JG
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NB
YR

Name: ____________________

Class: 12MT2__ or 12MTX__

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2012 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

Directions to candidates

- Attempt all questions
- Approved calculators may be used.
- Standard Integral Tables are provided at the back of this paper.
- Write your name and class in the space provided at the top of this question paper

Section I - TOTAL MARKS 10

- To be answered on the removable answer grid at the back of the exam paper
- Allow about 15 minutes for this section

Section II - TOTAL MARKS 90

- All answers to be completed on the writing paper provided. Each question is to be commenced on a new page clearly marked Question 11, Question 12, etc on the top of the page. Write your name and class at the top of each page.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Allow about 2 hours and 45 minutes for this section

YOUR ANSWERS WILL BE COLLECTED IN ONE BUNDLE. THE MULTIPLE CHOICE SECTION I ON TOP AND THEN WRITTEN ANSWERS TO SECTION II AND THEN THE QUESTION PAPER.

SECTION I 10 MARKS

INSTRUCTIONS

- Attempt all questions
- Allow about 15 minutes for this section
- Section I answers are to be completed on the multiple-choice answer sheet attached to the back of this question paper.
- Select the alternative A, B, C or D that best answers the question

1. Evaluate $\sqrt{6} + 7$ correct to 3 significant figures.

(A) 3.60

(B) 3.61

(C) 9.44

(D) 9.45

2. If the roots of the quadratic equation, $3x^2 - 9x + 5 = 0$, are α and β, then

(A) $\alpha\beta = -\frac{5}{3}$ and $\alpha + \beta = -3$

(B) $\alpha\beta = \frac{5}{3}$ and $\alpha + \beta = -3$

(C) $\alpha\beta = \frac{5}{3}$ and $\alpha + \beta = 3$

(D) $\alpha\beta = 3$ and $\alpha + \beta = \frac{5}{3}$

3. A parabola has as its focus $(4,8)$ and directrix $y = -2$. Find the coordinates of the vertex.

(A) $(4, 3)$

(B) $(8, 1)$

(C) $(2, 8)$

(D) $(1, 8)$

4. Solve the pair of simultaneous equations.

$$3x + y = 3 \quad (1)$$
$$5x + 2y = 4 \quad (2)$$

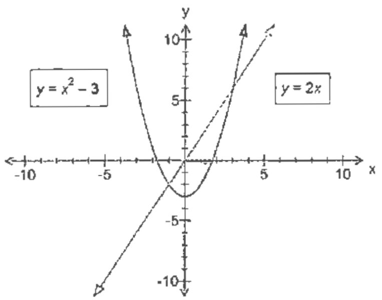
(A) $x = 2$ and $y = -3$

(B) $x = 2$ and $y = 3$

(C) $x = 4$ and $y = -3$

(D) $x = 4$ and $y = 3$

5. The shaded region shown in the diagram below is bounded by the functions, $y = x^2 - 3$ and $y = 2x$.



Which of the following is used to calculate the area of the shaded region

(A) $\int_{-1}^{3}(x^2 - 3 - 2x)dx$

(B) $\int_{-1}^{3}(x^2 - 3 + 2x)\,dx$

(C) $\int_{-1}^{3}(2x - x^2 - 3)\,dx$

(D) $\int_{-1}^{3}(2x - x^2 + 3)\,dx$

6. Evaluate

$$\sum_{r=3}^{5}(2r + 4)$$

(A) 24

(B) 36

(C) 50

(D) 72

7. Solve $|2a + 7| > 9$

(A) $a > -8, a < 1$

(B) $a < -8, a > 1$

(C) $a > -8 \ \ and \ \ a < 1$

(D) $a < -8 \ \ and \ \ a > 1$

8. If $\ln g = 5$ and $\ln h = 9$ find the value of $\ln(gh^2)$

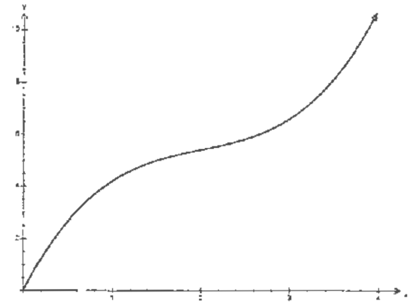
(A) 6.003887067

(B) 23

(C) 86

(D) 90

9. For the part of the function which is shown, which of the following properties are true?



(A) $f'(x) > 0 \ and \ f''(x) > 0$

(B) $f(x) \geq 0 \ and \ f'(x) > 0$

(C) $f'(x) < 0 \ and \ f''(x) \neq 0$

(D) $f(x) \geq 0 \ and \ f''(x) > 0$

10. The equation of any line that passes through the point of intersection of the lines, $3x + 2y + 1 = 0$ and $5x + y + 3 = 0$, can be written in the form

$$3x + 2y + 1 + k(5x + y + 3) = 0$$

Using this equation, an expression for the gradient of such a line is

(A) $\frac{8k}{3}$

(B) $\frac{-8k}{3}$

(C) $\frac{3+5k}{2+k}$

(D) $-\frac{3+5k}{2+k}$

SECTION II 90 MARKS

INSTRUCTIONS

- Answer all questions on the writing paper provided
- Allow about 2 hours and 45 minutes for this section
- Begin each question on a new page.
- Show all necessary working.

Question 11 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Simplify $\frac{x^2+10x+21}{3x^2-27}$ 2

(b) (i) Show that the sequence below is arithmetic 1
$4, 9, 14, 19, \ldots$

(ii) Hence find the sum of the first 100 terms of the series 1

(c) If $\frac{4}{\sqrt{2}+1} = a\sqrt{2} - b$, find the values of a and b. 2

(d) Find the equation of the tangent to the curve $y = x^2 - 4$ at the point where $x = 2$. Express your answer in gradient intercept form. 3

(e) The points A(2,0), B(6,2) and C(7,0) lie on a number plane.

(i) Draw a neat labelled diagram that shows this information. 1

(ii) Find the gradient of side AB and the gradient of side BC. 2

(iii) Explain why ΔABC is a right angled triangle. 1

(iv) Hence or otherwise, find the area of ΔABC. 2

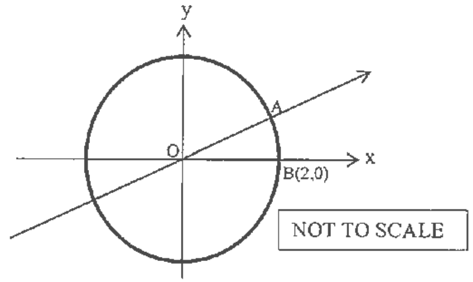
Question 12 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Evaluate $\int_1^3 (2x^3 - 9)dx$ 2

(b) What is the exact value of $cos\frac{\pi}{4} + sin\frac{\pi}{3}$? 2
Answer as a single fraction.

(c) Solve $5cos\alpha = -4$ where $0 \le \alpha \le 2\pi$. Answer correct to 2 decimal places. 2

(d) In the diagram below, the circle $x^2 + y^2 = 4$ and the line $y = \sqrt{3}x$ are shown. The point $B(2,0)$ is one of the x-intercepts of the circle and the point A is the point of intersection of the line and circle, as shown on the diagram.



(i) Show that $\angle AOB = \frac{\pi}{3}$. 1

(ii) Hence find the area of sector AOB. 1

(iii) Find the perimeter of sector AOB. 1

(e) Differentiate

i) $cos^4 x$ 1

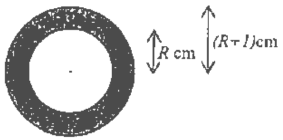
ii) $x^2 \tan x$ 1

(f) Find the area bounded by the x-axis and one period of the curve $y = \sin x$. 2

(g) Sketch the region $y > x^2$ and $y < 4$. 2

Question 13 (15 marks) BEGIN A NEW PAGE **Marks**

(a) An annulus is a 2 dimensional shape formed when a small circle is removed from the centre of a larger circle. In the diagram the radius of the inner circle is Rcm and the radius of the outer circle is 1cm larger, that is $(R+1)cm$.



(i) Show that the area, A, of the annulus can be given by $A = 2\pi R + \pi$. **2**

(ii) Find $\dfrac{dA}{dR}$ **1**

(iii) The radius of the inner circle is increasing at a rate of $2cm/s$. Find the rate of increase of the area. **1**

(b) During construction of a building the height of the building, is given by the equation $H = t - (t-10)^2 + 100$, where H is measured in metres and t is the number of months since construction began. **2**

Determine the rate that the height is increasing after *3 months*.

(c) A particle is subjected to external forces, such that its displacement, x, is given by the equation $x = 4t^3 - 3t$, where displacement is given in metres and time in seconds.

(i) Find expressions for the particle's velocity, $\dot{x}$, and acceleration, $\ddot{x}$. **2**

(ii) Find the acceleration of the object at $t = 5$ seconds. **1**

(iii) Find the time when the object is stationary and its position at this time. **2**

iv) When is the particle accelerating in the positive direction? **1**

Question 13 continues next page

Question 13 continued

(d) The velocity, $\dot{x}\ m/s$, of an object is given by the equation, $\dot{x} = 5t + 11$.

(i) Find an equation for the displacement of the object if it has an initial displacement of $-8\ m$. **2**

(ii) Hence find its displacement after 10 $seconds$. **1**

Question 14 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Differentiate with respect to x.

(i) $y = e^{2x}$ **1**

(ii) $y = \left(e^{x^2} - 7\right)^4$ **2**

(iii) $y = \ln\sqrt{3x - 9}$ **2**

(iv) $y = \log_4 5x$ **2**

(b) At the start of January 2008, Jin placed a number of fish in the dam on her farm. Over time the number of fish increases such that the rate of change of the population, $\frac{dP}{dt}$, is proportional to the population, P. That is $\frac{dP}{dt} = kP$, where k is a positive constant.

(i) Show that an expression for the population can be written in the form $P = Ae^{kt}$. **1**

(ii) If Jin initially placed 10 fish in the dam, find A. **1**

(iii) At the end of 2009, there were 300 fish. Find the value of k, correct to 3 significant figures. **2**

(iv) Hence, estimate to the nearest 1000, the number of fish that will be in the dam at the end of 2012. **1**

(c) Evaluate $\int_1^2 \frac{10}{5x+8}\,dx$. Leave your answer as an exact value. **3**

Question 15 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Trevor took out a loan of \$15000 to buy a car. Each month he is charged interest at 1.5% per month and makes a payment of \$M at the end of the month. A_n is the amount that Trevor still owes at the end of the n^{th} month.

(i) By first finding expressions for A_1 and A_2 show that $A_3 = 15000 \times 1.015^3 - M(1.015^2 + 1.015 + 1)$. **2**

(ii) Hence write an expression for A_n. **1**

(iii) How much does Trevor need to pay each month to repay the loan in 36 months? **2**

(b) (i) Differentiate $\frac{14}{7x^2+2}$ **1**

(ii) Hence find $\int \frac{14x}{(7x^2+2)^2}\,dx$ **1**

(c) For the function $y = 2x^3 - 3x^2 - 12x + 18$, find

(i) The first and second derivatives. **1**

(ii) Find any stationary points and determine their nature. **2**

(iii) Find the point of inflexion. **1**

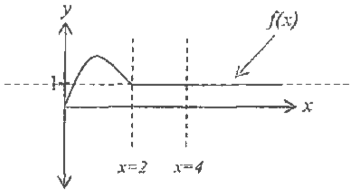
(iv) Find the intercepts. **2**

(v) Hence sketch the function showing these features. **2**

Question 16 (15 marks) BEGIN A NEW PAGE **Marks**

(a) Evaluate $\int_4^8 \frac{420}{x}\,dx$ by using Simpson's rule with 5 function values. **2**

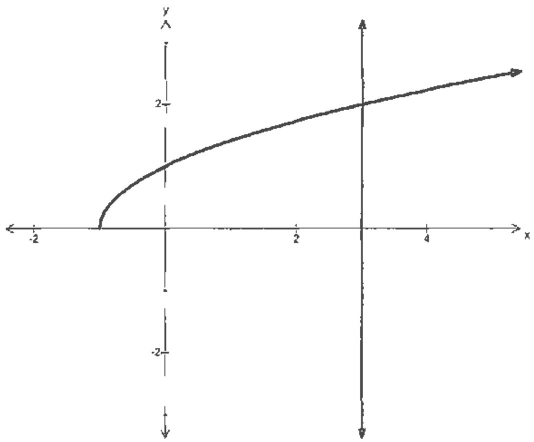
(b) The diagram below shows the right hand side of an odd function, $f(x)$.



(i) Copy or trace the diagram onto your writing paper and complete the left hand side of the graph. **1**

(ii) Hence evaluate $\int_{-2}^{4} f(x)dx$. **1**

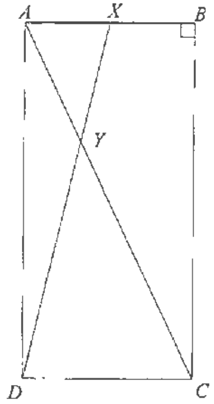
(c) The area enclosed by the curve $y = \sqrt{x+1}$, the x-axis and the line $x = 3$, is rotated around the x-axis **2**



Find the exact volume of the solid formed.

Question 16 continues next page

(d) In the diagram below, $ABCD$ is a rectangle. The point X is the midpoint of AB. The line AC meets XD at Y.



Not to scale

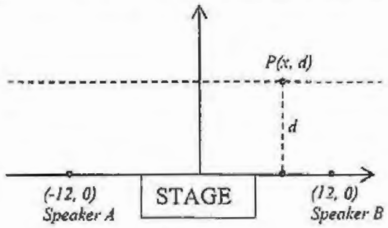
(i) Prove that ΔAXY and ΔCDY are similar. **2**

(ii) Show that $CY = 2AY$. **1**

Question 16 continues next page

Question 16 continued

(e) At a music festival, the speakers at the front are placed on either side of the main stage and are 24 $metres$ apart. The sound mixers are to be a distance of d $metres$ from the stage (in a particular row).



It is known that the total sound, S, from these speakers at point $P\,(x, d)$ is:

$$S = \frac{100}{d^2 + (x+12)^2} + \frac{100}{d^2 + (x-12)^2}$$

(i) Show that $\frac{dS}{dx} = -200\frac{M}{Q}$, where **2**

$M = (x+12)(d^2 + (x-12)^2)^2 + (x-12)(d^2 + (x+12)^2)^2$ and

$Q = (d^2 + (x+12)^2)^2(d^2 + (x-12)^2)^2$.

(ii) By noting that: **2**

$M = 2x(x^2 + 144 + d^2 + 24\sqrt{144 + d^2})(x^2 + 144 + d^2 - 24\sqrt{144 + d^2})$,

use $\frac{dS}{dx}$ to show that Mario the sound mixer, who moves along a row 20 $metres$ from the stage, measures the sound to be at a maximum when in line with the centre of the stage.

(iii) Another sound mixer (Aaron) decides it may be better to be closer to the stage. Aaron moves along a row 5 $metres$ from the stage. **2**

Describe how the sound level changes for Aaron as he moves along the row. Give clear reasons for your answer.

END OF PAPER

2012 Mathematics
2Unit AP4

MC

1	D	6	B
2	C	7	B
3	A	8	B
4	A	9	B
5	D	10	D

11 (a) $\frac{x^2+10x+21}{3x^2-27} = \frac{(x+7)(x+3)}{3(x+3)(x-3)}$ ✓

$= \frac{x+7}{3(x-3)}$ or $\frac{x+7}{3x-9}$ ✓

(b) (i) $\frac{19-4}{3} = 5$ ✓ or other working

(ii) $a=4$, $d=5$, $n=100$

$S_{100} = \frac{100}{2}(2\times4+(100-1)5)$

$= 25150$ ✓

(c) (i) $\frac{4}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{4\sqrt{2}-4}{2-1}$ ✓

$= 4\sqrt{2}-4$

$a=4$, $b=4$ ✓

(d) $y = x^2-4$

$\frac{dy}{dx} = 2x$

at $x=2$

$m = 2\times2$

$= 4$ ✓

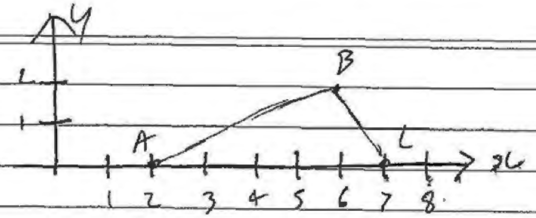
$y = 2^2-4$

$= 0$ ✓

$y-0 = 4(x-2)$

$y = 4x-8$ ✓

(e) (i)



point only required.

(ii) $m_{AB} = \frac{2-0}{6-2} = \frac{2}{4} = \frac{1}{2}$

$m_{BC} = \frac{0-2}{7-6} = \frac{-2}{1} = -2$

(iii) $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$

$= -1$

$\therefore AB \perp BC$ so ΔABC is right angled.

(iv) $AB = \sqrt{(2-0)^2 + (6-2)^2}$
$= 2\sqrt{5}$

$BC = \sqrt{1^2 + 2^2}$
$= \sqrt{5}$

Area $= \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5}$
$= 5u^2$

OR

Altitude through $B = 2$ units
$AC = 7 - 2 = 5$

Area $= \frac{1}{2} \times 2 \times 5$
$= 5u^2$

(12) (a) $\int_1^3 (2x^3 - 9)\,dx$

$= \left[\frac{2x^4}{4} - 9x\right]_1^3$

$= \left[\frac{x^4}{2} - 9x\right]_1^3$

$= \frac{81}{2} - 27 - (1 - 9)$

$= 22$

(b) $\cos\frac{\pi}{4} + \sin\frac{\pi}{3}$

$= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2} + \sqrt{3}}{2}$

(c) $5\cos\alpha = -4$

$\cos\alpha = -\frac{4}{5}$

S | A
T | C

reference angle $= 0.6435\ldots$

$\alpha = \pi \pm 0.6435\ldots$

$= 2.498091\ldots;\ 3.785093\ldots$

$= 2.50,\ 3.79$

(d) (i) $\tan\theta = m$

$\tan\theta = \sqrt{3}$ ✓ this line required - or similar.

$\theta = \tan^{-1}\sqrt{3}$

$= \pi/3$

$\angle AOB = \pi/3$

Note $\pi/3$ alone not accepted - Given in question.

(ii) $A = \dfrac{r^2\theta}{2}$

$= \dfrac{2^2 \pi/3}{2}$

$= \dfrac{2\pi}{3}\ u^2$ ✓

(iii) $2 + 2 + 2 \times \dfrac{\pi}{3}$

$= 4 + \dfrac{2\pi}{3}$ units ✓

ISE

(e) (i) $\dfrac{d}{dx}(\cos x)^4$ ✓

$= 4(\cos^3 x) \times (-\sin x)$ terms written in any order

(ii) $\dfrac{d}{dx}\, x^2 \tan x$

$= x^2 \sec^2 x + 2x \tan x$ ✓

ISE

(f) $A = 2\int_0^{\pi} \sin x\, dx$ ✓

$= 2\left[-\cos x\right]_0^{\pi}$

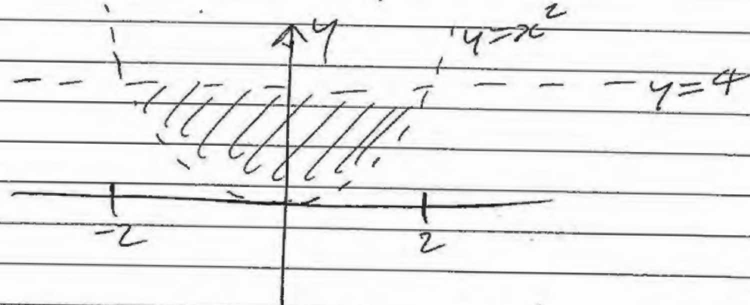
$= 2(1 - -1)$

$= 4u^2$ ✓

Note: It is also correct to start the period at a different place

eg $2\left|\int_{-\pi}^{0} \sin x\, dx\right|$

(g)



1 mark region shaded is enclosed by line & parabola.

2nd mark dotted boundary and at least one of the x coord of point of intersection.

Marking guidelines.

Question 12.

b) $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}$ ✓

$\frac{2+\sqrt{6}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{3}}{2}$ ✓

c) $\alpha = 2.50$ or 3.79 only ✓

Must show 2.50, $2\pi - 2.50$ to get the 2nd mark.

g) 1 mark for the region

2nd mark for dotted boundary and x intercept (s).

13 a (i) $A = \pi(R+1)^2 - \pi R^2$ ✓

$= \pi(R^2+2R+1) - \pi R^2$

$= \pi R^2 + 2\pi R + \pi - \pi R^2$ ✓

$= 2\pi R + \pi$

(ii) $\frac{dA}{dR} = 2\pi$ ✓

(iii) $\frac{dR}{dt} = 2$

$\frac{dA}{dt} = \frac{dR}{dt} \times \frac{dA}{dR}$

$= 2 \times 2\pi$

$= 4\pi \text{ cm}^2/\text{s}$ ✓

(b) $H = t - (t-10)^2 + 100$

$\frac{dH}{dt} = 1 - 2(t-10)$ ✓

OR

$$\left[\begin{aligned} H &= t - (t^2 - 20t + 100) + 100 \\ &= -t^2 - 19t \\ \frac{dH}{dt} &= -2t - 19 \end{aligned}\right]$$

at $t = 3$

$\frac{dH}{dt} = 1 - 2(3-10)$

$= 1 - 2 \times -7$

$= 15$ m/month ✓

(c) $x = 4t^3 - 3t$

(i) $\dot{x} = 12t^2 - 3$

$\ddot{x} = 24t$

(ii) At $t = 5$

$\ddot{x} = 24 \times 5$

$= 120\ m/s^2$

(iii) $12t^2 - 3 = 0$

$4t^2 - 1 = 0$

$4t^2 = 1$

$t^2 = 1/4$

$t = \pm 1/2$ s reject $-1/2$

stationary at $t = 1/2$ s at -1m

(d) (i) $\dot{x} = 5t + 11$

$x = \int (5t + 11)\,dt$

$x = \frac{5t^2}{2} + 11t + C$ ✓

at $t = 0$ $x = -8$

$-8 = 0 + 0 + C$

$C = -8$

$x = \frac{5t^2}{2} + 11t - 8$ ✓

(ii) at $t = 10$

$x = 352$ m ✓

14 a (i) $y = e^{2x}$

$y' = 2e^{2x}$

(ii) $y = (e^{x^2} - 7)^4$

$y' = 4(e^{x^2} - 7)^3 \times 2xe^{x^2}$ ✓

$= 8xe^{x^2}(e^{x^2} - 7)^3$ ✓

(iii) $y = \ln(3x - 9)^{1/2}$

$y' = \frac{\frac{1}{2}3(3x-9)^{-1/2}}{\sqrt{3x-9}}$ ✓

$= \frac{3}{2(3x-9)}$ ✓

(iv) $y = \log_4 5x = \frac{\ln 5x}{\ln 4}$ ✓

$y' = \frac{1}{\ln 4} \frac{5}{5x}$ ✓

$= \frac{1}{x \ln 4}$ ISE

b (i) $P = Ae^{kt}$

$\frac{dP}{dt} = kAe^{kt}$

$= kP$

(ii) at $t = 0$ $P = 10$

$10 = Ae^0$

$A = 10$ ✓

(iii) $P = 10e^{kt}$

at $t = 2$ $P = 300$

$300 = 10e^{2k}$ ✓

$30 = e^{2k}$

$2k = \ln 30$

$k = \frac{\ln 30}{2}$ ✓

$= 1.700598691\ldots$

$= 1.70$

(iv) $t = 5$

$P = 10 \times e^{1.70 \times 5}$

$= 49147.6884$

$= 49000$

Using rounded

OR

$P = 10 \times e^{\frac{\ln 30}{2} \times 5}$

$= 49295$

$= 49000$

using exact

accept either.

c $\int_1^2 \frac{10}{5x+8}\,dx = 2\int_1^2 \frac{5}{5x+8}\,dx$ ✓

$= 2\left[\ln(5x+8)\right]_1^2$ ✓

$= 2(\ln 18 - \ln 13)$

$= 2\ln\left(\frac{18}{13}\right)$ ✓

Q15

a) (i) $A_1 = 15000(1.015) - M$

$A_2 = \left(15000(1.015) - M\right)1.015 - M$

$= 15000(1.015)^2 - M(1.015) - M$

$= 15000(1.015)^2 - M(1.015 + 1)$ (1)

$A_3 = \left(15000(1.015)^2 - M(1.015+1)\right)1.015 - M$ (1)

$= 15000(1.015)^3 - M(1.015^2 + 1.015 + 1)$

(ii) $A_n = 15000(1.015)^n - M(1 + 1.015 + \dots + 1.015^{n-1})$ (1)

(accepted $A_n = 15000(1.015)^n - M\frac{(1.015^n - 1)}{1.015 - 1}$

(iii) $A_{36} = 0$ loan will be repaid in 36 mths

$0 = 15000(1.015)^{36} - M(1 + 1.015 + \dots + 1.015^{35})$

$0 = 15000(1.015)^{36} - M \cdot \frac{1(1.015^{36} - 1)}{1.015 - 1}$

$M = 15000(1.015)^{36} \times \frac{0.015}{1.015^{36} - 1}$

AW1 for CFm → incorrect formula with full working shown.

$M = 542.285933$

$M = \$542.29$

Common mistake
$0 = 15000 \times 1.015^{36} - \frac{M(1.015^{35} - 1)}{1.015 - 1}$
$M = \$562.31$. AW1 only.

b) i) $\frac{d}{dx}\left(14(7x^2+2)^{-1}\right) = -1 \times 14(7x^2+2)^{-2} \times 14x$ (marks not awarded for this line).

$= \frac{-14 \times 14x}{(7x^2+2)^2}$

$= \frac{-196x}{(7x^2+2)^2}$ (1)

(ii) $\int \frac{14x}{(7x^2+2)^2}\,dx = \frac{-1}{14}\int \frac{-14 \times 14x}{(7x^2+2)^2}\,dx$

$= -\frac{1}{14} \times \frac{14}{7x^2+2} + C$ Ignore C

$= -\frac{1}{7x^2+2} + C$ (1) ← must simplify to

(c) $y = 2x^3 - 3x^2 - 12x + 18$

(i) $\frac{dy}{dx} = 6x^2 - 6x - 12$

$\frac{d^2y}{dx^2} = 12x - 6$

both for (1)

(ii) $\frac{dy}{dx} = 0$ $\quad 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, -1$

*must have x values, y values + correct tests for min/max to get 2 marks

When $x = 2$ $\quad \frac{d^2y}{dx^2} = 24 - 6 = 18 > 0 \quad \therefore$ min. $\quad y = -2$

$x = -1$ $\quad \frac{d^2y}{dx^2} = -12 - 6 = -18 < 0 \quad \therefore$ max $\quad y = 25$

$\therefore$ minimum stat. pt at $(2, -2)$ (1) and maximum stat pt at $(-1, 25)$ (1)

iii) $\frac{d^2y}{dx^2} = 0$ $\quad 12x - 6 = 0$

$x = \frac{1}{2}$

When $x = \frac{1}{2}$ $\quad y = 2(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 12(\frac{1}{2}) + 18$

$y = 10\frac{3}{4}$

If test not shown → AW0

x	0	$\frac{1}{2}$	1
$\frac{d^2y}{dx^2}$	-6	0	6

Test

$\therefore$ change in concavity (1)

$\therefore$ point of inflexion at $(\frac{1}{2}, 11\frac{1}{2})$
$(\frac{1}{2}, \frac{23}{2})$

iv) x intercepts $y = 0$ $\quad 0 = 2x^3 - 3x^2 - 12x + 18$

$0 = x^2(2x-3) - 6(2x-3)$

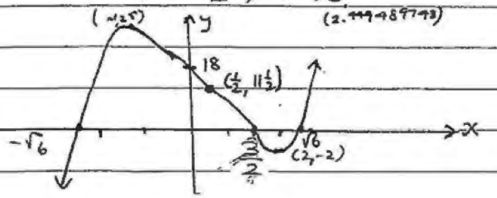
$0 = (2x-3)(x^2-6)$

$\therefore x = \frac{3}{2}, \pm\sqrt{6}$ (1) ← must have all 3

y intercept $x = 0$

$y = 18$ (1)

v)



AS PER ORIGINAL SOLUTIONS

* If pronounced Horizontal point of inflexion drawn → AW1 only

(16) (a)

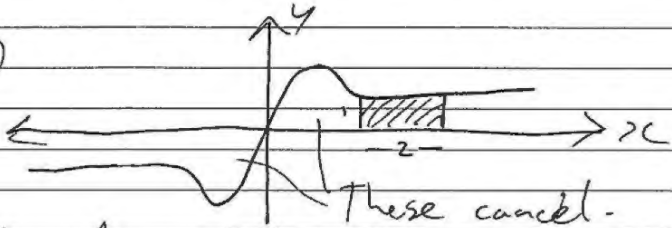
x	4	5	6	7	8
y	105	84	70	60	52.5

$A = \frac{1}{3}\left(105 + 52.5 + 4(84+60) + 2(70)\right)$

$= \frac{1}{3} \times 873.5$

≈ 291

(b) (i)



(ii) $\int_{-2}^{4} f(x)\,dx = 1 \times 2$

$= 2.$

(c) $V = \pi \int_{-1}^{3} (x+1)\,dx$

$= \pi \left[\frac{x^2}{2} + x\right]_{-1}^{3}$

$= \pi\left(\frac{9}{2} + 3 - \left(\frac{1}{2} - 1\right)\right)$

$= 8\pi\ u^3$ ISE

(d) (i) $\angle AYX = \angle CYD$ (vertically opposite $\angle$s)
$\angle XAY = \angle DCY$ (Alternate $\angle$s $AB \parallel DC$)

$\therefore \triangle AXY \parallel\!\!\!/ \triangle CDY$ (equiangular)

(ii) $\frac{AX}{DC} = \frac{1}{2} = \frac{AY}{CY}$

$\therefore\ CY = 2AY$

(i) $S = \frac{100}{d^2 + (x+12)^2} + \frac{100}{d^2 + (x-12)^2}$

$S = 100[d^2 + (x+12)^2]^{-1} + 100[d^2 + (x-12)^2]^{-1}$

$\frac{dS}{dx} = -100[d^2 + (x+12)^2]^{-2} \times 2(x+12)$

$-100[d^2 + (x-12)^2]^{-2} \times 2(x-12)$

$= \frac{-200(x+12)}{[d^2 + (x+12)^2]^2} + \frac{-200(x-12)}{[d^2 + (x-12)^2]^2}$

$= \frac{-200[(x+12)[d^2 + (x-12)^2]^2 + (x-12)[d^2 + (x+12)^2]^2]}{[d^2 + (x+12)^2]^2[d^2 + (x-12)^2]^2}$

$= -200\frac{M}{Q}$, as required

(ii) The maximum sound that Mario experiences occurs when $\frac{dS}{dx} = 0$.

$\therefore -200\frac{M}{Q} = 0$

i.e. when $M = 0$:

$2x(x^2 + 144 + d^2 + 24\sqrt{144 + d^2})$

$\times (x^2 + 144 + d^2 - 24\sqrt{144 + d^2}) = 0$

Putting $d = 20$, $\therefore d^2 = 400$

i.e. $2x(x^2 + 544 + 24\sqrt{544})(x^2 + 544 - 24\sqrt{544}) = 0$

The only possibility is if $x = 0$ as both second and third expressions are positive.

When $x = 0$, the sound is minimum or maximum, however on checking: MUST USE VALUES

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
S'	2437726[illegible] +	0	-24377642 −

$\therefore$ Sound level is maximum when $x = 0$.

(iii) For Aaron:

$2x(x^2 + (144 + 25) + 24\sqrt{169})(x^2 + 169 - 24\sqrt{169}) = 0$

$x = 0, x^2 + 169 + 24\sqrt{169} = 0$

or $x^2 + 169 - 24\sqrt{169} = 0$

$\therefore x = 0$ or $x^2 = 24 \times 13 - 169$

$x^2 = 143$

$x = \pm\sqrt{143}$

Checking each of these:

x	<	$-\sqrt{143}$	>
S^1	+	0	−

$\therefore$ maximum

−	0	+
−	0	+

$\therefore$ minimum

x	<	$+\sqrt{143}$	>
S^1	+	0	−

$\therefore$ maximum

these are not always accepted You should use values!

Hence, for Aaron the sound level will peak at two locations, when $x = \pm\sqrt{143} = \pm 11.96$m