CRANBROOK
SCHOOL

Term 3, 2008

Year 12 Mathematics

Trial Examination

Wednesday July 23, 2008

Time Allowed: 3 hours, plus 5 minutes reading time

Total Marks: 120

There are 10 questions, all of equal value

Submit your work in ~~twelve~~ ten 4 Page Booklets.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Board of Studies approved calculators may be used.

A list of standard integrals is attached to the back of this paper.

Total marks available – 120
Attempt all questions

Question 1 (12 marks) **Marks**

(a) Evaluate $\frac{2}{8+2\times(8-1)}$ correct to 4 significant figures. **2**

(b) Solve $|x-1|\le 2$ and graph the solution on a number line. **2**

(c) Simplify $\frac{2}{x(x-3)}-\frac{1}{x}$. **2**

(d) Solve $x^2-3=3x+1$. **2**

(e) Integrate $\frac{-1}{\sqrt{x}}$. **2**

(f) Sketch the graph of $y=-x+2$ on a set of axes, showing any x and y intercepts. **2**

Question 2 (12 marks) **Marks**

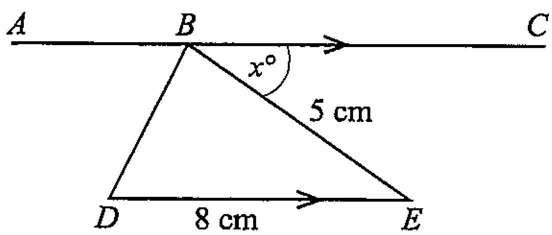
(a) Differentiate:

(i) $\cos(1-x^3)$ **2**

(ii) $\dfrac{x+1}{e^x}$. **2**

(b) If α and β are the roots of $2x^2-3x+7=0$, find the value of $\alpha^2+\beta^2$ **2**

(c)



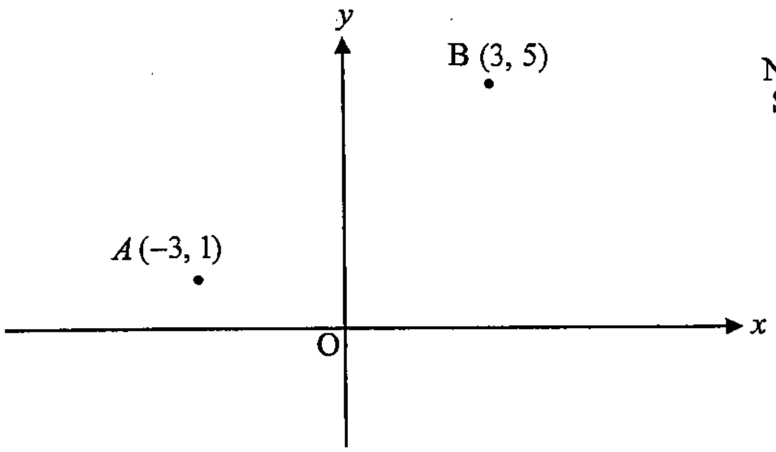
In the diagram, AC is parallel to DE, $BE = 5$ cm, $DE = 8$ cm and $\angle CBE = x°$. The area of triangle BDE is 10 square cm. Find the value of x, giving reasons for your answer. **3**

(d) Find the equation of the tangent to the curve $y = 2\sqrt{x}$ at the point (1, 2). **3**

Question 3 (12 marks) **Marks**

(a) Find the exact value of $\int_2^4 \frac{2}{x-1}\,dx$. **2**

(b)



The diagram shows the points $A(-3,1)$ and $B(3,5)$ on the Cartesian plane. Copy or trace this diagram onto your writing page.

(i) Show that the equation of AB is $2x-3y+9=0$. **2**

(ii) Show that the point C, which is the midpoint of AB is the y-intercept of AB. **1**

(iii) Calculate the perpendicular distance from the point $D(2,0)$ to the line AB and mark the point D on your diagram. **2**

(iv) The point E, lies on the line $y=-1$ and the line BE is perpendicular to the line AB. Show that E has the coordinates $(7,-1)$ and mark point E on your diagram. **2**

(v) Show that $BCDE$ is a trapezium. **1**

(vi) Find the area of $BCDE$. **2**

Question 4 (12 marks) **Marks**

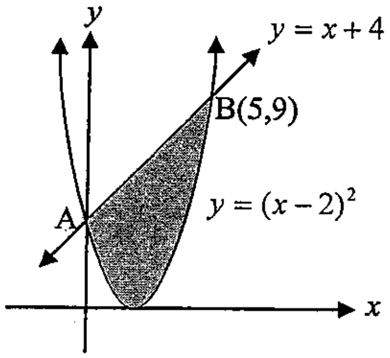
(a) If $\log_x 128 = \frac{7}{3}$, find x. 1

(b) (i) Sketch the graph of $y = 5\cos\frac{x}{2}$ for $-360° \le x \le 360°$. 2

(ii) Mark clearly on your graph the point or points where $5\cos\frac{x}{2} = -1$. 1

(iii) Calculate the value(s) of x which satisfy the equation $5\cos\frac{x}{2} = -1$. Express your answer(s) to the nearest minute. 2

(c)



The graphs of $y = (x-2)^2$ and $y = x + 4$ intersect at the point A and the point $B(5,9)$.

(i) Show that the point A lies on the y-axis. 2

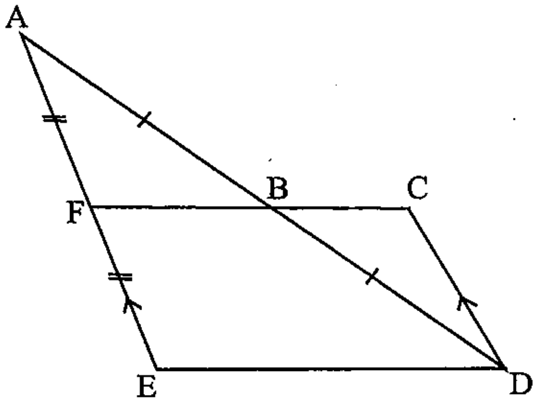
(ii) Write down the two inequalities whose intersection describes the shaded area shown in the diagram above. 1

(iii) Find the area of the shaded region bounded by the graphs of $y = (x-2)^2$ and $y = x + 4$. 3

Question 5 (12 marks) **Marks**

(a) Sketch the graph of the function $y = \dfrac{1}{x+1}$ and state the domain and the range of the function. 3

(b)



NOT TO SCALE

In the diagram, the line FC bisects AE at F and AD at B.
The line AE is parallel to CD.
Copy the diagram onto your working page.

(i) Explain why $ED = 2BF$. 1

(ii) Prove that $\Delta ABF \equiv \Delta DBC$. 3

(c) Organizers of a music festival issued 750 tickets in the first year of the festival. The number of tickets issued increased by 150 each year after that.

(i) How many tickets were issued in the fifteenth year of the festival? 1

(ii) In the first 20 years that the festival ran, what was the total number of tickets issued? 2

(iii) In which year of the festival did the number of tickets issued for that year first exceed 5000? 2

Question 6 (12 marks) **Marks**

(a) Find the equation of the parabola which has its vertex at (2,0) and its directrix is given by $x = 5$. **2**

(b) The number of subscribers S, to a pay-TV company t years after its launch is given by

$$S = S_0 e^{kt}$$

where S_0 and k are constants. Initially the pay TV company had 50 000 subscribers and after 3 years it had 200 000.

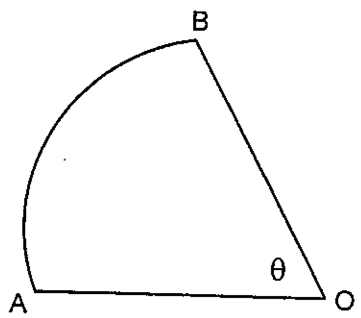
(i) Find the value of S_0. **1**

(ii) Find the value of k. Express your answer correct to 4 decimal places. **2**

(iii) After how many years will the number of subscribers first exceed one million? Express your answer correct to 1 decimal place. **2**

(iv) After 3 years, what is the rate at which the number of subscribers is increasing? Express your answer to the nearest whole number. **1**

(c) The area of a sector AOB of a circle centre O, radius 8cm, is $56\,cm^2$.



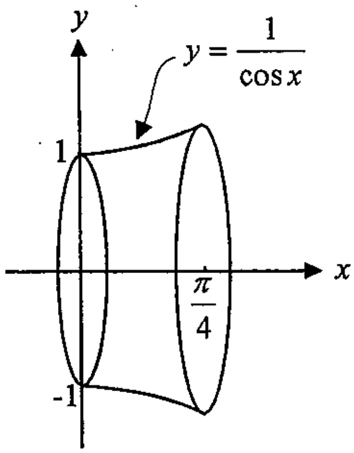
(i) Calculate the length of the minor arc AB **2**

(ii) The straight edges OA and OB are joined to form a cone. Find the exact value of the base radius of the cone. **2**

Question 7 (12 marks) **Marks**

(a) For the function $f(x)$ over the domain $0 \le x \le 5$, it is the case that $f'(x) > 0$ and $f''(x) < 0$. **2**
Sketch a graph which could be that of $y = f(x)$ over this domain.

(b) **4**



The graph of $y = \frac{1}{\cos x}$ between $x = 0$ and $x = \frac{\pi}{4}$ is rotated around the x-axis.
Find the volume of the solid of revolution.

(c) A particle moves in a straight line so that its displacement x, in metres from a fixed origin at time t seconds is given by

$$x = \log_e(t+1), \qquad t \ge 0$$

(i) Find the initial position of the particle. **1**

(ii) Explain how many times the particle is at the origin. **1**

(iii) Find an expression for the velocity and the acceleration of the particle. **2**

(iv) Explain whether or not the particle is ever at rest. **2**

Question 8 (12 marks) **Marks**

(a) Use Simpson's rule with 3 function values to find an approximate value of **2**

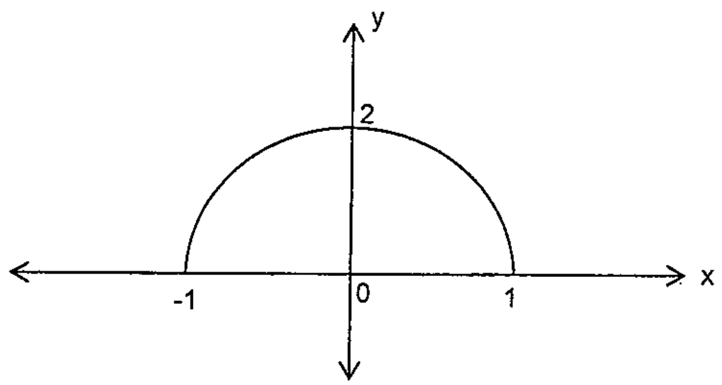
$$\int_0^2 \frac{5}{9-x^2}\,dx.$$

(b) Consider the function $y = x\ln x - x$, for $x > 0$.

(i) Find the x-intercept of the graph of the function. **1**

(ii) Find the coordinates of the turning point of the graph of the function. **3**

(c) An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of either a cosine curve or a parabola as illustrated on the axes below.



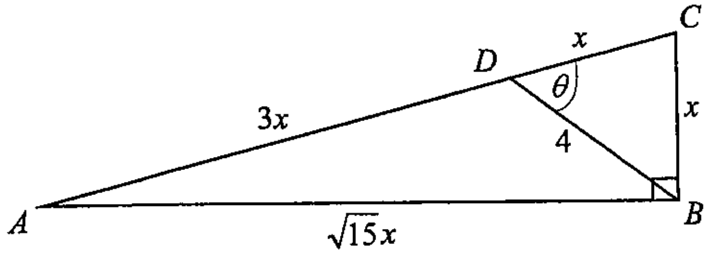
(i) If the arch is made in the shape of the curve $y = 2\cos\dfrac{\pi x}{2}$, find the exact area of the window. **2**

(ii) If the arch is made in the shape of a parabola, find the equation of the parabola. **2**

(iii) Hence find the area of this parabolic window. **2**

Question 9 (12 marks) **Marks**

(a)



In the diagram, ABC is a right angled triangle where $AB = \sqrt{15}x$ cm and $BC = x$ cm. The point D lies on AC and $CD = BC = x$ cm, $AD = 3x$ cm and $BD = 4$ cm. Let $\angle BDC = \theta$.

(i) Use the cosine rule to show that $\cos\theta = \dfrac{2}{x}$. **1**

(ii) Use the sine rule in triangle BCD to show that $\sin\theta = \dfrac{\sqrt{15}x}{16}$. **2**

(iii) Hence show that $15x^4 - 256x^2 + 1024 = 0$. **1**

(iv) Explain why one of the solutions to the equation in part (iii), namely $x = 2.53$ (to 2 decimal places), could not be the value of x indicated in the diagram above. **1**

(b) Gayle has a superannuation fund, which pays 5% per annum interest compounding annually. Gayle pays \$12 000 into the fund on 1 July each year.

(i) What is the value of Gayle's superannuation fund on 30 June one year after she makes her first payment? **1**

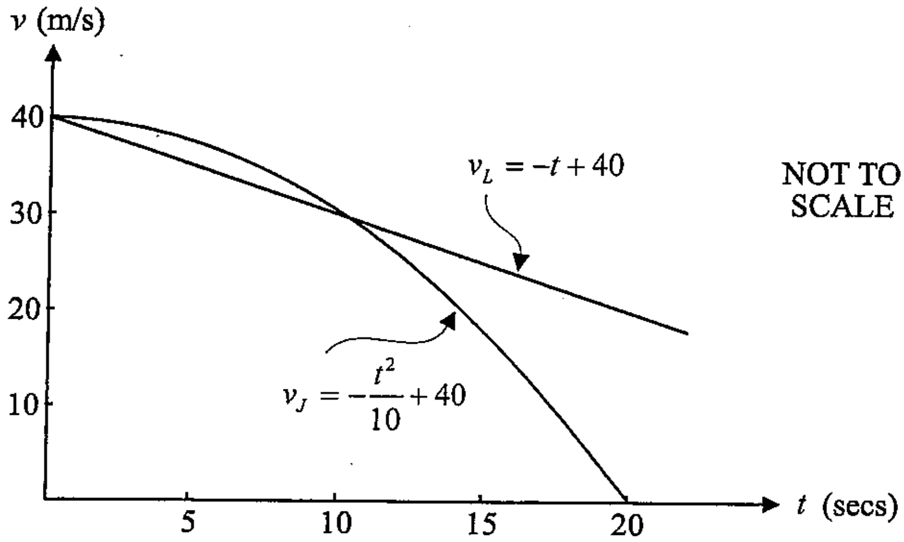
(ii) What is the value of Gayle's superannuation fund on 30 June ten years after she makes her first payment? **3**

(iii) After making her tenth payment, Gayle considers increasing her payment to M dollars per year. **3**
Show that if Gayle does this, then the value of her superannuation fund twenty years after her first payment of \$12 000 was made, would be approximately given by

$$13\cdot 2068(12\,000 \times 1\cdot 05^{10} + M).$$

Question 10 (12 marks) **Marks**

(a) Larry and Jack are each speeding down a straight stretch of freeway and are side by side, when they spot a police car. They each brake. The velocity of Larry's car during this braking phase is given by $v_L = -t + 40$ and the velocity of Jack's car during this phase is given by $v_J = \dfrac{-t^2}{10} + 40$.



(i) When are the velocities the same during this braking phase? **1**

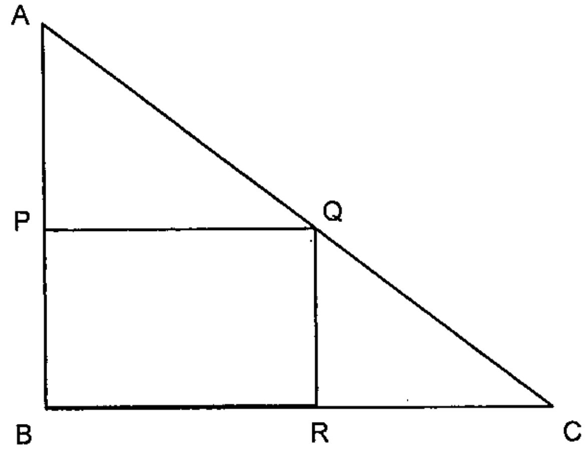
(ii) When are the two cars level with one another during this braking phase? **2**

(iii) State the times when Larry's car is further ahead of Jack's car during this braking phase? Give reasons for your answer. **1**

(b) Find the values of m for which the equation $x^2 + (m-2)x + 4 = 0$ has real roots. **3**

Question 10 continues on the next page.

(c) In ΔABC, AB = $20m$, BC = $15m$ and $\angle ABC = 90^0$. BPQR is a rectangle inscribed in ΔABC. PQ = x metres.
Copy and complete the diagram, showing all information given.



(i) Using similar triangles, or otherwise, find an expression for the length of AP in terms of x. **1**

(ii) Show that the area of the rectangle BPQR is given by **1**

$$A = x\left(20 - \frac{4}{3}x\right) \text{ square metres.}$$

(iii) Hence find the minimum possible area of the rectangle BPQR. **3**

END OF PAPER

12 2U Trial 2008

Question 1

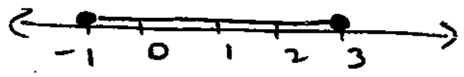
a) 0.09091 ✓✓ (Give 1 mark if not correct to 4 sig figs) (to 4 sig figs)

b) $x-1 \leq 2 \qquad -x+1 \leq 2$

$x \leq 3 \qquad -x \leq 1$

$x \geq -1$

$\therefore -1 \leq x \leq 3$ ✓



✓

c) $\dfrac{2}{x(x-3)} - \dfrac{1}{x}$

$= \dfrac{2-(x-3)}{x(x-3)}$ ✓

$= \dfrac{2-x+3}{x(x-3)}$

$= \dfrac{5-x}{x(x-3)}$ ✓

.) $x^2 - 3 = 3x + 1$

$x^2 - 3x - 4 = 0$ ✓

$(x-4)(x+1) = 0$

$x = 4,\ x = -1$ ✓

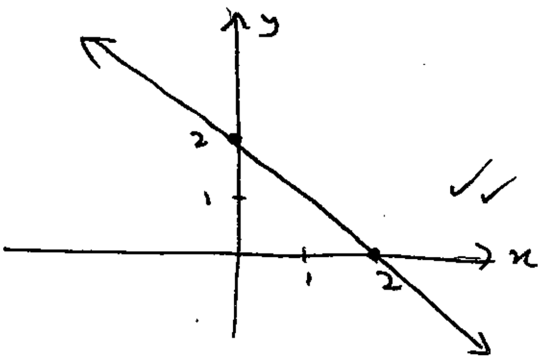
' $\int \frac{-1}{\sqrt{x}}\,dx = \int -x^{-\frac{1}{2}}\,dx$ ✓

$= -2x^{\frac{1}{2}} + c$

$= -2\sqrt{x} + c$ ✓

(+c is not necessary)

) $y = -x + 2$



✓✓

Question 2

a) (i) $y' = 3x^2 \sin(1-x^3)$ ✓ ✓

(ii) $y' = \dfrac{e^x \cdot 1 - (x+1)\cdot e^x}{e^{2x}}$ ✓

$= \dfrac{1-x-1}{e^x}$

$= \dfrac{-x}{e^x}$ ✓

b) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{7}{2}\right)$

$= -4.75$ ✓

c) $\angle BED = x^\circ$ alternate ∠'s are equal as $AC \parallel DE$ ✓

Area $= \frac{1}{2}ab\sin C$

$10 = \frac{1}{2} \times 8 \times 5 \times \sin x^\circ$ ✓

$\sin x^\circ = \frac{1}{2}$

$x^\circ = 30^\circ$ ✓

d) $y = 2\sqrt{x}$

$y = 2x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = x^{-\frac{1}{2}} = \dfrac{1}{\sqrt{x}}$ ✓

at $x = 1$ $\dfrac{dy}{dx} = 1$ ✓

$\therefore$ eqn. of tangent is:

$y - 2 = 1(x-1)$

$y = x + 1$ ✓

Question 3

i) $\int_2^4 \frac{2}{x-1}\,dx$

$= \left[2\ln(x-1)\right]_2^4$ ✓

$= 2\ln 3 - 2\ln 1$

$= 2\ln 3$ ✓

2) (i) $m_{AB} = \frac{5-1}{3+3}$

$= \frac{4}{6} = \frac{2}{3}$ ✓

$y - 1 = \frac{2}{3}(x+3)$ ✓

$3y - 3 = 2x + 6$

$\therefore 2x - 3y + 9 = 0$

ii) M.P. of AB $= \left(\frac{-3+3}{2}, \frac{1+5}{2}\right)$

$= (0, 3)$ ✓

y int of $2x - 3y + 9 = 0$

sub $x = 0$ $\quad -3y + 9 = 0$

$3y = 9$

$y = 3$

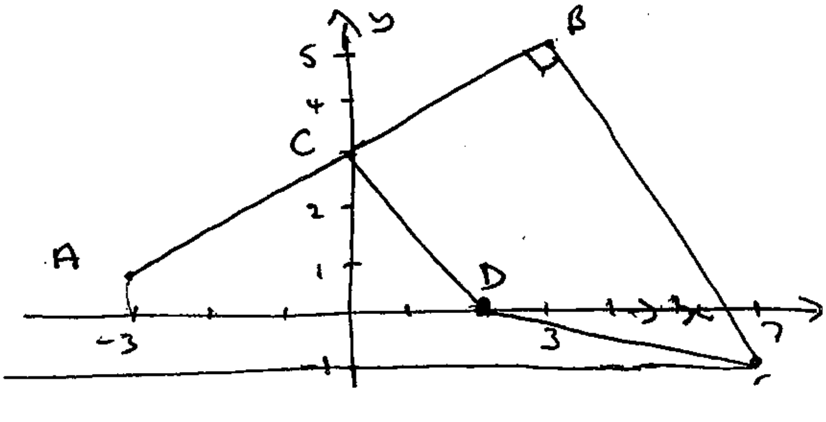
$\therefore$ C is midpt of AB and y int of line

iii) perp d $= \left|\frac{2x - 3y + 9}{\sqrt{2^2 + (-3)^2}}\right|$ ✓

$= \frac{|2(2) - 3(0) + 9|}{\sqrt{13}}$

$= \frac{13}{\sqrt{13}}$ ✓

$= \sqrt{13}$ units



(iv) m of BE $= -\frac{3}{2}$ ✓ $(m_1 m_2 = -1)$

Eqn. of BE

$y - 5 = -\frac{3}{2}(x - 3)$

$2y - 10 = -3x + 9$

$3x + 2y - 19 = 0$

For E sub $y = -1$

$\therefore 3x + 2(-1) - 19 = 0$ ✓

$3x - 21 = 0$

$3x = 21$

$x = 7$

$\therefore E\ (7, -1)$ as required.

(v) BCDE is a trapezium if BE || CD

$m_{BE} = -\frac{3}{2}$

$m_{CD} = \frac{0-3}{2-0} = -\frac{3}{2}$

$\therefore m_{BC} = m_{CD}$ ✓

$\therefore$ BC || CD

$\therefore$ BCDE is a trapezium

(vi) $CD = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

$BE = \sqrt{(7-3)^2 + (-1-5)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 2\sqrt{13}$

$CB = \sqrt{3^2 + (5-3)^2}$

$= \sqrt{13}$ ✓

Area $= \frac{1}{2}h(a+b)$

$= \frac{1}{2}\sqrt{13}(\sqrt{13} + 2\sqrt{13})$

$= 19.5$ units^2 ✓

Question 4

a) $\log_x 128 = \frac{7}{3}$

$128 = x^{\frac{7}{3}}$

$x = \sqrt[7]{128}^{\,3}$

$= 2^3$

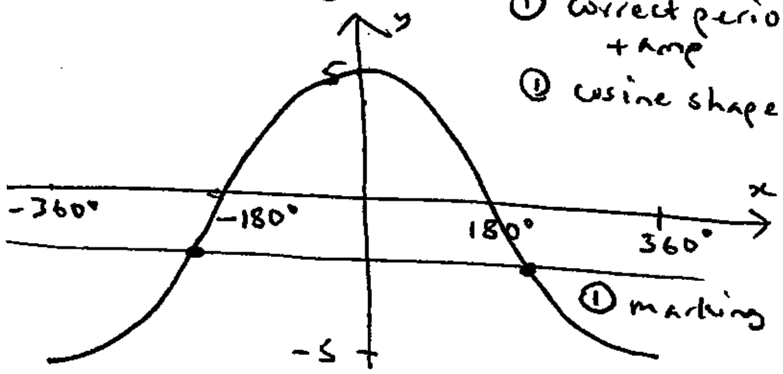
$= 8$ ✓

b) (i) $y = 5\cos\frac{x}{2} \quad -360° \le x \le 360°$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

① correct period + amp

① cosine shape



① marking pts

(iii) $5\cos\frac{x}{2} = -1$

$\cos\frac{x}{2} = -\frac{1}{5}$

$\frac{x}{2} = \cos^{-1}\left(-\frac{1}{5}\right)$ ✓

$\frac{x}{2} = 101°32', -101°32'$

$x = 203°4', -203°4'$ ✓

c) $(x-2)^2 = x+4$

(i) $x^2 - 4x + 4 = x + 4$

$x^2 - 5x = 0$ ✓

$x(x-5) = 0$

$x = 0, \quad x = 5$

$y = 4 \quad y = 9$

∴ A is $(0,4)$ which is on y axis ✓ as x coord is zero.

(ii) $y \le x+4$ and $y \ge (x-2)^2$

① for both correct

(iii) $A = \int_0^5 x+4-(x-2)^2\,dx$ ✓

$= \int_0^5 x+4-(x^2-4x+4)\,dx$

$= \int_0^5 5x - x^2\,dx$

$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$ ✓

$= 62\frac{1}{2} - \frac{125}{3}$

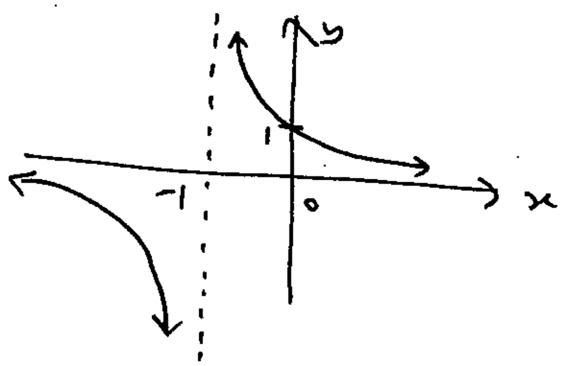
$= 20\frac{5}{6}$ units^2 ✓

Question 5

a) $y = \frac{1}{x+1}$

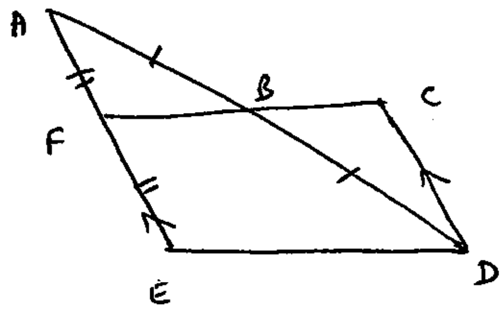
D: all x, $x \ne -1$ ✓

R: all y, $y \ne 0$ ✓



✓

b)



B is midpt of AD

F is midpt of AE

∴ BF joins 2 midpts

∴ BF is parallel to ED and half its length ✓

∴ $BF = \frac{1}{2}ED$

∴ $ED = 2BF$

In ΔABF and ΔDBC

$AB = BD$ given

$\angle FAB = \angle BDC$ alt $\angle$'s equal $AE \parallel CD$ ✓

$\angle ABF = \angle CBD$ vert. opp ✓

$\therefore \Delta ABF \equiv \Delta DBC$ by AAS ✓

c) 750, 900, 1050 ....

$a = 750$

$d = 150$

i) $T_{15} = ?$

$T_n = a + (n-1)d$

$T_{15} = 750 + 14 \times 150$

$= 2850$ ✓

$\therefore$ 2850 tickets were issued

ii) $S_n = \frac{n}{2}(2a + (n-1)d)$ ✓

$S_{20} = 10(1500 + 19 \times 150)$

$= 43\ 500$ ✓

iii) $T_n > 5000$

$\therefore 750 + (n-1)150 > 5000$

$750 + 150n - 150 > 5000$

$600 + 150n > 5000$

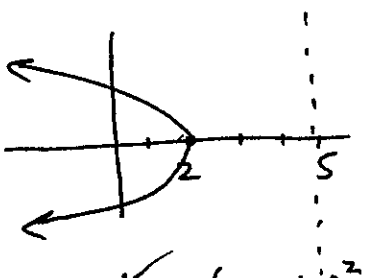
$150n > 4400$

$n > 29.3$ ✓

$\therefore$ in the 30th year ✓

Question 6

a)



$a = 3$ ✓ $(y-k)^2 = -4a(x-h)$

$(y-0) = -12(x-2)$ ✓

$y^2 = -12x + 24$

b) $S = S_0 e^{kt}$

(i) $S_0 = 50\ 000$ ✓

(ii) $200\ 000 = 50\ 000 e^{3k}$ ✓

$4 = e^{3k}$

$3k = \ln 4$

$k = \frac{1}{3} \ln 4$

$k = 0.4621$ (to 4 d.p.) ✓

(iii) $1\ 000\ 000 = 50\ 000 e^{0.4621t}$ ✓

$20 = e^{0.4621t}$

$0.4621t = \ln 20$

$t = 6.48289$

$t = 6.5$ years. ✓

(iv) $\frac{dS}{dt} = 50\ 000k\, e^{3k}$

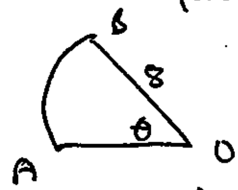
$= 92419.6$

$= 92420$ subscribers / year ✓

(also give mark for 92421)

c)

(i)



$A = 56$

$A = \frac{1}{2} r^2 \theta$

$56 = \frac{1}{2}(8)^2 \theta$

$\theta = 1.75$ ✓

$l = r\theta$

$= 8 \times 1.75$

$= 14$ cm ✓

(ii) arc length = circum of base ✓

$14 = 2\pi r$

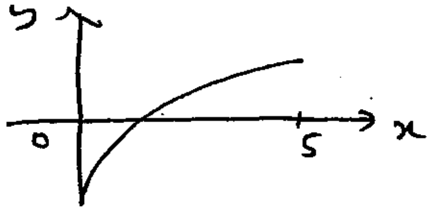
$r = \frac{14}{2\pi}$

$r = \frac{7}{\pi}$ cm ✓

Question 7

a) $f'(x) > 0$ increasing ✓

$f''(x) < 0$ concave down



✓

b) $V = \pi \int y^2 \, dx$

$= \pi \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos x} \right)^2 dx$ ✓

$= \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ ✓

$= \pi \left[\tan x \right]_0^{\frac{\pi}{4}}$ ✓

$= \pi \left(\tan \frac{\pi}{4} - \tan 0 \right)$

$= \pi$ units^3 ✓

c) $x = \log_e (t+1)$

i) $t = 0 \quad x = \ln 1$

$x = 0$ ✓

∴ initial position is at the origin

ii) $x = 0 \quad \log_e (t+1) = 0$

$t + 1 = e^0$

$t + 1 = 1$

$t = 0$

∴ particle only at origin once ✓

iii) $v = \frac{1}{t+1} = (t+1)^{-1}$ ✓

$a = -(t+1)^{-2}$

$= \frac{-1}{(t+1)^2}$ ✓

v) at rest when $v = 0$

∴ $\frac{1}{t+1} = 0$ ✓

$1 = 0$!!

∴ particle never at rest ✓

Question 8

a)

x	0	1	2
y	$\frac{5}{9}$	$\frac{5}{8}$	1

$h = 1$

∴ $\int_0^2 \frac{5}{9 - x^2} dx \doteqdot \frac{1}{3} \left(\frac{5}{9} + 1 + 4 \left(\frac{5}{8} \right. \right.$ ✓

$= \frac{1}{3} \left(4 \frac{1}{18} \right)$

$= \frac{73}{54}$ ✓

b) $y = x \ln x - x$

(i) x int when $y = 0$

∴ $x \ln x - x = 0$

$x(\ln x - 1) = 0$

$x = 0, \quad \ln x = 1$

$x = e$

but $x > 0$

∴ x int is $(e, 0)$ ✓

(ii) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1 - 1$

$= 1 + \ln x - 1$

$= \ln x$ ✓

turning pt when $\frac{dy}{dx} = 0$

∴ $\ln x = 0$

$x = e^0$

$x = 1$ ✓

$y = -1$ ✓

∴ turning pt is $(1, -1)$

c) $A = 2 \int_0^1 2 \cos \frac{\pi x}{2} dx$ ✓

$= 4 \left[\frac{2}{\pi} \sin \frac{\pi x}{2} \right]_0^1$

$= 4 \left[\frac{2}{\pi} \sin \frac{\pi}{2} - 0 \right]$

$= \frac{8}{\pi}$ units^2 ✓

ii) Vertex $(0,2)$

$y = ax^2 + 2$ ✓

sub $(1,0)$

$0 = a + 2$

$a = -2$

$\therefore y = -2x^2 + 2$ ✓

iii) $A = 2\int_0^1 -2x^2 + 2 \, dx$ ✓

$= 2\left[-\frac{2x^3}{3} + 2x\right]_0^1$

$= 2\left(-\frac{2}{3} + 2\right)$

$= \frac{8}{3}$ units^2 ✓

Question 9

a)(i) $\cos\theta = \frac{x^2 + 4^2 - x^2}{2(x)(4)}$ ✓

$= \frac{16}{8x}$

$= \frac{2}{x}$

(ii) let $\angle BCD = \alpha$

$\therefore \frac{\sin\theta}{x} = \frac{\sin\alpha}{4}$

$\sin\theta = \frac{x\sin\alpha}{4}$ ✓

In $\triangle ABC$

$\sin\alpha = \frac{\sqrt{15}x}{4x} = \frac{\sqrt{15}}{4}$

$\therefore \sin\theta = \frac{x}{4} \cdot \frac{\sqrt{15}}{4}$ ✓

$\sin\theta = \frac{\sqrt{15}x}{16}$ as required

(iii) Now $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{\sqrt{15}x}{16}\right)^2 + \left(\frac{2}{x}\right)^2 = 1$ ✓

$\frac{15x^2}{256} + \frac{4}{x^2} = 1$

$15x^4 + 1024 = 256x^2$

$\therefore 15x^4 - 256x^2 + 1024 = 0$ as required

(iv) Now $\cos\theta = \frac{2}{x} = \frac{2}{2.53}$

$\therefore \theta = 37°46'$

Also $\sin\alpha = \frac{\sqrt{15}}{4}$

$\alpha = 75°31'$

In $\triangle DCB$: $\theta + \theta + \alpha$ should be $180°$

But $37°46' + 37°46' + 75°31'$

$= 151°3'$ ✓

$\neq 180°$

$\therefore x \neq 2.53$

b) (i) $A_1 = 12000(1.05)$

$= \$12\,600$ ✓

(ii) $A_1 + A_2 + \cdots + A_{10}$

$= 12000(1.05 + 1.05^2 + \cdots + 1.05^{10})$ ✓

$a = 1.05$

$r = 1.05$

$n = 10$ ✓

$S_n = \frac{1.05(1.05^{10} - 1)}{0.05}$

$= 12000 \times S_n$

$= \$158\,481.45$ ✓

(iii) $A_1 + A_2 + \cdots + A_{19} + A_{20}$

$= 12000(1.05^{20} + 1.05^{19} + \cdots + 1.05^{11})$

$+ M(1.05^{10} + 1.05^9 + \cdots + 1.05)$ ✓

$= 12000(1.05)^{10}(1.05^{10} + 1.05^9 + \cdots + 1.05)$

$+ M(1.05^{10} + 1.05^9 + \cdots + 1.05)$

$= \left(12000(1.05)^{10} + M\right)(1.05^{10} + \cdots + 1.05)$ ✓

$n = 10$

$a = 1.05$

$r = 1.05$

$S_{10} = \frac{1.05(1.05^{10} - 1)}{0.05}$

$= \left(12000(1.05)^{10} + M\right)(13.2068)$ ✓

Question 10

a) (i) $-t+40 = -\frac{t^2}{10}+40$

$t = \frac{t^2}{10}$

$t^2 = 10t$

$t^2 - 10t = 0$

$t(t-10) = 0$

$t=0,\ t=10$

elocities are same initially and fter 10 seconds ✓

ii) The cars are level when their isplacements are equal. Let his time be T secs

$\therefore \int_0^T -t+40\,dt = \int_0^T -\frac{t^2}{10}+40\,dt$ ✓

$\left[-\frac{1}{2}t^2+40t\right]_0^T = \left[-\frac{t^3}{30}+40t\right]_0^T$

$-\frac{1}{2}T^2+40T = -\frac{T^3}{30}+40T$

$30T^2 = 2T^3$

$T^3 - 15T^2 = 0$

$T^2(T-15) = 0$

$T=0,\ T=15$

after 15 seconds cars are level ✓

) Cars are at same place after 15 seconds. By looking at the graph we can see there s more area under the curve nder Larry's graph after $t=15$ Larry is further ahead after 5 secs. ✓

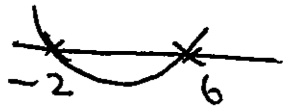
b) Real roots $\Delta \geq 0$

$(m-2)^2 - 4(1)(4) \geq 0$ ✓

$m^2 - 4m + 4 - 16 \geq 0$

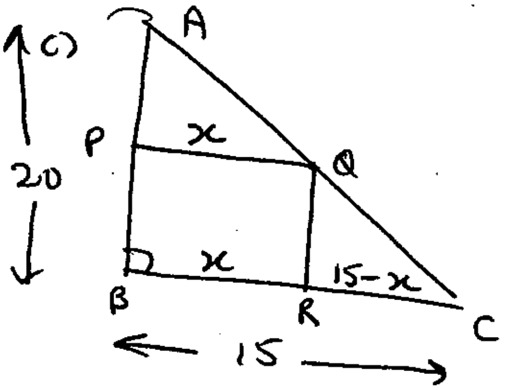
$m^2 - 4m - 12 \geq 0$

$(m-6)(m+2) \geq 0$ ✓



$\therefore m \leq -2,\ m \geq 6$ ✓

c)



(i) $\frac{AP}{20} = \frac{x}{15}$

$AP = \frac{20x}{15} = \frac{4x}{3}$ ✓

(ii) Area of rect $= PQ \cdot PB$ ✓

$= x \cdot \left(20 - \frac{4x}{3}\right)$

(iii) $A = 20x - \frac{4x^2}{3}$

$\frac{dA}{dx} = 20 - \frac{8x}{3}$

maximum when $\frac{dA}{dx} = 0$ and $\frac{d^2A}{dx^2} < 0$

$\therefore 20 - \frac{8x}{3} = 0$

$60 - 8x = 0$

$8x = 60$

$x = 7.5$ ✓

$\frac{d^2A}{dx^2} = -\frac{8}{3} < 0$ ∴ max as required ✓

$\therefore A = 20(7.5) - \frac{4}{3}(7.5)^2$

$= 75\ m^2$ ✓