EW
KL
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AB/KW

Name: ____________________

Class: 12MT2__ or 12MTX__

Teacher: ____________________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2014 AP4

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS PAPER

Time allowed - 3 HOURS
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- Multiple choice questions are to be answered on the multiple choice sheet provided
- Each question in Section 2 is to be commenced in a new booklet clearly marked Question 11, Question 12, etc
- All necessary working should be shown in every question in Section 2. Full marks may not be awarded for careless or badly arranged work.
- Board of Studies approved calculators may be used.
- A Standard Integral sheet is provided.
- Write your name and class in the space provided at the top of this question paper.

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section
Use the multiple choice answer sheet for Questions 1 – 10.

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1 Which of the following is a solution to the equation $x^3 = 2x^2$?

(A) $x = 0$ *or* $x = 2$

(B) $x = 2$

(C) $x = -2$

(D) $x = 0$ *or* $x = \frac{1}{2}$

2 Which of the following represents the domain of $y = \frac{1}{\sqrt{2-x}}$?

(A) $x > 2$

(B) $x < 2$

(C) $x \le 2$

(D) $x \ne 2$

3 If $\tan\theta = \frac{12}{5}$ and $\cos\theta < 0$, which of the following would $\sin\theta$ equate to?

(A) $67°23'$

(B) $\frac{-12}{13}$

(C) $\frac{-3}{5}$

(D) $\frac{5}{13}$

4 Which of the following equations describes the locus of all points with vertex $(3,3)$ and directrix $x = 1$?

(A) $(x-3)^2 = 4(y-3)$

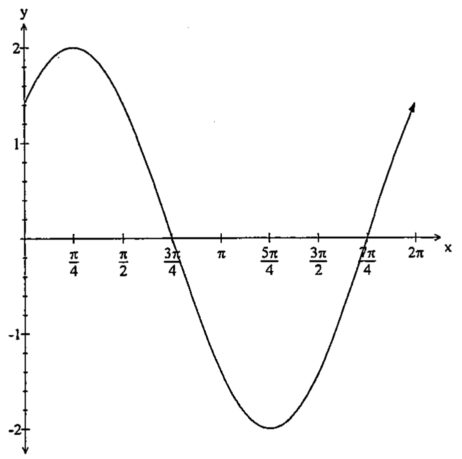
(B) $(y-3)^2 = 8(x-3)$

(C) $(y-3)^2 = -8(x-3)$

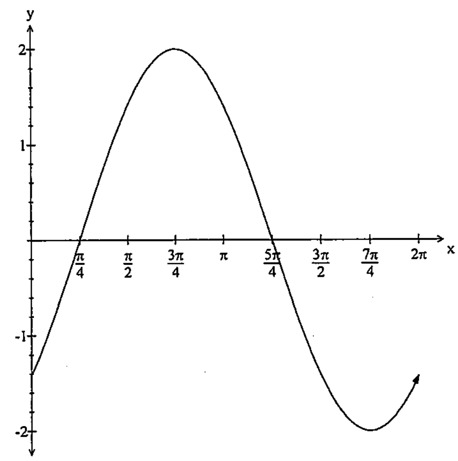
(D) $(x+3)^2 = -8(y-3)$

5 Which of the following graphs represents $y = 2\sin\left(x - \frac{\pi}{4}\right)$?

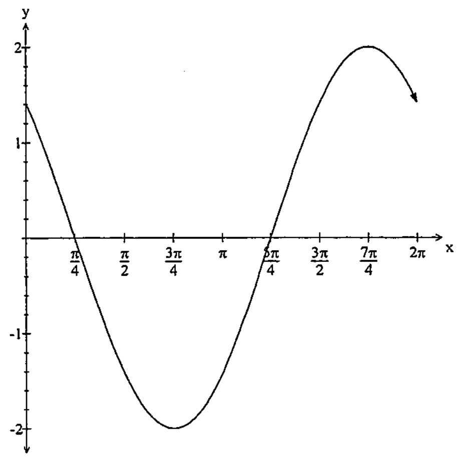
(A)



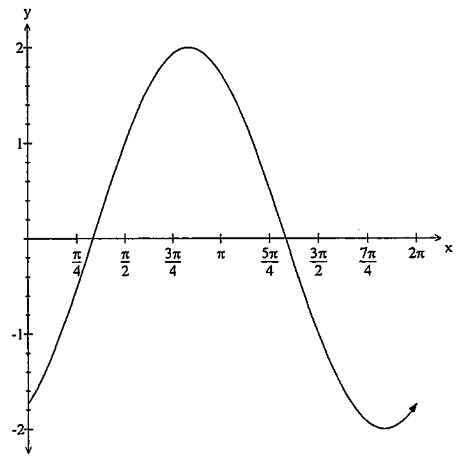
(B)



(C)



(D)



6 In a geometric series, the second term is 12 and the third term is -18. Which of the following is the value of the first term?

(A) -8

(B) 8

(C) 18

(D) $\frac{-3}{2}$

7 What is the value of $\lim_{x \to 3}\left(\frac{x^3 - 27}{x^2 - 9}\right)$?

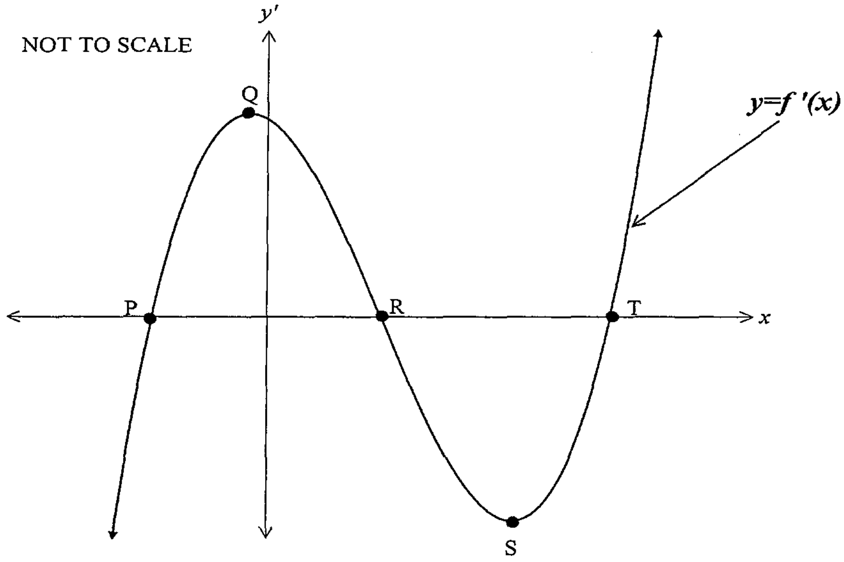
(A) undefined

(B) 0

(C) 1.5

(D) 4.5

8 The diagram below represents a sketch of the **gradient function** of the curve $y = f(x)$. Which of the following points have $f''(x) = 0$ and $f'(x) < 0$?



(A) R

(B) Q

(C) T

(D) S

9 Given that $y = \log_{10} x$, which of the following statements is correct?

(A) $10^x = y$

(B) $\dfrac{dy}{dx} = \dfrac{\ln 10}{x}$

(C) $\dfrac{dy}{dx} = \dfrac{1}{x \ln 10}$

(D) $\dfrac{dy}{dx} = \dfrac{1}{x}$

10 Let α and β be the roots of the equation $4x^2 - 3x - 2 = 0$.
Find the value of $\alpha^2 + \beta^2$.

(A) $\dfrac{9}{16}$

(B) $\dfrac{-7}{16}$

(C) $2\dfrac{9}{16}$

(D) $1\dfrac{9}{16}$

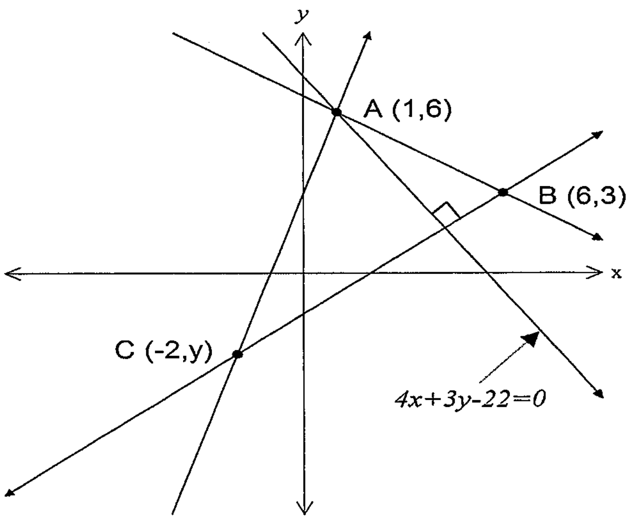
END OF SECTION 1

Section II
90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section
Answer each question in a separate writing booklet.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram above A is $(1,6)$, B is $(6,3)$ and C is $(-2,y)$. The line that passes through A and is **perpendicular** to line BC has the equation $4x+3y-22=0$.

(i) Find the equation of the line BC. **2**

(ii) Hence show that C is $(-2,-3)$. **1**

(iii) Find the length of BC. **1**

(iv) Show that the distance from A to the line BC is $\frac{27}{5}$ units. **2**

(v) Hence or otherwise, find the area of ΔABC. **1**

Question 11 continued next page

(b) Graph the region bounded by $x^2 + y^2 < 4$ and $y \le x^2 + 1$. **3**

(c) Find the equation of the tangent to the curve $y = \ln(2x+1)$, at $x = 0$. **3**

(d) Solve $|5x - 2| \ge 3$. **2**

End question 11

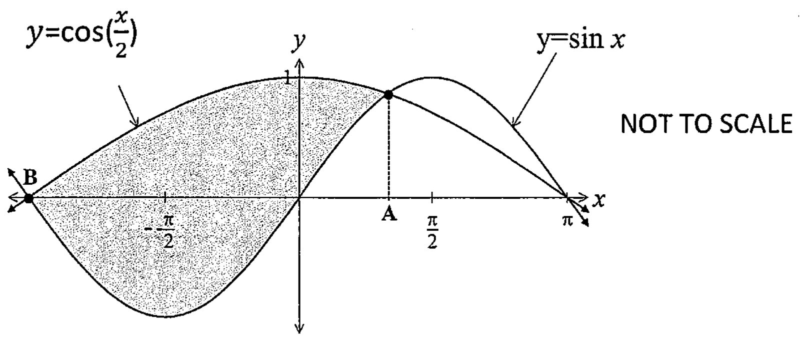
Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Convert 0.9 radians into degrees and minutes *(answer to the nearest minute).* **1**

(b) Use Simpson's Rule with all the values in the table to find an approximate value for $\int_0^3 f(x)dx$. **3**

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0.3	0	-1.3	-2.1	0	1.2	5

(c)



A section of the graphs of $y = \sin x$ and $y = \cos\left(\dfrac{x}{2}\right)$ are represented above.

(i) The curve meets at ***A*** and ***B***. Show by substitution, that the x values of point ***A*** and ***B*** are $\dfrac{\pi}{3}$ and $-\pi$ respectively. **2**

(ii) Hence or otherwise, find the exact area of the shaded region. **3**

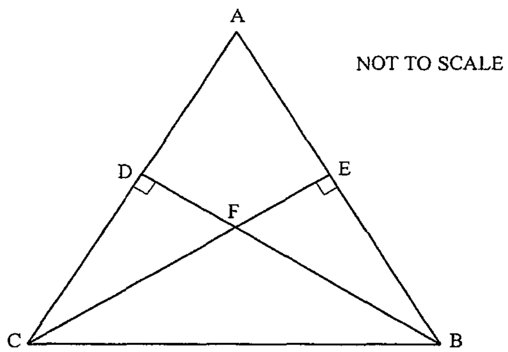
(d) Solve $3\sin\theta\tan^2\theta = \sin\theta$, for $0 \le \theta \le 2\pi$ leaving your answers in exact form. **3**

(e) Evaluate $\displaystyle\int_0^2 \frac{x^3}{2+2x^4}\, dx$, leaving your answer in simplified exact form. **3**

End question 12.

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a)



In diagram above, ΔABC is an **isosceles** triangle where $AB = AC$.
$CE \perp AB$ and $BD \perp AC$.

(i) Prove that ΔCDB is congruent to ΔBEC. **2**

(ii) Explain why ΔCFB is an isosceles triangle **1**

(iii) Hence or otherwise prove that $DF = FE$ **2**

(b) The blood-alcohol content (A) of a person after they have been drinking is given by $A = A_0 e^{-kt}$, where A_0 represents the blood-alcohol content at the time a person stops drinking, t is measured in *hours* and A in *mg/ml.*

Melita stops drinking at 11pm on Saturday night $(t = 0)$ and her blood alcohol level was measured as 0.24 *mg/ml.* It took 28 hours for Melita's blood-alcohol level to be 0.001 *mg/ml.*

(i) Find the value of k *correct to 4 decimal places.* **2**

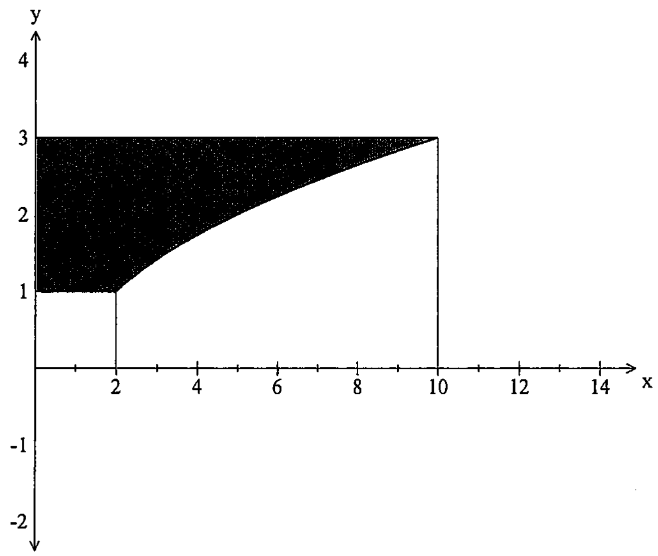
(ii) The allowable blood-alcohol limit for Melita to drive a car is 0.05 *mg/ml.* What is the earliest time on Sunday that Melita will be able to legally drive? *(leave your answer to the nearest hour)* **2**

(iii) What is the rate of decrease of the blood-alcohol level content in Melita's blood at 8.00am on Sunday? **1**

Question 13 continued next page

(c) Differentiate $\dfrac{5}{\sqrt{2-3x^2}}$ with respect to x **2**

(d) **3**



The diagram shows the shaded region enclosed by the curve $y=\sqrt{x-1}$, the y-axis and the lines $y=1$ and $y=3$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis (*leave your answer in exact form*).

End question 13

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) After a week of rain, the local dam starts to fill until at 10am Sunday the dam overflows into the river. At time ***t***, the height(***H***) of the river changes at the **rate** of

$\left(1-\frac{t}{20}\right)$ metres per hour.

Initially the height of the river is 5 metres.

(i) Show that the height of the river is given by the formula **1**

$$H=-\frac{t^2}{40}+t+5$$

(ii) Find the maximum height of the river during this flood. **2**

(iii) A bridge over this river will be flooded and closed once the height of the river reaches 12.5 metres. **3**

At what time(s) and day(s) will the bridge be closed and then reopened.

(b) Rachel borrowed \$35000 from a credit union to purchase a new car. Interest on the loan is calculated monthly at the rate of 0.7% per month and is charged immediately before each monthly repayment of \$$R$ is made.

Let A_n be the amount in dollars owing on the loan after the n^{th} repayment.

(i) Show that $A_3=35000\times1.007^3-R(1+1.007+1.007^2)$ **1**

(ii) Show that $A_n=35000\times1.007^n-\dfrac{1000R(1.007^n-1)}{7}$ **2**

(iii) If the loan is to be paid out after 5 years what would the value of R be? **2**

(iv) If Rachel decides to pay \$800 per month in repayments, how long would it take to pay out her loan? **2**

(c) Differentiate $\left(x^2-1\right)e^{3x-1}$ with respect to x. **2**

End question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $\sqrt{2}+\sqrt{18}+\sqrt{50}+\ldots\ldots$ is an arithmetic series. **2**

(ii) How many terms of this series give a sum of $100\sqrt{2}$? **2**

(b) The acceleration of a particle is given by $a = 1-2t$.
Where a is measured in cm/s^2 and t is measured in seconds.
Initially the particle is at rest 2 cm to the right of the origin.

(i) At what time is the particle next at rest? **2**

(ii) What is the position of the particle at this time? **2**

(c) Consider the curve $y = 3x^4 - 16x^3 + 24x^2$.

(i) Show that the curve cuts the x- axis only at the origin. **1**

(ii) Find the turning point(s) and determine their nature. **2**

(iii) Find the point(s) of inflexion. **2**

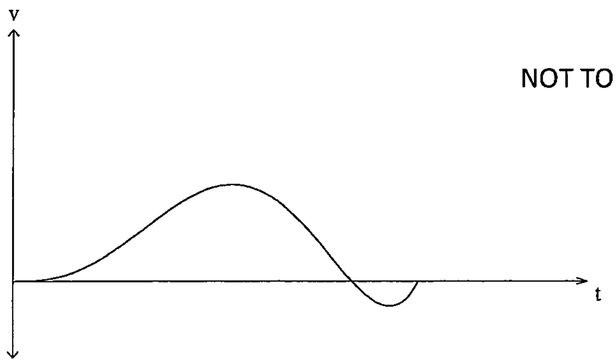
(iv) Sketch the curve, showing the intercepts with axes, turning point(s) and the point(s) of inflexion. **2**

End question 15

Question 16 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) A particle is observed as it moves in a straight line. Its velocity, v, at time t, is shown on the graph below. At time $t = 0$ the particle is at rest at origin.

Copy this graph into your Writing Booklet.



(i) On the time axis, mark and clearly label with the letter $\boldsymbol{K}$ when the acceleration of the particle is zero. **1**

(ii) On the time axis, mark and clearly label with the letter $\boldsymbol{L}$ when the acceleration of the particle is greatest. **1**

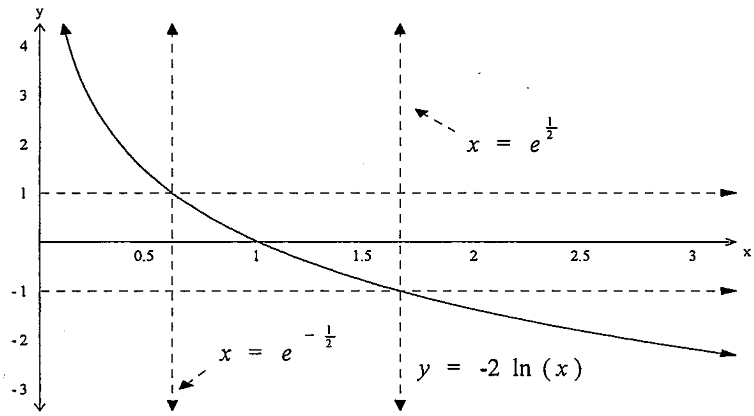
(iii) On the time axis, mark and clearly label with the letter $\boldsymbol{M}$ when the particle is furthest from origin. **1**

(b) Consider the series

$$\log_e x - 2(\log_e x)^2 + 4(\log_e x)^3 - 8(\log_e x)^4 + \ldots$$

(i) Show that this series is geometric. **1**

(ii) By using the graph below, find the values of x for which the series has a limiting sum. **2**



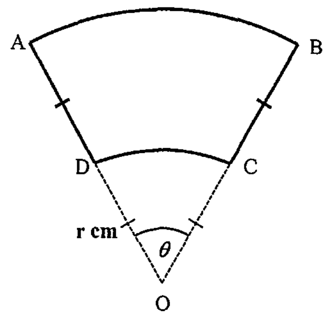
(iii) Find the limiting sum of this series in terms of x. **1**

Question 16 continued next page

(c) (i) Show that $\frac{d}{dx}(\frac{1}{3}\tan^3 x) = \tan^4 x + \tan^2 x$ **2**

(ii) Hence, evaluate $\int_0^{\frac{\pi}{3}} (\tan^4 x + \tan^2 x)dx$ **1**

(d)



NOT TO SCALE

In the diagram above, $AD = OD = OC = CB = r\,cm$ and $\angle AOB = \theta$ radians.

The perimeter of $ABCD = 12\,cm$. AB and CD are arcs of circles with centre O.

(i) Find an expression for r in terms of θ. **1**

(ii) Show that A, the area of the $ABCD$ in cm^2 is given by **2**

$$A = \frac{216\theta}{(2+3\theta)^2}.$$

(iii) Hence, find the value of θ which produces the maximum area for $ABCD$. **2**

End of paper

SOLUTION

2014 CTHS Mathematics Section I - Answer Sheet

Student Name ______________________________

Class _______________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

Correct

A ● B ⊗ C ○ D ○

1.	A ●	B ○	C ○	D ○
2.	A ○	B ●	C ○	D ○
3.	A ○	B ●	C ○	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ●
8.	A ○	B ○	C ○	D ●
9.	A ○	B ○	C ●	D ○
10.	A ○	B ○	C ○	D ●

Section 1.

1. $x^3 = 2x^2$

$x^3 - 2x^2 = 0$

$x^2(x-2) = 0$

$x = 0$ and $x = 2$

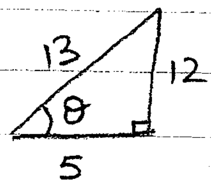
(A)

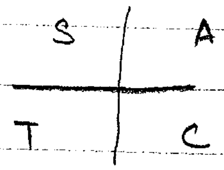
2. $2 - x > 0$

$x < 2$

B

3



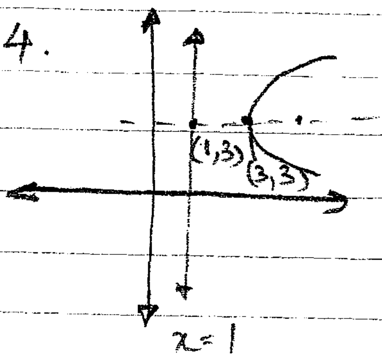


III rd Quadrant

$\sin\theta = -\frac{12}{13}$

B

4.



$a = 2$

$(y-3)^2 = 4 \times 2(x-3)$

$(y-3)^2 = 8(x-3)$

B

5 B

6 $ar = 12$

$ar^2 = -18$

$r = \frac{-3}{2}$

$a \times \frac{-3}{2} = 12$

$a = \frac{-24}{3} = -8$

A

7 $\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$

$\lim_{x \to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}(x+3)}$

$= \frac{3^2 + 3 \times 3 + 9}{3+3}$

$= \frac{\cancel{27}\ 9}{\cancel{6}\ 2}$

$= 4.5$

D

8. D

9. $y = \log_{10} x$

$10^y = x$

$y = \dfrac{\log_e x}{\log_e 10}$

$y = \dfrac{\ln x}{\ln 10}$

$\dfrac{dy}{dx} = \dfrac{1}{x \ln 10}$

(C)

10 $\alpha + \beta = \dfrac{3}{4}$

$\alpha\beta = \dfrac{-2}{4} = \dfrac{-1}{2}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\dfrac{3}{4}\right)^2 - 2 \times \left(\dfrac{-1}{2}\right)$

$= \dfrac{9}{16} + 1$

$= 1\dfrac{9}{16}$

D

12

a) 0·9 radians

$= \frac{0\cdot 9 \times 180}{\pi}$

$= 51°34'$

(1 mark)

b) $h = 0\cdot5$ (1 mark)

$\int f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)\right]$

$\therefore$

$= \frac{0\cdot5}{3}\left[0\cdot3 + 5 + 4(0 + -2 + 1\cdot2) + 2(-1\cdot3 + 0)\right]$

(1 mark)

$= -0\cdot15$ (1 mark)

c) Substituting $x = \frac{\pi}{3}$ in both the equations

$y = \sin x$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

$y = \cos\left(\frac{x}{2}\right)$

$= \cos\left(\frac{\pi}{3\times2}\right)$

$= \cos\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$.

So $x = \frac{\pi}{3}$ is a solution to both the equations

$\therefore$ both the curves will meet at pt A

(1 mark)

Substituting $x = -\pi$

$y = \sin x$

$= \sin(-\pi)$

$= 0$

$y = \cos\left(\frac{x}{2}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

$= 0$

$\therefore$ the curves meet at pt B as well.

(ii) $A = \int_{-\pi}^{\frac{\pi}{3}} \left(\cos \frac{x}{2} - \sin x \right) dx$

(1 mark)

$= \left[2\sin \frac{x}{2} + \cos x \right]_{-\pi}^{\frac{\pi}{3}}$

$= \left[2\sin\left(\frac{\pi}{3\times 2}\right) + \cos \frac{\pi}{3} \right] -$

$\left[2\sin\left(-\frac{\pi}{2}\right) + \cos(-\pi) \right]$

(1 mark)

$= \left[2\sin\left(\frac{\pi}{6}\right) + \cos \frac{\pi}{3} \right] - \left[-2\sin \frac{\pi}{2} - \cos\pi \right]$

$= \left[\cancel{2} \times \frac{1}{\cancel{2}} + \frac{1}{2} \right] - \left[-2\times 1 - (-1) \right]$

$= 1 + \frac{1}{2} - \left[-2+1 \right]$

$= 1 + \frac{1}{2} - \left[-1 \right]$

$= 1 + \frac{1}{2} + 1 = 2\frac{1}{2}$ units^2

(1 mark)

d) $3\sin\theta \tan^2\theta = \sin\theta$

$3\sin\theta \tan^2\theta - \sin\theta = 0$

$\sin\theta \left(3\tan^2\theta - 1 \right) = 0$

1 mark

$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$

1 mark

$3\tan^2\theta - 1 = 0$

$\tan\theta = \pm \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(1 mark)

final answer

$\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

e) $\int_0^2 \frac{x^3}{2+2x^4} dx$

$= \left[\frac{1}{8} \ln(2+2x^4) \right]_0^2$

1 mark

$= \frac{1}{8} \left[\ln(2+2\times(2)^4) - \ln(2+2(0)^4) \right]$

(1 mark

$= \frac{1}{8} \left(\ln 34 - \ln 2 \right)$

$= \frac{1}{8} \ln 17$

(1mark)

13 a)

In ΔCDB and ΔCEB

$\angle CDB = \angle CEB$ (90° given)

$\angle DCB = \angle EBC$

(base angles of an isosceles triangle are equal)
(1 mark)
for reasoning

$BC = BC$ (Common)

$\Delta CDB \cong \Delta CEB$ (AAS)
(1 mark)

ii). $\angle DCB = \angle ECB$

(corresponding angles of congruent triangles are equal)
$\therefore \Delta CFB$ is an isosceles Δ

ii) · $CE = DB$ (Corresponding sides of congruent Δs are equal
$FC = FB$ (sides of an Isosceles Δ CFB are equal)
(1 mark)

$\therefore CE - FC = DB - FB$

$\therefore DF = FE$ (1 mark)

13b

(i). $A = A_0 e^{-kt}$

At $t = 0$ $A = 0.24$

$\therefore A_0 = 0.24$
(1 mark)

now $t = 28$ $A = 0.001$

$0.001 = 0.24 e^{-k \times 28}$

$k = \dfrac{\ln(0.004167)}{-28}$

$k = 0.1957$
(1 mark)

(ii) $0.05 = 0.24 e^{-0.1957t}$
(1 mark

$t = \dfrac{\ln(0.2083)}{-0.1957}$

$t = 8.015$

$t = 8$

$\therefore$ time = Sunday 7am

(iii) $A = 0.24e^{-0.1957t}$

$\frac{dA}{dt} = -0.046968e^{-0.1957t}$

At 8:00 am
$t = 9$

$\frac{dA}{dt} = -0.00807$

Rate of decrease

$= 0.00807$ mg/mL per hour

(c) $\frac{d}{dx}\frac{5}{\sqrt{2-3x^2}}$

$= \frac{d}{dx} 5(2-3x^2)^{-\frac{1}{2}}$

$= -\frac{5}{2}(2-3x^2)^{-\frac{3}{2}} \times (-6x)$

1 mark

$= 15x(2-3x^2)^{-\frac{3}{2}}$

$= \frac{15x}{(2-3x^2)^{3/2}}$

(1 mark.

d)

$y = \sqrt{x-1}$

$y^2 = x-1$

$x = y^2+1$

$V = \pi \int_1^3 (y^2+1)^2 dy$

(1 mark)

$V = \pi \int_1^3 (y^4 + 2y^2 + 1) dy$

$= \pi \left[\frac{y^5}{5} + \frac{2y^3}{3} + y\right]_1^3$

(1 mark)

$= \pi \left\{ \left[\frac{3^5}{5} + \frac{2(3^3)}{3} + 3\right] - \left[\frac{1}{5} + \frac{2}{3} + 1\right] \right\}$

$= \frac{1016\pi}{15} u^3$

(1 mark

14 a

(i) $\frac{dH}{dt} = 1 - \frac{t}{20}$

$H = t - \frac{t^2}{40} + C$

At $t = 0$, $H = 5$

$\therefore C = 5$

$H = -\frac{t^2}{40} + t + 5$

(1 mark)

ii) $\frac{dH}{dt} = 0$

$1 - \frac{t}{20} = 0$

$t = 20$

$\frac{d^2H}{dt^2} = -\frac{1}{20}$ −ve

$\therefore$ maxima occurs at $t = 20$ (1 mark)

$H = -\left[\frac{20^2}{40}\right] + 20 + 5$

$= 15$ m

(1 mark)

(iii)

$12.5 = -\frac{t^2}{40} + t + 5$

1 mark

$\frac{t^2}{40} - t + 7.5 = 0$

$t^2 - 40t + 300 = 0$

$(t-10)(t-30) = 0$

$t = 10$ and 30 hrs

(1 mark)

Bridge blocked

10 am Sunday + 10 hours

= 8pm Sunday

Bridge Reopened

8pm Sunday + 20 hours

= 4pm Monday

1 mark

14 b

$A_1 = 35000(1.007) - R$

$A_2 = A_1(1.007) - R$

$= (35000(1.007) - R)1.007 - R$

$= 35000(1.007)^2 - R(1 + 1.007)$

$A_3 = A_2(1.007) - R$

$= (35000(1.007)^2 - R(1 + 1.007))1.007 - R$

$= 35000(1.007)^3 - R(1.007)^2 - R \times 1.007 - R$

$= 35000(1.007)^3 - R(1 + 1.007 + 1.007^2)$

(1 mark)

)

$A_n = 35000(1.007)^n - R(1 + 1.007 + \ldots + 1.007^{n-1})$

> (1 mark)

$= 35000(1.007)^n - R\left[\frac{1(1.007^n - 1)}{1.007 - 1}\right]$

$= 35000(1.007)^n - \frac{R(1.007^n - 1)}{0.007}$

$= 35000(1.007)^n - \frac{1000R(1.007^n - 1)}{7}$

(1 mark)

(iii)

$A_{60} = 0$

$A_{60} = 35000(1.007)^{60} - \frac{1000R(1.007^{60} - 1)}{7}$

$0 = 35000(1.007)^{60} - \frac{1000R(1.007^{60} - 1)}{7}$

(1 mark)

$\frac{1000R}{7}(1.007^{60} - 1) = 35000(1.007)^{60}$

$R = \frac{35000(1.007)^{60} \times 7}{1000(1.007^{60} - 1)}$

$= \$716.39$ (1 mark)

(iv)

$0 = 35000(1.007)^n - \frac{1000R(1.007^n - 1)}{7}$

$35000(1.007)^n = \frac{1000 \times 800(1.007^n - 1)}{7}$

1 mark

$245(1.007)^n = 800(1.007)^n - 800$

$555(1.007)^n = 800$

$1.007^n = \frac{160}{111}$

$n \ln 1.007 = \ln\left[\frac{160}{111}\right]$

$n = 52.4174$ months

(1 mark)

14 c)

$y = (x^2 - 1)e^{3x-1}$

$u = x^2 - 1$

$u' = 2x$

$v = e^{3x-1}$

$v' = 3e^{3x-1}$

(1 mark)

$y' = uv' + vu'$

$= 3(x^2-1)e^{3x-1} + 2xe^{3x-1}$

$= e^{3x-1}(3x^2 - 3 + 2x)$

(1 mark)

15 a

(i) $T_2 - T_1 = \sqrt{18} - \sqrt{2}$
$= 3\sqrt{2} - \sqrt{2}$
$= 2\sqrt{2}$

(1 mark)

$T_3 - T_2 = \sqrt{50} - \sqrt{18}$
$= 5\sqrt{2} - 3\sqrt{2}$
$= 2\sqrt{2}$

Since $T_3 - T_2 = T_2 - T_1 = 2\sqrt{2}$

$\therefore$ the series is an arithmetic

i):- $a = \sqrt{2}$ and $d = 2\sqrt{2}$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$100\sqrt{2} = \frac{n}{2}\left[2\sqrt{2} + 2\sqrt{2}(n-1)\right]$$

(1 mark)

$$100\cancel{\sqrt{2}} = \frac{\cancel{2\sqrt{2}}n}{\cancel{2}}\left[1 + 1(n-1)\right]$$

$$100 = n\{\cancel{1} + n - \cancel{1}]$$

$$100 = n^2$$

$$n = \pm 10$$

$n = 10$ as it cannot be negative. (1 mark).

15 b

$a = 1 - 2t$

$\frac{dv}{dt} = 1 - 2t$

$\int dv = \int (1-2t)\,dt$

$v = t - t^2 + C$

At $t = 0$, $v = 0$, $\therefore C = 0$

$v = t - t^2$ 1 mark

Particle at rest means $v = 0$

$0 = t - t^2$

$0 = t(1-t)$

$t = 0$ or $t = 1$ (1 mark)

(ii) $v = t - t^2$

$\frac{dx}{dt} = t - t^2$

$\int dx = \int (t - t^2)\,dt$

$x = \frac{t^2}{2} - \frac{t^3}{3} + 2$

(1 mark)

when $t = 1$

$x = \frac{1}{2} - \frac{1}{3} + 2$

$= 2\frac{1}{6}$

The particle is at $2\frac{1}{6}$ cm right of the origin.

(1 mark)

15c

(i) $y = 3x^4 - 16x^3 + 2x^2$

$y = x^2(3x^2 - 16x + 2)$

either $x^2 = 0$ or $3x^2 - 16x + 2 = 0$

but $3x^2 - 16x + 2$ will never be equal to 0 as discriminant $= -34$.

$\therefore$ the curve cuts the x-axis only at the origin.

(1 mark)

(ii).

$y' = 12x^3 - 48x^2 + 48x$

$= 12x(x^2 - 4x + 4)$

$= 12x(x-2)^2$

$y' = 0$ at $x = 0$ and $x = 2$.

for $x = 0$, $y = 0$ ~~for~~

$x = 2$, $y = 6$ (1 mark)

x	-1	0	1	2	3
y'	-12	0	12	0	36
	$\searrow$	$-$	$\nearrow$		$\nearrow$

$\therefore$ there is only one minimum turning point at $(0,0)$ (1 mark)

$(2,6)$ is a horizontal point of inflexion.

(iii):- $y'' = 36x^2 - 96x + 48$

$= 12(3x-2)(x-2)$

$y'' = 0$

at $x = \frac{2}{3}$ and $x = 2$.

When $x = \frac{2}{3}$, $y = 6\frac{14}{27}$

and when

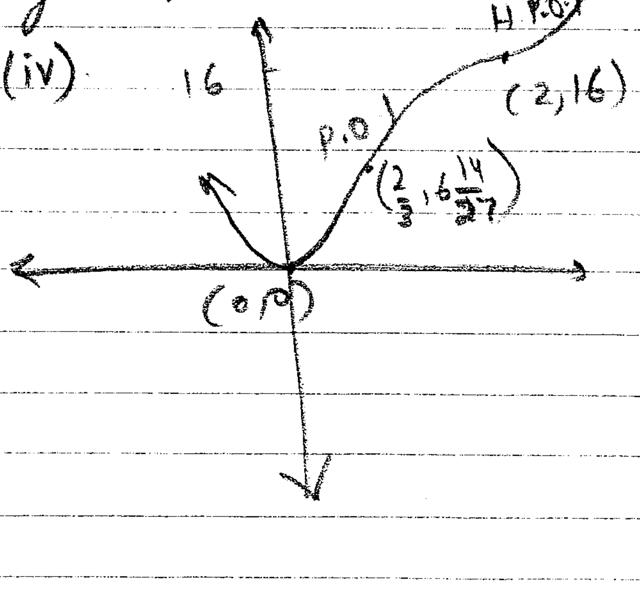
$x = 2$, $y = 16$

(1 mark)

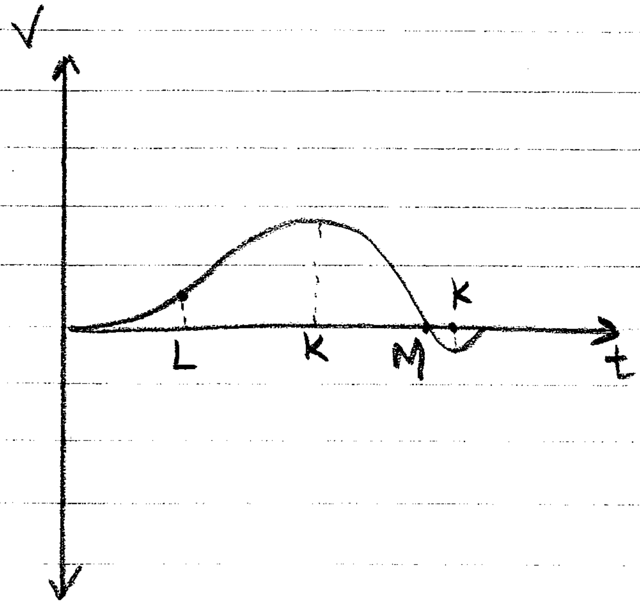
x	0	$\frac{2}{3}$	1	2	3
y''	$+48$	0	-12	0	84
	$\cup$		$\cap$		$\cup$

$\therefore \left(\frac{2}{3}, 6\frac{14}{27}\right)$ as well as $(2,16)$ are points of inflexion. (1 mark)

(iv)



16 a.



1 mark for both K.

1 mark for L

1 mark for M.

b) (i) $\frac{T_2}{T_1} = \frac{-2(\log_e x)^2}{\log_e x} = -2\log_e x$

$$\frac{T_3}{T_2} = \frac{4(\log_e x)^3}{-2(\log_e x)^2} = -2\log_e x$$

Since $\frac{T_3}{T_2} = \frac{T_2}{T_1}$, the series is geometric (1 mark)

(ii) $r = -2\log_e x$

$-1 < -2\log_e x < 1$ for limiting sum (1 mark)

i.e. $\frac{1}{e^{\frac{1}{2}}} < x < e^{\frac{1}{2}}$

(1 mark)

(iii) $S_\infty = \frac{a}{1-r}$

$$S_\infty = \frac{\log_e x}{1+2\log_e x}$$ (1 mark)

c) (i)

$$\frac{d}{dx}\frac{1}{3}\tan^3 x$$

$$= \frac{1}{\cancel{3}} \times \cancel{3}\tan^2 x \sec^2 x$$

$$= \tan^2 x \sec^2 x$$

(1 mark)

$\sec^2 x = 1 + \tan^2 x$

$$\therefore = \tan^2 x(1+\tan^2 x)$$
$$= \tan^2 x + \tan^4 x$$

1 mark

(ii) $\frac{d}{dx}\frac{1}{3}\tan^3 x \, dx = \tan^2 x + \tan^4 x$

$$\left[\frac{1}{3}\tan^3 x\right]_0^{\pi/3} = \int_0^{\frac{\pi}{3}} (\tan^2 x + \tan^4 x)\,dx$$

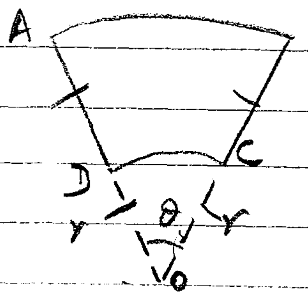
$$\left[\frac{1}{3}\tan^3\frac{\pi}{3} - \frac{1}{3}\tan^3 0\right] = \int_0^{\pi/3} (\tan^2 x + \tan^4 x)\,d$$

$$\left[\frac{1}{3}(\sqrt{3})^3 - 0\right] = \int_0^{\pi/3} (\tan^2 x + \tan^4 x)\,dx$$

$\sqrt{3} =$ "

16 d)

(i)



$\overset{\frown}{CD} = r\theta$

$\overset{\frown}{AB} = 2r\theta$

Perimeter ABCD = 12

$r + r + r\theta + 2r\theta = 12$

$2r + 3r\theta = 12$

$r(2 + 3\theta) = 12$

$r = \dfrac{12}{2+3\theta}$

1 mark

(ii) Aea (ABCD) = Area (AOB) − Area (COD)

$= \frac{1}{2}(2r)^2\theta - \frac{1}{2}r^2\theta$

$= \frac{1}{2}\theta(2r - r)(2r + r)$

$= \frac{1}{2}\theta(r)(3r)$

$= \frac{3}{2}\theta r^2$

$= \frac{3\theta}{2} \cdot \frac{12^2}{(2+3\theta)^2}$

$= \dfrac{216\theta}{(2+3\theta)^2}$

(iii). $A = \dfrac{216\theta}{(2+3\theta)^2}$

$u = 216\theta$

$u' = 216$

$v = (2+3\theta)^2$

$v' = 6(2+3\theta)$

$A' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(2+3\theta)^2\, 216 - 216\theta \times 6(2+3\theta)}{(2+3\theta)^4}$

$= \dfrac{216\,\cancel{(2+3\theta)}\left((2+3\theta) - 6\theta\right)}{(2+3\theta)^{\cancel{4}3}}$

$= \dfrac{216(2 + 3\theta - 6\theta)}{(2+3\theta)^3}$

$= \dfrac{216(2 - 3\theta)}{(2+3\theta)^3}$

$A' = 0$

$2 - 3\theta = 0$

$\theta = \frac{2}{3}$ radians.

1 mark.

Check

x	0	$\frac{2}{3}$	1
y'	54	0	-1.728
	/	—	\

Area is maximum when

$\theta = \frac{2}{3}$ radians

(1 mark)