log_a 3 = y$, find an expression for $\log_a 63$ in terms of x and y. **2**

Question 2 **BEGIN A NEW PAGE** **Marks**

(a) Differentiate with respect to x

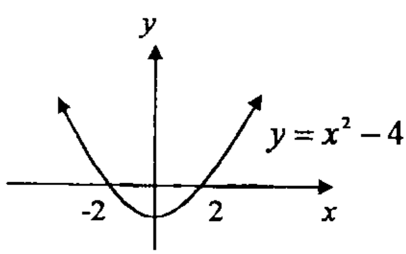
(i) $\dfrac{x+1}{e^x}$ **2**

(ii) $\tan^3 x$ **1**

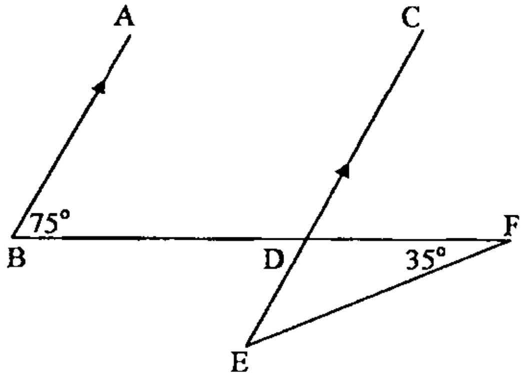
(b) Evaluate $\displaystyle\int_1^2 \frac{3x}{x^2+1}dx$ and simplify your answer. **3**

(c) Given that $f(x)=2x^3-3x^2+1$, find $f''(2)$. **2**

(d) Find the area bounded by the curve $y=x^2-4$ and the x-axis. **2**



(e) In the diagram below, $AB \parallel CE$, $\angle ABF = 75^0$ and $\angle BFE = 35^0$. **2**
Find the size of $\angle DEF$ giving reasons.



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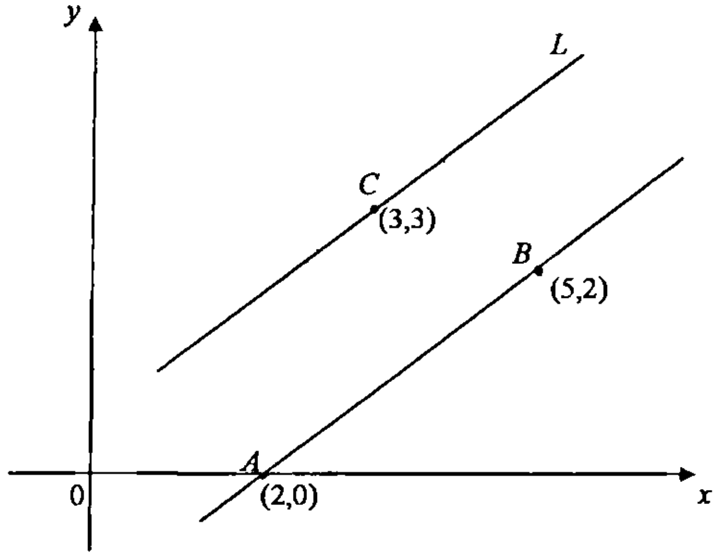
Question 3 **BEGIN A NEW PAGE** **Marks**

(a)



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In the diagram above the points $A(2,0)$, $B(5,2)$ and $C(3,3)$ are shown.
Copy the diagram onto your answer sheet.

(i) Find the exact length of AB. **1**

(ii) Show that the equation of AB is $2x-3y-4=0$. **2**

(iii) Find the exact perpendicular distance from C to AB. **2**

(iv) The line L passing through C has equation $2x-3y+3=0$. **1**
Show that L is parallel to AB.

(v) D is a point on L such that the length of DC is $\frac{\sqrt{13}}{2}$ units.
What type of quadrilateral is $ABCD$? Give reasons. **1**

(vi) Calculate the area of $ABCD$. **1**

Question 3 continued on page 4........

(b) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330° at a speed of 180 km/h and Quentin flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.

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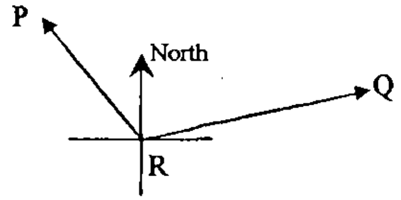


(i) How far apart are Peta and Quentin after 2 hours? (Answer correct to 1 decimal place). **2**

(ii) Find the bearing of Quentin from Peta after 2 hours. (Answer correct to the nearest degree). **2**

Question 4 **BEGIN A NEW PAGE**

(a) Find the equation of the tangent to the curve $y = e^{3x+1}$ at the point $(0, e)$. **2**

(b) (i) Find the derivative of $x\ln x - x$. **1**

(ii) Hence evaluate $\int_1^2 \ln x\,dx$. **2**

(c) (i) Sketch $y = 2e^{-x}$. **1**

(ii) Find the volume when the area bounded by curve $y = 2e^{-x}$, $x = 0$, $x = 1$ and the x-axis is rotated about the x-axis. **2**

(d) The velocity of a particle moving in a straight line at time t seconds is given by $v = \dfrac{4}{2t+1} ms^{-1}$.

Initially the particle is 2 metres to the left of the origin.

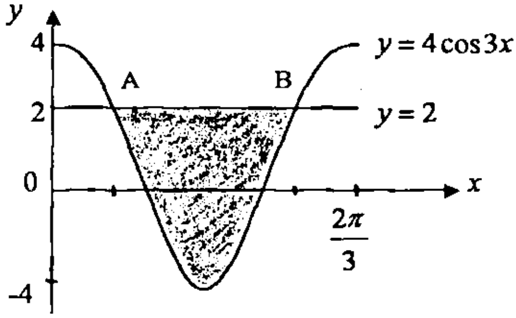
(i) Find an expression for the displacement x metres at time t seconds. **2**

(ii) Determine the velocity of the particle as it passes through the origin. **2**

Question 5 **BEGIN A NEW PAGE** **Marks**

(a) Find $\int \cot x dx$. **1**

(b) The diagram shows the graphs of $y = 4\cos 3x$ and $y = 2$, $0 \le x \le \frac{2\pi}{3}$.

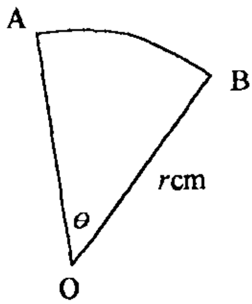


(i) Determine the x-coordinates of the two points of intersection A and B. **2**

(ii) Determine the shaded area. **3**

(c) Solve for x, $2\sin^2 x = 1 + \sin x$, where $0 \le x \le 2\pi$. **3**

(d) The sector OAB has an area of π cm^2. **3**



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The arc AB has length $\frac{\pi}{2}cm$.

Find the exact values of r and θ.

Question 6 **BEGIN A NEW PAGE** **Marks**

(a) A curve has the equation $y = 12x^2 - x^3$

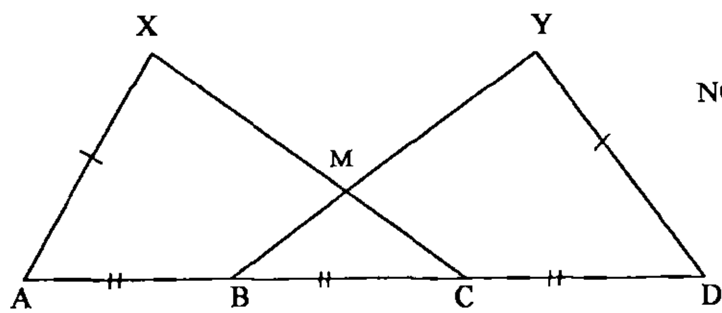
(i) Find the coordinates of the stationary points and determine their nature. **3**

(ii) Show that the coordinates of the point of inflexion is the midpoint of the stationary points. **3**

(iii) Sketch the curve $y = 12x^2 - x^3$ showing the stationary points and the x-intercepts. **1**

(iv) For what values of x is $y = 12x^2 - x^3$ concave down and increasing? **1**

(b) The diagram shows two triangles XAC and YDB, with $AX = DY$, and $\angle XAC = \angle YDB$. The lines XC and YB intersect at M, and the points A, B, C and D are collinear. The lengths AB, BC, CD are equal.



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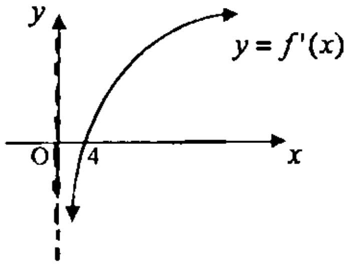
Copy the diagram onto your solution sheet.

(i) Prove that $\Delta XAC \equiv \Delta YDB$. **2**

(ii) Prove that XM=YM. **2**

Question 7 **BEGIN A NEW PAGE** **Marks**

(a) The sketch of the curve $y = f'(x)$ is given below. 2



Sketch the curve $y = f(x)$, given $f(4) = 0$.

(b) A plant nursery has a watering system which repeatedly fills a storage tank then empties its contents to water different sections of the nursery.

The volume of water (in cubic metres) in the tank at a time t is given by the equation

$V = 2 - \sqrt{3}\cos t - \sin t$, where t is measured in minutes.

(i) Give an equation for $\frac{dV}{dt}$, the rate of change of the volume at a time t. 1

(ii) Is the tank initially filling or emptying? 1

(iii) At what time does the tank first become completely full and what is its capacity when full? 3

(c) Gold is extracted from a mine at a rate that is proportional to the amount of gold remaining in the mine. Hence the amount M remaining after t years is given by

$$M = M_0 e^{-kt},$$

where k is a constant and M_0 is the initial amount of gold. After 10 years, 50% of the original amount of gold remains.

(i) Show that $M = M_0 e^{-kt}$ satisfies the equation $\frac{dM}{dT} = -kM$. 1

(ii) Find the value of k correct to 4 significant figures. 2

(iii) How many more years will elapse before only 20% of the original amount of gold remains? 2

Question 8 **BEGIN A NEW PAGE** **Marks**

(a) Evaluate $\sum_{n=2}^{\infty}\left(\frac{1}{\sqrt{3}}\right)^{n}$ **2**

(b) During the drought of the last few years, the water level in the local dam in the township of Wattamatta was reduced to $2\cdot5\%$ of its capacity.

In the first week of drought breaking rain, the inflow added 3% of capacity to the amount of water in the dam (i.e. the dam was $5\cdot5\%$ full).

In the next week the inflow added $3\cdot5\%$ of capacity to the amount of water in the dam.

In the third week 4% of capacity was added.

This pattern continued so that each week an extra $0\cdot5\%$ of capacity was added to the dam until it was full.

(i) What percentage of capacity was added to the dam in the 10^{th} week? **1**

(ii) What percentage of capacity was in the dam after 10 weeks? **2**

(iii) How many weeks would it have taken to fill the dam? **2**

(c) John has just started work and he is very keen to buy the latest 'Aussie Ute'. The price of the ute is $24000.

(i) John decides to borrow the money to buy the ute. To repay the money he will make monthly payments of $600 for 4 years. Calculate the total amount John will pay for the ute. **1**

(ii) John's father told him that the ute would lose 20% of its previous year's value each year. Show that the ute will lose $8640 in value during the first two years. **1**

(iii) John's father recommended that, instead of borrowing money to buy the ute, at the end of each month, John should deposit $600 into a special savings account that pays compound interest monthly at the rate of 6%p.a. **2**

John's father said that at the end of two years, John could buy a two year old ute.

Let A_n= the value of the account at the end of n months.

Show that if John invests $600 at the end of each month at 6% p.a compounded monthly then the amount in the savings account at the end of n months is given by

$$A_n = 120000(1\cdot005^n - 1).$$

(iv) If John decides to follow this savings plan for two years, will he have sufficient funds to buy a two year old ute? Use calculations to justify your answer. **1**

Question 9 **BEGIN A NEW PAGE** **Marks**

(a) The equation of a parabola is $2y = x^2 - 4x + 6$.

(i) Find the coordinates of the vertex V and the focus F. **2**

(ii) Find the equation of the directrix. **1**

(iii) Draw a neat sketch of the graph of this parabola showing the information obtained in (i) and (ii). **1**

(b) The quadratic equation $x^2 + Lx + M = 0$ has one root twice the other.

(i) Prove that $M = \frac{2L^2}{9}$. **2**

(ii) Prove the roots are rational whenever L is rational. **1**

(c) (i) Copy and complete the following table for $y = \sqrt{9 - x^2}$, expressing your answers correct to 2 decimal places. **1**

x	0	0·75	1·5	2.25	3
y					

(ii) Using Simpson's Rule and the five function values in the table above, find an approximation for the area bounded by $y = \sqrt{9 - x^2}$ and the x and y axes in the first quadrant. **2**

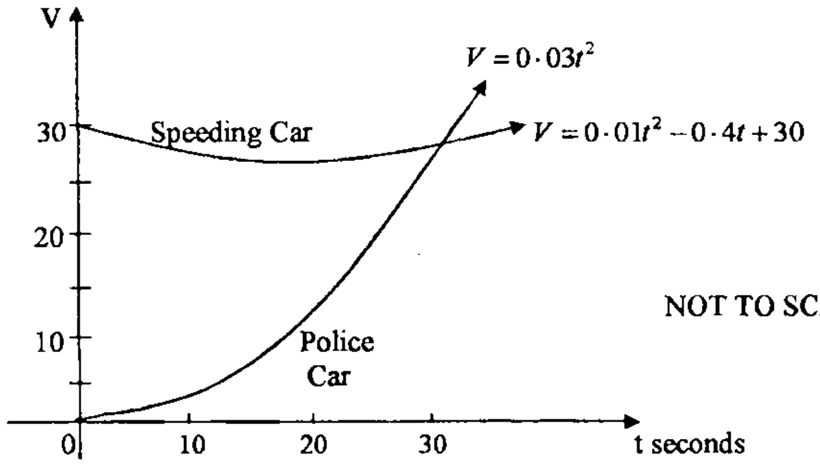
(iii) Calculate the exact area described in c(ii). **1**

(iv) Use your results from c(ii) and (iii) to find an approximation for π. **1**

Question 10 **BEGIN A NEW PAGE** **Marks**

(a) A speeding car travelling at a speed of 30ms^{-1} passes a police car waiting on the side of a straight road. Immediately, the police car starts to chase the speeding car reaching the same speed as the car in 30 seconds.

The velocity time graph shows the speeds of both cars.



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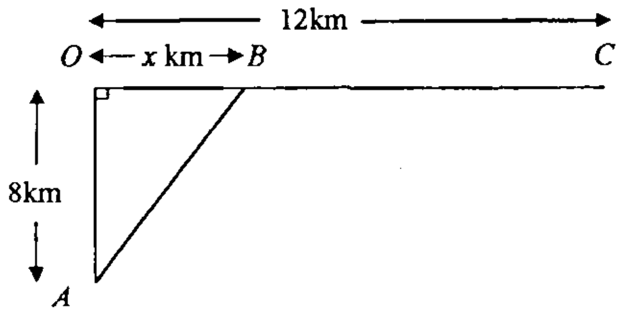
(i) How far is the police car behind the speeding car after 30 seconds? **3**

(ii) After how many seconds does the police car reach the speeding car? (Give your answer correct to 1 decimal place.) **2**

Question 10 continued on page 11........

(b) The diagram below represents two roads, AO and OC which meet at right angles at O. A man decides to walk from A through the forest and meet the road at B, and then continue along the road to C.



His walking speed through the forest is 3km/h and along the road 6km/h. OA =8 km, OC=12 km. Let OB=x km.

(i) Show that the time t hours taken for the journey is given by **2**

$$t = \frac{2\sqrt{x^2+64}+12-x}{6}.$$

(ii) Find the value of x such that the time taken for the journey is a minimum . (Give your answer correct to 3 significant figures.) **4**

(iii) Find the minimum time for the journey. (Give your answer correct to the nearest minute.) **1**

END OF EXAM

Final

Question 1

(a) $2\pi^2+3e = 27.89405\ldots$ ✓

$= 27.9$ (3 sig. fig.) ✓

(chosen question for rounding)

(b) $2\sqrt{75} - 3\sqrt{3} = 10\sqrt{3} - 3\sqrt{3}$

$= 7\sqrt{3}$ ✓

(c) $x^3 + 27y^3 = (x+3y)(x^2-3xy+9y^2)$ ✓

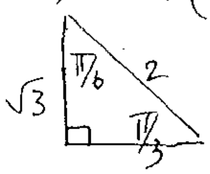
(d) $3^x \times 2^x = 1$

$6^x = 1$

$x = 0$ ✓

(e) $\tan\left(-\frac{2\pi}{3}\right) = \tan\frac{\pi}{3}$

$= \sqrt{3}$ ✓



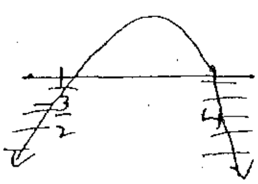
(quad 3)

(f) $f(x) = \frac{1}{2x^3}$

$= \frac{1}{2}x^{-3}$ ✓

$F(x) = -\frac{1}{4}x^{-2} + c$ or $-\frac{1}{4x^2} + c$

(g) (i) $(4-x)(2x+3) < 0$



$\therefore x < -\frac{3}{2}$ or $x > 4$ ✓

(ii) $\frac{2x+1}{5} - \frac{x-1}{3} = 1$

$\frac{15(2x+1)}{5} - \frac{15(x-1)}{3} = 1 \times 15$ ✓

$6x+3-5x+5 = 15$

$x + 8 = 15$

$x = 7$ ✓

OR

2nd method

$\frac{2x+1}{5} - \frac{x-1}{3} = 1$

$\frac{3(2x+1)-5(x-1)}{15} = 1$

$6x+3-5x+5 = 15$ ✓

$x + 8 = 15$

$\therefore x = 7$ ✓

(h) $\log_a 63 = \log_a 7 + \log_a 9$

$= \log_a 7 + 2\log_a 3$ ✓

$= x + 2y$ ✓

12 MARKS

~~Question 2~~

(a)(i) $f'(x) = \dfrac{e^x \times 1 - (x+1) \times e^x}{(e^x)^2}$ ✓

$= \dfrac{e^x - xe^x - e^x}{e^{2x}}$

$= \dfrac{-xe^x}{e^{2x}}$

$= \dfrac{-x}{e^x}$ ✓ or $-xe^{-x}$

or

2nd method (via product)

$f(x) = (x+1)e^{-x}$

$f'(x) = (x+1)\cdot -e^{-x} + e^{-x}\cdot 1$ ✓

$= -xe^{-x} - e^{-x} + e^{-x}$

$= -xe^{-x}$ ✓

(ii) $f'(x) = 3\tan^2 x \sec^2 x$ ✓

b) $\int_1^2 \dfrac{3x}{x^2+1}\,dx = \dfrac{3}{2}\int_1^2 \dfrac{2x}{x^2+1}\,dx$

(for primitive function) → ✓ $= \dfrac{3}{2}\left[\log_e(x^2+1)\right]_1^2$

$= \dfrac{3}{2}\left[\log_e 5 - \log_e 2\right]$ ✓

$= \dfrac{3}{2}\log_e \dfrac{5}{2}$ ✓

(must have exact form for 3rd mark)

c) $f'(x) = 6x^2 - 6x$

$f''(x) = 12x - 6$ ✓

$f''(2) = 24 - 6$

$= 18$ ✓

d) $\text{Area} = \left|\int_{-2}^{2}(x^2-4)\,dx\right|$

$= \left|\left[\dfrac{x^3}{3} - 4x\right]_{-2}^{2}\right|$ ✓

$= \left|\left(\dfrac{8}{3} - 8\right) - \left(\dfrac{-8}{3} + 8\right)\right|$

$= \left|-10\tfrac{2}{3}\right|$

2nd method

$\text{Area} = \left|2\int_0^2 (x^2-4)\,dx\right|$

$= \left|2\left[\dfrac{x^3}{3} - 4x\right]_0^2\right|$ ✓

$= \left|2\left[\left(\dfrac{8}{3} - 8\right) - 0\right]\right|$

$= \left|-10\tfrac{2}{3}\right|$

$= 10\tfrac{2}{3}$ units^2 ✓

(e) ✓ $\angle ABD = \angle CDF$ (corresponding $\angle$'s $=$, $AB \parallel CD$)
$= 75°$

✓ $\angle DEF + \angle DFE = \angle CDF$ (exterior $\angle$ of a Δ theorem)

$\angle DEF = 75 - 35$
$= 40°$

OR

✓ $\angle ABD = \angle BDE$ (alternate $\angle$'s $=$, $AB \parallel CE$)
$= 75°$

✓ $\angle DEF + \angle DFE = \angle BDE$ (exterior $\angle\Delta$ theorem)

$\angle DEF + 35 = 75$

$\angle DEF = 40$

OR

✓ $\angle ABD + \angle CDB = 180$ (cointerior angles are supplementary, $AB \parallel CE$)

$\angle CDB = 180 - 75$
$= 105°$

$\angle CDB = \angle EDF$ (vertically opposite $\angle$'s are $=$)
$= 105°$

✓ In ΔDEF

$\angle DEF = 180 - 35 - 105$ (angle sum Δ is $180°$)
$= 40°$

12 MARKS

students will use angle sum Δ + straight angle or angle sum Δ + adjacent supplementary angles instead of exterior $\angle$ Δ thm ...

Question 3

(a) (i) $AB = \sqrt{(5-2)^2 + (2-0)^2}$ (or Pythagoras' theorem)

$= \sqrt{13}$ units ✓

(ii) $m_{AB} = \frac{2}{3}$ (from diagram using $\frac{rise}{run}$) ✓

$y - 0 = \frac{2}{3}(x-2)$

$3y = 2x - 4$

$2x - 3y - 4 = 0$ ✓

(iii) $p = \left| \frac{Ax_1 + By_1 + c}{\sqrt{A^2 + B^2}} \right|$

$= \left| \frac{2 \times 3 - 3 \times 3 - 4}{\sqrt{2^2 + (-3)^2}} \right|$ ✓

$= \left| \frac{6 - 9 - 4}{\sqrt{13}} \right|$

$= \left| \frac{-7}{\sqrt{13}} \right|$

$= \frac{7\sqrt{13}}{13}$ units ✓ accept $\frac{7}{\sqrt{13}}$

(iv) $2x - 3y + 3 = 0$

$3y = 2x + 3$

$y = \frac{2}{3}x + 1$

slope of line L is $\frac{2}{3}$

$m_{\text{line } L} = m_{AB} = \frac{2}{3}$ ✓

$\therefore$ Line L is $\parallel AB$

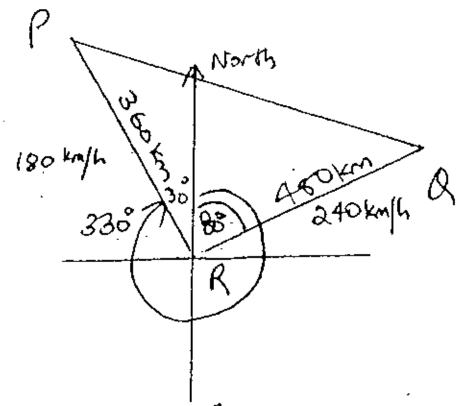
(v) ✓ Trapezium
DC $\parallel$ AB and DC $\neq$ AB

(vi) Area $= \frac{1}{2} \times \frac{7\sqrt{13}}{13}\left[\frac{\sqrt{13}}{2} + \sqrt{13}\right]$

$= \frac{1}{2} \times \frac{7\sqrt{13}}{13} \times \frac{3\sqrt{13}}{2}$

$= \frac{21}{4}$ units^2 ✓

(b)



(i)

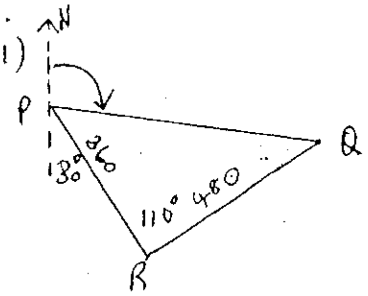
$PQ^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \times \cos 110°$ ✓

$= 478202.1615...$ ✓

$PQ = 691.5216...$

$= 691.5$ km (1 dec pl)

(ii)



$\frac{\sin \angle QPR}{480} = \frac{\sin 110°}{PQ}$

$\sin \angle QPR = \frac{480 \sin 110°}{PQ}$ (calculator readout used)

✓ $\angle QPR = 40.7122...$

$\angle QPR = 41°$ (nearest degree)

Solutions using 691.5 km get $\angle QPR = 40.71381...$ $= 41°$ (nearest degree)

$\therefore$ Bearing is 109° T or S 71° E ✓

12 MARKS

Question 4

a) $y = e^{3x+1}$

$\frac{dy}{dx} = 3e^{3x+1}$ ✓

Slope of tangent at $x = 0$ $= 3e^1$

∴ eqn of tangent is

$y - e = 3e(x - 0)$ ✓

$y = 3ex + e$

(b)(i) $\frac{d}{dx}\left[x\ln x - x\right]$

$= x \cdot \frac{1}{x} + \ln x \cdot 1 - 1$

$= \ln x$ ✓

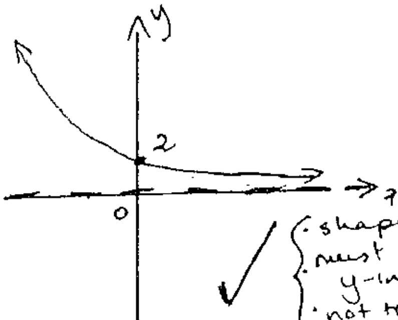
(ii) $\int_1^2 \ln x = \left[x \ln x - x\right]_1^2$ ✓

$= (2\ln 2 - 2) - (\ln 1 - 1)$

$= 2\ln 2 - 1$ ✓

(accept decimal 0.386)

(c) (i)



✓ { · shape · must show y-int of 2 · not touch x-axis

(ii)

$V = \pi \int_0^1 (2e^{-x})^2 \, dx$ ✓

$= \pi \int_0^1 4e^{-2x} \, dx$

$= \pi \left[-2e^{-2x}\right]_0^1$

$= \pi \left[(-2e^{-2}) - (-2e^0)\right]$

$= \pi \left[2 - \frac{2}{e^2}\right]$ units^3 ✓

(accept decimal 5.43)

(d) $v = \frac{4}{2t+1}$

(i) $x = \int \frac{4}{2t+1} \, dt$

$x = 2\int \frac{2}{2t+1} \, dt$

$x = 2\log_e(2t+1) + c$ ✓

$t = 0,\ x = -2$

$-2 = 2\log_e 1 + c$

$c = -2$

$\therefore x = 2\log_e(2t+1) - 2$ ✓

(ii) $x = 0 \quad v = ?$

find t

$0 = 2\log_e(2t+1) - 2$

$2\log_e(2t+1) = 2$

$\log_e(2t+1) = 1$

$2t + 1 = e$

$t = \frac{e-1}{2}$ ✓

$v = \frac{4}{2\left(\frac{e-1}{2}\right) + 1}$

$v = \frac{4}{e - 1 + 1}$

$v = \frac{4}{e}$ (or $4e^{-1}$) ✓

∴ velocity is $\frac{4}{e}$ ms^{-1} when it passes through the origin

12 Marks

Question 5

(a) $\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx$ ✓

$= \log_e(\sin x) + c$

(b) (i) $4\cos 3x = 2$

$\cos 3x = \frac{1}{2}$

$3x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \ldots$

$= \frac{\pi}{9}, \frac{5\pi}{9}, \ldots$

x coordinate of A is $\frac{\pi}{9}$ ✓

x coordinate of B is $\frac{5\pi}{9}$ ✓

(ii) Area $= \int_{\pi/9}^{5\pi/9} (2 - 4\cos 3x) \, dx$ ✓

$= \left[2x - \frac{4}{3}\sin 3x\right]_{\pi/9}^{5\pi/9}$

$= \left(\frac{10\pi}{9} - \frac{4}{3}\sin\frac{5\pi}{3}\right) - \left(\frac{2\pi}{9} - \frac{4}{3}\sin\frac{\pi}{3}\right)$ ✓

$= \left(\frac{10\pi}{9} + \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\right) - \left(\frac{2\pi}{9} - \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\right)$

$= \left(\frac{8\pi}{9} + \frac{4\sqrt{3}}{3}\right)$ units^2 ✓

(c) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$ ✓

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, \frac{\pi}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$ ✓ ✓

↑ any 2 solns ↑ 3rd solution

(d) $\frac{1}{2}r^2\theta = \pi$ ——— ①

$r\theta = \frac{\pi}{2}$ ——— ② ✓

$A = \pi$

$\frac{1}{2}r^2 \cdot \frac{\pi}{2r} = \pi$

$r = 4$ ✓

Sub into eqn ②

$\theta = \frac{\pi}{8}$ ✓

$\therefore r = 4, \theta = \frac{\pi}{8}$

12 Marks

Question 6

(a)(i) $y = 12x^2 - x^3$

$\frac{dy}{dx} = 24x - 3x^2$

$24x - 3x^2 = 0$

$3x(8 - x) = 0$ ✓

$x = 0$ or $x = 8$

$x=0, y=0$ ... $x=8, y=256$

$\frac{d^2y}{dx^2} = 24 - 6x$

$x = 0$, $\frac{d^2y}{dx^2} = 24 - 0 = 24 > 0$

∴ Relative minimum turning point at $(0,0)$ ✓

$x = 8$, $\frac{d^2y}{dx^2} = 24 - 6 \times 8 = -24 < 0$

∴ Relative maximum turning point at $(8, 256)$ ✓

(ii) $\frac{d^2y}{dx^2} = 24 - 6x$

$24 - 6x = 0$

$x = 4$

$x = 4$, $y = 128$ ✓

x	3	4	5
$\frac{d^2y}{dx^2}$	6	0	−6
	+		−

✓

change in sign

∴ change in concavity

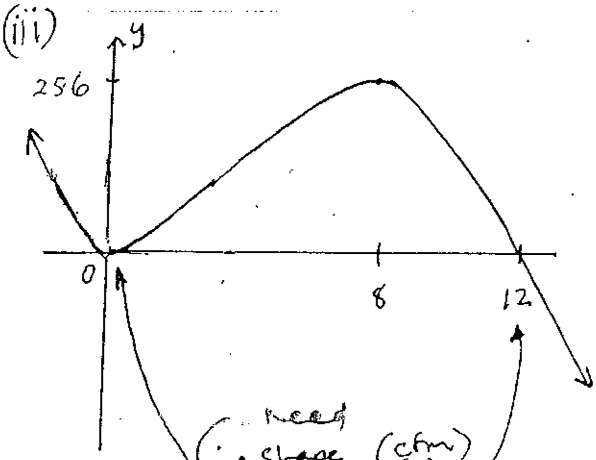
∴ P.O.I at $(4, 128)$

midpoint of stationary points

$\left(\frac{0+8}{2}, \frac{0+256}{2}\right) = (4, 128)$ ✓

which is the POI

(iii)



need
- shape (cfm (a))
- intercepts ✓

(iv) $4 < x < 8$ ✓

(b)(i) In $\triangle XAC$, YDB

$AB + BC = BC + CD$ (given $AB = BC = CD$)

∴ $AC = BD$ ✓

$\angle XAC = \angle YDB$ (given)

$XA = YD$ (given)

∴ $\triangle XAC \equiv \triangle YDB$ (SAS test) ✓

(ii) $\angle YBD = \angle XCA$ (corresponding ∠'s of congruent △'s)

$MC = MB$ (In any △, ∠'s opposite = sides are =) ✓

alternate reason involving proving isosceles △ then = sides

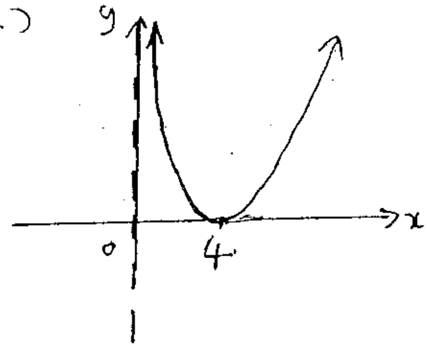
$XC = YB$ (corresponding sides of congruent △s)

$XM = YM$ ($XC - MC = YB - MB$) ✓

12 Marks

Question 7

(a)



✓ concave up and correct position

✓ x-intercept 4 and graph approaching asymptote

(b) $V = 2 - \sqrt{3}\cos t - \sin t$

(i) $\frac{dV}{dt} = \sqrt{3}\sin t - \cos t$ ✓

(ii) $t = 0$

$\frac{dV}{dt} = \sqrt{3}\sin 0 - \cos 0$
$= -1$
< 0 ∴ emptying ✓

(iii) $\frac{dV}{dt} = 0$

$\sqrt{3}\sin t = \cos t$

$\tan t = \frac{1}{\sqrt{3}}$

$t = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \ldots$ ✓

as tank initially emptying, it will be full when $t = \frac{7\pi}{6}$ minutes ✓ (or show $V = 0$ when $t = \frac{\pi}{6}$)

$V = 2 - \sqrt{3}\cos\frac{7\pi}{6} - \sin\frac{7\pi}{6}$

$= 2 - \sqrt{3} \cdot -\frac{\sqrt{3}}{2} - -\frac{1}{2}$

$= 2 + \frac{3}{2} + \frac{1}{2}$ ✓

Volume $= 4\,m^3$

(c) (i) $M = M_0 e^{-kt}$

$\frac{dM}{dt} = -k M_0 e^{-kt}$ ✓
$= -kM$

(ii) $\frac{1}{2}M_0 = M_0 e^{-10k}$

$e^{-10k} = \frac{1}{2}$ ✓

$\log_e \frac{1}{2} = -10k$

$k = -\frac{1}{10}\log_e \frac{1}{2}$

$= 0.069314\ldots$ ✓

(iii) $0.2M_0 = M_0 e^{-kt}$

$e^{-kt} = 0.2$

$-kt = \ln 0.2$

$t = -\frac{1}{k}\ln 0.2$ ✓

$t = 23.21928\ldots$

∴ It will take another 13.2 years before 20% remains $(23.2 - 10)$ ✓

12 Marks

Question 8

(a) $\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^3+\left(\frac{1}{\sqrt{3}}\right)^4+\dots$

$= \frac{1}{3}+\frac{1}{3\sqrt{3}}+\frac{1}{9}+\dots$

G.P $a=\frac{1}{3}$, $r=\frac{1}{\sqrt{3}}$ ✓

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{3}}{1-\frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}}{3\sqrt{3}-3}$ or equivalent ✓

$= \frac{\sqrt{3}(3\sqrt{3}+3)}{(3\sqrt{3}-3)(3\sqrt{3}+3)}$

$= \frac{3+\sqrt{3}}{6}$

(b) i) pattern for water added

$3+3.5+4+\dots$

arithmetic series, $a=3$, $d=0.5$

$T_{10} = 3+(10-1)0.5$

$= 7.5\%$ added ✓

(might do T_{11} with $a=2.5$ OK)

ii) Inflow $= S_{10}$

$S_{10}=\frac{10}{2}\left[2\times3+(10-1)0.5\right]$ ✓

$= 52.5\%$

Dam is 55% full after 10 weeks. ✓

(52.5% added + 2.5% initial)

iii) when full, 97.5% has been added

$97.5=\frac{n}{2}\left[2\times3+(n-1)0.5\right]$ ✓

$195 = 6n+0.5n^2-0.5n$

$n^2+11n-390=0$

$(n-15)(n+26)=0$

$n=15$ or $n=-26$

$n>0$

$\therefore n=15$ ✓

∴ It took 15 weeks to fill the dam

(c) (i) Cost via monthly payments $= \$600\times4\times12$ ✓

$= \$28\,800$ ✓

(ii) ✓ Depreciation $= 24\,000 - \left[24000\times(1-0.2)^2\right]$

$\$8640$; $\$15360$

(iii) forwards method

$A_1=600$

$A_2=600(1.005)+600$

$A_3=600(1.005)^2+600(1.005)+600$

(1st mark for demonstrating $A_1A_2A_3$ or see below) ✓

$A_n=600(1.005)^{n-1}+600(1.005)^{n-2}+600(1.005)^{n-3}+\dots+600$

$=600\left[(1.005)^{n-1}+(1.005)^{n-2}+(1.005)^{n-3}+\dots+600\right]$

$=600.1\cdot\frac{(1.005)^n-1}{1.005-1}$ ✓

$=120000\left((1.005)^n-1\right)$

as required

or in reverse

1st \$600 earns $600(1.005)^{n-1}$ (since invested at end of 1st month)

2nd \$600 earns $600(1.005)^{n-2}$

3rd \$600 earns $600(1.005)^{n-3}$ ✓

⋮

last \$600 earns \$600 (no interest on last \$600)

$A_n=600\left[\frac{1(1.005)^n-1}{1.005-1}\right]$ ✓

$=120000(1.005^n-1)$

(iv) $A_{24}=120000(1.005^{24}-1)$

$= \$15259.17$

As the ute costs \$15360 (from c(ii))

No, John will be \$100.83 short (accept \$100 or \$101) ✓

QUESTION 9

(a) $2y = x^2 - 4x + 6$

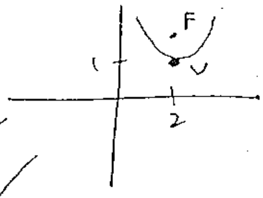
(i) $x^2 - 4x = 2y - 6$

$x^2 - 4x + 4 = 2y - 2$

$(x-2)^2 = 2(y-1)$

$4a = 2$

$a = \frac{1}{2}$



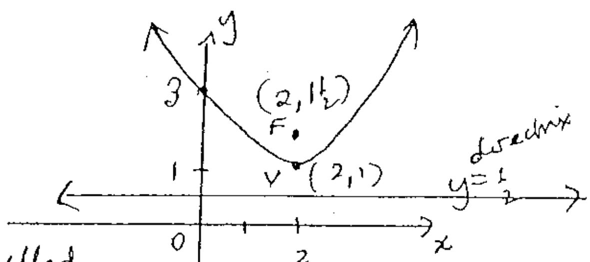
vertex $(2,1)$ ✓

focus $(2, \frac{3}{2})$ ✓

(ii) directrix eqn

$y = \frac{1}{2}$ ✓

(iii)



For mark
- shape
- F & V labelled with coordinates or no's on axes
- directrix with eqn / label.

✓

(b)(i) $\alpha + \beta = \frac{-b}{a} = -L$

$\alpha\beta = \frac{c}{a} = M$

$\alpha = 2\beta$

$\therefore 3\beta = -L$

$\beta = -\frac{L}{3}$ — ①

$2\beta^2 = M$ — ②

→ either ✓

Sub eqn ① into eqn ②

$2\left(-\frac{L}{3}\right)^2 = M$

$\therefore \frac{2L^2}{9} = M$

$M = \frac{2L^2}{9}$ ✓

(ii) To be rational, Δ is a perfect square

$\Delta = L^2 - 4 \times 1 \times M$

$= L^2 - 4 \times \frac{2L^2}{9}$

$= \frac{L^2}{9}$ ✓

If L is rational, $\frac{L^2}{9}$ is a perfect square

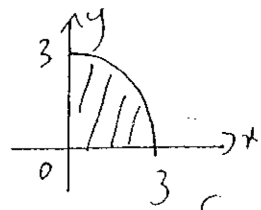
∴ roots are rational

(c) (i) $y = \sqrt{9 - x^2}$

x	0	0.75	1.5	2.25	3
y	3	2.90	2.60	1.98	0

✓

(ii)



✓

Area $\doteq \frac{0.75}{3}\left[3 + 4 \times 2.9 + 2 \times 2.6 + 4 \times 1.98 + 0\right]$

$= 6.93$ ✓

(iii) $A = \frac{1}{4} \times \pi \times 3^2$

$= \frac{9\pi}{4}$ units^2 ✓

(iv) $\frac{9\pi}{4} = 6.93$

$\pi = \frac{4}{9} \times 6.93$

$= 3.08$ ✓

12 Marks

Question 10

(i) Distance between cars = Distance travelled by speeding car − Distance travelled by police car

$= \int_0^{30} [(0.01t^2 - 0.4t + 30) - 0.03t^2] \, dt$ ✓

$= \int_0^{30} (-0.02t^2 - 0.4t + 30) \, dt$

$= \left[\frac{-0.02t^3}{3} - 0.2t^2 + 30t \right]_0^{30}$ ✓

$= \left(\frac{-0.02(30)^3}{3} - 0.2(30)^2 + 30^2 \right) - (0)$

$= -180 - 180 + 900$

$= 540$ m ✓

∴ The police car is 540 m behind the speeding car after 30 seconds

(ii) Let T seconds be the time when the police car reaches the speeding car and $T > 30$. When cars meet, area under each curve should be equal (equal distance travelled)

$\int_0^T 0.03t^2 \, dt = \int_0^T (0.01t^2 - 0.4t + 30) \, dt$

$\therefore \int_0^T (0.02t^2 + 0.4t - 30) \, dt = 0$

$\left[\frac{0.02t^3}{3} + 0.2t^2 - 30t \right]_0^T = 0$

$0.02T^3 + 0.6T^2 - 90T = 0$ ✓

$T^3 + 60T^2 - 9000T = 0$

$T^2 + 30T - 4500 = 0$

$T = \frac{-30 \pm \sqrt{900 - 4 \times 1 \times -4500}}{2}$

$= \frac{-30 \pm \sqrt{18900}}{2}$

$= 53.7386\ldots$ or $-83.73\ldots$

$t > 30$

$\therefore T = 53.7786\ldots$ ✓

∴ Police car reaches speeding car after 53.7 sec.

(b)(i) $AB^2 = OB^2 + OA^2$ (Pythagoras' thm)

$AB^2 = x^2 + 64$

$AB = \sqrt{x^2 + 64}$ km ✓

time = distance / speed

time taken

(ii) $t = \frac{\sqrt{x^2+64}}{3} + \frac{12 - x}{6}$ ✓

$= \frac{2\sqrt{x^2+64} + 12 - x}{6}$ as required

(ii) $\frac{dt}{dx} = \frac{\frac{1}{2}(x^2+64)^{-\frac{1}{2}} \times 2x}{3} - \frac{1}{6}$ ✓

$= \frac{x}{3\sqrt{x^2+64}} - \frac{1}{6}$

$\frac{dt}{dx} = 0$

$\frac{1}{6} = \frac{x}{3\sqrt{x^2+64}}$

$\sqrt{x^2+64} = 2x$ ✓

note
- $x^2 + 64 > 0$
- also $x > 0$
- $12 - x > 0$, $x < 12$

$x^2 + 64 = 4x^2$

$3x^2 = 64$

$x^2 = \frac{64}{3}$

$x = \pm\sqrt{\frac{64}{3}}$

$x = \frac{8}{\sqrt{3}}$ $(x > 0)$

$= 4.6188\ldots$

$= 4.62$ (3 sig. fig.)

} any for mark ✓

x	4	$\frac{8}{\sqrt{3}}$	5
dt/dx	-0.02	0	0.0099...
slope	− \	0 —	+ /

✓ or 2nd derivative test

∴ $x = 4.62$ for time to be minimum

(iii) $x = 4.62$ gives ↓ $t = 4.30940\ldots$ or 4 h 19 min

$x = \frac{8}{\sqrt{3}}$ gives ↓ $t = 4.30940\ldots$ or 4 h 19 min ✓