# CRANBROOK SCHOOL

## YEAR 12 MATHEMATICS – 2 UNIT

**Term 3 2003** **Time : 3 h / GC, CGH, HRK and SKB**

**All questions are of equal value.**
**All necessary working should be shown in every question.**
**Full marks may not be awarded if work is careless or badly arranged.**
**Approved silent calculators may be used.**
**Submit your work in ten 4 Page Booklets.**

**1.** **(12marks) (Begin a 4 page booklet.)** **GC**

**(a)** Calculate $\dfrac{432\cdot5}{18\cdot9\times4\cdot6}$ correct to two decimal places. **2**

**(b)** Factorise $3x^2-x-10$ **2**

**(c)** Solve for $x$: $3-\dfrac{2x}{3}<4$ **2**

**(d)** If $(2\sqrt{3}-1)(\sqrt{3}+2)=a+\sqrt{b}$ find the values of $a$ and $b$. **2**

**(e)** Solve and graph on a number line the values of $x$ for which $|x-2|\le8$. **2**

**(f)** Express $\dfrac{3^{-1}a^2}{2a^{-3}}$ with positive indices in its simplest form. **2**

**2.** **(12marks) (Begin a 4 page booklet.)** **GC**

The points A (-3,2) and B (5,8) lie on a number plane.

**(a)** Find the equation of AB in general form. **2**

**(b)** Find the midpoint of AB. **1**

**(c)** Find the length of AB. **1**

**(d)** Show that the equation of the circle with diameter AB is

$$x^2 + y^2 - 2x - 10y + 1 = 0$$ **2**

**(e)** Prove that the point C (1,10) lies on this circle. **1**

**(f)** Prove that $AC \perp CB$ **1**

**(g)** Prove that the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$ are parallel. **1**

**(h)** The point A(1, $k$ ) lies on the line $12x - 5y + 3 = 0$. Find the value of $k$. **1**

**(i)** Hence find the perpendicular distance between the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$. **2**

**3.** **(12marks) (Begin a 4 page booklet.)** **GC**

**(a)** Differentiate the following with respect to $x$. Leave your answer in its simplest form.

(i) $(7x^2 - 2)^5$ **2**

(ii) $\dfrac{3x}{2x+5}$ **2**

**(b)** The tangent to the curve $y = 3x^3 - 8x^2$ at the point of contact, P (2,-8), cuts the $x$-axis at A, and the normal to the curve at the same point of contact cuts the y-axis at B.

(i) Find the equation of the tangent at P. **2**

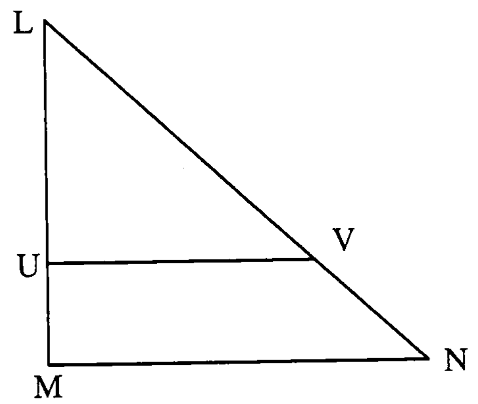
(ii) Find the equation of the normal at P. **2**

(iii) Find the coordinates of A and B. **2**

**(c)** Find $\lim_{x \to 1} \frac{3x^2 + x - 4}{x - 1}$ **2**

**4.** **(12marks) (Begin a 4 page booklet.)** **CGH**

**(a)**



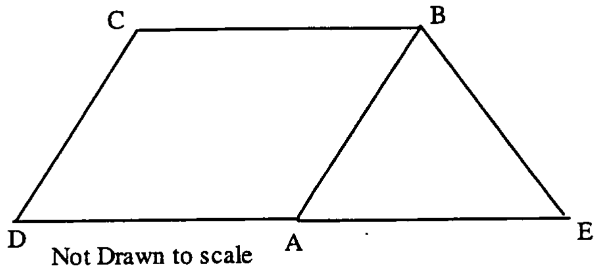

(i) Draw this diagram in your book. **1**

(ii) Consider triangles LMN and LUV where UV is parallel to MN and UM=3m, MN=7m and UV=5m. LM$\perp$MN.

(A) Find the length of LU **2**

(B) Find the length of VN **1**

**(b)**




Not Drawn to scale

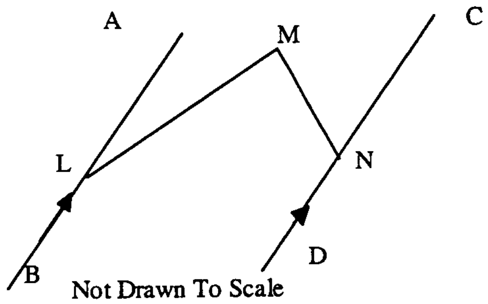
ABCD is a rhombus with $\angle BCD = 120^0$ while ABE is an equilateral triangle.

(i) In your examination booklet draw a neat sketch showing this information.

(ii) Find the size of $\angle EBC$ giving reasons for your answer.

(iii) Find the size of $\angle ECB$ giving reasons for your answer. **4**

**(c)**




Not Drawn To Scale

(i) In your examination booklet draw a neat sketch showing this information.

(ii) If $\angle MLB = 130^0$ and $\angle MND = 110^0$ find, giving reasons for your answers:

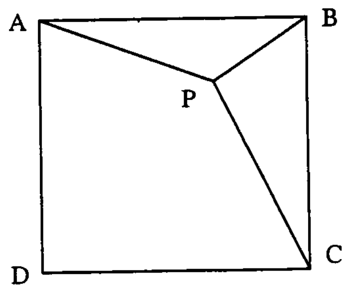
(A) $\angle LMN$

(B) $\angle MNC$ **4**

**5.** **(12marks) (Begin a 4 page booklet.)** **CGH**

**(a)** Sketch the graph of $y = 3\sin 2x$ *for* $|x| \le \pi$ and find the area between the x- axis and the curve $y = 3\sin 2x$ *for* $|x| \le \pi$. **4**

**(b)** ABCD is a square of side $x$ cm, with P a point within the square such that PC=6cm,PB=2cm and AP= $2\sqrt{5}$ cm. Let $\angle PBC = \alpha$.




Not to Scale

(i) Using the cosine rule in the triangle PBC show that $\cos\alpha = \dfrac{x^2 - 32}{4x}$. **3**

(ii) By considering triangle PBA, show that $\sin\alpha = \dfrac{x^2 - 16}{4x}$. **2**

(iii) Hence or otherwise show that the value of $x$ is a solution of $x^4 - 56x^2 + 640 = 0$ **1**

(iv) Find $x$. Give reasons for your answer. **2**

**6.** **(12marks) (Begin a 4 page booklet.)** **CGH**

**(a)** If $y=(x-3)^3$ find all stationary points and determine their nature. **3**

**(b)** If $y=x^3-9x$:

(i) Find any $x$ intercepts. **1**

(ii) Find both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **1**

(iii) Determine whether $y=x^3-9x$ is an odd or an even function. **1**

(iv) Find any maximum or minimum turning points. **3**

(v) Find any points of inflexion. **2**

(vi) Graph $y=x^3-9x$ **1**

**7.** **(12marks) (Begin a 4 page booklet.)** **HRK**

**(a)** To calculate the area of the region bounded by the curve $y=x^2-2x$ and the $x$ axis and between the lines $x = 0$ and $x = 4$, Ernie used $\int_0^4 (x^2-2x)\,dx$.

(i) Explain why Ernie's method of calculating this area is incorrect. **1**

(ii) Find the area of the required region. **3**

**(b)** Use Simpson's Rule with 3 function values to approximate the area enclosed between the curve $y=\frac{1}{(x+1)^2}$ and the lines $x = 0$ and $x = 4$ correct to 2 significant figures. **4**

**(c)** Find the volume of the solid formed when the area bounded by the curve $y=5-x^2$ for $x\geq 0$, the $y$ axis and the line $y = 1$ is rotated about the $y$ axis. **4**

**8.** **(12marks) (Begin a 4 page booklet.)** **HRK**

**(a)** For the equation $x^2+(k+6)x-2k=0$ find the:

(i) discriminant in terms of $k$; **2**

(ii) values of $k$ for which this equation has real roots. **2**

**(b)** Given the equation $3x^2+4x-3=0$ has roots $\alpha$ and $\beta$, evaluate the following without finding $\alpha$ or $\beta$:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $2\alpha^2+2\beta^2$ **2**

**(c)** A parabola has equation $x^2+6x-33=12y$.

(i) Find the coordinates of its vertex. **1**

(ii) Find the focal length. **1**

(iii) Find the equation of its directrix. **1**

(iv) Show that the line $x-2y+7=0$ is not a focal chord of this parabola. **1**

**9.** **(12marks) (Begin a 4 page booklet.)** **SKB**

**(a)** Differentiate the following with respect to $x$ (do not simplify your answer):

(i) $x^3e^{5x-1}$ **2**

(ii) $\log_e\left[\dfrac{3x^5-4}{6x^3-5}\right]$ **2**

**(b)** Evaluate the following integral:

$$\int_0^1 \frac{24x^2-14}{4x^3-7x-5}\,dx$$ **4**

**(c)** Find the exact area bounded by the curve $y=2\ln x$, the $x$-axis and line $x=2$. **4**

**10.** **(12marks) (Begin a 4 page booklet.)** **SKB**

(a) If $x$, $y$ and 9 are the first three terms of a geometric series and $y$, $x$ and 2 are the first three terms of an arithmetic sequence, find the values of $x$ and $y$. **4**

(b) An investor wants to borrow \$1 000 000 to purchase a block of units at Penrith from financial institution X which offers an interest rate of 6% p.a. reducible. The investor is to repay the loan in equal monthly instalments $M$, over 10 years.

**(i)** If $A_n$ is the amount owing after n instalments develop expressions for $A_1, A_2, A_3$ and $A_n$ .Hence show that the monthly instalment, $M$ is given by : $M = \dfrac{5000(1.005)^{120}}{1.005^{120} - 1}$ **3**

**(ii)** Calculate the value of the monthly instalment, $M$ to the nearest cent. **1**

**(iii)** Determine the amount still owing to institution X after 5 years, to the nearest cent. **2**

**(iv)** If the investor borrows the same amount of money over 10 years from F.B.Knightly Investments Ltd. which offers simple interest at 4.5% p.a., would the investor have been worse off? Explain your answer. **2**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

① (a) $\dfrac{432.5}{18.9 \times 4.6} = 4.974695\ldots$
$= 4.97$ (2 d.p.)

(b) $3x^2 - x - 10 = (3x+5)(x-2)$

(c) $3 - \dfrac{2x}{3} < 4$
$\therefore -\dfrac{2x}{3} < 1$
$\therefore x > -1\frac{1}{2}$

(d) $(2\sqrt{3}-1)(\sqrt{3}+2) = a + \sqrt{b}$
$\therefore 6 + 4\sqrt{3} - \sqrt{3} - 2 = a + \sqrt{b}$
$\therefore 4 + 3\sqrt{3} = a + \sqrt{b}$
$\therefore 4 + \sqrt{27} = a + \sqrt{b}$
$\therefore a = 4,\ b = 27$

(e) $|x-2| \le 8$
$\therefore -8 \le x - 2 \le 8$
$\therefore -6 \le x \le 10$


(f) $\dfrac{3^{-1}a^2}{2a^{-3}} = \dfrac{a^5}{6}$

② $A = (-3, 2),\ B = (5, 8)$.

(a) Eq'n of AB is: $\dfrac{y-2}{x+3} = \dfrac{8-2}{5+3}$
$8(y-2) = 6(x+3)$
$8y - 16 = 6x + 18$
$6x - 8y + 34 = 0$
ie $3x - 4y + 17 = 0$.

(b) $M_{AB} = \left(\dfrac{-3+5}{2}, \dfrac{2+8}{2}\right)$
$= (1, 5)$

(c) length $AB = \sqrt{(5--3)^2 + (8-2)^2}$
$= \sqrt{64+36}$
$= 10$ units.

(d) If AB is the diameter of the circle $\Rightarrow$ centre $= C(1,5)$
radius $= 5$ units
$\therefore$ Eq'n of circle is: $(x-1)^2 + (y-5)^2 = 5^2$
$x^2 - 2x + 1 + y^2 - 10y + 25 = 25$
ie $x^2 + y^2 - 2x - 10y + 1 = 0$.

(e) Sub $C(1,10)$ into $x^2 + y^2 - 2x - 10y + 1 = 0$
$\therefore$ LHS $= 1 + 100 - 2 - 100 + 1$
$= 0$
$=$ RHS
$\Rightarrow C(1,10)$ lies on the given circle

(f) $m_{AC} = \dfrac{10-2}{1--3} = 2$
$m_{CB} = \dfrac{10-8}{1-5} = -\frac{1}{2}$
As $m_{AC} \cdot m_{CB} = -1$
$\Rightarrow AC \perp CB$.

(g) For $12x - 5y + 3 = 0$ $\quad m_1 = \dfrac{-12}{-5} = \dfrac{12}{5}$
For $24x - 10y - 7 = 0$ $\quad m_2 = \dfrac{-24}{-10} = \dfrac{12}{5}$
As $m_1 = m_2 \Rightarrow$ lines are parallel

(h) Sub $(1,k)$ into $12x - 5y + 3 = 0$
$\therefore 12 - 5k + 3 = 0$
$\therefore 5k = 15$
$\therefore k = 3$

(i) perp. distance $= \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$
where $(x_1, y_1) = (1, 3)$; $(A,B,C) = (24, -1$
$\therefore$ perp. distance $= \dfrac{|24 - 30 - 7|}{\sqrt{24^2 + (-10)^2}} = \dfrac{|-13|}{26} = \frac{1}{2}$

3 (a)(i) Let $y = (7x^2-2)^5$

$\therefore \frac{dy}{dx} = 5(7x^2-2)^4 \cdot 14x$

$= 70x(7x^2-2)^4$

(ii) Let $y = \frac{3x}{2x+5}$

$\therefore \frac{dy}{dx} = \frac{(2x+5).3 - 3x.2}{(2x+5)^2}$

$= \frac{6x+15-6x}{(2x+5)^2}$

$= \frac{15}{(2x+5)^2}$

(b) $y = 3x^3 - 8x^2$

(i) $\frac{dy}{dx} = 9x^2 - 16x$

At $P(2,-8)$ $\frac{dy}{dx} = 4 = m_{tangent}$

$\therefore$ Eq'n of tangent at P is:

$y - -8 = 4(x-2)$

$\therefore y+8 = 4x-8$

$\therefore 4x - y - 16 = 0$.

(ii) At $P(2,-8)$ $m_{normal} = -\frac{1}{4}$

$\therefore$ Eq'n of normal at P is:

$y - -8 = -\frac{1}{4}(x-2)$

$\therefore 4y + 32 = -x + 2$

$\therefore x + 4y + 30 = 0$.

(iii) At A tangent cuts x-axis

$\therefore y = 0$ $\therefore 4x - 16 = 0$

$\therefore x = 4$

$\Rightarrow A = (4,0)$.

At B normal cuts y-axis

$\therefore x = 0$ $\therefore 4y + 30 = 0$

$\therefore y = -7\frac{1}{2}$

$\Rightarrow B = (0, -7\frac{1}{2})$.

(c) $\lim_{x \to 1} \frac{3x^2+x-4}{x-1}$

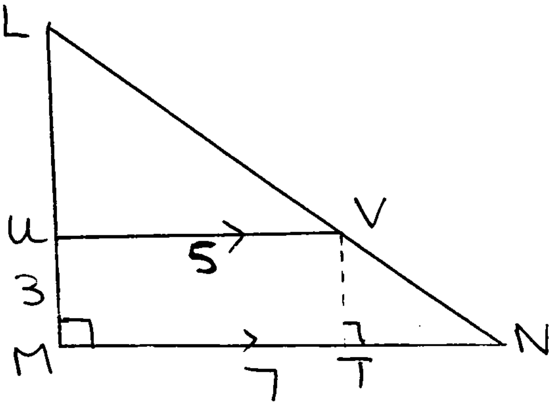
$= \lim_{x \to 1} \frac{(3x+4)(x-1)}{(x-1)}$ $(x \neq 1)$

$= \lim_{x \to 1} (3x+4)$

$= 7$

4 (a)

(i)




(ii) (A) In $\Delta$s LUV and LMN

$\angle LUV = \angle LMN$ (corr. $\angle$s in ∥ lines are equal)

Similarly $\angle LVU = \angle LNM$

$\angle L$ is common

$\therefore \Delta LUV ||| \Delta LMN$ ($\Delta$s are equiangular)

$\therefore \frac{LU}{LU+3} = \frac{5}{7}$ (corr. sides of similar $\Delta$s are in the same ratio.)

$\therefore 7LU = 5LU + 15$

$\therefore 2LU = 15$

$\therefore LU = 7.5$ m

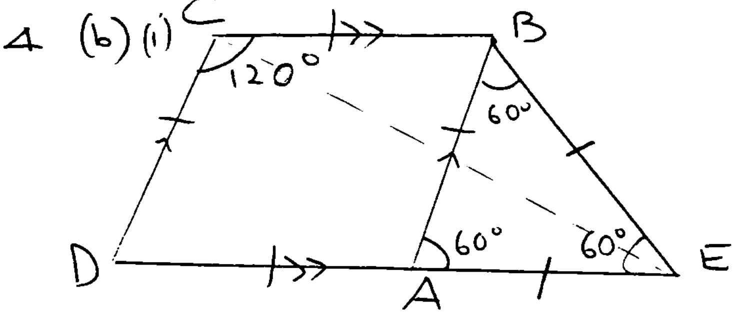
(B) Construct $VT \perp MN$

$\therefore VT = 3$, $TN = 2$

$\therefore VN = \sqrt{3^2+2^2}$

$= \sqrt{13}$ m

4 (b)(i)




(ii) $DC \parallel BA$ (property of rhombus)

$\angle CBA = 60^\circ$ (coint. ∠s in ∥ lines are supp.)

$\angle ABE = 60^\circ$ (property of equil. Δ.)

$\therefore \angle EBC = 120^\circ$ (∠sum of adj. ∠s)

(iii) As $CB = AB$ (property of rhombus)

and $AB = BE$ (property of equil. Δ)

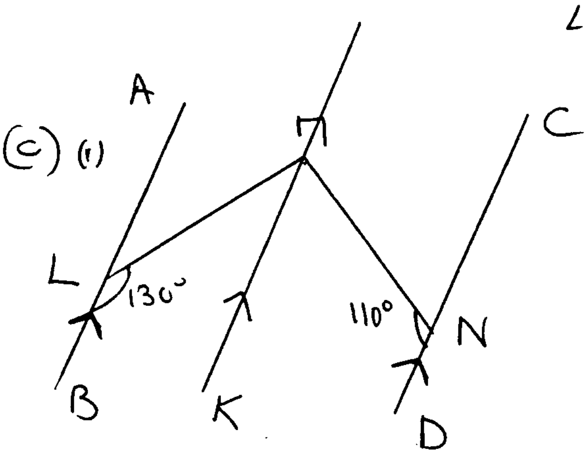
$\therefore CB = BE$

$\Rightarrow \triangle CBE$ is isosceles.

$\therefore \angle ECB = \dfrac{180^\circ - 120^\circ}{2}$

$= 30^\circ$ (base ∠s of isos. Δ are equal; ∠sum of Δ=180°)

(c) (i)




(ii) (A) Construct $MK \parallel BA \parallel DC$

$\therefore \angle LMK = 50^\circ$ (coint. ∠s in ∥ lines are supp.)

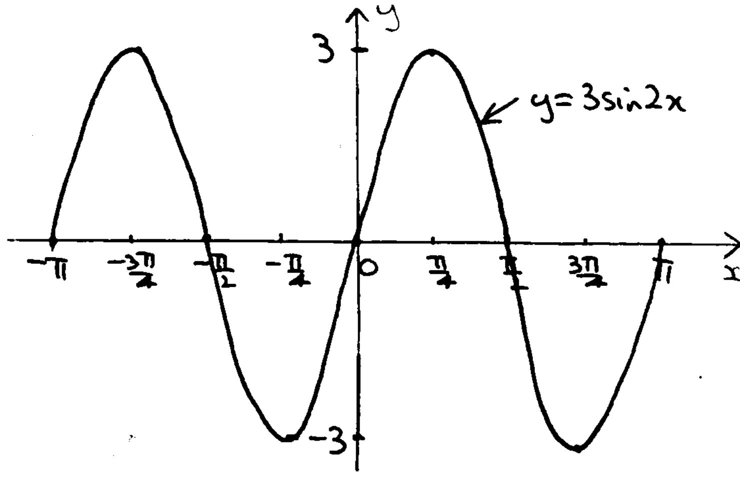
Similarly $\angle KMN = 70^\circ$

$\therefore \angle LMN = 120^\circ$ (∠sum of adj. ∠s)

(B) $\angle MNC = 70^\circ$ (∠sum on st. line = 180°)

5. (a) $y = 3\sin 2x$, $|x| \le \pi$

Period $= \frac{2\pi}{2} = \pi$ ; Amplitude = 3 units



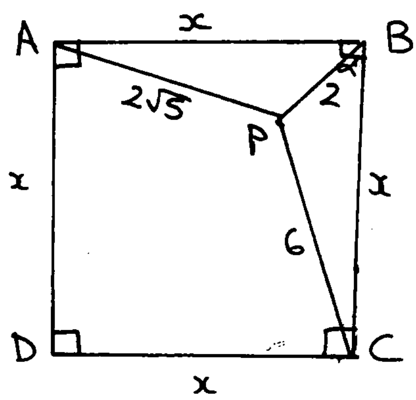

Required Area $= 4\int_0^{\frac{\pi}{2}} 3\sin 2x \, dx$

$= 12\left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$

$= 6\left[-\cos\pi + \cos 0\right]$

$= 12$ units$^2$

(b)




(i) In Δ PBC : $\cos\alpha = \dfrac{2^2 + x^2 - 6^2}{2 \times 2 \times x}$

$= \dfrac{x^2 - 32}{4x}$ —(1)

(ii) In Δ PBA $\angle ABP = \left(\frac{\pi}{2} - \alpha\right)$

$\therefore \cos\left(\frac{\pi}{2} - \alpha\right) = \dfrac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times 2 \times x}$

$= \dfrac{x^2 - 16}{4x} = \sin\alpha$ —(2)

(iii) Now as $\sin^2\alpha + \cos^2\alpha = 1$

sub. ① and ②: $\left(\frac{x^2-16}{4x}\right)^2 + \left(\frac{x^2-32}{4x}\right)^2 = 1$

$\therefore x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$

$\therefore 2x^4 - 112x^2 + 1280 = 0$

$\therefore x^4 - 56x^2 + 640 = 0$

(iv) $x^2 = \frac{56 \pm \sqrt{(-56)^2 - 4.1.640}}{2}$

$= \frac{56 \pm 24}{2}$

$= 40$ or $16$

$\therefore x = 2\sqrt{10}$ ($x^2 > 32$ otherwise ① would not yield an acute value for $\alpha$)

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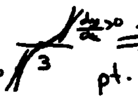
6 (a) $y = (x-3)^3$

$\frac{dy}{dx} = 3(x-3)^2$, $\frac{d^2y}{dx^2} = 6(x-3)$

For a stat. pt $\frac{dy}{dx} = 0$ $\therefore x = 3$

| $x$ | $3^-$ | $3$ | $3^+$ |
|---|---|---|---|
| $\frac{dy}{dx}$ | + | 0 | + |

→ 


$\Rightarrow$ horiz. pt. of inflexion on a rising curve at $(3,0)$

ie the only stationary point is a horizontal point of inflexion at $(3,0)$.

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(b) $y = x^3 - 9x$

(i) For x-intercepts $y=0$

$\therefore 0 = x(x^2-9)$ $\therefore x = 0, \pm 3$.

(ii) $\frac{dy}{dx} = 3x^2 - 9$, $\frac{d^2y}{dx^2} = 6x$

(iii) Let $y = f(x) = x^3 - 9x$

Now $f(-x) = -x^3 + 9x = -f(x)$

$\Rightarrow y = x^3 - 9x$ is an odd function

(iv) For a stat. pt $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 9 = 0$ $\therefore x = \pm\sqrt{3}$

when $x = \sqrt{3}$ $\frac{d^2y}{dx^2} > 0$ $\Rightarrow$ min. turning pt at $(\sqrt{3}, -6\sqrt{3})$

As the function is odd $\Rightarrow$ max. turning pt at $(-\sqrt{3}, 6\sqrt{3})$

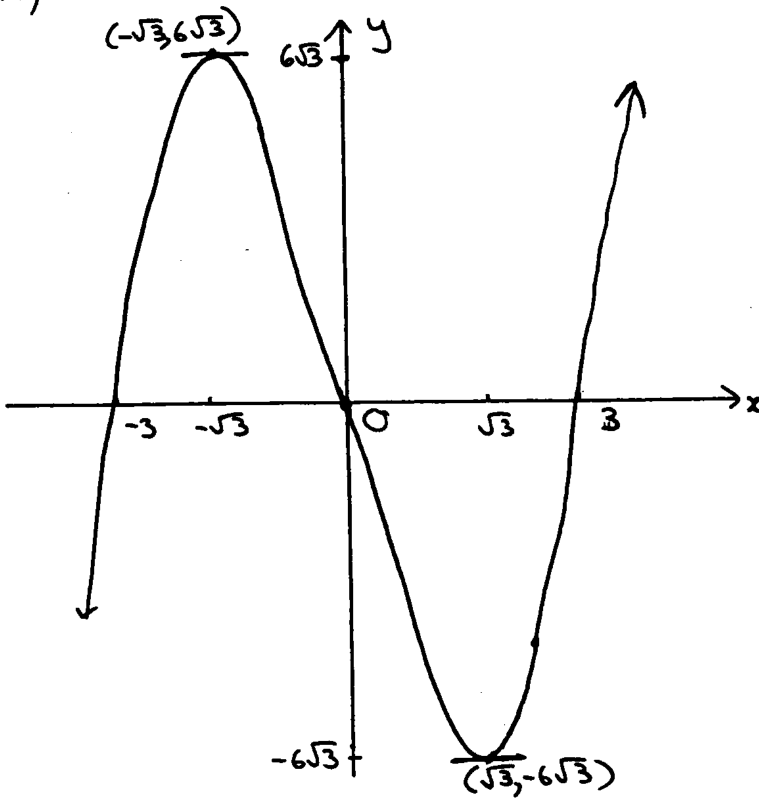
(v) For a possible pt of inflexion $\frac{d^2y}{dx^2} = 0$

$\therefore 6x = 0$ $\therefore x = 0$

| $x$ | $0^-$ | $0$ | $0^+$ |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | − | 0 | + |

concavity change $\Rightarrow$ pt. of inflexion at $(0,0)$.

(vi)

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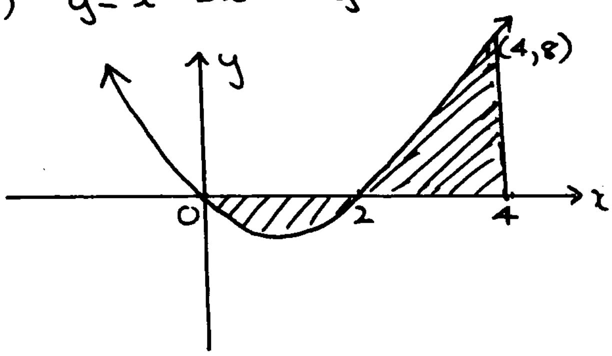
⑦ (a) $y = x^2 - 2x$ $\therefore y = x(x-2)$

(i)




Ernie's method to find the area was wrong as $\int_0^4 (x^2-2x)\,dx$ did not take into account that part of the area was below the x-axis.

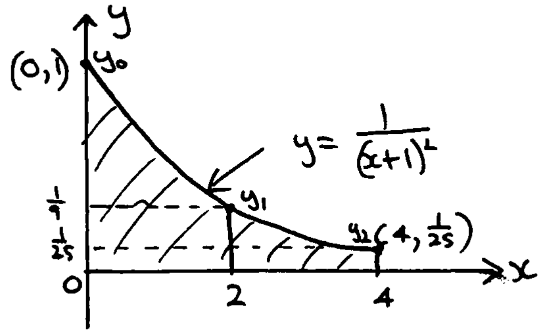
(ii) Req'd area $= \left|\int_0^2 (x^2-2x)\,dx\right| + \int_2^4 (x^2-2x)\,dx$

$= \left|\left[\frac{x^3}{3} - x^2\right]_0^2\right| + \left[\frac{x^3}{3} - x^2\right]_2^4$

$= \left|\left(\frac{8}{3}-4\right)-0\right| + \left[\left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-4\right)\right]$

$= 8$ units$^2$

(b)




By Simpson's Rule, $\int_0^4 \frac{1}{(x+1)^2}\,dx \doteqdot \frac{h}{3}\left[y_0 + y_2 + 4y_1\right]$

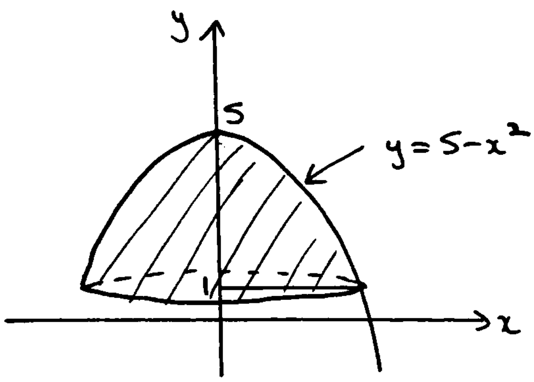
$h = \frac{4-0}{2} = 2$

$\therefore$ Area $= \frac{2}{3}\left[1 + \frac{1}{25} + 4\left(\frac{1}{9}\right)\right]$

$= \frac{668}{675}$

$= 0.99$ units$^2$ (2 sig. figs)

(c)




Volume $= \pi \int_1^5 x^2\,dy$

$= \pi \int_1^5 (5-y)\,dy$

$= \pi \left[\frac{(5-y)^2}{-2}\right]_1^5$

$= \frac{-\pi}{2}\left[0 - 16\right]$

$= 8\pi$ units$^3$

(8) (a) $x^2 + (k+6)x - 2k = 0$

(i) $\Delta = b^2 - 4ac$

$= (k+6)^2 - 4.1.(-2k)$

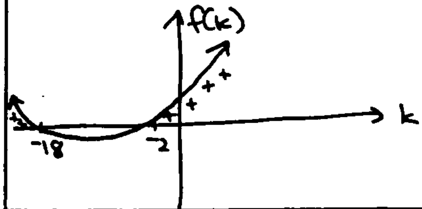
$= k^2 + 12k + 36 + 8k$

$= k^2 + 20k + 36$

(ii) For real roots $\Delta \geqslant 0$

$\therefore k^2 + 20k + 36 \geqslant 0$

$\therefore (k+18)(k+2) \geqslant 0$




$\Rightarrow k \leqslant -18$ or $k \geqslant -2$

(b) $3x^2 + 4x - 3 = 0$

(i) $\alpha + \beta = \frac{-b}{a} = \frac{-4}{3}$

(ii) $\alpha\beta = \frac{c}{a} = \frac{-3}{3} = -1$

(iii) $2\alpha^2 + 2\beta^2 = 2(\alpha^2 + \beta^2)$

$= 2\left((\alpha+\beta)^2 - 2\alpha\beta\right)$

$= 2\left(\left(\frac{-4}{3}\right)^2 + 2\right)$

$= 7\frac{5}{9}$

(c) $x^2 + 6x - 33 = 12y$

$\therefore (x+3)^2 - 9 - 33 = 12y$

$\therefore (x+3)^2 = 12y + 42$

$\therefore (x+3)^2 = 12(y + 3\frac{1}{2})$

ie of form $(x-h)^2 = 4a(y-k)$

(i) vertex $= (-3, -3\frac{1}{2})$

(ii) focal length $= 3$ units

(iii) Directrix is: $y = -6\frac{1}{2}$

(iv) Now focus is: $(-3, -\frac{1}{2})$

$\therefore$ sub $(-3, -\frac{1}{2})$ into $x - 2y + 7 = 0$

LHS $= -3 + 1 + 7$

$= 5$

$\neq 0$

$\neq$ RHS

$\therefore x - 2y + 7 = 0$ is not a focal chord of the parabola.

9 (a) (i) let $y = x^3 e^{5x-1}$

$$\therefore \frac{dy}{dx} = x^3 . 5e^{5x-1} + e^{5x-1} . 3x^2$$

(ii) let $y = \log_e \left[\frac{3x^5-4}{6x^3-5}\right]$

$$\therefore y = \log_e[3x^5-4] - \log_e[6x^3-5]$$

$$\therefore \frac{dy}{dx} = \frac{15x^4}{3x^5-4} - \frac{18x^2}{6x^3-5}$$

(b) $I = \int_0^1 \frac{24x^2-14}{4x^3-7x-5}\,dx$

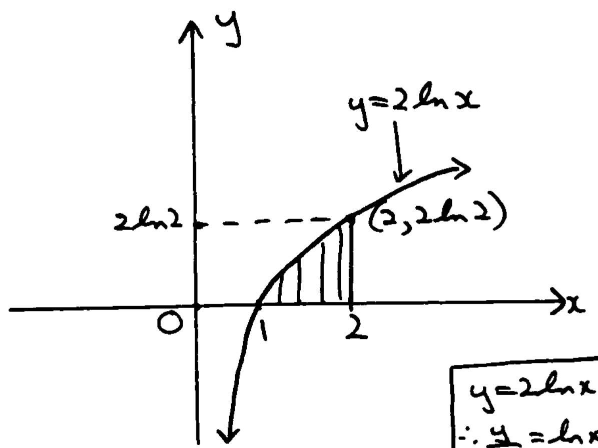
$$= \int_0^1 \frac{2(12x^2-7)}{4x^3-7x-5}\,dx$$

$$= 2\left[\ln|4x^3-7x-5|\right]_0^1$$

$$= 2\left[\ln|-8| - \ln|-5|\right]$$

$$= 2\ln\frac{8}{5}$$

(c)




$y = 2\ln x$
$\therefore \frac{y}{2} = \ln x$
$\therefore x = e^{\frac{y}{2}}$

$$\text{Area} = \int_1^2 2\ln x\,dx$$

$$= \text{Area rectangle} - \int_0^{2\ln 2} x\,dy$$

$$= 2\times 2\ln 2 - \int_0^{2\ln 2} e^{y/2}\,dy$$

$$= 4\ln 2 - \left[\frac{e^{y/2}}{1/2}\right]_0^{2\ln 2}$$

$$= 4\ln 2 - 2\left[e^{\ln 2} - e^0\right]$$

$$= 4\ln 2 - 2[2-1]$$

$$= (4\ln 2 - 2)\text{ units}^2$$

(10) (a) If $x, y$ and $9$ are in G.P.

$\therefore \frac{y}{x} = \frac{9}{y}$ $\quad\therefore y^2 = 9x$ —(1)

If $y, x$ and $2$ are in A.P.

$\therefore x - y = 2 - x$ $\quad\therefore 2x - 2 = y$ —(2)

sub (2) into (1): $(2x-2)^2 = 9x$

$$\therefore 4x^2 - 8x + 4 = 9x$$

$$\therefore 4x^2 - 17x + 4 = 0$$

$$(4x-1)(x-4) = 0$$

$$\therefore x = \tfrac{1}{4} \text{ or } 4$$

when $x = \frac{1}{4}$ $\quad y = -1\frac{1}{2}$

$x = 4$ $\quad y = 6$

$\Rightarrow (x,y) = (\frac{1}{4}, -1\frac{1}{2})$ or $(4, 6)$.

(b)(i) 6% p.a. $= \frac{6}{12}$% p.month

$= 0.005$ p.month

Let $A_n$ be the amount owing after n instalments

Let $P = 1\,000\,000$

After 1 instalment amount owing, $A_1 = P(1.005) - M$

" 2 instalments " ", $A_2 = A_1(1.005) - M$

$= (P(1.005) - M)1.005 - M$

$= P(1.005)^2 - M(1 + 1.005)$

" 3 instalments, " ", $A_3 = A_2(1.005) - M$

$= (P(1.005)^2 - M(1+1.005))1.005 - M$

$$\therefore A_3 = P(1.005)^3 - M(1.005^2 + 1.005 + 1)$$

$\therefore$ continuing this process the amount owing after n instalments, $A_n = P(1.005)^n - M(1.005^{n-1} + \dots + 1.005 + 1)$

Now after n instalments, $A_n = 0$.

After 10 years $n = 10 \times 12 = 120$

$$\therefore 0 = 1000000(1.005)^{120} - M(1 + 1.005 + \dots + 1.005^{119})$$

$$\therefore M = \frac{1\,000\,000(1.005)^{120}}{1 + 1.005 + \dots + 1.005^{119}}$$

G.P. $a=1, r=1.005, n=120$

$$\therefore M = \frac{1000000(1.005)^{120}}{1\left[\frac{1.005^{120}-1}{0.005}\right]}$$

$$\therefore M = \frac{5000(1.005)^{120}}{1.005^{120}-1}$$

(ii) $\therefore$ Monthly instalment, $M = \$11102.05$
(to the nearest cent)

(iii) After 5 years, $n = 5 \times 12 = 60$.

$\therefore$ Amount owing, $A_{60} \doteqdot 1000000(1.005)^{60} - 11102.05\left[1 + 1.005 + \ldots + 1.005^{59}\right]$

$$\therefore \quad A_{60} = 1000000(1.005)^{60} - 11102.05\left[\frac{1.005^{60} - 1}{0.005}\right]$$

$= \$574259.79$ (to nearest cent)

(iv) Interest paid over 10 years

$= 120 \times \$11102.05 - 1000000$

$= \$332246$

Now Simple Interest @ 4.5% p.a over 10 years $= \frac{P.R.T}{100}$

$= \frac{1000000 \times 4.5 \times 10}{100}$

$= \$450000$

$\therefore$ The investor would have been worse off by some \$117754 if he had chosen to borrow from F.B. Knightly Investments Ltd. instead of financial institution X.