AW
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KP
FH

Name: ____________________

Class: 12MT2__ or 12MTX__

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2009 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 10.

***Each page must show your name and your class. ***

Question 1 (12 marks) **Marks**

(a) Expand and simplify $8x^2 + 2xy - 2x(4x + y)$. **2**

(b) Completely factorise $5x^2 - 20$ **2**

(c) Find the value of $\tan\frac{8\pi}{7}$ correct to three significant figures. **2**

(d) Find the values of x for which $|2 - 3x| > 11$. **2**

(e) Express $3\sqrt{32} - \sqrt{128}$ in simplest surd form. **2**

(f) If $k = \frac{1}{3}m\left(v^2 - u^2\right)$ find the value of m when $k = 1224$, $v = 14{\cdot}2$ and $u = 7{\cdot}4$. **2**

Question 2 (12 marks) (START A NEW PAGE) **Marks**

(a) Differentiate with respect to x.

(i) $(2x^2 + 1)^8$ **2**

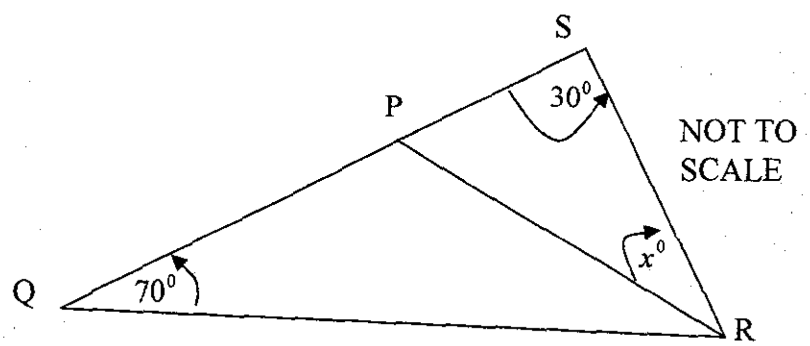
(ii) $x^2 e^{2x}$ **2**

(iii) $\dfrac{x}{\ln x}$ **2**

(b) (i) Find $\int (2x+1)^5 \, dx$ **2**

(ii) Evaluate $\int_0^{\frac{\pi}{8}} 4\cos 4x \, dx$ **2**

(c)



Given PQ = PR, $\angle PQR = 70^0$, $\angle PSR = 30^0$ and $\angle PRS = x^0$, **2**

find the value of x giving reasons.

Question 3 (12 marks) (START A NEW PAGE) **Marks**

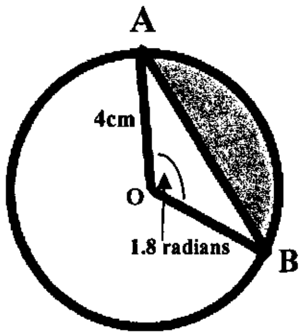
(a) Given the points A(1,2), B(3,1), C(-1,4)

(i) Find the equation of the line BC. **1**

(ii) Find the perpendicular distance from point A to the line BC. **2**

(iii) Hence, or otherwise, find the area of ΔABC. **2**

(b) A chord AB subtends an angle of $1\cdot 8$ radians at the centre of a circle of radius 4 cm. Find the area of the minor segment cut off by the chord AB.(Give your answer correct to 1 decimal place.) **2**



(c) For the function $y = 2\sqrt{9 - x^2}$

state the

(i) domain and **2**

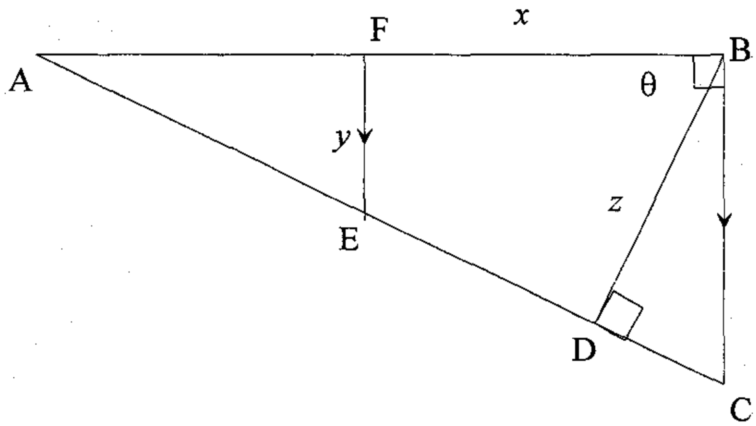
(ii) range. **1**

(d) Find the radius and centre of the circle with equation **2**

$$x^2 - 4x + y^2 + 6y + 12 = 0.$$

Question 4 **(12 Marks)** (START A NEW PAGE) **Marks**

(a) The right triangle ABC is shown below. $BC \parallel FE$, $BD \perp AC$, $\angle FBD = \theta$, $BF = x$, $EF = y$ and $BD = z$.



Prove

(i) $\Delta BDA \parallel\!| \Delta EFA$ **2**

Show that

(ii) $\angle FEA = \theta$ **1**

(iii) $AB = x + y\tan\theta$ **1**

(iv) $z = x\cos\theta + y\sin\theta$ **1**

(b) Find the equation of the normal to the curve $y = \sqrt{x}$ at the point (4,2). **2**

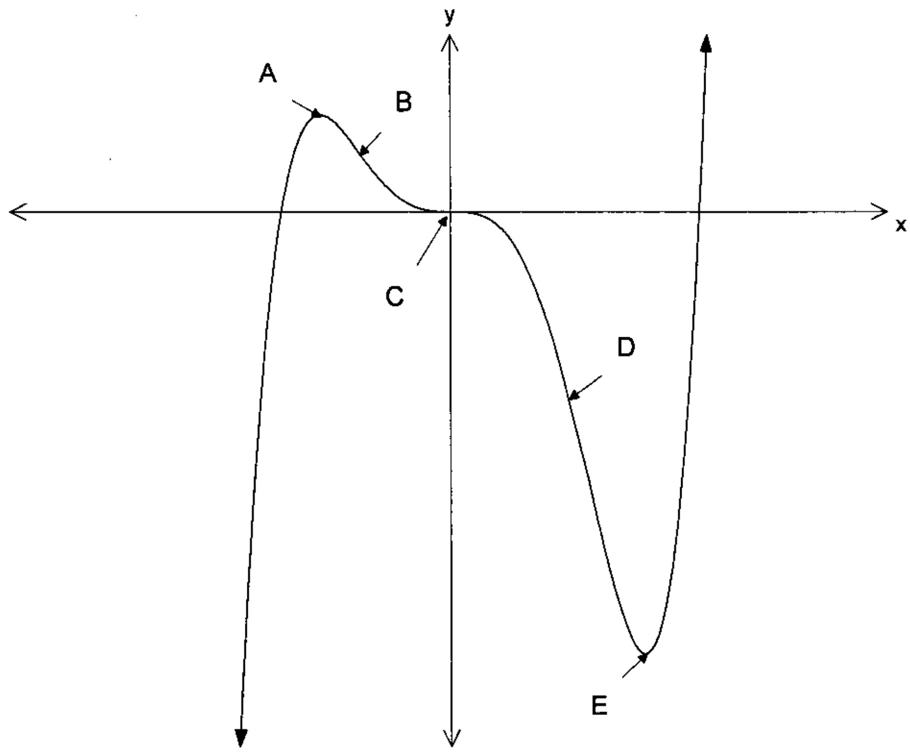
(c) A ship sails from port A, 60 nautical miles due west, to a port B. It then proceeds a distance of 50 nautical miles on a bearing of 210° to a port C.

(i) Draw a diagram to illustrate the information given. **1**

(ii) Find the distance (nearest nautical mile) and bearing of C from A. **4**

Question 5 **(12 Marks)** (START A NEW PAGE) **Marks**

(a) The graph of the curve $y = f(x)$ is drawn below.



(i) Name the points of inflexion. **1**

(ii) Name the stationary points. **1**

(iii) Sketch the gradient function. **1**

(b) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\frac{dV}{dt} = \frac{1}{100}\left(30t - t^2\right)$ where V cm^3 is the volume of gas produced after t minutes.

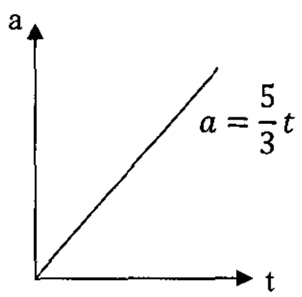
(i) At what rate is the gas being produced 15 minutes after the experiment begins? **1**

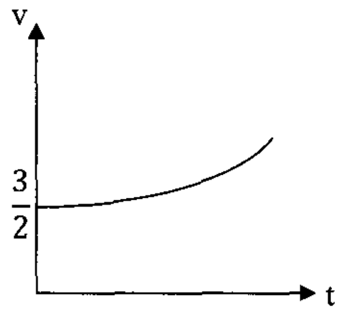
(ii) How much Carbon Dioxide has been produced during these 15 minutes? **2**

Question 5 is continued on page 6.....

Question 5 continued.... **Marks**

(c) The graphs below describe the flight of a sugar glider after it jumps from a tall tree. Assume that the displacement at time $t = 0$ is 0 metres.





(i) Show that the velocity of the sugar glider for $t \geq 0$ is given by $v = \frac{1}{6}(5t^2 + 9)$. **2**

(ii) Determine the expression for the displacement (x) for $t \geq 0$ and hence calculate the displacement of the sugar glider at $t = 2$. **2**

(d) The student population "C" at Cherrybrook Technology High School in the first 6 months of 2009 was increasing at a decreasing rate.
What does this tell you about

$\frac{dC}{dt}$ and $\frac{d^2C}{dt^2}$. **2**

Question 6 **(12 Marks)** (START A NEW PAGE) **Marks**

(a) For the curve $y = 2x^3 - 9x^2 + 12x$,

(i) Show that $\frac{dy}{dx} = 6(x-1)(x-2)$ **1**

(ii) Find the coordinates of the stationary points and determine their nature. **3**

(iii) Graph the function showing clearly the stationary points. **2**

(iv) Where is $y=f(x)$ decreasing? **1**

(b) Consider the parabola $y^2 = 8(x+2)$

(i) Find the coordinates of the vertex. **1**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

(c) Find the value(s) of m for which the equation $4x^2 - mx + 9 = 0$ has exactly one real root. **2**

Question 7 **(12 Marks)** (START A NEW PAGE) **Marks**

(a) The population, P, of a certain town grows at a rate proportional to the population, ie. $\frac{dP}{dt} = kP.$

If the population grows from 20 000 to 25 000 in two years, find:

(i) the value of k (the growth constant) correct to 4 significant figures. **2**

(ii) the population of the town, to the nearest hundred, after a <u>further</u> 8 years. **1**

(iii) Calculate the rate of change of the population at this time (ie after these further 8 years) correct to the nearest hundred. **1**

(b) A particle moves in a straight line so that its displacement, x metres from a fixed point O on a line, is given by

$$x = t + \frac{16}{t+1}$$

where t is measured in seconds.

(i) Find the particle's initial position **1**

(ii) Find expressions for the velocity and acceleration of the particle in terms of t. **2**

(iii) Find when and where the particle is at rest. **2**

(iv) Find the limiting velocity of the particle. **1**

(v) Find the total distance travelled by the particle in the first 4 seconds. **2**

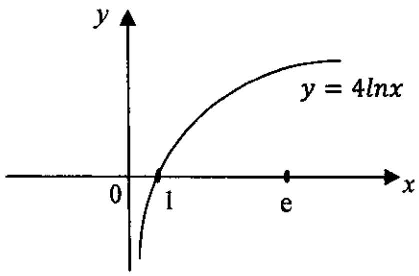
Question 8 **(12 Marks)** (START A NEW PAGE) **Marks**

(a) The gradient of a curve is given by $\dfrac{dy}{dx} = \dfrac{2x}{x^2 + e}$. **2**

The curve passes through the point (0,2).

What is the equation of the curve?

(b) The arc of the curve $y = 4\ln x$, between $x = 1$ and $x = e$, is rotated about the y-axis.



(i) Show that the volume formed is given by

$$V = \pi \int_0^4 e^{0.5y}\, dy$$ **2**

(ii) Hence, find the exact volume. **2**

(c) (i) Is the series $\log 3 + \log 9 + \log 27 + \ldots\ldots.$ arithmetic or geometric? **2**
Give reasons for your answer.

(ii) Find the sum of the first 10 terms of the series in exact form. **2**

(d) Use the trapezoidal rule with four function values. to find an approximate value of **2**

the area under the curve $y = 3^x$, bounded by the x axis, $x = 1$ and $x = 4$

Question 9 (12 marks) (START A NEW PAGE) **Marks**

(a) Solve $2\cos x + \sqrt{3} = 0$ for $-\pi \le x \le \pi$ **2**

(b) (i) Differentiate $e^{\tan x}$ **1**

(ii) Hence or otherwise, evaluate $\int_0^{\frac{\pi}{4}} 3\, e^{\tan x} \sec^2 x \; dx$ **2**

(c) The minute hand on a clock face is 12 centimetres long.

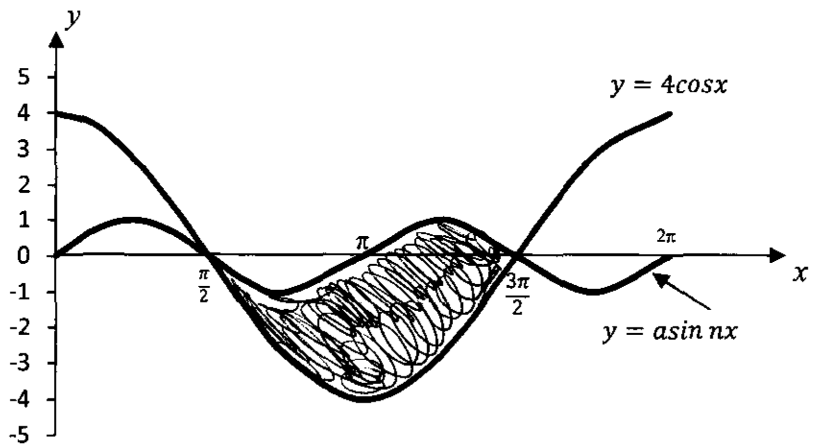
In 40 minutes,

(i) through what angle does the hand move (in radians)? **1**

(ii) how far does the tip of the hand move? **1**

(iii) what area does the hand sweep through in this time? **1**

(d)



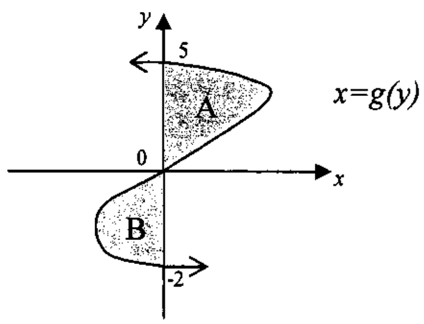
The graph above shows $y = 4cosx$ and $y = asin\, nx$.

(i) Find the values of a and n. **1**

(ii) Find the area of the shaded region in the diagram. **3**

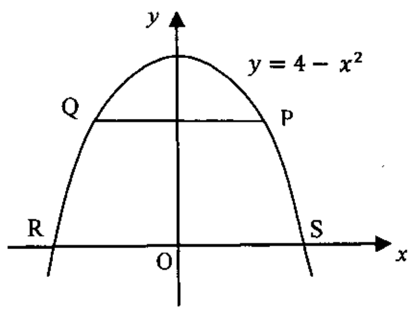
Question 10 (12 marks) (START A NEW PAGE) **Marks**

(a) The area A is equal to 7 square units and area B is equal to 9 square units. **1**



Evaluate $\int_{-2}^{5} g(y)\,dy$.

(b)



Not to scale

The parabola $y = 4 - x^2$ cuts the x-axis at R and S. The point P(x,y) lies on the parabola in the first quadrant. Q also lies on the parabola such that PQ is parallel to the x-axis.

(i) Write down the co-ordinates of R and S. **1**

(ii) Show that the area of trapezium PQRS is given by: **2**

$$A = (2 + x)(4 - x^2)$$

(iii) Hence, find the value of x which gives a maximum value of A, justifying your answer. **3**

Question 10 is continued on page 12.....

 Marks

(c) A small oil company has 50 000 barrels of oil in stock. In its first month of operation, the company increased its stock by 5%, and then sold 1500 barrels.

Each subsequent month, the company continued to increase its previous month's stock by 5%, and then also increased the number of barrels of oil sold by 3%.

(i) Show that the number of barrels of oil in stock after 3 months can be expressed as: **3**

$$B_3 = 50000 \times 1.05^3 - 1500 \times (1.05^2 + 1.05 \times 1.03 + 1.03^2)$$

(ii) Show that the number of barrels of oil in stock after n months can be expressed as: **2**

$$B_n = 50000 \times 1.05^n - 75000 \times 1.05^n \left(1 - \left(\frac{1.03}{1.05}\right)^n\right)$$

End of Examination.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ \ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax dx, = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}),\ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

YR12 MATHEMATICS TRIAL 2009

SOLUTIONS

Question 1

(a) $8x^2 + 2xy - 2x(4x+y)$

$= 8x^2 + 2xy - 8x^2 - 2xy$ ✓ ✓

$= 0$

(b) $5x^2 - 20 = 5(x^2 - 4)$ ✓

$= 5(x-2)(x+2)$ ✓

(c) $\tan \frac{8\pi}{7} = 0.4815746\ldots\ldots$ ✓

$= 0.482$ ✓

(d) $|2-3x| > 11$

$2-3x < -11$ or $2-3x > 11$ ✓

$-3x < -13$ or $-3x > 9$

$x > \frac{13}{3}$ or $x < -3$ ✓

(e) $3\sqrt{32} - \sqrt{128} = 12\sqrt{2} - 8\sqrt{2}$ ✓ or ✓

$= 4\sqrt{2}$ ✓

(f) $k = \frac{1}{3}m(v^2 - u^2)$

$1224 = \frac{1}{3}m(14.2^2 - 7.4^2)$ ✓

$1224 = 48.96m$

$m = \frac{1224}{48.96}$

$m = 25$ ✓

Question 2

(a) (i) $= 8(2x^2+1)^7 \times 4x$ ✓

$= 32x(2x^2+1)^7$ ✓ (award ② for answer only)

(ii) $x^2 \times 2e^{2x} + e^{2x} \times 2x$ ✓

$= 2x^2e^{2x} + 2xe^{2x}$ ✓

or $= 2xe^{2x}(x+1)$ } either (award ② answer only)

(iii) $\dfrac{\ln x \times 1 - x \times \frac{1}{x}}{(\ln x)^2}$ ✓

$= \dfrac{\ln x - 1}{(\ln x)^2}$ ✓ (do not accept $\ln x^2$; ie correct with $\ln x^2$ give ①) (award ② answer only)

(b) (i) $\int (2x+1)^5 dx = \dfrac{(2x+1)^6}{6 \times 2} + c$ ✓ ✓ (ignore missing +C)

$= \frac{1}{12}(2x+1)^6 + c$ (award ② answer only)

(ii) $\int_0^{\pi/8} 4\cos 4x \, dx$

$= \left[\sin 4x\right]_0^{\pi/8}$ ✓

$= \sin\frac{\pi}{2} - \sin 0$

$= 1$ ✓

(c) PQ = QR

$\therefore \angle PQR = \angle PRQ = 70^\circ$ (In any Δ, angles opposite = sides are =) ✓ → or ΔPQR is isosceles (PQ = PR) $\therefore \angle PQR = \angle PRQ = 70^\circ$ (equal ba... angles of isosceles Δ)

In ΔQSR

$70 + 70 + x + 30 = 180$ (angle sum Δ) ✓

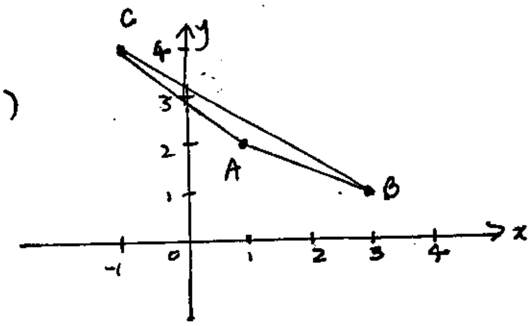
$\therefore x = 10$

(for 2nd mark → need to see =180 somewhere + words angle sum Δ then x=10)

(for 1st mark → most will do this way; need to see • isos because of = sides; • ∠PRQ = 70 (equal ∠s ...))

QUESTION 3

(a) (i)



slope BC $= \frac{-3}{4}$

equation BC

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-3}{4}(x+1)$

$4y - 16 = -3x - 3$

$3x + 4y - 13 = 0$ ✓ (or equivalent expression)

(ii) $p = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$ x_1, y_1 $A(1, 2)$

$= \left| \frac{3 \times 1 + 4 \times 2 - 13}{\sqrt{3^2 + 4^2}} \right|$ ✓

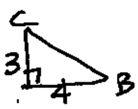
$= \left| \frac{3 + 8 - 13}{\sqrt{25}} \right|$

$= \left| \frac{-2}{5} \right|$

$= \frac{2}{5}$ units ✓

(correct but no absolute signs AW ①)

(iii) Find BC using Pythagoras' Thm or distance formula



BC = 5 units ✓

(Pythagorean Triad {3,4,5})

Area $= \frac{1}{2} \times \frac{2}{5} \times 5$

$= 1$ sq. unit ✓

(b) Area $= \frac{1}{2} r^2 (\theta - \sin\theta)$

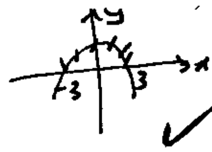
$= \frac{1}{2} \times 4^2 \times (1.8 - \sin 1.8)$ ✓

$= 6.60921\ldots$ ✓

$= 6.6 \text{ cm}^2$ (1 dec pl)

(c) (i) $9 - x^2 \geq 0$ ✓

$(3 - x)(3 + x) \geq 0$



$-3 \leq x \leq 3$ ✓

(or $x \geq -3$ and $x \leq 3$)

(ii) $0 \leq y \leq 6$ ✓

(or $y \geq 0$ and $y \leq 6$)

(d) $x^2 - 4x + y^2 + 6y + 12 = 0$

$x^2 - 4x + 4 + y^2 + 6y + 9 = -12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 1$ ✓

$\therefore$ radius = 1 unit; centre = $(2, -3)$ ✓

QUESTION 4

(a) (i) In ΔBDA, EFA

$\angle BDA = 90°$ (given $BD \perp AC$)

$\angle AFE = \angle ABC$ (corresponding angles =, $FE \parallel BC$) ✓

$= 90°$

- $\angle AFE = \angle BDA = 90°$ ✓
- $\angle BAD$ is common

$\therefore \Delta BDA \parallel\mid \Delta EFA$ (equiangular)

(ii) $\angle DBA = \angle FEA = \theta$ (corresponding angles of similar Δ's) ✓

(or might use angle sum and complementary $\angle$'s)

(iii) In ΔAFE

$\tan\theta = \dfrac{AF}{y}$

$AF = y\tan\theta$ ✓

$\therefore AB = y\tan\theta + x$

(iv) In ΔABD

$\cos\theta = \dfrac{z}{AB}$

$= \dfrac{z}{y\tan\theta + x}$

$z = y\tan\theta\cos\theta + x\cos\theta$ ✓

$= y\dfrac{\sin\theta}{\cos\theta}\cos\theta + x\cos\theta$

$= y\sin\theta + x\cos\theta$

(b) $\dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

when $x = 4$

slope of tangent $= \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$\therefore$ slope of normal $= -4$ ($m_1 m_2 = -1$) ✓

$x = 4$, $y = 2$

$y - 2 = -4(x - 4)$

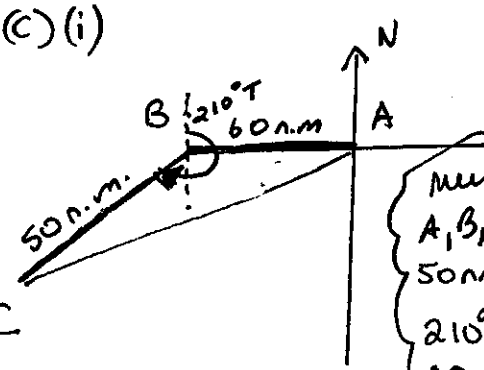
$y - 2 = -4x + 16$

$4x + y - 18 = 0$ ✓

or $y = -4x + 18$

or (equivalent eqn)

(c) (i)



Must have A, B, C, 50nm, 60nm, 210° at B or show $\angle ABC = 120°$ ✓

(d) $AC^2 = 60^2 + 50^2 - 2 \times 60 \times 50 \times \cos 120°$ ✓

$AC = 95.3939\ldots$ ✓

$AC = 95$ n.m. (nearest nm)

$\dfrac{\sin\angle BAC}{50} = \dfrac{\sin 120°}{AC}$ ✓

$\sin\angle BAC = \dfrac{50\sin 120°}{AC}$

$\angle BAC = 27°$

Bearing of C from A is 243°T or S63°W ✓

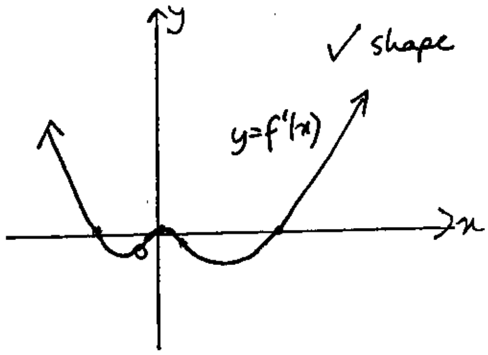
(answer using 95nm rounded is same → Accept)

QUESTION 5

(a) (i) B, C and D ✓

(ii) A, C and E ✓

(iii)



(b) $\frac{dV}{dt} = \frac{1}{100}(30t - t^2)$

(i) $\frac{dV}{dt} = \frac{1}{100}(30 \times 15 - 15^2)$

$= 2\frac{1}{4}$ ✓

$2\frac{1}{4}$ cm³/min

(ii) $\int_0^{15} \frac{1}{100}(30t - t^2)\,dt$

$= \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right]_0^{15}$ ✓

$= \frac{1}{100}\left[\left(15 \times 15^2 - \frac{15^3}{3}\right) - (0)\right]$

$= \frac{1}{100}\left[3375 - \frac{3375}{3}\right]$

$= 22.5$ ✓

∴ 22.5 cm³ is produced in 15 minutes

OR $V = \int \frac{1}{100}(30t - t^2)\,dt$

$= \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right] + c$ (✓)

$t=0 \quad V=0$

$0 = \frac{1}{100}[0 - 0] + c$

$c = 0$

$\therefore V = \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right]$

$t = 15 \quad V = \frac{1}{100}\left[15 \times 15^2 - \frac{15^3}{3}\right]$ (✓)

$= 22.5$ cm³

} alternate method and corresponding mark allocation

(c) (i) $a = \frac{5}{3}t$

$v = \frac{5}{6}t^2 + c$ ✓

$t=0, \; v = 3/2$ (diagram)

$t=0 \quad \frac{3}{2} = \frac{5}{6} \times 0^2 + c$

$c = \frac{3}{2}$

$v = \frac{5}{6}t^2 + \frac{3}{2}$ ✓

$v = \frac{1}{6}(5t^2 + 9)$

(ii) $x = \frac{5}{18}t^3 + \frac{3}{2}t + c$

$t=0, \; x=0$

$\therefore c = 0$

$x = \frac{5}{18}t^3 + \frac{3}{2}t$ ✓

when $t = 2$

$x = \frac{5}{18}(2)^3 + \frac{3}{2}(2)$

$= 5\frac{2}{9}$

∴ The displacement is $5\frac{2}{9}$ m ✓ (in the positive direction)

(d) $\frac{dC}{dt} > 0$ ✓

$\frac{d^2C}{dt^2} < 0$ ✓

QUESTION 6

$y = 2x^3 - 9x^2 + 12x$

(a) (i) $\frac{dy}{dx} = 6x^2 - 18x + 12$ ✓

$= 6(x^2 - 3x + 2)$

$= 6(x-2)(x-1)$

(ii) $\frac{dy}{dx} = 0$ for stationary points

$6(x-2)(x-1) = 0$

$\therefore x = 1$ or $x = 2$ ✓

(2nd derivative method)

$\frac{d^2y}{dx^2} = 12x - 18$

$x = 1$

$\frac{d^2y}{dx^2} = 12 \times 1 - 18$

$= -6$

< 0 $\therefore$ relative maximum turning point at $(1, 5)$ ✓

$x = 2$

$\frac{d^2y}{dx^2} = 12 \times 2 - 18$

$= 6$

> 0 $\therefore$ relative minimum turning point at $(2, 4)$ ✓

→ or (1st derivative test)

x	0	1	1.5	2	3
$f'(x)$	12	0	−1.5	0	12
slope	/	−	\	−	/

note they need to be careful testing to the right of $x=1$ and to the left of $x=2$

$\therefore$ relative maximum turning point at $(1,5)$

relative minimum turning point at $(2,4)$

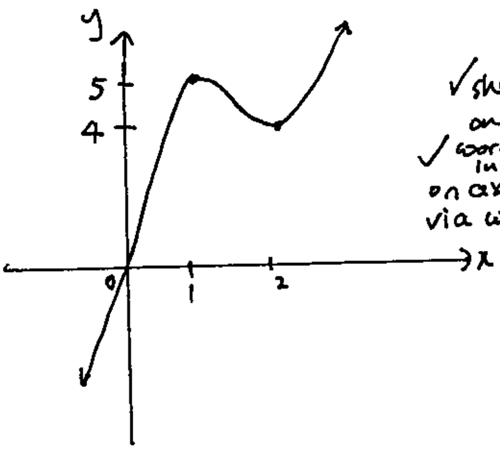
1 mark for a correct test and a pair of coordinates (each time)

OR

1 mark for 2 correct tests (ie mistake only in y-values)

but not 1 mark for 2 y values

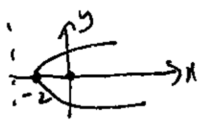
(iii)



✓ shape and ✓ coordinates indicated on axes or via coordinates

(iv) $1 < x < 2$ ✓

(b) (i) $(-2, 0)$ ✓



(ii) $8 = 4a$

$a = 2$

Focus $(0, 0)$ ✓

(iii) $x = -4$ ✓

(c) $\Delta = 0$

$m^2 - 4 \times 4 \times 9 = 0$ ✓

$m^2 = 144$

$m = \pm 12$ ✓ must have both

QUESTION 7

(a)

$\frac{dP}{dt} = KP$

(i) $P = 20000e^{kt}$

$25000 = 20000e^{2K}$ ✓

$e^{2K} = \frac{25}{20} = \frac{5}{4}$

$2K = \log_e\left(\frac{5}{4}\right)$

$K = \frac{1}{2}\log_e\left(\frac{5}{4}\right)$ ✓

$= 0.111571\ldots$

$= 0.1116$ (4 sig. fig)

(ii) $P = 20\,000e^{kt}$

$t = 10$, $K = \frac{1}{2}\log_e\left(\frac{5}{4}\right)$

$P = 20000e^{5\ln\frac{5}{4}}$

$= 61\,035.156\ldots$ ✓

∴ Population is 61 000 (correct to nearest hundred)

(accept this: Using K = 0.1116 gives P = 61052.38.. and 61100 is population)

(iii) $\frac{dP}{dt} = KP$

$= \frac{1}{2}\ln\left(\frac{5}{4}\right) \times 61035.156\ldots$ ✓

$= 6809.80076\ldots$

alternate variations:

or using (ii) answer

$= \frac{1}{2}\ln\frac{5}{4} \times 61000$ (✓)

$= 6805.878\ldots$

or using (ii) answer + rounded K

$= 0.1116 \times 61000$ (✓)

$= 6807.6$

Answer for various "K"
Answer is correct to nearest hundred so → 6800 people/year.

(accept bald answer)

Students might also do

$P = 20000e^{kt}$

$\frac{dP}{dt} = 20000ke^{kt}$ (✓) accept with a sub for (K)

$= \ldots\ldots$ (answers same as above)

(b) $x = t + \frac{16}{t+1}$

(i) $t = 0$

$x = 0 + \frac{16}{0+1}$

$x = 16$ ✓ → 16 m in the positive direction

(ii) $v = 1 - 16(t+1)^{-2}$

$= 1 - \frac{16}{(t+1)^2}$ ✓

$a = 32(t+1)^{-3}$

$= \frac{32}{(t+1)^3}$ ✓

(iii) $0 = 1 - \frac{16}{(t+1)^2}$

$\frac{16}{(t+1)^2} = 1$

$(t+1)^2 = 16$

$t + 1 = \pm 4$

$t = 3$ or -5

$t \geq 0$ ∴ $t = 3$ ✓

$t = 3$

$x = 3 + \frac{16}{3+1}$

$= 7$ ✓

at rest after 3 seconds at 7m in the positive direction

(iv) $t \to \infty$

$v \to 1$ since $\left\{ \frac{16}{(t+1)^2} \to 0 \text{ as } t \to \infty \right.$

$\therefore$ limiting velocity is $1\ ms^{-1}$ ✓ (accept 1)

(v) Total Distance travelled $= \left| \int_0^3 1 - \frac{16}{(t+1)^2}\, dt \right| + \int_3^4 1 - \frac{16}{(t+1)^2}\, dt$

$= \left| \left[t + \frac{16}{t+1} \right]_0^3 \right| + \left[t + \frac{16}{t+1} \right]_3^4$ ✓

$= \left| (3 + 4) - (0 + 16) \right| + \left[\left(4 + \frac{16}{5}\right) - \left(3 + \frac{16}{4}\right) \right]$

$= |-9| + \frac{1}{5}$

$= 9\frac{1}{5}$ metres ✓ (accept $9\frac{1}{5}$)

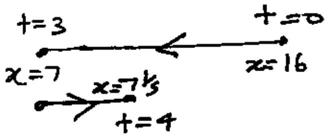
alternate method and mark allocation:

OR

$t = 0$, $x = 16$

$t = 3$, $x = 7$

$t = 4$, $x = 4 + \frac{16}{4+1} = 7\frac{1}{5}$



(✓)

$\therefore$ Total distance travelled $= 9 + \frac{1}{5}$

$= 9\frac{1}{5}\, m$ (✓)

QUESTION 8

(a) $\frac{dy}{dx} = \frac{2x}{x^2+e}$

$y = \int \frac{2x}{x^2+e}\,dx$

$y = \log_e(x^2+e) + C$ ✓

(0,2)

$2 = \log_e e + C$

$C = 1$

$\therefore y = \log_e(x^2+e) + 1$ ✓

(b)(i) $y = 4\ln x$

$\frac{y}{4} = \log_e x$

$x = e^{y/4}$ ✓

$V = \pi\int_0^4 (e^{y/4})^2\,dy$ ✓

for 2nd mark look for squaring of function and calculation of 4 ln e = 4

$x=1$, $y=0$

$x=e$, $y=4\ln e = 4$

$= \pi\int_0^4 e^{0.5y}\,dy$

(ii) $V = \pi\left[\frac{e^{\frac{1}{2}y}}{1/2}\right]_0^4$ ✓

$= \pi(2e^2 - 2\times 1)$

$= 2\pi(e^2-1)$ units^3 ✓

(c)(i) $\log 3 + \log 9 + \log 27 + \ldots$

$= \log 3 + 2\log 3 + 3\log 3 + \ldots$

$T_2 - T_1 = 2\log 3 - \log 3$
$= \log 3$

$T_3 - T_2 = 3\log 3 - 2\log 3$
$= \log 3$ ✓

$T_3 - T_2 = T_2 - T_1 \quad \therefore$ A.P. ✓

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$ ✓

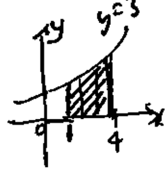
$S_{10} = \frac{10}{2}[2\times\log 3 + (10-1)\times\log 3]$

$= \frac{10}{2}[2\log 3 + 9\log 3]$

$= 55\log 3$ ✓

(d)

x	1	2	3	4
$f(x)$	3^1	3^2	3^3	3^4

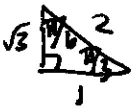


Area $\doteq \frac{1}{2}[3^1 + 2\times 3^2 + 2\times 3^3 + 3^4]$ ✓ or equivalent

$\doteq 78$ units^2 ✓

QUESTION 9

(a) $\cos x = -\frac{\sqrt{3}}{2}$



$x = \frac{5\pi}{6}, -\frac{5\pi}{6}$

✓ ✓

(b)(i) $f(x) = e^{\tan x}$

$f'(x) = \sec^2 x \, e^{\tan x}$ ✓

(ii) $\therefore \int 3e^{\tan x}\sec^2 x \, dx = 3e^{\tan x} + c$

$\int_0^{\pi/4} 3e^{\tan x}\sec^2 x \, dx = \left[3e^{\tan x}\right]_0^{\pi/4}$ ✓

$= 3\left(e^{\tan \pi/4} - e^0\right)$

$= 3(e - 1)$ ✓

(c) (i) $240°$

$= 240° \times \frac{\pi}{180}$

$= \frac{4\pi}{3}$

OR $\frac{2}{3} \times 2\pi$

$= \frac{4\pi}{3}$ ✓

(ii) $l = r\theta$

$= 12 \times \frac{4\pi}{3}$

$= 16\pi$ cm ✓

(iii) $a = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 12^2 \times \frac{4\pi}{3}$

$= 96\pi$ cm^2 ✓

(d) (i) $a = 1$, $n = 2$ } ✓ need both

(d)(ii) Area $= \int_{\pi/2}^{3\pi/2} (\sin 2x - 4\cos x)\,dx$ ✓

$= \left[-\frac{1}{2}\cos 2x - 4\sin x\right]_{\pi/2}^{3\pi/2}$ ✓

$= \left(-\frac{1}{2}\cos 3\pi - 4\sin\frac{3\pi}{2}\right) - \left(-\frac{1}{2}\cos\pi - 4\sin\frac{\pi}{2}\right)$

$= \left(-\frac{1}{2} \times -1 - 4 \times -1\right) - \left(-\frac{1}{2} \times -1 - 4 \times 1\right)$

$= \left(\frac{1}{2} + 4\right) - \left(\frac{1}{2} - 4\right)$

$= 4\frac{1}{2} - \left(-3\frac{1}{2}\right)$

$= 8$ units^2 ✓

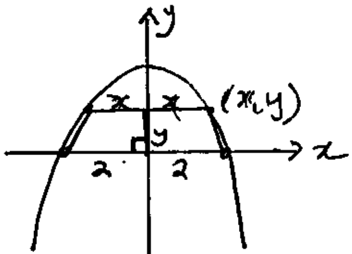
QUESTION 10

(a) $\int_{-2}^{5} g(y)\, dy = 7 - 9$

$= -2$ ✓

(b) (i) $R(-2,0)$, $S(2,0)$ ✓

(ii)



Area $= \frac{1}{2} \times y \times (2x + 4)$ ✓ as required

$= \frac{1}{2} \times (4 - x^2)(2x+4)$ ✓

$= (x+2)(4-x^2)$

as required

(iii) $A = 4x - x^3 + 8 - 2x^2$

$= -x^3 - 2x^2 + 4x - 8$

$\frac{dA}{dx} = -3x^2 - 4x + 4$ ✓

$\frac{dA}{dx} = 0$

$3x^2 + 4x - 4 = 0$

$(3x-2)(x+2) = 0$

$x = 2/3$ or -2

$x \neq -2$

$x = \frac{2}{3}$ ✓

either test

$\frac{d^2A}{dx^2} = -6x - 4$

$x = 2/3$

$\frac{d^2A}{dx^2} = -6 \times \frac{2}{3} - 4$

$= -8$

< 0 ✓ ∴ maximum value of A when $x = 2/3$

OR

x	$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{dA}{dx}$	$\frac{7}{3}$	0	-3
slope	/	—	\

(c) $B_1 = 50\,000 \times 1.05 - 1500$ ✓

$B_2 = (50\,000 \times 1.05 - 1500) \times 1.05 - 1500 \times 1.03$ ✓

$= 50\,000 \times 1.05^2 - 1500(1.05 + 1.03)$

✓ $B_3 = [50\,000 \times 1.05^2 - 1500(1.05 + 1.03)]1.05 - 1500(1.03)^2$

$= 50000 \times 1.05^3 - 1500(1.05^2 + 1.03 \times 1.05 + 1.03^2)$

Note B_2 and B_3

Marks awarded for indicating the method on previous "B" and not just the answer (especially B_3 as it is given in the question!)

(ii) Following this pattern

$B_n = 50000 \times 1.05^n - 1500(1.05^{n-1} + 1.03 \times 1.05^{n-2} + 1.03^2 \times 1.05^{n-3} + \ldots + 1.03^{n-1})$

GP $a = 1.05^{n-1}$

$r = \frac{1.03}{1.05}$

'n' terms

Using $S_n = \frac{a(r^n - 1)}{r - 1}$

this line ✓

$B_n = 50000 \times 1.05^n - 1500\left[\frac{1.05^{n-1}\left(\left(\frac{1.03}{1.05}\right)^n - 1\right)}{\frac{1.03}{1.05} - 1}\right]$

showing work!! ✓

$= 50000 \times 1.05^n - 1500\left[\frac{1.05^{n-1}\left(\left(\frac{1.03}{1.05}\right)^n - 1\right)}{\frac{-0.02}{1.05}}\right]$

$= 50000 \times 1.05^n - \frac{1500}{0.02}\left[1.05^n\left(1 - \left(\frac{1.03}{1.05}\right)^n\right)\right]$

$= 50\,000 \times 1.05^n - 75000 \times 1.05^n\left(1 - \left(\frac{1.03}{1.05}\right)^n\right)$

given in question

AS REQUIRED!