CRANBROOK TRIAL 2001

1. (Use an 8 page booklet) [Marks 1, 2, 2, 2, 2, 3]

(a) A coin is tossed at the same time that a die is thrown.
What is the probability of getting a 3 on the die and a head on the coin?

(b) Factor $3x^2 - x - 4$

(c) Find the exact value of cosec $60° + \tan 30°$

(d) Find integers a and b such that $\left(3-\sqrt{5}\right)^2 = a + b\sqrt{5}$

(e) Solve $|2x-1| = 5$

(f) Solve the simultaneous equations: $3x - y = 5$
$5x + 3y = -8$

2. (Use the same booklet as for question 1) [Marks 2, 5, 5]

(a) A die is rolled n times. Find an expression to describe the probability of rolling n sixes.

(b) In a certain factory a machine manufactures bicycle components, of which 3% are faulty. I buy 3 such components. By drawing a tree diagram, or otherwise, find the probability that :

(i) exactly one component is faulty.

(ii) at least one is faulty.

(c) A ball is dropped from a height of 3 m on to the floor. After each bounce the maximum height reached by the ball is 60% of the previous maximum height. Thus, after the first bounce it reaches a height of 180 cm, before falling again.

(i) What is the height reached after the second bounce?

(ii) What kind of sequence is formed by the successive heights?

(iii) What is the total distance travelled by the ball from the time it was dropped until it comes to rest?

3. (Use a new 4 page booklet) [Marks 2, 2, 3, 2, 1, 2]

If A, B and C are the points (5, 3), (-2, 5) and (4, -3) respectively, find the :

(a) exact distance from A to C.

(b) gradient of the join of A and C.

(c) equation of the line passing through A and C, in general form.

(d) perpendicular distance from B to the line passing through A and C.

(e) area of the triangle ABC.

(f) co-ordinates of the point D such that ABCD is a parallelogram.

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4. (Use a new 4 page booklet) [Marks 4, 3, 5]

(a) For the equation $x^2 + (m-3)x + m = 0$ find the values of m for which the equation has

(i) two equal roots

(ii) two distinct real roots

(b) Find all real roots of the equation $4^x - 6\times 2^x + 8 = 0$

(c) For the parabola $2y^2 - 4x - 8 = 0$, find

(i) the co-ordinates of the vertex

(ii) the co-ordinates of the focus

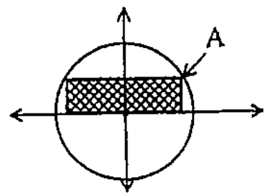
(iii) the equation of the directrix

5. (Use a new 8 page booklet) [Marks 2, 4, 6]

(a) Differentiate $\log_e\left(\dfrac{2x-3}{x+1}\right)$ with respect to x leaving your answer without simplifying it.

(b) Show that the graph of $y = e^x \cos x$ has a stationary point at $x = \dfrac{\pi}{4}$ and determine its nature.

(c)



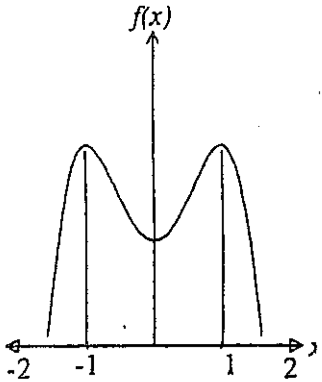
The diagram shows a rectangle inscribed in a semicircle whose equation is $y = \sqrt{4-x^2}$.

(i) Let the x coordinate of point A be x. Show the area of the rectangle is given by $2x\sqrt{4-x^2}$.

(ii) Find the maximum area of the rectangle.

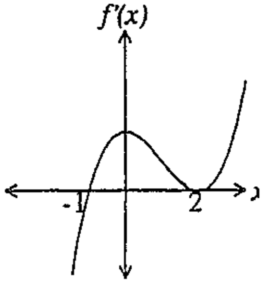
6. (Use the same booklet as for question 5) [Marks 5, 4, 3]

(a) The graph shows part of the function $y = 2\cos(1 - x^2)$. Using 5 function values, find an approximation using the trapezoidal rule for the area between this graph, the x-axis and the lines $x = 1$ and $x = -1$.



(b) The area enclosed by the graph of $y = \dfrac{1}{\sqrt{1-x}}$, the coordinate axes and the line $x = \dfrac{1}{2}$ is rotated around the x axis. Calculate the volume of the solid so formed.

(c) The diagram below shows the graph of the derived function $f'(x)$, for the function $y = f(x)$.



(i) Explain the behaviour of the graph of $y = f(x)$ at $x = -1$ and at $x = 2$.

(ii) Explain what you know about the graph of $y = f(x)$ when x is between these values.

7. (Use a new 8 page booklet) [Marks 2, 2, 2, 4, 2]

(a) Find the derivatives, without simplifying, of each of :

(i) $e^{\cos 3x}$ (ii) $\sqrt{xe^{2x}}$ (iii) $\tan^3(5x+4)$

(b) (i) Sketch, on the same set of axes, graphs of $y = 3\sin 2x$ and $y = \dfrac{x}{2}$

(ii) State the number of solutions of the equation $6\sin 2x - x = 0$

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8. (Use the same booklet as for question 7) [Marks 3, 3, 3, 3]

(a) Show that the graph of $y = \log_e x$ is concave down and increasing for all values of $x > 0$

(b) Show that if $y = A\cos 4x + B\sin 4x$ then $\dfrac{d^2y}{dx^2} + 16y = 0$.

(c) The area under the curve $y = \dfrac{1}{x}$ between $x = 1$ and $x = b$ is equal to 1 square unit. What is the value of b ?

(d) If the gradient of the tangent at the point (x, y) on a particular curve is $2\sin 3x$ and the curve passes through the point $\left(\dfrac{\pi}{3}, \dfrac{8}{3}\right)$, find the equation of the curve.

9. (Use a new 8 page booklet) [Marks 1, 2, 2, 3, 4]

(a) Given $F(x) = \dfrac{x+1}{x-1}$, $G(x) = x^2$, $H(x) = \sqrt{x}$

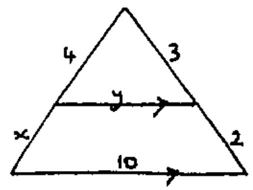
(i) State the domain of $F(x)$.

(ii) Evaluate, where possible, HG(-3) and GH(-3).
[If either is not possible, explain why.]

(iii) Show that $F\left(\dfrac{1}{x}\right) = -F(x)$.

(iv) Show that $F^{-1}(x) = F(x)$.

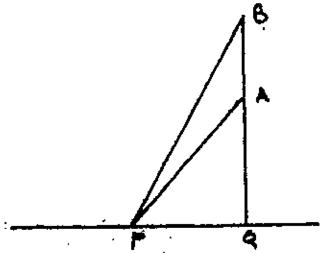
(b) From the diagram evaluate x and y.



10. (Use the same booklet as for question 9) [Marks 4, 3, 5]

(a) A point $P(x, y)$ moves so that its distance from $(-1,1)$ is equal to its distance from the straight line $3x-4y+4=0$.
Show that the cartesian equation of the locus is $16x^2+26x+24xy+9y^2-18y+34=0$.

(b) AB is a 12 m flag pole on the top of a building. Find the length of BP given that $\angle QPB = 56°37'$ and $\angle QPA = 43°19'$.



(c) ABCD is a parallelogram. J and K are points of trisection of the diagonal AC (i.e. AJ = JK = KC).

(i) Draw a clear diagram to represent this situation.

(ii) Prove that $\Delta ADJ \equiv \Delta CBK$.

(iii) Hence explain why $DJ \parallel BK$.

STANDARD INTEGRALS

$\int x^n dx = \frac{1}{n+1}x^{n+1} \quad (n \neq -1; x \neq 0 \text{ if } n<0)$

$\int \frac{1}{x}dx = \log_e x \quad (x>0)$

$\int e^{ax}dx = \frac{1}{a}e^{ax} \quad (a \neq 0)$

$\int \cos ax\,dx = \frac{1}{a}\sin ax \quad (a \neq 0)$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax \quad (a \neq 0)$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax \quad (a \neq 0)$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax \quad (a \neq 0)$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \quad (a \neq 0)$

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \quad (a>0, -a<x<a)$

$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log_e\left\{x+\sqrt{x^2-a^2}\right\} \quad (|x|>|a|)$

$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log_e\left\{x+\sqrt{x^2+a^2}\right\}$

Question 1

(a) $P(3) = \frac{1}{6}$ $\quad P(H) = \frac{1}{2}$

$P(3 \text{ AND } H) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

(b) Factorize $3x^2 - x - 4 = (3x-4)(x+1)$

$3x \quad -4$
$x \quad 1$

(c) $\operatorname{Cosec} 60^\circ + \tan 30^\circ = \frac{1}{\sin 60^\circ} + \tan 30^\circ$

$= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{\sqrt{3}}$

$= \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{3}{\sqrt{3}}$

$= \frac{3\sqrt{3}}{3}$

$= \boxed{\sqrt{3}}$

(d) $(3-\sqrt{5})^2 = (3)^2 - 2(3)(\sqrt{5}) + (\sqrt{5})^2$

$= 9 - 6\sqrt{5} + 5$

$= 14 - 6\sqrt{5}$

$\therefore a = 14 \quad b = -6$

(e) $|2x-1| = 5$

$2x - 1 = 5$ OR $-(2x-1) = 5$

$2x = 6$ $\qquad 2x - 1 = -5$

$x = 3$ $\qquad 2x = -4$

$x = -2$

Check Solutions

$\therefore x = 3$ AND $x = -2$ ARE Solutions.

(f) Solving

$3x - y = 5 \ \ldots (1)$

$5x + 3y = -8 \ \ldots (2)$

Elimination

$(1) \times 3 \quad 9x - 3y = 15 \ \ldots (3)$

$5x + 3y = -8 \ \ldots (2) \ +$

$14x = 7$

$\therefore x = \frac{1}{2}$

Sub $x = \frac{1}{2}$ into equ (1) to find x

ie $3\left(\frac{1}{2}\right) - y = 5$

$\frac{3}{2} - y = 5$

$y = -5 + \frac{3}{2}$

$y = \frac{-10}{2} + \frac{3}{2}$

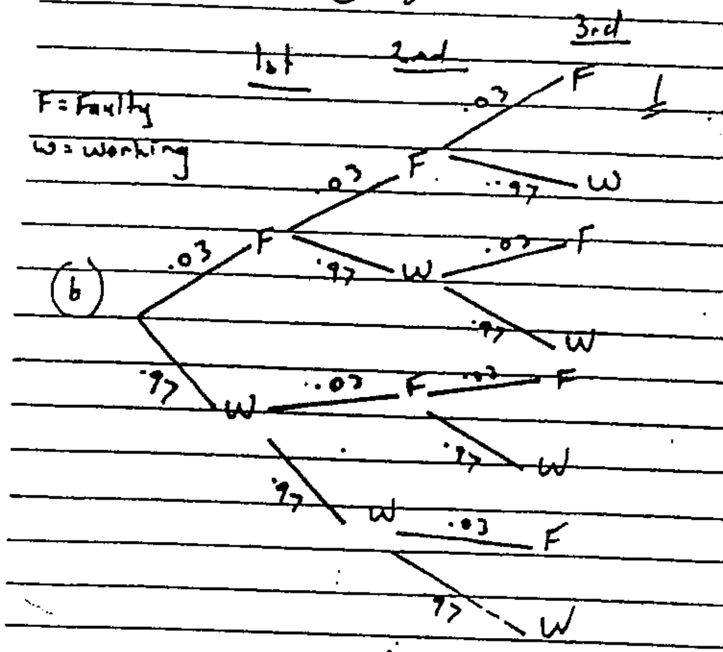
$\therefore y = \frac{-7}{2}$

Hence Solution to Simult equations is $\left(\frac{1}{2}, \frac{-7}{2}\right)$

Question 2

(a) $\left(\frac{1}{6}\right)^n$

(b)



SAMPLE SPACE

FFF
FFW
FWF
FWW
WFF
WFW
WWF
WWW

(i) $P\left(\text{EXACTLY ONE FAULTY}\right) = P(FWW) + P(WFW) + P(WWF)$

$= (.03 \times .97 \times .97) + (.97 \times .03 \times .97) + (.97 \times .97 \times .03)$

$= (0.028) + (0.028) + (0.028)$

$= \underline{0.084}$

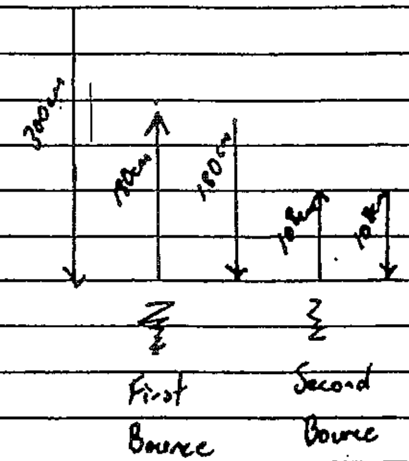
(ii) $P\left(\text{at least one Faulty}\right) = 1 - P(WWW)$

$= 1 - (.97 \times .97 \times .97)$

$= 1 - 0.913$

$= \underline{0.087}$

(c)



(i) 1·8m OR 108cm 1

(iii) accept Infinite Sequence OR Geometric OR limiting Geometric 1

(iii) Total Distance $= 3 + 2(1{\cdot}8 + 1{\cdot}08 + 0{\cdot}648)$ 1

$= 3 + 2\left(\frac{a}{1-r}\right)$

$= 3 + 2\left(\frac{1{\cdot}8}{1-0{\cdot}6}\right)$ 1

$= 3 + 9$

So BALL TRAVELS $= 12$m 1 OR 1200cm

Q3. $A(5,3)$, $B(-2,5)$, $C(4,-3)$

(a) $d_{AC} = \sqrt{(5-4)^2 + (3--3)^2}$ 1

$= \sqrt{1+36} = \sqrt{37}$ 1 (2)

(b) $m_{AC} = \frac{6}{1} = 6$ 2 (2)

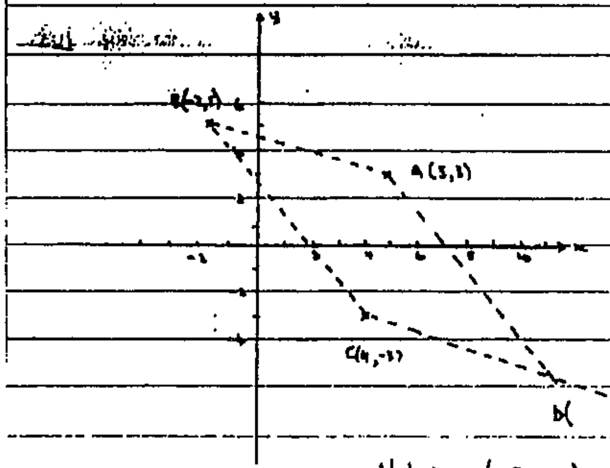
(c) $y - 3 = 6(x-5)$ 1 $y - 3 = 6x - 30$ 1

$6x - y - 27 = 0.$ 1 (3)

(d) $d = \frac{|6(-2) - 5 - 27|}{\sqrt{6^2+1^2}}$ 1 $= \frac{44}{\sqrt{37}}$ 1 (2)

(e) $A = \frac{1}{2} \cdot \frac{44}{\sqrt{37}} \cdot \sqrt{37} = 22\ u^2$ 1 (1)

(f)



B → A

x: +7

y: −2

∴ C → D

x: 4+7 = 11 1

y: −3−2 = −5

D pt: (11,−5) 1

Note: (−3,−1) gives a point in the parallelogram ABDC

You were asked for ABCD

4. (a) $\Delta = (m-3)^2 - 4m$

$= m^2 - 10m + 9$

(i) Two equal roots if $m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

$m = 1, 9$ 2

(ii) Two distinct roots if $m^2 - 10m + 9 > 0$

$\therefore m < 1$ or $m > 9$

(b) $4^x - 6 \times 2^x + 8 = 0$

$(2^x - 2)(2^x - 4) = 0$ 1

$\therefore 2^x = 2, 4$ 1

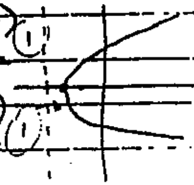
$x = 1, 2$ 1

(c) $2y^2 - 4x - 8 = 0$

$y^2 = 2(x+2)$ 1 $a = \frac{1}{2}$

(i) Vertex $= (-2, 0)$ 1

(ii) Focus $= (-\frac{3}{2}, 0)$ 1



(iii) Directrix

$x = -\frac{5}{2}$ 1

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QUESTION 5

(a) Outcomes assessed: H3, P7

Marking Criteria

Criteria	Marks
• 1mark for correct differentiation procedure • 1 mark for correct answer	2

Answer

$$y = \ln\left(\frac{2x-3}{x+1}\right)$$

$$= \ln(2x-3) - \ln(x+1)$$

$$\therefore \frac{dy}{dx} = \frac{2}{2x-3} - \frac{1}{x+1} \quad \text{OR} \quad \frac{x+1}{2x-3} \times \frac{2(x-1)-(2x+3)}{(x+1)^2}$$

(b) Outcomes assessed: H5, P6, P7

Criteria	Marks
• 1 mark for derivative • 1 mark for showing derivative 0 at $x = \frac{\pi}{4}$ • 1 mark for either sign diag for derivative or second derivative • 1 mark for correct conclusion from the above	4

Answer

$y = e^x \cos x$

$y' = e^x \cos x - e^x \sin x$

$at\ x = \frac{\pi}{4},\ y' = e^x (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = 0$

$\therefore$ *stationary pt at* $x = \frac{\pi}{4}$

$y'' = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x) = -2e^x \sin x$

$at\ x = \frac{\pi}{4}\ y'' = -2e^{\frac{\pi}{4}} \times \frac{1}{\sqrt{2}} < 0$

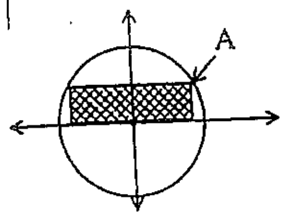
$\therefore$ *maximum turning point*

(c) Outcomes assessed: H1, H2, H5

Criteria	Marks
• 1 mark for identifying dimensions of rectangle • 1 mark for area • 1 mark for derivative of area • 1 mark for solution of $a' = 0$ • 1 mark for justifying maximum • 1 mark for max area = 4	6

P4

Answer:



A's coordinates are $(x, \sqrt{4-x^2})$

Thus dimensions of rectangle are $2x \times \sqrt{4-x^2}$

Ie, area $= 2x\sqrt{4-x^2}$

$$a = 2x\sqrt{4-x^2}$$

$$\frac{da}{dx} = 2 \times \sqrt{4-x^2} + 2x \times \frac{1}{\sqrt{4-x^2}} \times -2x$$

$$= \frac{8-4x^2}{\sqrt{4-x^2}}$$

$$= 0 \text{ if } x^2 = 2$$

$x = \sqrt{2}$ *(must be positive)*

$\frac{da}{dx} > 0$ *if* $x < \sqrt{2}$ *and* $\frac{da}{dx} < 0$ *if* $x > \sqrt{2}$

ie, max turning point

and $a_{max} = 2\sqrt{2} \times \sqrt{2} = 4$

QUESTION 6

(a) Outcomes assessed: P3, P4, H5,

Criteria	Marks
• 2 marks for correct function values • 2 marks for substituting correctly into a correct formula • 1 mark for evaluation from correct formula	5

Answer:

x	-1	-0.5	0	0.5	1
y	2	1.463...	1.081...	1.463...	2

$$area = \frac{0.5}{2}(2 + 2 + 2(1.463 + 1.081 + 1.463))$$

$= 3.0036...$

$= 3.00$ *units of area (2d.p.)*

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(b) Outcomes assessed: P3, H8

Criteria	Marks
• 1 mark for correct volume formula • 1 mark for integral of correct x function • 1 mark for correct integral • 1 mark for evaluation	4

Answer

$$V = \pi \int_0^{\frac{1}{2}} y^2\, dx$$

$$= \pi \int_0^{\frac{1}{2}} \frac{1}{1-x}\, dx$$

$$= \pi\left[-\ln(1-x)\right]_0^{\frac{1}{2}}$$

$$= \pi \ln 2$$

(c) Outcomes assessed: P7, P8, H2, H5, H6, H7

Criteria	Marks
• 1 mark for identifying the minimum at $x = -1$ • 1 mark for identifying the stationary inflexion at $x = 2$ OR 1 mark for identifying that these is a stationary point at each of $x = -1$ and $x = 2$ • 1 mark for identifying the inflexion between $x = -1$ and $x = 2$ OR identifying that $f(x)$ is increasing between these values	3

Answer:

(i)

at $x = -1$ $f'(x) = 0$

$f'(-1^-) < 0$

$f'(-1^+) > 0$

$\therefore$ *minimum turning point*

at $x = 2$, $f'(x) = 0$

$f'(2^+) > 0$ *and* $f'(2^-) > 0$

$\therefore$ *stationary inflexion*

(ii)

for $-1 < x < 2$ $f'(x) > 0$

$\therefore$ *f(x) is increasing*

also, in this interval the gradient of $f'(x)$ changes from positive to negative, so there is a [illegible] of cur[illegible] ie. an [illegible]) at [illegible]

QUESTION 7

(a) (i)

Outcomes assessed: P7 H3

Marking Criteria

Criteria	Marks
• 1 mark for correct differentiation procedure • 1 mark for correct answer	2

Answer

$$\frac{d}{dx} e^{\cos 3x} = e^{\cos 3x} \times -\sin 3x \times 3$$

(ii) Outcomes assessed: P7

Marking Criteria

Criteria	Marks
• 1 mark for correct differentiation procedure • 1 mark for correct answer	2

Answer

(ii) $\sqrt{xe^{2x}} = x^{\frac{1}{2}}(e^{2x})^{\frac{1}{2}}$

$\frac{d}{dx}\, x^{\frac{1}{2}} e^{x}$

$= x^{\frac{1}{2}} e^{x} + e^{x} \frac{1}{2} x^{-\frac{1}{2}}$ ✓✓

$= e^{x}\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$

OR $\frac{d}{dx}\left(xe^{2x}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\left(xe^{2x}\right)^{-\frac{1}{2}} \times \left[x \cdot e^{2x} \cdot 2 + e^{2x}\right]$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \frac{1}{e^{x}} \times e^{2x}(2x+1)$

$= e^{x}\left(\frac{1}{2\sqrt{x}} \cdot 2x + \frac{1}{2\sqrt{x}}\right)$

$= e^{x}\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$ as before

(iii) Outcomes assessed: P7

Marking Criteria

Criteria	Marks
• 1 mark for correct differentiation procedure • 1 mark for correct answer	2

Answer

$\frac{d}{dx} \tan^3(5x+4)$

$= \frac{d}{dx}\left[\tan(5x+4)\right]^3$

$= 3\left[\tan(5x+4)\right]^2 \cdot \sec^2(5x+4) \times 5$ ✓✓

$= 15 \tan^2(5x+4)\sec^2(5x+4)$

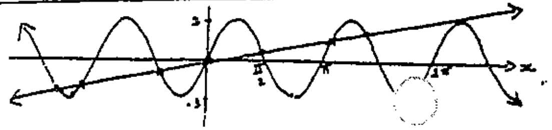
(b)

(i) Outcomes assessed: P4 H5

Marking Criteria

Criteria	Marks
• 1 mark for sine graph • 1 mark for correct range & period • 1 mark for straight line graph • 1 mark for correct gradient	4

Answer



$y = \frac{x}{2}$

Criteria	Marks
• 1 mark for linking given equation to the graph • 1 mark for correct answer according to graph	2

Answer (ii) $6\sin 2x - x = 0$

ie $\frac{6\sin 2x}{2} = \frac{x}{2}$

ie $3\sin 2x = \frac{x}{2}$

Since $y = 3\sin 2x$ and $y = \frac{x}{2}$ intersect 7 times

$6\sin 2x - x = 0$ has 7 SOLNS.

QUESTION 8

(a) Outcomes assessed: P5 H6

Marking Criteria

Criteria	Marks
• 1 mark for 1st and 2nd derivatives • 1 mark for correct link to increasing curve • 1 mark for correct link to concave down curve	3

(b) Outcomes assessed: P7 H5

Criteria	Marks
• 1 mark for 1st derivative • 1 mark for 2nd derivative • 1 mark for proof	3

Answers

(a) $\frac{dy}{dx} = \frac{1}{x}$

$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$

When $x > 0$, $\frac{1}{x} > 0$ ∴ INCREASING

$-\frac{1}{x^2} < 0$ ∴ CONCAVE DOWN

Hence $y = \log_e x$ is concave down + increasing for all $x > 0$

b) $y = A\cos 4x + B\sin 4x$

$\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x$ ✓

$\frac{d^2y}{dx^2} = -16A\cos 4x - 16B\sin 4x$ ✓

$\frac{d^2y}{dx^2} + 16y = -16A\cos 4x - 16B\sin 4x + 16(A\cos 4x + B\sin 4x)$

$= 0$ ✓

(c) Outcomes assessed: P8 H8

Criteria	Marks
• 1 mark for identifying the integral • 1 mark for integration • 1 mark for substitution to correct answer	3

(d) Outcomes assessed: P8 H5

Criteria	Marks
• 1 mark for integration • 1 mark for substitution • 1 mark for correct answer	3

Answers

c) $\int_1^b \frac{1}{x}\,dx = 1$ ✓

$\left[\log_e x\right]_1^b = 1$ ✓

$\log_e b - \log_e 1 = 1$

$\log_e b - 0 = 1$ ✓

∴ $b = e$

OR $\int_b^1 \frac{1}{x}\,dx = 1$

$\left[\log_e x\right]_b^1 = 1$

$\log_e 1 - \log_e b = 1$

$-\log_e b = 1$

$b = e^{-1}$

(d) $\frac{dy}{dx} = 2\sin 3x$

$y = -\frac{2}{3}\cos 3x + c$ ✓

$\frac{8}{3} = -\frac{2}{3}\cos\left(3 \times \frac{\pi}{3}\right) + c$ ✓

$8 = -2\cos\pi + 3c$

$8 = -2 \times -1 + 3c$

$c = \frac{6}{3} = 2$

∴ $y = -\frac{2}{3}\cos 3x + 2$ ✓

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Cranbrook School 2001 July trial

2 unit Q9 & Q10.

(1)

9(a) $F(x) = \frac{x+1}{x-1}$, $G(x) = x^2$, $H(x) = \sqrt{x}$ (i) Domain $F(x)$ $\forall x \in \mathbb{R}$ except $x = 1$

(ii) $H(G(-3)) = H(9) = 3$. (1) $G(H(-3)) = G(\sqrt{-3})$ But $\sqrt{-3}$ is not a Real Number $\therefore$ Value $\sqrt{-3}$ not useable. ie $G(\sqrt{-3})$ not possible. (1)

(iii) $F\left(\frac{1}{x}\right) = \dfrac{\frac{1}{x}+1}{\frac{1}{x}-1}$

$= \frac{1+x}{1-x}$ or $-\left(\frac{1+x}{x-1}\right)$ & $-F(x) = -\left(\frac{1+x}{x-1}\right)$ (2)

$\therefore F\left(\frac{1}{x}\right) = -F(x)$

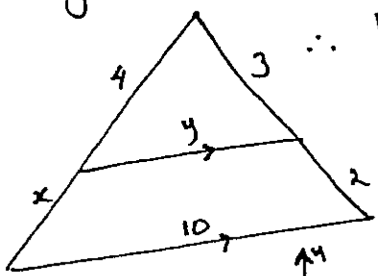
(iv) For inverse of $F(x)$ rewrite as $y = \frac{x+1}{x-1}$ & for inverse (x, y) ie $x = \frac{y+1}{y-1}$

$\therefore x(y-1) = y+1$ & $xy - y = x+1$

so $y(x-1) = x+1$ or $y = \frac{x+1}{x-1}$ But this y is $f^{-1}(x)$ (3)

$\therefore F^{-1}(x) = \frac{x+1}{x-1}$

(b)



From the diagram & considering similar Δs

$\frac{4}{x} = \frac{3}{2} \Rightarrow x = \frac{8}{3}$ or $2\frac{2}{3}$ (2)

& $\frac{y}{10} = \frac{3}{5} \Rightarrow y = 6.$ (2)

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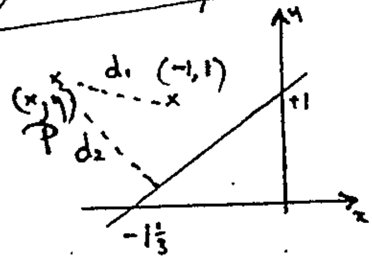
Q10 (a) 4 marks



(1)

Let d_1 be distance from P to $(-1,1)$ ie $d_1 = \sqrt{(x+1)^2 + (y-1)^2}$

& let d_2 be distance P to line ie $d_2 = \frac{|3x - 4y + 4|}{\sqrt{9+16}}$ (1)

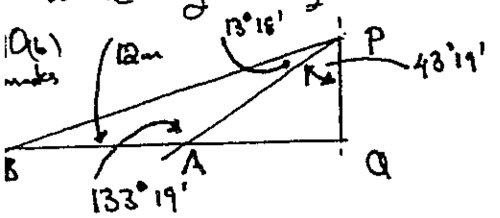
But $d_1 = d_2 \Rightarrow d_1^2 = d_2^2 \Rightarrow (x+1)^2 + (y-1)^2 = \frac{(3x-4y+4)^2}{25}$ (1)

$\therefore 25(x^2 + 2x + 1) + 25(y^2 - 2y + 1) = (3x - 4y + 4)^2$

$\therefore 25x^2 + 50x + 25 + 25y^2 - 50y + 25 = 9x^2 + 24x + 16y^2 - 32y - 24xy + 16.$

So the required equation of the locus is $16x^2 + 26x + 24xy + 9y^2 - 18y + 34$ (1)

10(b) marks



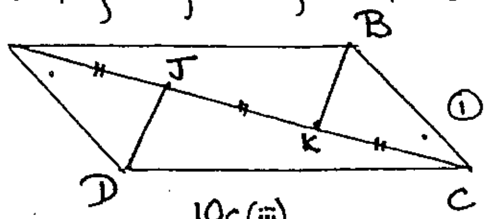
From the diagram

$\frac{BP}{\sin 133°19'} = \frac{12}{\sin 13°15'}$ (2)

$\therefore BP = 37.95213536$

ie $BP = 37.95$ m (1)

10(c) 5 marks



(1)

10c(ii)

Consider ΔADJ & ΔCBK

$AJ = CK$ given

$\angle DAJ = \angle BCK$ alt $\angle$ AD // BC

(3) $AD = BC$ opp sides //gm

$\therefore \Delta ADJ \equiv \Delta CBK$ SAS

& $\angle AJD = \angle CKB$ Corresp $\angle$s Congruent Δ

10c(iii)

Since $\angle AJD = \angle CKB$

then $\angle DJK = \angle BKJ$ Adj $\angle$s St line

$\therefore DJ \parallel BK$

Alternate $\angle$s equal.

(1)

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