Section I (10 Marks)
Attempt Questions 1-10
Allow about 15 minutes for this section
Use the multiple choice answer sheet for Questions 1 - 10

---

1 What is the value of $8e^{-2}$ correct to 3 significant figures?

(A) 1.08
(B) 1.082
(C) 1.083
(D) 1.10

2 What is the focus of the parabola $x^2 = 12(y + 3)$

(A) $(0, -3)$
(B) $(0, 0)$
(C) $(0, 3)$
(D) $(0, 12)$

3 What is the radius of the circle $x^2 + y^2 - 4x + 8y + 11 = 0$

(A) 2
(B) 3
(C) 4
(D) 9

4 What is the value of $\int_0^{\ln 7} e^{-x}dx$?

(A) $\frac{6}{7}$
(B) $\frac{9}{7}$
(C) $e^{-\ln 7}$
(D) $\ln 7$

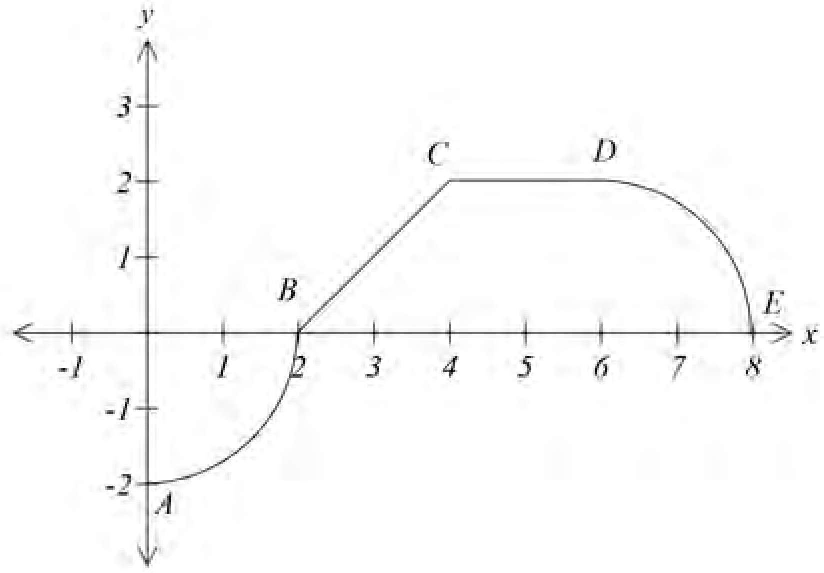
5 How many solutions are there to the equation $2 - x = 4\cos x$? ($0 \le x \le 2\pi$)

(A) 0
(B) 1
(C) 2
(D) 3

6 Which of the following is NOT an even function?

(A) $f(x) = x^2 - 2$
(B) $f(x) = x$
(C) $f(x) = \cos(x)$
(D) $f(x) = e^x + e^{-x}$

7



The graph of the function $y = f(x)$ consists of quarter of a circle *AB*, straight line segment *BC*, a horizontal straight line segment *CD*, and a quarter circle *DE*.

The area bounded by $y = f(x)$, the x axis, and the lines $x = 0$ and $x = 8$ is

(A) $6 + 6\pi$
(B) $6 + 4\pi$
(C) $6 + 2\pi$
(D) 6

8 $\lim\limits_{x \to -3} \dfrac{2x^2 + 6x - 3a - ax}{x + 3}$

(A) $6a$
(B) $a - 6$
(C) $-6 - a$
(D) $6 - a$

9 Find the values of m for which the expression $-4x^2+3x+m$ is negative definite.

(A) $m<-\frac{9}{16}$

(B) $m<-\frac{16}{9}$

(C) $m\geq-\frac{9}{16}$

(D) $m\geq-\frac{16}{9}$

10 A water tank holds 800 litres of water. Water is let out of the tank at a rate of R litres per minute where $R = 100\,t$ after t minutes. How long does it take the tank to be empty?

(A) 2 minutes
(B) 4 minutes
(C) 6 minutes
(D) 8 minutes

End of Section I

Section II (90 Marks)
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section
Answer each question in a separate writing booklet

In questions 11 -16, your response should include relevant mathematical reasoning and / or calculations

---

Question 11 (15 marks) **Marks**

(a) Differentiate the following with respect to x.

(i) $(e^{2x} + 1)^3$ **2**

(ii) $\dfrac{\cos 3x}{x}$ **2**

(b) Find the value of k (where $k > 2$) such that $\int_2^k \frac{2t}{3t^2 - 1}\, dt = \frac{1}{3}\log_e 13$ **3**

(c) The roots of the quadratic equation $6x^2 - px - 3 = 0$ are α and β.

(i) Find the value of α β. **1**

(ii) Given that $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = 8$, find the value of p. **2**

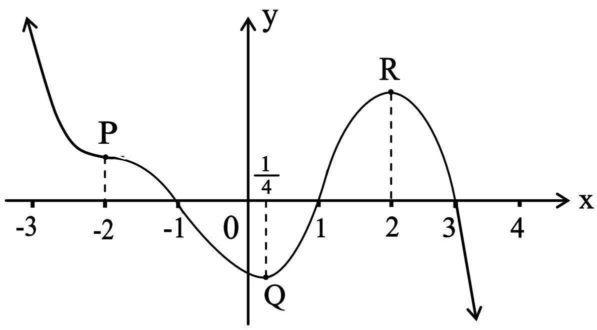
(d) Graph the region bounded by $x - y + 1 \geq 0$ and $y \geq x^2 - 1$ **2**

(e) Find the equation of the normal to the curve $y = 2\,tanx$ at the point where $x = \frac{\pi}{4}$ **3**

End of Question 11

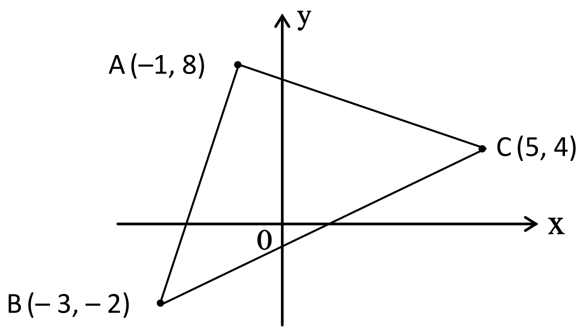
Question 12 (15 marks) **Marks**

(a) The diagram shows the graph of a function g(x). The graph has a horizontal point of inflexion at P, a minimum turning point at Q and a maximum turning point at R. **3**



Sketch the graph $y = g'(x)$.

(b) The points A (–1, 8), B (– 3, – 2), C (5, 4) form a triangle as shown in the diagram



(i) Find the length of BC **1**

(ii) Show that the equation of the line BC is $3x - 4y + 1 = 0$. **2**

(iii) Find the perpendicular distance from A to the line BC **2**

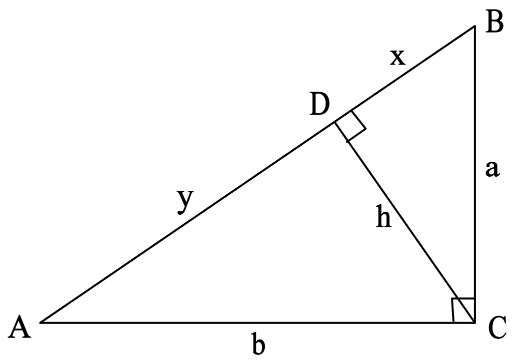
(iv) Hence or otherwise, find the area of the triangle ABC **1**

Question 12 continues on following page

Question 12 (continued) **Marks**

(c) In the diagram ABC is a right angle triangle at C. The perpendicular from C to AB meets AB at D.

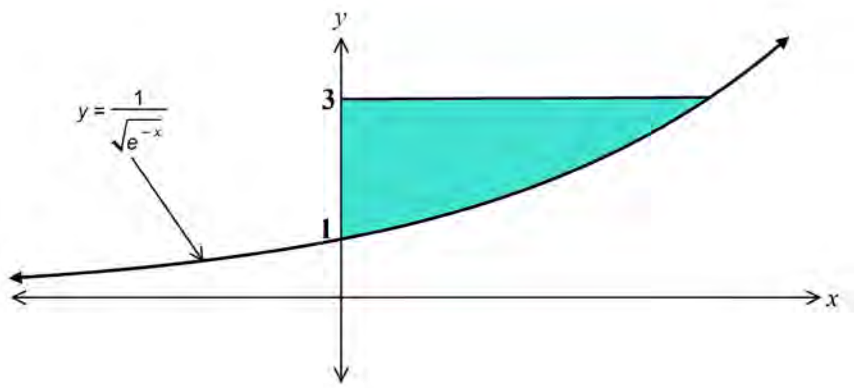
BC = a, AC = b, DC = h, DB = x and AD = y.



(i) Show that $h\,(x + y) = ab$. **2**

(ii) Show that $h = \dfrac{ab}{\sqrt{a^2 + b^2}}$. **1**

(d) **3**

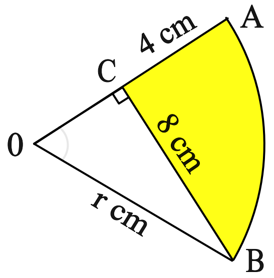


The diagram above shows the area bounded by the curve $y = \dfrac{1}{\sqrt{e^{-x}}}$, $y = 3$ and the $y-$axis. Use Simpson's rule with three y function values to find an approximation of the volume when this area is rotated about the $y-axis$. *(leave your answer to 2 decimal places)*

End of Question 12

Question 13 (15 marks) **Marks**

(a) The diagram shows the sector OAB with centre O and radius r. The perpendicular from B meets the radius OA at C. BC = 8 cm and AC = 4 cm.



NOT TO SCALE

(i) Show that the radius r of the circle is 10 cm. **2**

(ii) Find the area of the shaded region in this sector. **2**

(b) Find $\int sec^2 \frac{x}{2}\ dx$ **2**

(c) **2**

Consider the geometric series $\frac{1}{2}+\frac{(x+1)}{4}+\frac{(x+1)^2}{8}+\ldots$

Find the values of x for which this geometric series has a limiting sum.

(d) (i) Solve the equation $1-2\cos x=0$ for $0\leq x\leq 2\pi$ **2**

(ii) Sketch the graph of the curve $y=1-2\cos x$ for $0\leq x\leq 2\pi$, showing clearly the coordinates of the endpoints and the maximum turning point. **2**

(iii) Find in simplest exact form the area of the region bounded by the curve $y=1-2\cos x$ and the x axis between $x=0$ and $x=\pi$ **3**

End of Question 13

Question 14 (15 marks) **Marks**

(a) The curve $y = f(x)$ passes through the origin and has a gradient function given by $f'(x) = x(x-2)^2$.

(i) Find the equation of the curve $y = f(x)$. **2**

(ii) Find the coordinates of the stationary points of the curve. **1**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve $y = f(x)$, clearly labelling the stationary points and the y-intercept. **2**

(b) Raakhi's grandparents have set up a fund with a single investment of \$400,000 to provide financial support for her. She is granted an annual payment of \$25,000 from this fund at the end of each year. The fund accrues interest at a rate of 5% per annum compounded annually.

(i) Calculate the balance in the fund at the end of the first year. **1**

(ii) Let $\$A_n$ be the balance of the fund at the end of n years (after Raakhi receives her payment). Show that $A_n = 500{,}000 - 100{,}000(1.05)^n$. **2**

(iii) If this fund began at the beginning of 2000, in what year will the fund run out of money? **3**

(c) The seventh term of a geometric progression is $\frac{1}{32}$and the eleventh term is $\frac{1}{512}$ **2**
Find the first three terms.

End of Question 14

Question 15 (15 marks) **Marks**

(a) A particle moves in a straight line so that its displacement x, in metres from a fixed origin at time t seconds is given by

$$x = \log_e(t+1), \qquad t \geq 0$$

(i) Find the initial position of the particle. **1**

(ii) Find an expression for the velocity and the acceleration of the particle. **2**

(iii) Explain whether or not the particle is ever at rest **1**

(b) In a laboratory, a food product is heated in an oven set at 210°C in an effort to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After ten minutes the number of bacteria recorded is 550.

It is found that the number N, of bacteria remaining t minutes after being in the oven is given by

$$N = N_0 e^{-kt}$$

where N_0 and k are constants.

(i) Find the value of N_0. **1**

(ii) Calculate the value of k correct to 4 decimal places. **2**

(iii) If an acceptable number of bacteria present is 100 or less, For how long should the food product be in the oven? **3**
Express your answer to the nearest minute.

(c) A particle is moving in a straight line. Initially, it is travelling to the left at 1 cm/min. Its acceleration as a function of time (t) is given by

$$a = \pi \cos \pi t + \pi \sin \pi t \quad \text{for } 0 \leq t \leq 2$$

where time and displacement are measured in minutes and cm respectively.

(i) Find when the particle first changes its direction. **2**

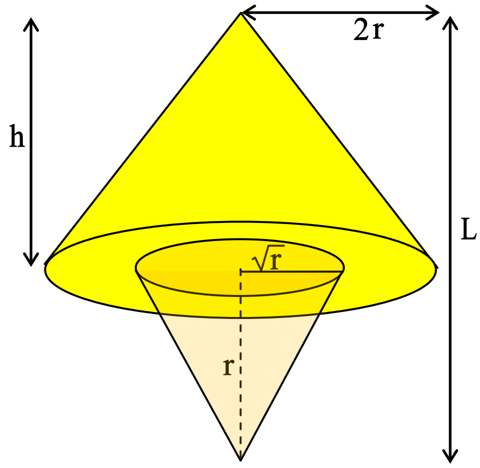
(ii) Find the exact total distance travelled in the first half a minute. **3**

End of Question 15

Question 16 (15 marks) **Marks**

(a) The line $y = kx + 1$ is the tangent to the curve $y = \ln x$ at a point A. Find the coordinates of A. **2**

(b) The diagram shows a pendant consisting of two solids. The top solid is a metallic cone with radius 2r cm, height h cm and density d g/cm ^{3}. The bottom is a precious stone also in the shape of a cone, but with radius $\sqrt{r}$ cm, height r cm and density 4d g/cm ^{3}. The total length of the pendant is L cm, where L is constant.



Using the formula mass = volume × density

(i) Show that the mass of the top cone is $\dfrac{4\pi d}{3}(Lr^2 - r^3)$. **3**

(ii) Show that the mass of the pendant is $M = \dfrac{4\pi d}{3}\left[r^2(L+1) - r^3\right]$. **2**

(iii) Find the values of r and h in terms of L that maximise M the mass of the pendant. **3**

Question 16 continues on following page

Question 16 continued **Marks**

(c) At the start of a month, Henry deposited M dollars into a new bank account and kept depositing M dollars at the start of each of the following months. The money in the account was earning interest at the rate of 0.4% per month, compounding monthly.

At the same time, his son started saving $10 at the start of that month, and each month after that he saved $5 more than the previous month.

(i) Show that the son has saved $3510 after 3 years. **2**

(ii) If the total money saved by Henry and his son is $18035 after 3 years, find the value of M to the nearest dollar. **3**

End of Paper

Section I

1. 1.08 (3 SF) $\boxed{A}$

2. $(x-0)^2 = 12(y+3)$

$4a = 12$

$a = 3$

Vertex $(0, 0)$ $\boxed{B}$

3. $(x-2)^2 + (y-4)^2 = -11+4+16$

$= 9$

$r = 3$ $\boxed{B}$

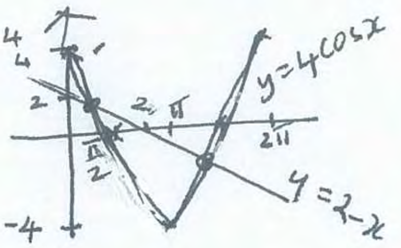
4. $\int e^{-x}\,dx = \left[-e^{-x}\right]_0^{\ln 7}$

$= -\left[e^{-\ln 7} - e^{0}\right]$

$= -\left[+\frac{1}{7} - 1\right]$

$= \frac{6}{7}$ $\boxed{A}$

5.



2 $\boxed{C}$

6. $\boxed{B}$

7. $6 + 2\pi$ $\boxed{C}$

$\frac{1}{4} \times \pi \times 2^2 + \frac{1}{2} \times 2 \times 2$

$+ 2 \times 2 + \frac{1}{2} \times \pi \times 2^2$

$= 6 + 2\pi$

8. $\lim_{x \to -3} \frac{(x+3)(2x-a)}{x+3}$ $\boxed{C}$

$= -6 - a$

9. $\Delta < 0$

$(3)^2 - 4(-4m) < 0$

$9 + 16m < 0$

$16m < -9$

$m < -\frac{9}{16}$ $\boxed{A}$

10. $R = -100t$

$\frac{dV}{dt} = -100t$

$V = -50t^2 + C$

when $t = 0$ $V = 800$

$800 = -50t^2 + C$

$800 = 0 + C$

$\therefore C = 800$

$V = -50t^2 + 800$

$V = 0$, find t

$0 = -50t^2 + 800$

$50t^2 = 800$

$t^2 = 16$ $\boxed{B}$

$t = 4$

Section 2

Question 11

a (i) $y = (e^{2x}+1)^3$

$y' = 3(e^{2x}+1)^2 \cdot e^{2x} \cdot 2$

$= 6e^{2x}(e^{2x}+1)^2$

ii) $y = \dfrac{\cos 3x}{x}$ $\quad u = \cos 3x$

$u' = -3\sin 3x$

$v = x$

$v' = 1$

$y' = \dfrac{x(-3\sin 3x) - (\cos 3x)\times 1}{x^2}$

$= \dfrac{-3x\sin 3x - \cos 3x}{x^2}$

b $\displaystyle\int_2^K \frac{2t}{3t^2-1}\,dt = \frac{1}{3}\log_e(13)$

$\displaystyle\int_2^K \frac{2t}{3t^2-1}\,dt$

$= \displaystyle\frac{1}{3}\int_2^K \frac{6t}{3t^2-1}\,dt$

$= \frac{1}{3}\left[\ln(3t^2-1)\right]_2^K$

$= \frac{1}{3}\left[\ln(3K^2-1) - \ln 11\right]$

$= \frac{1}{3}\ln\left(\frac{3K^2-1}{11}\right)$

$= \frac{1}{3}\log_e 13$

$\therefore \frac{3K^2-1}{11} = 13$

$3K^2 - 1 = 143$

$3K^2 = 143+1$

$3K^2 = 144 \qquad K^2 = 48$

$K = \pm\sqrt{48}$

$K = \pm 4\sqrt{3}$

$K > 2 \qquad \therefore K = 4\sqrt{3}$

c) $6x^2 - px - 3 = 0$

(i) $\alpha\beta = -\frac{3}{6}$

$\alpha\beta = -\frac{1}{2}$

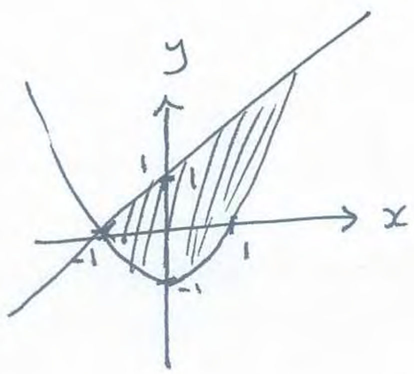
(ii) $\alpha+\beta = -\frac{(-p)}{6} = \frac{p}{6}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$8 = \frac{p}{6} \div -\frac{1}{2}$

$\frac{p}{6} = -4$

$p = -24$

d) 

1 e

$y = 2\tan x$

$y' = 2\sec^2 x$

$y'_{x=\frac{\pi}{4}} = \dfrac{2}{\cos^2 \pi/4} = 4$

$m_{normal} = -\dfrac{1}{4}$

when $x = \dfrac{\pi}{4}$, $y = 2\tan\dfrac{\pi}{4} = 2$

Equation of the normal is

$y - 2 = -\dfrac{1}{4}\left(x - \dfrac{\pi}{4}\right)$

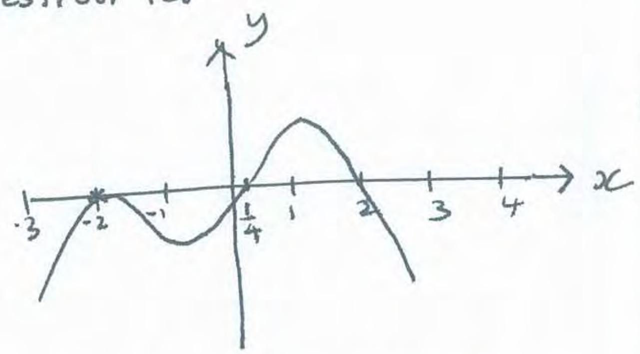
$4y - 8 = -x + \dfrac{\pi}{4}$

$16y - 32 = -4x + \pi$

$4x + 16y - 32 - \pi = 0$

Question 12

a.



Turning points at $x = 1/4, 2$

Point of inflexion at $x = -2$

b

i) $BC = \sqrt{(-3-5)^2 + (-2-4)^2}$

$= 10$

(ii) $m_{BC} = \dfrac{-2-4}{-3-5} = \dfrac{3}{4}$

Eqn of BC

$y - 4 = \dfrac{3}{4}(x-5)$

$4y - 16 = 3x - 15$

$3x - 4y + 1 = 0$

iii) $\perp$ dist $= \dfrac{|3(-1) - 4(8) + 1|}{\sqrt{3^2 + (-4)^2}}$

$= \dfrac{34}{5}$

(iv) Area$(\Delta ABC) = \dfrac{1}{2} \times 10 \times \dfrac{34}{5}$

$= 34$ units^2

c) In Δles ACB and DCB

$\angle B$ is common

$\angle ACB = \angle CDB = 90^\circ$

$\therefore \Delta ACB \ ||| \ \Delta DCB$ (Triangles are equiangular)

$\dfrac{x+y}{a} = \dfrac{b}{h}$ [Corresponding sides of similar triangles are in the same ratio]

$\therefore h = \dfrac{ab}{x+y}$

2 d)

$$y = \frac{1}{\sqrt{e^{-x}}}$$

$$\sqrt{e^{-x}} = \frac{1}{y}$$

$$e^{-x} = \frac{1}{y^2}$$

$$-x = \ln\left(\frac{1}{y^2}\right)$$

$$x = \log(y^2)$$

$$= 2\log y$$

$$x^2 = 4(\log y)^2$$

y	1	2	3
$x^2 = (2\ln y)^2$	$(2\ln 1)^2$	$(2\ln 2)^2$	$(2\ln 3)^2$

$$V = \frac{1}{3}\pi\left[1 \times (2\ln 1)^2 + 4(2\ln 2)^2 + 1(2\ln 3)^2\right]$$

$= 13.11 \,/\, 13.12$ cubic units

Question 13

1 i)

$$(r-4)^2 + 8^2 = r^2$$

$$r^2 - 8r + 16 + 64 = r^2$$

$$-8r = -80$$

$$r = 10$$

ii)

$$\sin\theta = \frac{8}{10}$$

$$\theta = 0.9272 \text{ (radians)}$$

Shaded area

$$= \frac{1}{2} \times 10^2 \times 0.9272 - \frac{1}{2} \times 8 \times 6$$

$$= 22.36 \text{ cm}^2$$

b)

$$\int \sec^2 \frac{x}{2}\, dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan\frac{x}{2} + C$$

c) For S_∞ $|r| < 1$

$$\left|\frac{x+1}{2}\right| < 1$$

$$-1 < \frac{x+1}{2} < 1$$

$$-3 < x < 1$$

d. (i)

$$1 - 2\cos x = 0$$

$$\cos x = \frac{1}{2}$$

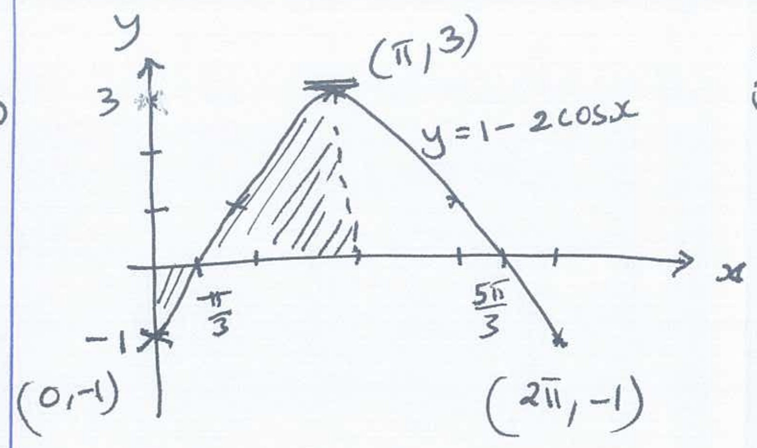
$$x = \frac{\pi}{3}$$

$$x = \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3}$$

$$x = \frac{\pi}{3}, \quad \frac{5\pi}{3}$$

Question 13

d (ii)



(iii)

$$\text{Area} = \int_0^{\pi} (1-2\cos x)\,dx$$

$$= \left|\int_0^{\pi/3} 1-2\cos x\,dx\right| + \int_{\pi/3}^{\pi}(1-2\cos x)\,dx$$

$$= \left|\left(x-2\sin x\right)_0^{\pi/3}\right| + \left(x-2\sin x\right)_{\pi/3}^{\pi}$$

$$= \left|\left[\;-2\sin\tfrac{\pi}{3}\right)-0\right| + \left[\left(\pi-2\sin\pi\right)-\left(\tfrac{\pi}{3}-2\sin\tfrac{\pi}{3}\right)\right]$$

$$= \left|\tfrac{\pi}{3}-2\sin\tfrac{\pi}{3}\right| + \left[(\pi-0)-\left(\tfrac{\pi}{3}-2\tfrac{\sqrt{3}}{2}\right)\right]$$

$$= \left|\tfrac{\pi}{3}-2\times\tfrac{\sqrt{3}}{2}\right| + \pi-\tfrac{\pi}{3}+\sqrt{3}$$

$$= \left|\tfrac{\pi}{3}-\sqrt{3}\right| + \tfrac{2\pi}{3}+\sqrt{3}$$

$$= \sqrt{3}-\tfrac{\pi}{3}+\tfrac{2\pi}{3}+\sqrt{3}$$

$$= 2\sqrt{3}+\tfrac{\pi}{3}$$

Question 14

a (i)

$$f'(x) = x(x-2)^2$$

$$f(x) = \int x(x-2)^2\,dx$$

$$= \int x^3-4x^2+4x\,d$$

$$f(x) = \frac{x^4}{4}-\frac{4x^3}{3}+2x^2+c$$

When $x=0$, $f(x)=0$

$$0 = c$$

$$\therefore f(x) = \frac{x^4}{4}-\frac{4x^3}{3}+2x^2$$

(ii) For stationary points, $f'(x)=0$

$$x(x-2)^2=0$$

$$x=0\,;\quad x=2$$

When $x=0$, $y=0$ $(0,0)$

When $x=2$,

$$y = \left(\frac{2^4}{4}-4\frac{(2)^3}{3}+2(2)^2\right)$$

$$= \frac{4}{3}$$

$$\left(2,\tfrac{4}{3}\right)$$

Stationary points are

$(0,0)$ and $\left(2,\tfrac{4}{3}\right)$

Question 14

a iii

$$f'(x) = x(x-2)^2$$

$$f''(x) = x \cdot 2(x-2) + (x-2)^2 \cdot 1$$

$$= (x-2)(3x-2)$$

$$f''(x)_{x=0} = 4 > 0$$

$\therefore f(x)$ has a mini at $(0,0)$

$$f''(x)_{x=2} = 0$$

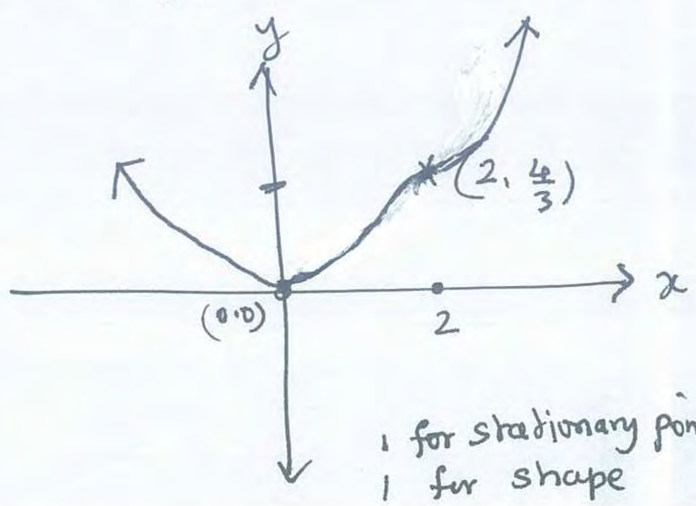
There is a possible point of inflexion.

x	1.5	2	2.5
$f''(x)$	−1.25	0	.75
	−	0	+

Concavity changes

$\therefore (2, \frac{4}{3})$ is a point of inflexion.

iv)



1 for stationary points
1 for shape

14

14 c) $t_7 = \frac{1}{32}$ $\quad t_{11} = \frac{1}{512}$

$ar^6 = \frac{1}{32}$ $\quad ar^{10} = \frac{1}{512}$

$$\frac{ar^{10}}{ar^6} = \frac{1}{512} \div \frac{1}{32}$$

$$r^4 = \frac{1}{16}$$

$$r = \pm \frac{1}{2}$$

when $r = \pm \frac{1}{2}$,

$$a(\pm\tfrac{1}{2})^6 = \frac{1}{32}$$

$$a = 2$$

The first 3 terms are

$2, 1, \frac{1}{2}$

or

$2, -1, \frac{1}{2}$

14 b)

(i) $A_1 = 400000(1.05)^1 - 25000$

$= \$395000$

$A_2 = A_1(1.05) - 25000$

$= 400000(1.05)^2 - 25000(1 + 1.05^1)$

$A_3 = A_2(1.05) - 25000$

$= [400000(1.05^2) - 25000(1+1.05^1)]1.05 - 25000$

$= 400000(1.05^3) - 25000(1 + 1.05^1 + 1.05^2)$

$A_n = 400000(1.05)^n - 25000[1 + 1.05^1 + \cdots + 1.05^{n-1}]$

$= 400000(1.05)^n - 25000 \times 1 \left(\dfrac{1.05^n - 1}{1.05 - 1}\right)$

$= 400000(1.05)^n - 500000(1.05^n - 1)$

$= -100000(1.05)^n + 500000$

(iii) $A_n = 0$

$0 = 500000 - 100000(1.05)^n$

$1.05^n = \dfrac{500000}{100000}$

$n = \dfrac{\ln 5}{\ln(1.05)} \approx 32.927$

The fund will run out in 2032.

Question 15

a (i)

$$x = \log_e (t+1)$$

when $t=0$

$$x = \log_e (0+1)$$

$$= 0$$

ii)

$$v = \frac{dx}{dt} = \frac{1}{t+1}$$

$$a = \frac{dv}{dt} = -\frac{1}{(t+1)^2}$$

iv) For a particle to be at rest,

$$v = 0$$

$$\frac{1}{t+1} = 0$$

$$(t+1)0 = 1$$

No solution.

$\therefore$ The particle will never be at rest

b) -i)

$$N = N_0 e^{kt}$$

$$N_0 = 1800$$

ii)

$$N = 1800 e^{-kt}$$

$t = 10, \quad N = 550$

$$550 = 1800 e^{-k(10)}$$

$$e^{-10k} = \frac{550}{1800}$$

$$-10k = \ln\left(\frac{550}{1800}\right)$$

$$k = 0.11856$$

$$= 0.1186$$

iii)

$$N \leq 100$$

$$1800 e^{-0.1186t} \leq 100$$

$$e^{-0.1186t} \leq \frac{1}{18}$$

$$-0.1186t \leq \ln\left(\frac{1}{18}\right)$$

$$t \geq \frac{\ln\left(\frac{1}{18}\right)}{-0.1186}$$

$$t \geq 24.37$$

The food should be kept in the oven for 25 minutes.

15 c (i) Question 15

The particle changes its direction when $v = 0$

$a = \pi \cos \pi t + \pi \sin \pi t$

$v = \int (\pi \cos \pi t + \pi \sin \pi t)\, dt$

$v = \sin \pi t - \cos \pi t + c$

when $t = 0$, $v = -1$

$-1 = 0 - 1 + c$

$c = 0$

$\therefore v = \sin \pi t - \cos \pi t$

$v = 0$

$\sin \pi t - \cos \pi t = 0$

$\tan \pi t = 1$

$\pi t = \frac{\pi}{4}$

$t = \frac{1}{4}$

$\therefore$ The particle first changes its direction when $t = \frac{1}{4}$ min

$t = 15$ sec

Total distance

$=$

15 c

(ii) Total distance $= \left| \int_0^{\frac{1}{4}} v\,dt \right| + \int_{\frac{1}{4}}^{\frac{1}{2}} v\,dt$

$$= \left| \int_0^{\frac{1}{4}} (\sin \pi t - \cos \pi t)\,dt \right| + \int_{\frac{1}{4}}^{\frac{1}{2}} (\sin \pi t - \cos \pi t)\,dt$$

$$= \left| -\frac{1}{\pi} \Big[\cos \pi t + \sin \pi t \Big]_0^{\frac{1}{4}} \right| + -\frac{1}{\pi} \Big[\cos \pi t + \sin \pi t \Big]_{\frac{1}{4}}^{\frac{1}{2}}$$

$$= -\frac{1}{\pi} \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (1+0) \right] - \frac{1}{\pi} \left[(0+1) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right]$$

$$= \left| \frac{1}{\pi} \left[1 - \frac{2}{\sqrt{2}} \right] \right| - \frac{1}{\pi} \left[1 - \sqrt{2} \right)$$

$$= \left| \frac{1}{\pi} (1 - \sqrt{2}) \right| + \frac{1}{\pi} (\sqrt{2} - 1)$$

$$= \frac{1}{\pi} (\sqrt{2} - 1) + \frac{1}{\pi} (\sqrt{2} - 1)$$

$$= \frac{2}{\pi} (\sqrt{2} - 1) \text{ cm}$$

Question 16

a $y = kx + 1$

$m_{\text{tangent}} = k$

$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

$k = \frac{1}{x}$

$x = \frac{1}{k}.$

$y_{(x=\frac{1}{k})} = k(\frac{1}{k}) + 1$

$= 2$

$y = 2$

When $y=2$, find x

$\ln x = 2$

$x = e^2$

$\therefore$ A has coordinates

$(e^2, 2)$

b Volume of the top cone

(i) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi (2r)^2 (L-r)$

$= \frac{4\pi}{3} r^2 (L-r)$

$= \frac{4\pi}{3}\left[Lr^2 - r^3\right]$

$m = V \times d$

$m = \frac{4}{3}\pi\left[Lr^2 - r^3\right]d$

$= \frac{4}{3}\pi d\left[Lr^2 - r^3\right]$

(ii) Volume of the bottom cone

$V = \frac{1}{3}\pi (r)^2 \times r$

$= \frac{1}{3}\pi r^2$

$m = V \times d$

$= \frac{1}{3}\pi r^2 \times 4d$

$= \frac{4\pi d}{3} r^2$

16 Total mass

(ii)

$$= \frac{4\pi d}{3}\left[Lr^2 - r^3\right] + \frac{4\pi d}{3} r^2$$

$$= \frac{4\pi d}{3}\left[(L+1) r^2 - r^3\right]$$

(i) $M = \frac{4\pi d}{3}\left[(L+1) r^2 - r^3\right]$

$$\frac{dM}{dr} = \frac{4\pi d}{3}\left[2(L+1) r - 3r^2\right]$$

$$\frac{dM}{dr} = 0$$

$$\Rightarrow \quad 2(L+1)r - 3r^2 = 0$$

$$r\left[2(L+1) - 3r\right] = 0$$

$$r = 0, \quad r = \frac{2(L+1)}{3}$$

since $r > 0$

$$r = \frac{2(L+1)}{3}$$

$$\frac{d^2M}{dr^2} = \frac{4\pi d}{3}\left[2(L+1) - 6r\right]$$

$$= \frac{4\pi d}{3}\left[2(L+1) - 6 \cdot \frac{2(L+1)}{3}\right]$$

$$= \frac{4\pi d}{3}\left[2(L+1) - 4(L+1)\right]$$

$$= \frac{4\pi d}{3}\left[-4(L+1)\right]$$

$$= -\frac{8\pi d}{3}(L+1)$$

$$< 0$$

$\therefore$ M is maximum

when $r = \frac{2(L+1)}{3}$

and

$$h = L - r$$

$$= L - \frac{2(L+1)}{3}$$

$$= \frac{L-2}{3}.$$

16 Son's savings

c) $10 + 15 + \cdots$

(i) $n = 36$

$$S_{36} = \frac{36}{2}\left[2 \times 10 + 35 \times 5\right]$$

$$= \$\,3510.$$

ii) Henry's earnings

$$= \$\,18035 - \$\,3510$$

$$= \$\,14525$$

Value of Henry's Deposit.

1st month's deposit is

$$M(1.004)^{36}$$

2nd month's deposit is

$$M(1.004)^{35}$$

$\vdots$

36th month's deposit is

$$M(1.004)^{1}$$

Total saving is (13)

$$= M[1.004)^{36} + \cdots + M(1.004)^{1}$$

$$= M\left[1.004^{1} + \cdots + 1.004^{36}\right]$$

$$= M\,(1.004)\,\frac{(1.004^{36} - 1)}{1.004 - 1}$$

$$14525 = M \times 38.7926$$

$$M = \$\,374.43$$