AT
KW
KL
JG
FH
NB
YR
MB
HK

Name: WHITE

Class: 12MT2__ or 12MTX__

Teacher: ____________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2010 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- All questions are of equal value.
- Each question is to be commenced on a new page clearly marked Question 1, Question 2, etc on the top of the page. **.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used. Standard Integral Tables are provided
- Your solutions will be collected in one bundle stapled in the top left corner.

Please arrange them in order, Q1 to 10.

Each page must show your name and your class. *

Question 1 12 MARKS **Start a NEW page** **MARKS**

a) Evaluate $\frac{\sqrt{5^2+144}}{13-6}$ correct to **two** decimal places. 2

b) Find the primitive of $4x^2+2$. 1

c) If $\frac{1}{a}=\sqrt{10}-3$, show that $a=\sqrt{10}+3$. 1

d) Factorise $3x^2+6x-9$. 2

e) Express the following as a single fraction 2

$$\frac{5}{2a+6}+\frac{a}{a^2-9}$$

f) Solve $|2x+5|<3$. 2

g) On a number plane, shade the region where $y\leq 0$ and $y\geq x^2+3x$ hold simultaneously. 2

End of Question 1

Question 2 12 MARKS **Start a NEW page** **MARKS**

a) The coordinates of the points A, B and C are $(0,-2)$, $(4,0)$ and $(6,-4)$ respectively.

(i) Find the length AB. 1

(ii) Find the gradient of AB. 1

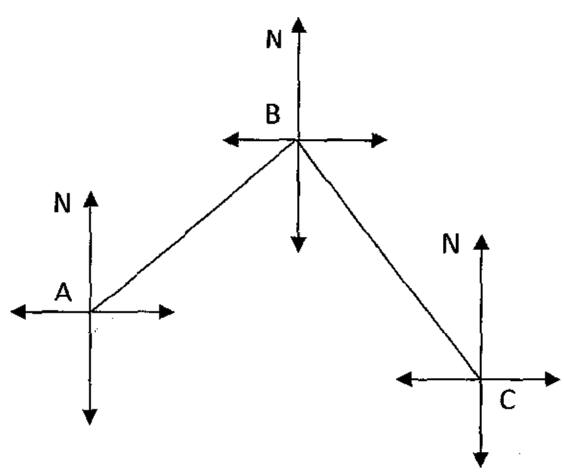
(iii) Show that the equation of the line L, drawn through C parallel to AB is $x-2y-14=0$ 2

(iv) Find the coordinates of D, the point where L intersects the x-axis. 1

(v) Find the perpendicular distance of the point A from the line L. 2

(vi) Find the area of the quadrilateral $ABDC$. 2

b) A ship sails 150 *km* from Appleville (A) to Brooktown (B) on a bearing of 050^{0}T. It then sails on a bearing of 130^{0}T to Cook (C) which is 300 *km* from Brooktown.



(i) Copy the above diagram onto your writing paper showing all given information. 1

(ii) Find the distance from Appleville to Cook. Give your answer correct to two decimal places. 2

End of Question 2

Question 3 **12 MARKS** **Start a NEW page** **MARKS**

a) Evaluate $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$ 2

b) Differentiate with respect to x

(i) $4x^2 e^{3x^3}$ 2

(ii) $\frac{\pi \cos x}{x^2}$ 2

(iii) $\ln\sqrt{3x^2 - 1}$ 2

c) Find the following indefinite integrals.

(i) $\int \frac{8x + 10}{2x^2 + 5x} dx$ 2

(ii) $\int \frac{x + 1}{\sqrt{x}} dx$ 2

End of Question 3

Question 4 12 MARKS **Start a NEW page** **MARKS**

a) The roots of the equation $2x^2 - 7x + 12 = 0$ are α and β

Find

(i) $\alpha + \beta$ and $\alpha\beta$ 1

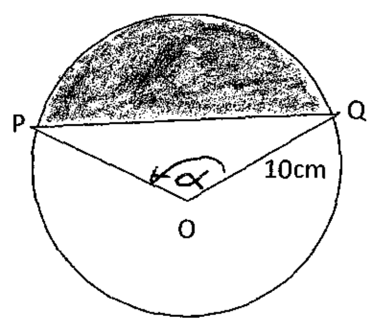
(ii) $\alpha^2 + \beta^2$ 1

b) For the parabola $4y = x^2 - 6x + 1$ find

(i) the coordinates of the vertex. 2

(ii) the coordinates of the focus 1

c)



(i) The area of sector *OPQ* is $\frac{100\pi}{3} cm^2$. Given that the radius of the circle is 10 cm, find the angle α, leaving your answer in exact form. 2

(ii) Hence, or otherwise, find the area of the shaded segment. 2

d) To calculate the area of the region bounded by the curve $y = x^2 - 2x$, the lines $x = 0$, $x = 4$ and the x-axis, Fiona used

$$\int_0^4 (x^2 - 2x)dx.$$

(i) Explain why Fiona's method of calculating this area is incorrect. 1

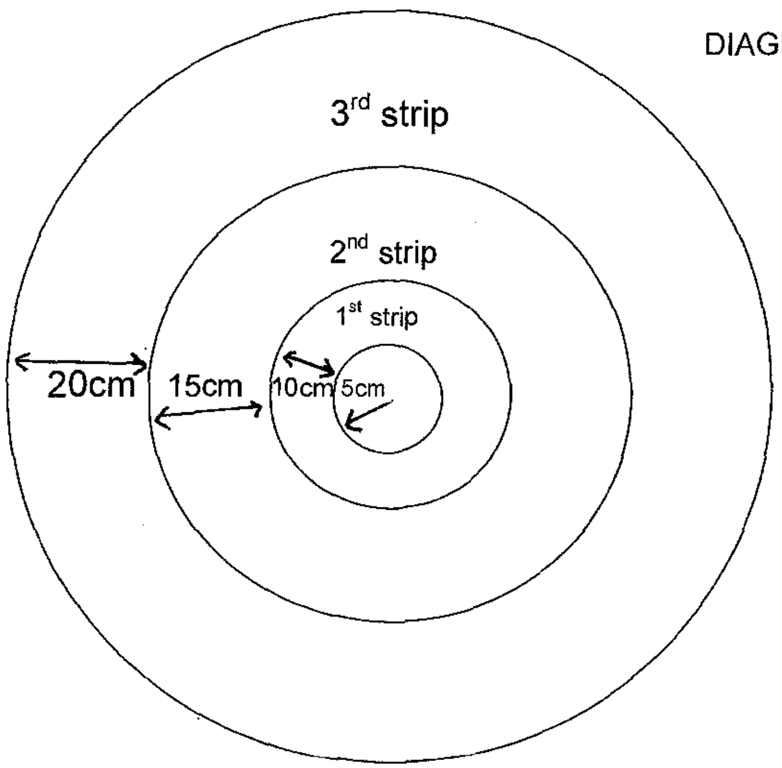
(ii) Find the correct area of the region. 2

End of Question 4

Question 5 12 MARKS **Start a NEW page** **MARKS**

a) Beginning with a circular piece of fabric with radius $5\,cm$, Kevin sewed together circular strips of different fabrics which increased in width to make a circular parachute. The finished width of the 1st strip was 10cm, the second was 15cm and the third was 20cm and so on .

DIAGRAM NOT TO SCALE



(i) Show that the width of the 10th strip was 55 cm. 2

(ii) The radius of the parachute was 455 cm. How many strips were sewn to the edge of the first circular piece? 3

QUESTION 5 CONTINUED ON PAGE 6...........

b) Tiarn borrows \$500 000 to buy a house. An interest rate of 9% p.a. compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly installments (R) over a 25 year period.

(i) Show the amount owing after 3 months is: 2

$$A_3 = 500000(1.0075)^3 - R\left[1 + 1.0075 + 1.0075^2\right]$$

(ii) Assuming this pattern continues the monthly repayment can be calculated using 3

$$A_n = 500000(1.0075)^n - R\left[1 + 1.0075 + 1.0075^2 + \ldots\ldots\ldots + 1.0075^{n-1}\right]$$

How much should Tiarn be paying each month?

(iii) How much interest does Tiarn pay over the 25 years? 1

(iv) What is the equivalent simple interest rate of this loan? 1

End of Question 5

Question 6 12 MARKS **Start a NEW page** **MARKS**

a) For what value of x is the tangent to the curve $y = \log_e(x^2 - 9)$ parallel to the line $x - 4y = 0$? 3

b) Consider the function $y = (x^2 + 1)e^{-x}$.

The first derivative and second derivatives of this function are

$\frac{dy}{dx} = -e^{-x}(x-1)^2$ and $\frac{d^2y}{dx^2} = e^{-x}(x^2 - 4x + 3)$ (You don't need to show these)

(i) Find any stationary points and determine their nature. 2

(ii) Find any points of inflexion. 2

(iii) State the equation of the horizontal asymptote 1

(iv) Sketch the function clearly showing all of the above information and the y-intercept . 1

c) (i) Copy and complete the table of values for $y = (\log_e x)^2$. Express your answers correct to 2 decimal places. 1

x	2	2·5	3
y			

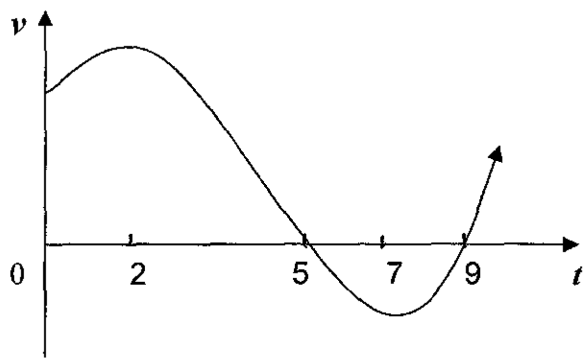
(ii) Using your results from Q6c) (i) and one application of Simpson's Rule, find, correct to one decimal place the volume when the area bounded by $y = \log_e x$, $x = 2$, $x = 3$ and the x-axis is rotated about the x-axis. 2

End of Question 6

Question 7 12 MARKS **Start a NEW page** **MARKS**

a) A particle is moving in a straight line so that at time t seconds its displacement from the origin is x metres. Initially the particle is 6 metres to the left of the origin.

Consider the graph below illustrating the particle's velocity as a function of time.



(i) During the first two seconds is the particle moving towards or away from the origin? 1

(ii) Given that $\int_0^9 v\,dt = 4$, find the displacement of the particle after 9 seconds 1

(iii) Sketch the graph of the particle's acceleration as a function of time. 2

b) The acceleration of a particle moving in a straight line is given by $a = 6 - 6t$ ms^{-2}. The particle is initially 2 metres to the right of the origin and moves towards the origin with an initial velocity of $-3ms^{-1}$.

(i) Find an expression for the velocity of the particle in terms of time t. 2

(ii) Find an expression for the displacement of the particle in terms of time t. 2

(III) Find when and where the particle comes to rest. 2

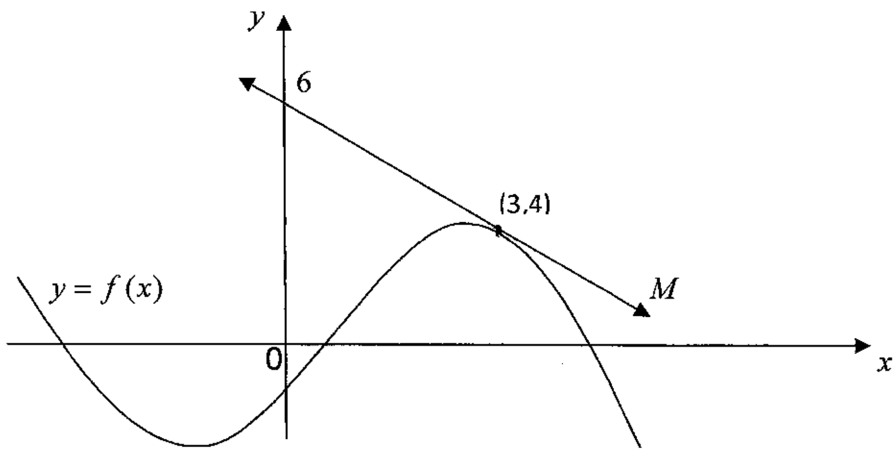
(iv) Show the particle passes through the origin after 2 seconds of motion. 1

(v) Show the particle never changes direction. 1

End of Question 7

Question 8 12 MARKS **Start a NEW page** **MARKS**

a) The line M is a tangent to $y = f(x)$ at the point $(3,4)$. The tangent has a y-intercept of 6. Find the value of $f'(3)$. 1



b) A swimming pool is to be emptied for maintenance. The quantity of water Q litres, remaining in the pool at anytime, *t* minutes, after it starts to empty is given by:

$$Q(t) = 2000(25 - t)^2, \quad t \geq 0$$

(i) At what rate is the pool being emptied at any time (*t*) 1

(ii) How long will it take to half empty the pool to the nearest minute? 3

(iii) At what time is the water flowing out at 20 kL / minute. 1

(iv) What is the average rate of water flow in the first 10 minutes in L/min? 2

c) The size of a colony of bees is given by the equation $P = 5000e^{kt}$ where *P* is the population after *t* weeks.

(i) If there are 6000 bees after one week, find the value of *k* correct to 2 decimal places 1

(ii) When will the colony (to the nearest day) triple in size? 1

(iii) Find the growth rate of the population after two weeks. 2

End of Question 8

Question 9 12 MARKS **Start a NEW page** **MARKS**

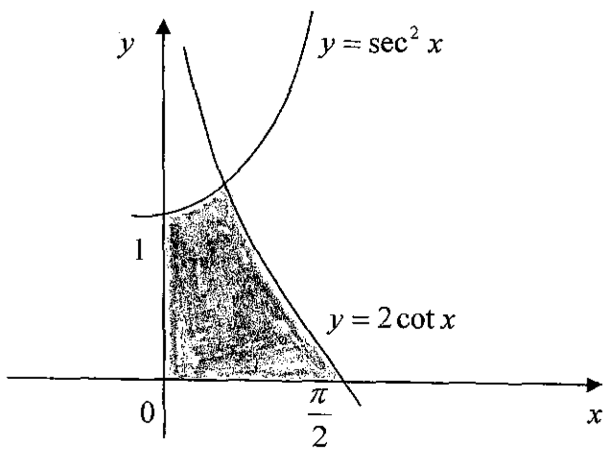
a) (i) State the period of the curve $y = 3\sin \pi x$. 1

(ii) Sketch $y = 3\sin \pi x$ for $-1 \le x \le 1$. 2

b) Solve the equation $4\cos^2 \theta - 4\sin \theta = 1$ for $0 \le \theta \le 2\pi$. 2

c) The diagram below shows the region bounded by the curves

$y = \sec^2 x$, $y = 2\cot x$ and the coordinate axes.



(i) Verify, by substitution, that the point $\left(\frac{\pi}{4}, 2\right)$ lies on both

$y = \sec^2 x$ and $y = 2\cot x$. 1

(ii) Differentiate $y = \log_e(\sin x)$. 1

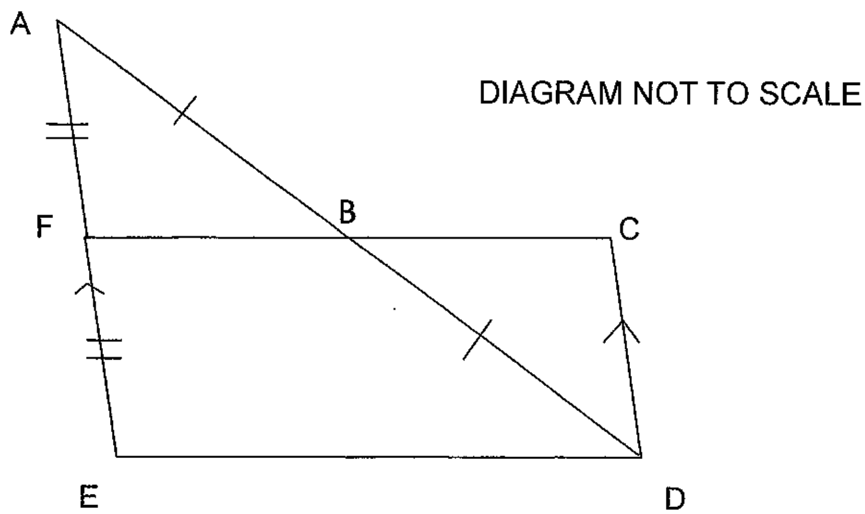
(iii) Hence, or otherwise, find the exact area of the shaded region. 3

d) For the function $y = 2\sin 3x + 4\cos 2x$ find p if $\frac{d^2y}{dx^2} + 4y = p\sin 3x.$ 2

End of Question 9

Question 10 **12 MARKS** **Start a NEW page** **MARKS**

a) In the diagram, the line FC bisects AE at F and AD at B. The line AE is parallel to CD. Copy the diagram onto your writing paper.



(i) Explain why ED=2BF. 1

(ii) Prove that $\Delta ABF \equiv \Delta DBC$. 3

b) (i) Given that $m^2 + n^2 = 14mn$, show that $\left[\frac{(m+n)}{4}\right]^2 = mn$. 1

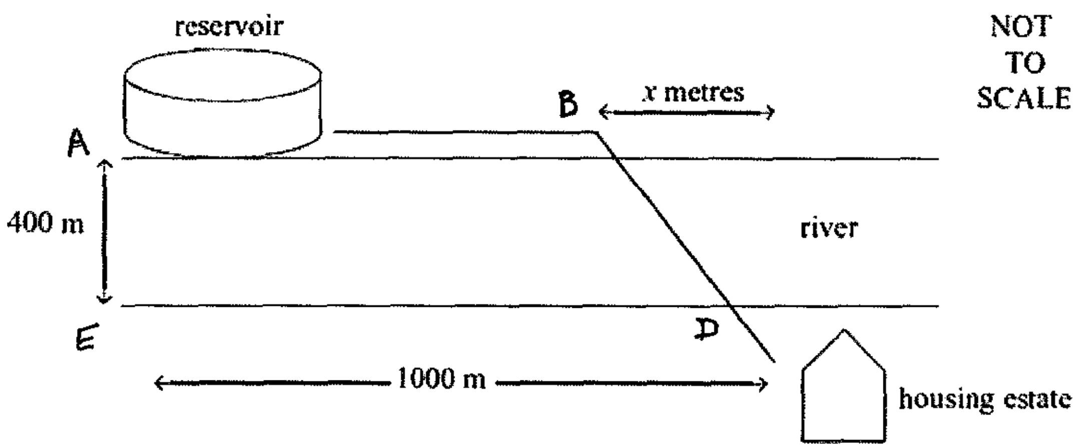
(II) Hence show that $\log\left[\frac{(m+n)}{4}\right] = \frac{1}{2}[\log m + \log n]$. 2

QUESTION 10 CONTINUED ON PAGE 12

c) An underground water pipe is being built from a reservoir to a new housing estate on the other side of a river, 1000 metres downstream. The river is 400 metres wide and has straight banks.

The diagram shows the proposed route of the water pipe. It follows the river bank for some distance before crossing the river to the housing estate.

The cost of laying the pipe underground is \$800 per metre and the cost of laying the pipe underwater is \$1000 per metre.



(i) Show that the cost (C) of the pipeline, in terms of x, is given by **1**

$$C = 800(1000 - x) + 1000\sqrt{400^2 + x^2}\,.$$

(ii) Find the value of x that gives the minimum cost. **3**

(iii) What is the minimum cost of the pipeline? **1**

End of Question 10

End of Paper

Q1

a) $= 1.857142\ldots$ ✓

$= 1.86$ (2 dec pl) ✓

b) $F(x) = \frac{4x^3}{3} + 2x + c$ ✓

c) $\frac{1}{a} = \sqrt{10} - 3$

$a = \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$ ✓

$= \frac{\sqrt{10}+3}{1}$

$= \sqrt{10} + 3$

d) $3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$ ✓

$= 3(x+3)(x-1)$ ✓

e) $\frac{5}{2a+6} + \frac{a}{a^2-9}$

$= \frac{5}{2(a+3)} + \frac{a}{(a-3)(a+3)}$

$= \frac{5(a-3) + 2 \times a}{2(a-3)(a+3)}$ ✓

$= \frac{5a - 15 + 2a}{2(a-3)(a+3)}$

$= \frac{7a - 15}{2(a-3)(a+3)}$ ✓

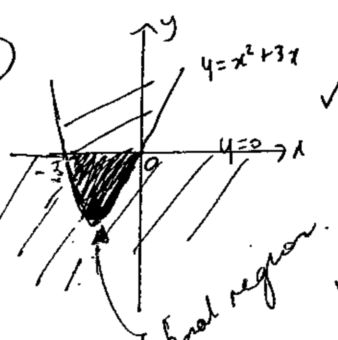
f) $|2x+5| < 3$

$-3 < 2x + 5 < 3$ ✓

$-8 < 2x < -2$

$-4 < x < -1$ ✓

g)



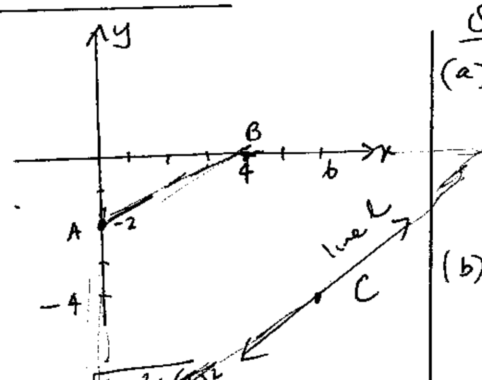
✓ parabola + shade above

✓ final region

12

Q2

(a)



(i) $AB = \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ units ✓

(ii) $m_{AB} = \frac{2}{4} = \frac{1}{2}$ ✓

(iii) m of line $l = \frac{1}{2}$ as $l \parallel AB$, $m_1 = m_2$ ✓

$y + 4 = \frac{1}{2}(x - 6)$ ✓

$2y + 8 = x - 6$

$x - 2y - 14 = 0$

(iv) $y = 0$

$x - 2 \times 0 - 14 = 0$

$x = 14$

$\therefore$ D is $(14, 0)$ ✓

(v) $p = \left|\frac{1 \times 0 - 2 \times -2 - 14}{\sqrt{1^2 + (-2)^2}}\right|$ ✓

$= \left|\frac{-10}{\sqrt{5}}\right|$

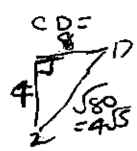
$= \frac{10}{\sqrt{5}}$ or $2\sqrt{5}$ units ✓

(vi) Area trapezium $= \frac{1}{2} \times 2\sqrt{5}\,(2\sqrt{5} + 4\sqrt{5})$ ✓

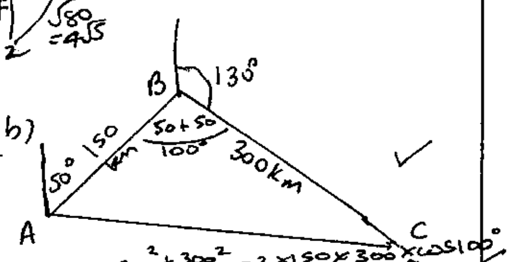
$= \frac{1}{2} \times 2\sqrt{5} \times 6\sqrt{5}$

$= 30$ units^2 ✓

CD =



(b)



✓

$AC^2 = 150^2 + 300^2 - 2 \times 150 \times 300 \times \cos 100°$ ✓

$= 128128.336\ldots$

$AC = 357.9501\ldots$

$= 357.95$ km (2 dec pl) ✓

12

Q3

(a) $\lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{x-3}$ ✓✓

$= 9 + 9 + 9$

$= 27$ ✓

(b) (i) $f'(x) = 4x^3 \cdot 9x^2 e^{3x^3} + e^{3x^3} \cdot 8x$ ✓

$= 36x^4 e^{3x^3} + 8x e^{3x^3}$ ✓

$= 4x e^{3x^3}[9x^3 + 2]$

(ii) $f'(x) = \frac{x^2 \times -\pi \sin x - \pi \cos x \times 2x}{x^4}$ ✓

$= \frac{-\pi x^2 \sin x - 2\pi x \cos x}{x^4}$

$= \frac{-\pi x \sin x - 2\pi \cos x}{x^3}$ ✓

$= \frac{-\pi[x \sin x + 2\cos x]}{x^3}$

(iii) $\ln\sqrt{3x^2 - 1}$

$= \frac{1}{2}\ln(3x^2 - 1)$ ✓

$f'(x) = \frac{1}{2} \cdot \frac{6x}{3x^2 - 1}$

$= \frac{3x}{3x^2 - 1}$ ✓

(c) i) $\int \frac{8x + 10}{2x^2 + 5x}\,dx$

$= 2\int \frac{4x+5}{2x^2+5x}\,dx$ ✓

$= 2\log_e(2x^2 + 5x) + c$ ✓

ii) $\int \frac{x+1}{\sqrt{x}}\,dx$

$= \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}})\,dx$ ✓

$= \frac{2x^{3/2}}{3} + 2x^{\frac{1}{2}} + c$ ✓

$= \frac{2}{3}x\sqrt{x} + 2\sqrt{x} + c$

12

Q4 a) i) $\alpha+\beta=-\frac{b}{a}=\frac{7}{2}$

$\alpha\beta=\frac{c}{a}=\frac{12}{2}=6$ } ✓ for both

ii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{7}{2}\right)^2-2\times 6$

$=\frac{49}{4}-12$

$=\frac{1}{4}$ ✓

b) i) $4y=x^2-6x+1$

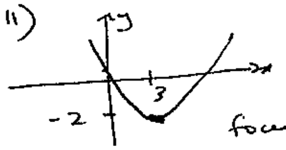
$x^2-6x+9=4y-1+9$ ✓

$(x-3)^2=4y+8$

$(x-3)^2=4(y+2)$

$\therefore$ vertex is $(3,-2)$. ✓

ii)



focal length $(a)=1$

focus $=(3,-1)$ ✓

(c) i) $A=\frac{1}{2}r^2\theta$

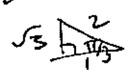
$\frac{100\pi}{3}=\frac{1}{2}\times 100\times\theta$ ✓

$\frac{1}{2}\theta=\frac{\pi}{3}$

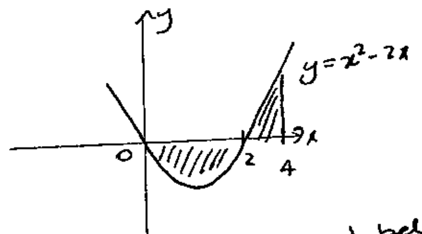
$\theta=\frac{2\pi}{3}$ ✓

ii) Area $=\frac{1}{2}\times 10^2\times\left(\frac{2\pi}{3}-\sin\frac{2\pi}{3}\right)$ ✓

$=50\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ cm² ✓



(d) i)



The area is above and below the x-axis, so it needs to be split (and the integral representing the area below the x-axis needs absolute value brackets!) ✓

ii) $A=\left|\int_0^2(x^2-2x)\,dx\right|+\int_2^4(x^2-2x)\,dx$

$=\left|\left[\frac{x^3}{3}-x^2\right]_0^2\right|+\left[\frac{x^3}{3}-x^2\right]_2^4$ ✓

$=\left|\left(\frac{8}{3}-4\right)-(0-0)\right|+\left[\left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-4\right)\right]$

$=\frac{4}{3}+\frac{20}{3}$

$=8$ units² ✓

/12

Q5

a) i) $10, 15, 20, \ldots$

A.P. $a=10$ $d=5$ → ✓

$T_{10}=10+9\times 5$

$=55$ ✓

$\therefore$ The 10^{th} strip is 55cm

ii) total sum of "width of strips" = 450 cm

$S_n=\frac{n}{2}[2a+(n-1)d]$

$450=\frac{n}{2}[2\times 10+(n-1)\times 5]$ ✓

$450=\frac{n}{2}[20+5n-5]$

$900=n[5n+15]$ ✓

$5n^2+15n-900=0$

$n^2+3n-180=0$

$(n+15)(n-12)=0$

$n=12$ or -15

$\therefore n=12$ as $n>0$

$\therefore$ 12 strips were sewn together ✓

(b) i) $A_1=500\,000\left(1+\frac{9}{100\times 12}\right)^1$

$=500\,000(1.0075)-R$ } ✓

$A_2=[500\,000(1.0075)-R]\,1.0075-R$ ✓

$=500\,000(1.0075)^2-R(1+1.0075)$

$A_3=[500\,000(1.0075)^2-R(1+1.0075)]\,1.0075-R$

$=500\,000(1.0075)^3-R(1+1.0075+1.0075^2)$

⋮

ii) $A_n=500\,000(1.0075)^n-R(1+1.0075+1.0075^2+\ldots\ 1.0075^{n-1})$

G.P.
$a=1$
$r=1.0075$
n terms

$A_n=500\,000(1.0075)^n-R\left[\frac{1.(1.0075)^n-1}{1.0075-1}\right]$

$25\times 12=300$

$A_{300}=500\,000(1.0075)^{300}-R\frac{(1.0075^{300}-1)}{1.0075-1}$ ✓

paid off when $A_{300}=0$

$0=500\,000(1.0075)^{300}-R\frac{(1.0075^{300}-1)}{0.0075}$ ✓

$R\frac{(1.0075^{300}-1)}{0.0075}=500\,000(1.0075)^{300}$

$R=\frac{500\,000(1.0075)^{300}}{(1.0075^{300}-1)}\times 0.0075$

$=4195.981818\ldots$

$R=\$4195.98$ ✓

iii) Interest = Amount via payments − cash price $=1258794-500000$

$=\$758794$ ✓

iv) $I=P\times\frac{R}{100}\times T$

$758794=500\,000\times\frac{R}{100}\times 25$

$R=6.0703\ldots$ $=6.07\%$ ✓

/12

Q6 (a) $\frac{dy}{dx} = \frac{2x}{x^2-9}$ ✓ $x - 4y = 0$, $y = \frac{1}{4}x$

$\frac{2x}{x^2-9} = \frac{1}{4}$ $(m_1 = m_2)$

$x^2 - 9 = 8x$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$ ✓

$x = -1$ or 9 as $x^2 - 9 > 0$

$x = 9$ only ✓

(b) $y = (x^2+1)e^{-x}$

(i) $\frac{dy}{dx} = -e^{-x}(x-1)^2$

$\frac{dy}{dx} = 0$ for stat pts

$\therefore x = 1$

$y = 2e^{-1}$

$\frac{d^2y}{dx^2} = e^{-x}(x^2 - 4x + 3)$ ✓

$x = 1$

$= e^{-1}(1 - 4 + 3)$

$= 0$ Possible HPOI

check concavity

x	0	1	2
$\frac{d^2y}{dx^2}$	3 >0	0	$-1e^{-2}$ <0

change in sign $\therefore$ change in concavity

$\therefore$ HPOI at $(1, 2e^{-1})$ ✓

OR

x	0	1	2
$f'(x)$	-1	0	$-e^{-2}$
slope	\	–	\

$\therefore$ HPOI at $(1, 2e^{-1})$ (✓)

(ii) $\frac{d^2y}{dx^2} = 0$ for POI

$\therefore e^{-x}(x^2 - 4x + 3) = 0$

$e^{-x}(x-3)(x-1) = 0$

$\therefore x = 1$ or 3

$x = 1$, $y = 2e^{-1}$ } already found HPOI

$x = 3$, $y = 10e^{-3}$ } ✓

test

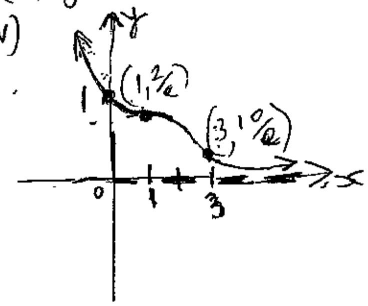
x	2	3	4
y''	$-e^{-2}$	0	$3e^{-4}$
	$-$	0	$+$

change in sign ✓

$\therefore$ POI $(3, 10e^{-3})$

(iii) $y = 0$ ✓

(iv)



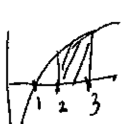
✓

(c) (i)

x	2	2.5	3
y	0.48	0.84	1.21

✓

(ii)



$V = \pi \int_2^3 (\log_e x)^2 \, dx$ ✓

$= \pi \cdot \frac{0.5}{3}\left[0.48 + 4 \times 0.84 + 1.21\right]$

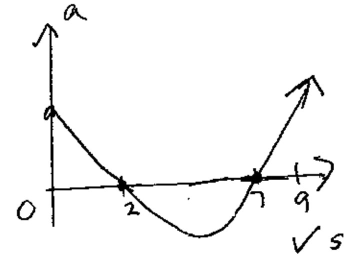
$= 2.644$ ✓

$= 2.6$ units^3 (1 dec pl)

Q7 (a) (i) towards the origin ✓

(ii) 2m to left of origin ✓

(iii)



✓ shape

✓ $t = 2, 7$ labelled

(b) $a = 6 - 6t$

(i) $v = 6t - 3t^2 + c$ ✓

$v = -3$, $t = 0$

$-3 = 0 + c$

$c = -3$

$v = 6t - 3t^2 - 3$ ✓

(ii) $x = 3t^2 - t^3 - 3t + c$ ✓

$t = 0$, $x = 2$

$2 = 0 - 0 - 0 + c$

$c = 2$

$\therefore x = 3t^2 - t^3 - 3t + 2$ ✓

(iii) $v = 0$

$0 = 6t - 3t^2 - 3$

$-3(t^2 - 2t + 1) = 0$

$-3(t-1)^2 = 0$

$t = 1$ ✓

$t = 1$

$x = 3 \times 1^2 - 1^3 - 3 + 2$

$= 1$ ✓

$\therefore$ at rest 1m to right of origin after 1 sec.

(iv) $t = 2$

$x = 3 \times 2^2 - 2^3 - 3 \times 2 + 2$

$= 12 - 8 - 6 + 2$

$= 0$

$\therefore$ passes through origin after 2 seconds ✓

(v) $v = -3(t-1)^2$

the particle is at rest at $t = 1$ but does not change direction as $v \leq 0$ for all values of t. (ie the velocity is always negative (when not at rest) hence always moving in the negative direction) ✓

Q8 (a) $f'(3) = -\frac{2}{3}$ ✓

(b) $Q(t) = 2000(25-t)^2$

(i) $\frac{dQ}{dt} = -4000(25-t)$ litres/min ✓

$\therefore$ emptying at rate $4000(25-t)$ L/min

(ii) $t = 0$

$Q(0) = 2000 \times 25^2$

$= 1250\,000$ L

$\therefore$ half empty $\Rightarrow 625\,000$ L

$625\cancel{000} = 2\cancel{000}(25-t)^2$ ✓

$625 = 2(625 - 50t + t^2)$

$2t^2 - 100t + \cancel{1}250 - 625 = 0$

$2t^2 - 100t + 625 = 0$ ✓

$t = \frac{100 \pm \sqrt{10000 - 4 \times 2 \times 625}}{4}$

$= \frac{100 \pm \sqrt{5000}}{4}$

$= 42.6776\ldots, 7.3223\ldots$

$\therefore$ 7 minutes to $\frac{1}{2}$ empty pool ✓

(iii) $-20\,000 = -4\,000(25-t)$

$25 - t = \frac{20\,000}{4000}$

$25 - t = 5$

after 20 minutes ✓

(iv) $\int_0^{10} -4000(25-t)\,dt = \left[2000(25-t)^2\right]_0^{10}$

$= 2000(25-10)^2 - 2000 \times 25^2$

$= 800\,000$ L ✓

Total water lost in 1st 10 min = 800 000 L

Average flow $= \frac{800\,000}{10}$ L/min ✓

$= 80\,000$ L/min

c) i) $P = 5000e^{kt}$

$6000 = 5000e^{1k}$

$e^k = \frac{6}{5}$

$k = \ln\frac{6}{5}$

$= 0.18232\ldots$

$= 0.18$ (2 dec pl) ✓

ii) $3 = e^{kt}$

$kt = \ln 3$

$t = \frac{\ln 3}{k}$

$= \frac{\ln 3}{\ln 6/5}$ $\left(\text{or } \frac{\ln 3}{0.18}\right)$

$= 6.0256\ldots$ $= 6.10340\ldots$

6 wks (42 days) ✓

iii) P at 2 weeks $= 5000e^{2\ln 6/5} = 7200$ ✓

$dP/dt = \ln(6/5) \times 7200 = 1312.7152\ldots$ ✓

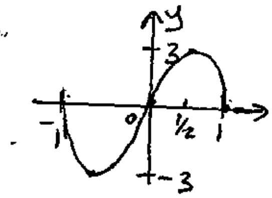
1313 bees/wk

$5000 e^{2 \times 0.18}$

Q9 (a)(i) $y = 3\sin \pi x$

$\frac{2\pi}{n} = \frac{2\pi}{\pi} = 2$ ✓

(ii)



✓ range and shape

✓ domain

(b) $4\cos^2\theta - 4\sin\theta = 1 \qquad 0 \le \theta \le 2\pi$

$4(1-\sin^2\theta) - 4\sin\theta = 1$

$4 - 4\sin^2\theta - 4\sin\theta = 1$

$4\sin^2\theta + 4\sin\theta - 3 = 0$

✓ any of these

$(2\sin\theta + 3)(2\sin\theta - 1) = 0$

$\sin\theta = -\frac{3}{2}$ or $\frac{1}{2}$

no soln $\qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ ✓

$\sqrt{3}$ $\frac{\pi}{6}$ 2 1

(c) i) $y = \sec^2 x \qquad y = 2\cot x$

$x = \frac{\pi}{4} \qquad x = \frac{\pi}{4}$

$y = \sec^2\frac{\pi}{4} \qquad y = 2\cot\frac{\pi}{4}$

$= (\sqrt{2})^2 \qquad = 2 \times 1$

$= 2 \qquad = 2$

must show both ✓

$\therefore \left(\frac{\pi}{4}, 2\right)$ lies on both curves

ii) $y = \log_e(\sin x)$

$\frac{dy}{dx} = \frac{\cos x}{\sin x}$

$= \cot x$ ✓

iii) Area $= \int_0^{\pi/4} \sec^2 x\,dx + \int_{\pi/4}^{\pi/2} 2\cot x\,dx$ ✓

$= \left[\tan x\right]_0^{\pi/4} + 2\left[\log_e \sin x\right]_{\pi/4}^{\pi/2}$ ✓

$= \left(\tan\frac{\pi}{4} - \tan 0\right) + 2\left(\log_e \sin\frac{\pi}{2} - \log_e \sin\frac{\pi}{4}\right)$

$= (1 - 0) + 2\left(\ln 1 - \ln\frac{1}{\sqrt{2}}\right)$

$= 1 + \ln 2$ units² ✓ or equivalent

(d) $y = 2\sin 3x + 4\cos 2x$

$\frac{dy}{dx} = 6\cos 3x - 8\sin 2x$

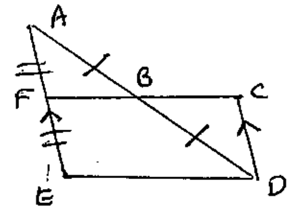
$\frac{d^2y}{dx^2} = -18\sin 3x + 16 - 16\cos 2x$ ✓

$\frac{d^2y}{dx^2} + 4y = -18\sin 3x - 16\cos 2x + 4(2\sin 3x + 4\cos 2x)$

$= -10\sin 3x$

$\therefore p = -10$ ✓

Q16 (a)



(i) ED = 2 BF (the line joining the midpoints of two sides of a Δ is ∥ to 3rd side and half its length) ✓

Note It cannot be assumed that FC ∥ ED or FCED is a parallelogram

(ii) In Δ ABF, DBC

$\angle AFB = \angle BCD$ (alternate ∠'s =, AE ∥ CD) ✓

$\angle FAB = \angle BDC$ (alternate ∠'s =, AE ∥ CD) ✓

AB = BD (given)

∴ Δ ABF ≡ Δ DBC (AAS test) ✓ → other proofs are possible BUT "handle with care" It must not involve an assumption that FC ∥ ED - this must be proven if used...

(b)

(i) $\left[\frac{m+n}{4}\right]^2 = \frac{m^2+2mn+n^2}{16}$ ✓

$= \frac{14mn+2mn}{16}$

$= \frac{16mn}{16}$

$= mn$

(ii) $\left[\frac{m+n}{4}\right]^2 = mn$

take logs both sides

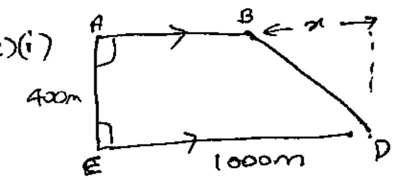
$\log\left(\frac{m+n}{4}\right)^2 = \log mn$

$2\log\frac{m+n}{4} = \log mn$ ✓

$\log\frac{m+n}{4} = \frac{1}{2}\log mn$

$= \frac{1}{2}(\log m + \log n)$ ✓

(c)(i)



AB = (1000 − x) m

$BD^2 = x^2 + 400^2$ (Pythagoras' Thm)

$BD = \sqrt{x^2+400^2}$ ← must show how

$C = 800\times(1000-x) + 1000\sqrt{x^2+400^2}$ ✓

(ii) $\frac{dC}{dx} = -800 + \frac{1}{2}\times 1000(400^2+x^2)^{-\frac{1}{2}}\times 2x$

$= -800 + \frac{1000x}{\sqrt{400^2+x^2}}$ ✓

$\frac{dC}{dx} = 0$ for min cost

$800 = \frac{1000x}{\sqrt{400^2+x^2}}$

$\sqrt{400^2+x^2} = \frac{5}{4}x$

$400^2 + x^2 = \frac{25}{16}x^2$

$400^2 = \frac{9x^2}{16}$

$x^2 = \frac{400^2\times 4^2}{3^2}$

$x = \pm\frac{1600}{3}$

x > 0

∴ $x = \frac{1600}{3}$ ✓

x	500	$\frac{1600}{3}$	600
$\frac{dC}{dx}$	−19.13	0	32.050...
slope	╲	—	╱

✓

∴ minimum when $x = \frac{1600}{3}$

(iii) minimum cost of pipeline

$= 800\times\left(1000-\frac{1600}{3}\right) + 1000\sqrt{\left(\frac{1600}{3}\right)^2+400^2}$

= $1040000 ✓