+ Solns

CRANBROOK
SCHOOL

Year 12 (2U) Mathematics

Trial HSC Examination

Wednesday July 20, 2011

Time Allowed: 3 hours *plus* 5 minutes reading time

Total Marks: 120

There are 10 questions, each of equal value.

Start a new booklet for each question.

All necessary working should be shown in every question.

Full marks may not be awarded if work is careless or badly arranged.

Only approved calculators may be used.

Question 1 (12 marks) **START A NEW BOOKLET** **Marks**

a. Simplify: $8a^3 - 27$ **2**

b. Find the exact value of $\sin 120^\circ$ **2**

c. Solve $|x+7| < 2$ **2**

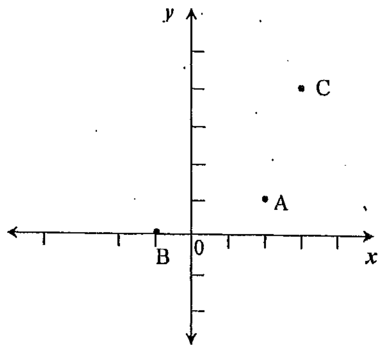
d. Find the values of a and b if $\dfrac{4}{\sqrt{3}-1} = a\sqrt{3} + b$ **3**

e. Solve for x: $8^x = \dfrac{1}{16}$ **2**

f. Find the domain of the function $f(x) = \log_e(x-2)$ **1**

Question 2 (12 marks) **START A NEW BOOKLET** **Marks**

a. The co-ordinates of points A,B and C are A(2, 1), B(–1, 0) and C(3, 4).



i. Find the gradient of BC. **1**

ii. Show that the equation of the line BC is $x - y + 1 = 0$ **1**

iii. Find the equation of the line l, through A, perpendicular to BC. **2**

iv. Find the co-ordinates of the point D where the line l intersects BC. **1**

v. Find the distances AD and BC. **2**

vi. Find the area of the triangle ABC. **1**

b. A pendulum is 60 cm long and it bob swings through an arc of length 15 cm. Find the angle through which it swings. Leave your answer in radians. **1**

c. Find the equation of the normal to the curve $y = 2\log_e(3-x)$ at the point (–2,0). **3**

Question 3 (12 marks) **START A NEW BOOKLET** **Marks**

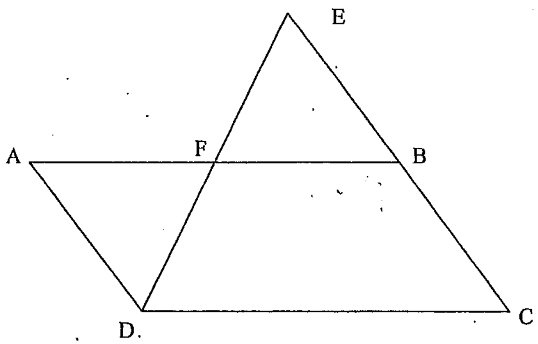
a. Differentiate with respect to x:

i. $y=-2\left(1+x^3\right)^4$ **2**

ii. $y=4e^{\cos x}$ **2**

iii. $y=\dfrac{3x^2}{e^x}$ **2**

b. ABCD is a parallelogram. DF is produced to E so that BE=AD.



i. Prove $\Delta AFD \equiv \Delta EFB$ **3**

ii. Prove FB = ½ DC **1**

c. Evaluate: $\displaystyle\int_{\pi/4}^{\pi/2} \cos 2x\,dx$ **2**

Question 4 (12 marks) **START A NEW BOOKLET** **Marks**

a. The n^{th} term of the sequence 5, 10, 16,.... is given by $2^{n-1}+4n$
Find the 9^{th} term of the sequence. **1**

b. Find the first three terms of a geometric sequence in which the 5^{th} term is 80 and the 8^{th} term is 640. **3**

c. For what values of k will the equation $x(x-2k)=k-2x-3$ have no real roots? **3**

d. The area of a sector AOB of a circle, centre 0, is $\dfrac{25\pi}{6}cm^2$. The angle AOB is $\dfrac{\pi}{3}$ radians.
Find the radius of the sector. **2**

e. If $a(x-1)^2+b(x-1)+c=x^2$, find the values of a, b and c. **3**

Question 5 (12 marks) **START A NEW BOOKLET** **Marks**

a. Consider the function $f(x)=x^2\left(\frac{1}{3}x-1\right)$

i. Show that $f'(x)=x^2-2x$ **1**

ii. Find any turning points and determine their nature **3**

iii. Find any points of inflexion. **2**

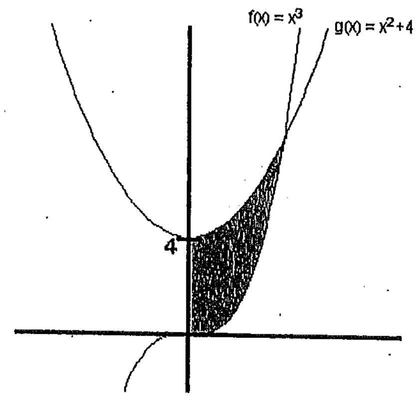
iv. Sketch the curve in the domain $-1\le x\le 3$ **2**

b. Evaluate $\int_0^{\ln 9} e^{2x}\,dx$. **2**

c. Solve $2\cos\theta+1=0$ for $0\le\theta\le 2\pi$ **2**

Question 6 (12 marks) **START A NEW BOOKLET** **Marks**

a. The diagram shows the curves $f(x)=x^3$ and $g(x)=x^2+4$.



i. Verify that the curves will intersect at the point (2, 8). **2**

ii. Calculate the area between the two curves. **3**

b. i. Copy and complete the table showing corresponding x and y values for the function $y=e^{2x}-1$. Giving your answers correct to 1 decimal place. **2**

x	0	0.5	1	1.5	2
y					

ii. Use Simpson's Rule with 5 function values to find an approximate value for $\int_0^2 \left(e^{2x}-1\right)dx$. Give your answer correct to 1 decimal place. **2**

c. Find the value of k for which $\int_2^{\sqrt{k}} \frac{x}{x^2-3}\,dx=1$ **3**

Question 7 (12 marks) **START A NEW BOOKLET** **Marks**

a. The velocity (v *m/s*) of a moving particle moving in a straight line at time t, is given by:

$$v = 8t + \sin 2t$$

i. Find the initial acceleration. **1**

ii. Find the displacement of the particle after $\frac{\pi}{8}$ seconds. **2**

b. The population of a particular country at any one time can be given by $P = P_0 e^{kt}$, where the rate of population growth of the country is proportional to the population size. The population of the country grew from 350 000 in 2001 to 460 000 in 2005

i. Find the growth rate, k in exact form. **2**

ii. Find the population of the country in 2015, correct to the nearest person. **1**

iii. Calculate the rate of change of the population in 2021. **2**

c. A particle is moving along a straight line so that its acceleration is a constant $6ms^{-2}$. If the initial velocity of the particle is $\log_e 7\ m/s$, find the velocity when $t = 5$. Leave your answer in exact form. **2**

d. Simplify: $\dfrac{\sin(\pi-\theta)\cos(\pi-\theta)}{\sin\left(\frac{\pi}{2}-\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)}$, where θ is acute. **2**

Question 8 (12 marks) **START A NEW BOOKLET** **Marks**

a. The 5 angles of a pentagon are in arithmetic progression. Given the size of the largest angle is three times the size of the smallest angle, find the size of all the angles. **3**

b. A = (1, 1), B = (4, 7) and P = (x, y).

i. Write expressions for the length PA and PB. **1**

ii. P moves so that $PA = 2 \times PB$.
Show that the locus of P is the circle: $x^2 - 14x + y^2 - 26y + 128 = 0$. **2**

iii. Find the centre and radius of this circle. **4**

c. Find the value of $\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$ **2**

Question 9 (12 marks) **START A NEW BOOKLET** **Marks**

a. State the period for $y = \tan 3x$, where x is in radians. **1**

b. Show that $\sec\theta - \cos\theta = \tan\theta \sin\theta$ **3**

c. Geoff borrows \$350 000 from a bank for 20 years at 6% p.a. compounded monthly.
The conditions of the loan are that there will be no interest charged for the first three months but repayments will be required at the end of every month. The first interest charged is at the end of the fourth month.
Let A_n be the amount owing at the end of n months and let m be the monthly repayments.

i. Build an expression for A_4 **2**

ii. Show that $A_5 = (350000 - 3m)1.005^2 - m(1 + 1.005)$ **2**

iii. Find the monthly repayments if the loan is to be repaid in 20 years. **4**

Question 10 (12 marks) Use a separate page/booklet **Marks**

a. i. Show that $\frac{d}{dx}(x\ln x - x) = \ln x$ **1**

ii. Hence evaluate $\int_1^{e^3} \ln x \, dx$. Leave your answer in exact form. **2**

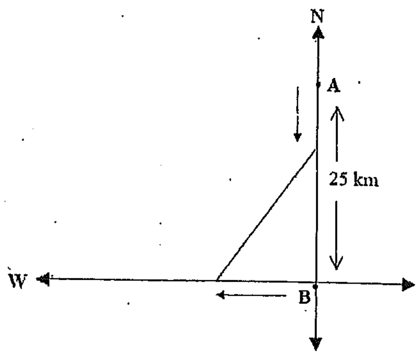
b. ABC is a triangle in which $AB = AC = x$ *metres* and AB+BC+CA= 1 *metre* and $\angle ABC = \theta$ *radians*. D is the mid-point of BC

i. Show that the altitude $AD = \frac{\sqrt{4x-1}}{2}$ *metres* **2**

ii. Hence or otherwise show that $\sin\theta = \frac{\sqrt{x - \frac{1}{4}}}{x}$ **1**

QUESTION CONTINUED ON NEXT PAGE

c. On the compass diagram below, Mary is at position A, 25km due north of position B. John is at B. Mary walks towards B at 4km/h. John moves due west at 6km/h.



i. Show that the distance between Mary and John after t hours is given by:

$$d^2 = 52t^2 - 200t + 625$$ **2**

ii. Letting $L = d^2$, find the time when L is minimum. **3**

iii. Hence find the minimum distance between John and Mary, correct to the nearest kilometre. **1**

END OF PAPER
GO BACK AND CHECK!

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1}, n \neq -1;\ x \neq 0,$ if $n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax dx$	$= \frac{1}{a}\sin ax,\ a \neq 0$
$\int \sin ax dx$	$= -\frac{1}{a}\cos ax,\ a \neq 0$
$\int \sec^2 ax dx,$	$= \frac{1}{a}\tan ax,\ a \neq 0$
$\int \sec ax \tan ax dx$	$= \frac{1}{a}\sec ax,\ a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln(x + \sqrt{x^2 - a^2}),\ x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln(x + \sqrt{x^2 + a^2})$

NOTE : $\ln x = \log_e x, \quad x > 0$

4R 12 2U HSC TRIAL SOL'NS 2011

QUESTION 1

a) $8a^3 - 27 = (2a-3)(4a^2+6a+9)$ ✓ ✓

b) $\sin 120^\circ = \sin(180^\circ - \theta)$

$\therefore \theta = 60^\circ$ ✓WORING

IN Q2, POSITIVE

$\therefore \sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$ ✓

c) $|x+7| < 2$

$x + 7 < 2$ $\quad$ $x + 7 > -2$

$x < -5$ ✓ $\quad$ $x > -9$ ✓

$-9 < x < -5$

d) $\frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4\sqrt{3}+4}{3-1}$ ✓

$= \frac{4\sqrt{3}+4}{2}$

$= 2\sqrt{3}+2$

$\therefore a = 2$ ✓

$b = 2$ ✓

e) $8^x = \frac{1}{16}$

$\log_8 \frac{1}{16} = x$ ✓

$\frac{\log_e \frac{1}{16}}{\log_e 8} = x$

$x = -\frac{4}{3}$ ✓

OR

$8^x = \frac{1}{4^2}$

$8^x = \frac{1}{\sqrt[3]{8^4}}$ ✓

$8^x = 8^{-4/3}$

$x = -\frac{4}{3}$ ✓

MOST STUDENTS USED THE LOGARITHM METHOD

f) $f(x) = \log_e(x-2)$

$\log_e 0$ DOES NOT EXIST

$\log_e$ (-VE NO.) DOES NOT EXIST

$\therefore$ DOMAIN: $x > 2$ ✓

OVERALL, QUITE GOOD. NO MAJOR PROBS IN ANY PART.

Question 2

Solutions

(a)(i) Gradient of BC $= \dfrac{4-0}{3-(-1)}$

$= \dfrac{4}{4}$

$= 1$

(ii) Point $C(3,4)$, $m = 1$

$\therefore (y-4) = 1(x-3)$

$y - 4 = x - 3$

$\therefore x - y + 1 = 0$

(iii) Perpendicular gradient $= -1$ Point $A(2,1)$

$(y-1) = -1(x-2)$

$y - 1 = -x + 2$

$\therefore y = 3 - x$

(iv) BC is $x - y + 1 = 0$ — ①

l is $y = 3 - x$ — ②

Solve simultaneously ② into ①

$\therefore x - (3-x) + 1 = 0$

$x - 3 + x + 1 = 0$

$\therefore x = 1$ $\quad \therefore y = 3 - 1$

$= 2$

$\therefore D(1,2)$

(v) $AD = \sqrt{(2-1)^2 + (1-2)^2}$

$= \sqrt{1^2 + (-1)^2}$

$= \sqrt{2}$

$BC = \sqrt{(3 - -1)^2 + (4-0)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

Comments

- One mark for perpendicular gradient (many students were awarded e.c.f. from part (ii)).

Question 2 - continued...

Solutions

(vi) ΔABC

$A = \frac{1}{2} \times \text{base} \times \text{height}$

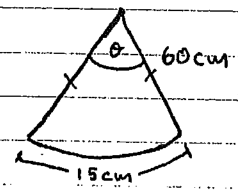
$= \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times \sqrt{32} \times \sqrt{2}$

$= \frac{1}{2} \times 8$

$= 4 \text{ units}^2$

(b)



$l = r\theta$

$15 = 60 \times \theta$

$\therefore \theta = 0.25^c$

(c) $y = 2\log_e(3-x)$

$y' = 2\left(\dfrac{-1}{3-x}\right)$

$\therefore y' = \dfrac{-2}{3-x}$ when $x = -2$,

$m = \dfrac{-2}{3-2}$

$\therefore m = -2$ $\quad \therefore M_n = 5/2$

$\therefore (y-0) = \frac{5}{2}(x+2)$

$\therefore y = \frac{5x}{2} + 5$

$\therefore 2y = 5x + 10$

Comments

- Use result from part (v) to solve.
- e.c.f awarded on many papers.

- Use result for remainder of question (e.c.f. awarded on many papers)

2UNIT TRIAL 2011

Q3 a)

✓✓ i) $y = -2(1+x^3)^4$

$y' = -8(1+x^3)^3 \times 3x^2$

$= -24x^2(1+x^3)^3$

A simple chain rule application is all that is needed here

NOTE IT IS NOT A PRODUCT OF 2 FUNCTIONS IN x just 1

(ii) $y = 4e^{\cos x}$

✓✓ $y' = 4e^{\cos x} \times -\sin x$

$= -4(\sin x)e^{\cos x}$

An exponential function so recall $\frac{d}{dx} e^{f(x)} = e^{f(x)} \times f'(x)$

(ITSELF × differentiation of the power)

(iii) ✓✓ $y = \frac{3x^2}{e^x}$ $\frac{u}{v}$ a quotient $y' = \frac{vu' - uv'}{v^2} = \frac{e^x 6x - 3x^2 e^x}{(e^x)^2}$

$= \frac{6x - 3x^2}{e^x}$

b)

(i) In Δ's AFD, EFB

✓✓✓ 1, BE = AD (given)

2, ∠EFB = ∠AFD (vert opp ∠'s are equal)

3, ∠EBF = ∠FAD (alt. ∠'s on ∥ lines are equal)

∴ ΔAFD ≡ ΔEFB (AAS test)

More practice with proofs in geometry is needed

Some students wasted time writing too many unnecessary facts.

(ii) AB = DC (opp sides of parm are equal)

AF = FB (corresp sides in congr. Δ's)

✓ ∴ FB = ½AB = ½DC

∴ FB = ½DC

c) ✓✓ $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \, dx = \left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= 0 - \frac{1}{2}$

$= -\frac{1}{2}$

LEARN THE RULES! APPLY CAREFULLY!!

Q4

✓ = 1 mark

a) $T_9 = 2^8 + 4 \times 9$

✓ $= 292$

Simply substitute!

b) $T_5 = ar^4 = 80$

✓✓✓ $T_8 = ar^7 = 640$

GP terms DIVIDE

$\frac{ar^7}{ar^4} = \frac{640}{80}$

$r^3 = 8$

$\therefore r = 2$

$ar^4 = 80$

$a(2^4) = 80$

$a = 5$

then $T_1 = 5$

$T_2 = 5(2) = 10$

$T_3 = 5 \cdot 2^2 = 20$

c) NO ROOTS ie $\Delta < 0$

✓✓✓ 1ST Rearrange eqn

$x^2 - 2kx - k + 2x + 3 = 0$

$x^2 + x(-2k+2) - k + 3 = 0$

$\Delta = b^2 - 4ac$

$= (-2k+2)^2 - 4 \times 1 \times (-k+3)$

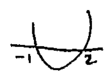
$= 4k^2 - 8k + 4 + 4k - 12$

$= 4k^2 - 4k - 8$

$\Delta < 0$

$4(k^2 - k - 2) < 0$

$(k-2)(k+1) < 0$



$-1 < k < 2$

∴ equation has no real roots for $-1 < k < 2$

e) EXPAND LHS and equate ✓✓✓ coefficients of like terms.

LHS $= ax^2 - 2ax + a + bx - b + c$

$ax^2 + x(b - 2a) + a - b + c$

$= 1x^2 + 0x + 0$

$\therefore a = 1$

$b - 2a = 0$

$b - 2 = 0$

$\therefore b = 2$

$a - b + c = 0$

$1 - 2 + c = 0$

$\therefore c = 1$

$\therefore a = 1 \; b = 2 \; c = 1$

Many need to practise solving quadratic inequalities - it is very likely that one will appear in your HSC paper.

d) $A = \frac{1}{2} r^2 \theta$ LEARN the BASIC RULES!!

✓✓ $\frac{25\pi}{6} = \frac{1}{2} r^2 \frac{\pi}{3}$

$25\pi = \pi r^2$

$\therefore r = 5$

Q5.

a) $f(x) = x^2(\frac{1}{3}x - 1)$

i) $u = x^2 \qquad v = \frac{1}{3}x - 1$

$u' = 2x \qquad v' = \frac{1}{3}$

$$f'(x) = \frac{x^2}{3} + \frac{2x^2}{3} - 2x$$

$$= \frac{3x^3}{3} - 2x$$

$$= x^2 - 2x \qquad (1)$$

Was well done.

ii) $x^2 - 2x = 0 \qquad f(0) = 0$

$x(x-2) = 0 \qquad f(2) = -\frac{4}{3} \qquad (1)$

$x = 0, 2$

$f''(x) = 2x - 2$

(1) $\therefore f''(0) = -2$ as $f''(0) < 0$, $(0,0)$ is a max (1)

$f''(2) = 2$ as $f''(2) > 0$, $(2, -\frac{4}{3})$ is a min

should make this statement!

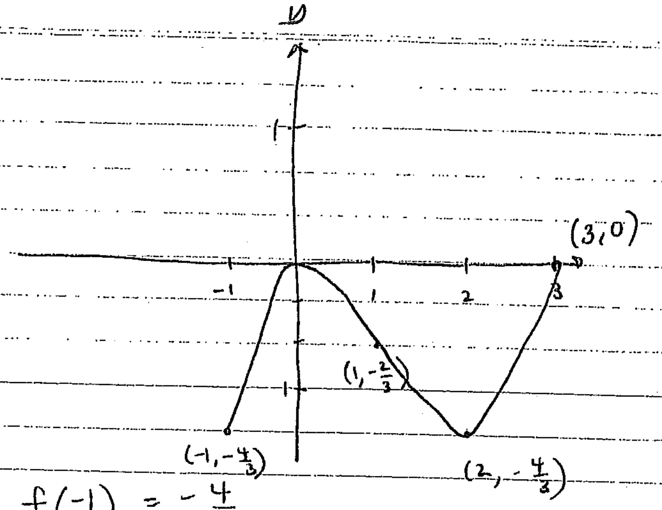
iii) when $f''(x) = 0 \qquad 2x - 2 = 0$

$x = 1 \qquad f(1) = -\frac{2}{3} \quad (1)$

Must test the concavity. Many students didn't

x	0	1	2
$f''(x)$	$-$	0	$+$

(1) pt of inf. $(1, -\frac{2}{3})$



$f(-1) = -\frac{4}{3}$

$f(3) = 0$

* End points needed to be plotted correctly

• Some labelling was not clear.

(1) End points and points labelled

(2) Shape.

b) $\int_0^{\ln 9} e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_0^{\ln 9} \qquad (1)$

$$= \frac{1}{2}\left[e^{2\ln 9} - e^{2(0)}\right]$$

$$= \frac{1}{2}[81 - 1]$$

$$= 20 \qquad (1)$$

c) $2\cos\theta + 1 = 0$

$2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

Some students gave answers

Question 6

Solution

ⓐ $f(x) = x^3 \qquad g(x) = x^2 + 4$

(i) Sub $(2,8)$ into both equations...

$8 = 2^3 \qquad\qquad 8 = 2^2 + 4$

$8 = 8 \qquad\qquad 8 = 8$

LHS = RHS $\qquad\qquad$ LHS = RHS

$\therefore$ lies on $f(x)$ $\qquad$ $\therefore$ lies of $g(x)$

$\therefore$ The curves will intersect at $(2,8)$ ✓✓

(ii) $A = \int_0^2 x^2 + 4 \, dx - \int_0^2 x^3 \, dx$ ✓

$= \int_0^2 x^2 + 4 - x^3 \, dx$

$= \left[\frac{x^3}{3} + 4x - \frac{x^4}{4} \right]_0^2$ ✓

$= \left(\frac{2^3}{3} + 4(2) - \frac{2^4}{4} \right) - \left(\frac{0^3}{3} + 4(0) - \frac{0^4}{4} \right)$

$= \left(\frac{8}{3} + 8 - 4 \right)$

$= 6\frac{2}{3}$ units^2 ✓

ⓑ (i)

x	0	0.5	1	1.5	2
y	0	1.7	6.4	19.1	53.6

✓✓

(ii) $A = \frac{0.5}{3} \left[0 + 4(1.7) + 2(6.4) + 4(19.1) + 53.6 \right]$ ✓

$= 24.93\ldots$

$= 24.9$ ✓

Comments

- Many tried to solve simultaneously which proved too difficult.
- Sub $(2,8)$ into both equations to prove.
- Many students were confused by the ~~bounds~~ bounds to find area in.

Question 6 - continued...

Solution

ⓒ $\int_2^{\sqrt{k}} \frac{x}{x^2 - 3} \, dx = 1 \qquad \left(\text{if } y = x^2 - 3, \; y' = 2x \right.$

$\therefore \frac{1}{2} \int_2^{\sqrt{k}} \frac{2x}{x^2 - 3} \, dx = 1$ ✓

$\therefore \frac{1}{2} \left[\ln(x^2 - 3) \right]_2^{\sqrt{k}} = 1$

$\left(\ln(k-3) - \ln(1) \right) = 2$ ✓

$\therefore \ln(k-3) = 2$

$e^2 = k - 3$

$\therefore k = e^2 + 3$ ✓

Comments

- Plenty of students still need to revise their integration laws.

Question 7

Solutions

(a) $V = 8t + \sin 2t$

(i) $a = \frac{dv}{dt} = 8 + 2\cos 2t$

$\therefore$ at $t=0$, $a = 8 + 2\cos 2(0)$
$= 8 + 2(1)$
$= 10\ m.s^{-2}$ ✓

(ii) $\therefore x = \frac{8t^2}{2} + \left(\frac{-\cos 2t}{2}\right) + c$ ✓

$x = 4t^2 - \frac{\cos 2t}{2} + c$

↳ no info given to find c.

$\therefore t = \pi/8$, $x = 4\left(\frac{\pi}{8}\right)^2 - \frac{\cos 2\left(\frac{\pi}{8}\right)}{2} + c$

$\therefore x = \frac{\pi^2}{16} - \frac{1/\sqrt{2}}{2} + c$

$\therefore x = \frac{\pi^2}{16} - \frac{1}{2\sqrt{2}} + c$ ✓

(b) $P = P_0 e^{kt}$

(i) $\therefore 460000 = 350000 e^{4k}$ ✓

$\frac{46}{35} = e^{4k}$

$\therefore \log_e\left(\frac{46}{35}\right) = 4k$

$\therefore k = \frac{\ln\left(\frac{46}{35}\right)}{4}$ ✓

(ii) $P = 350000 e^{14k}$ (since $k = \frac{\ln\left(\frac{46}{35}\right)}{4}$)

$\therefore P = 910924$ persons. ✓

Comments

- No information was given to find the constant c. ∴ Marks were given for leaving "+c" or assuming $t=0, x=0$ to find c.

- May students for (b) lost marks for getting time wrong

Question 7 - continued.

Solutions

(iii) $P = 350000 e^{kt}$

$\therefore \frac{dP}{dt} = k \times 350000 e^{kt}$ ✓

when $t=20$, $\frac{dP}{dt} = k \times 350000 e^{20k}$

$\therefore \frac{dP}{dt} = 93775$ people per year ✓

(to the nearest person)

(c) $a = 6$

$\therefore v = 6t + c$

@ $t=0$, $v = \ln 7$

$\therefore \ln 7 = 6\times 0 + c$

$\therefore c = \ln 7$

$\therefore v = 6t + \ln 7$ ✓

when $t=5$, $v = 6(5) + \ln 7$

$\therefore v = (30 + \ln 7)$ m/s ✓

(d) $\frac{\sin(\pi-\theta)\cos(\pi-\theta)}{\sin(\pi/2-\theta)\cos(\pi/2-\theta)}$

$= \frac{\cancel{\sin\theta}\cdot -\cos\theta}{\cancel{\cos\theta}\cdot \sin\theta}$ ✓

$= -1$ ✓

Comments

- Many students differentiated with respect to k, not t as required.

QUESTION 8

a) ANGLE SUM $= (5-2) \times 180°$
$= 540°$

$S_n = 540°$ $\qquad S_n = \frac{n}{2}(a+\ell)$
$n = 5$
$a = a$ $\qquad 540° = \frac{5}{2}(a+3a)$
$\ell = 3a$ $\qquad 1080° = 5(4a)$

$1080° = 20a$
$a = 54°$ ✓

$\underset{T_5}{a+4d} = 3a$ (LAST ∠ = 3 × FIRST ∠)

$4d = 2a$
$d = \frac{a}{2}$ $\qquad \therefore d = 54° \div 2$
$= 27°$ ✓

∴ ANGLES ARE: 54°, 81°, 108°, 135°, 162° ✓

COULD HAVE BEEN ANSWERED BETTER. MANY STUDENTS DID NOT RECOGNISE HOW TO USE SUM OF ARITHMETIC SERIES TO SOLVE THE PROB.

b)
(i) $PA = \sqrt{(x-1)^2 + (y-1)^2}$ ✓

$PB = \sqrt{(x-4)^2 + (y-7)^2}$

(ii) $PA = 2\,PB$

YOU HAVE TO SQUARE THE 2

$(x-1)^2 + (y-1)^2 = 4\left[(x-4)^2 + (y-7)^2\right]$ ✓

$x^2 - 2x + 1 + y^2 - 2y + 1 = 4\left[x^2 - 8x + 16, + y^2 - 14y + 49\right]$

$x^2 - 2x + 1 + y^2 - 2y + 1 = 4x^2 - 32x + 64 + 4y^2 - 56y + 196$

$0 = 3x^2 - 30x + 63 + 3y^2 - 54y + 195$
$0 = 3x^2 - 30x + 3y^2 + -54y + 258$ ✓

WHILE THE SOLUTION PROVIDED COULD ONLY BE ACHIEVED BY (INCORRECTLY) NOT SQUARING THE 2, MARKS WERE ALLOCATED FOR...

- ANSWERING THE QUESTION CORRECTLY
- WORKING THROUGH, CORRECTLY EXPANDING & GROUPING ALGEBRAIC TERMS.

WE WERE MORE CONCERNED WITH STUDENTS' METHODS THAN THE ANSWER ITSELF.

(iii) USING THE GIVEN SOLUTION, COMPLETE THE SQUARE:

✓ WORKING.

$$x^2 - 14x + 49 + y^2 - 26y + 169 = -128 + 49 + 169$$

$$(x-7)^2 + (y-13)^2 = 90 \checkmark$$

CENTRE $= (7, 13)$ ✓
RADIUS $= \sqrt{90}$ ✓

MOSTLY ANSWERED QUITE WELL.
ANY ERRORS FROM (ii) WERE CARRIED FWD.

c) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \Rightarrow a = \frac{1}{3}, \; r = \frac{1}{3}$ ✓ (CHECK BY FINDING T_1, T_2, etc..)

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1/3}{2/3} = \frac{1}{2} \checkmark$$

QUITE A FEW STUDENTS FORGOT THE LIMITING SUM FORMULA

Question 1

Solutions		Comments
(a) Period $= \pi/3$	✓	
(b) $\sec\theta - \cos\theta = \tan\theta\sin\theta$		
LHS $= \sec\theta - \cos\theta$		• Multiple ways this could be proved.
$= \frac{1}{\cos\theta} - \cos\theta$		
$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$	✓	
$= \frac{1-\cos^2\theta}{\cos\theta}$		
$= \frac{\sin^2\theta}{\cos\theta}$	✓	
$= \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$	✓	
$= \tan\theta\sin\theta = $ RHS.		
$\therefore \sec\theta - \cos\theta = \tan\theta\sin\theta$		
(c) (i) $A_1 = 350000 - m$ $(r = 0.005)$	✓	• Had to build A_4 using A_1 to gain full marks.
$A_2 = 350000 - 2m$		
$A_3 = 350000 - 3m$		
$A_4 = (350000 - 3m)(1.005) - m$	✓	
(ii) $A_5 = A_4(1.005) - m$		• Had to <u>show</u>! not just give A_5.
$\therefore A_5 = (350000-3m)(1.005)^2 - m(1.005) - m$	✓	
$\therefore A_5 = (350000-3m)(1.005)^2 - m(1 + 1.005)$	✓	
(iii) 20 years = 240 months.		
$A_5 = (350000-3m)(1.005)^2 - m(1+1.005)$		• One mark for A_{240} (not A_{20})
$\therefore A_{240} = (350000-3m)(1.005)^{237} - m(1 + 1.005 + \dots + 1.005^{236})$	✓	
$\therefore A_{240} = (350000-3m)(1.005)^{237} - m\left[\frac{1(1.005^{237}-1)}{1.005-1}\right]$	✓	• One mark for correct sum of GP.

QUESTION 7 - CONTINUED

Solutions		Comments
$A_{240} = (350000 - 3m)(1.005)^{237} - m\left[\frac{1.005^{237} - 1}{0.005}\right]$		
$0 = (350000)(1.005)^{237} - (3m)(1.005)^{237} - m\left[\frac{1.005^{237} - 1}{0.005}\right]$		
$3m(1.005)^{237} + m\left[\frac{1.005^{237} - 1}{0.005}\right] = 350000(1.005)^{237}$		
$m\left[3(1.005)^{237} + \left(\frac{1.005^{237} - 1}{0.005}\right)\right] = 350000(1.005)^{237}$	✓	• One mark for factorising "m".
$\therefore m = \frac{350000(1.005)^{237}}{\left[3(1.005)^{237} + \left(\frac{1.005^{237}-1}{0.005}\right)\right]}$		
$\therefore m = \$2470.53$	✓	• One mark for final answer.
		↳ e.c.f (error carried forward) was applied to numerous papers.

QUESTION 10

a)

(i) If $y = x\ln x - x$

$$\frac{dy}{dx} = x.\frac{1}{x} + \ln x - 1 \qquad u = x \quad v = \ln x \qquad u' = 1 \quad v' = 1/x$$

$$= 1 + \ln x - 1$$

$$= \ln x$$ ✓ WORKING

MANY DID NOT RECOGNISE THE PRODUCT RULE!

(ii) $$\int_1^{e^3} \ln x \, dx = \left[x\ln x - x\right]_1^{e^3}$$

FROM (i)!

$$= (e^3 \ln e^3 - e^3) - (1\ln 1 - 1)$$ ✓

*REM: $\ln e^3 = 3$

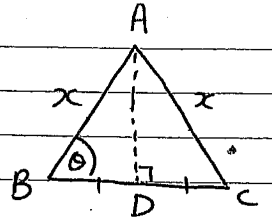
$$= (3e^3 - e^3) - (-1)$$

$$= 2e^3 + 1$$ ✓

MANY DID NOT RECOGNISE THAT YOU HAD TO USE THE RESULT FROM (i)

MANY LEFT THE ANSWER HALF FINISHED, ie: $e^3\ln e^3 - e^3 + 1$
THIS IS INCOMPLETE & WOULD NOT HAVE RECEIVED FULL MARKS.

b)



(i) $x + x + BC = 1$

$\therefore BC = 1 - 2x$

$\therefore BD$ or $CD = \dfrac{1-2x}{2}$ ✓

$AD^2 = x^2 - BD^2$ (PYTHAG!)

$AD = \sqrt{x^2 - \left(\dfrac{1-2x}{2}\right)^2}$

$= \sqrt{\dfrac{4x^2}{4} - \dfrac{(1-2x)^2}{4}}$ ✓WORKING.

$= \sqrt{\dfrac{4x - (1-4x+4x)^2}{4}}$

$\therefore AD = \sqrt{\dfrac{4x-1}{2}}$

TOO MANY STUDENTS GOT STUCK WITH THEIR WORKING & SIMPLY WROTE THE EXPECTED SOLUTION.

MANY DID NOT RECOGNISE THE NEED FOR PYTHAGORAS' THEOREM.

(ii) $\sin\theta = {}^{o}/_{a}$

$= \dfrac{\sqrt{\dfrac{4x-1}{2}}}{x}$

$= \dfrac{\sqrt{4x-1}}{2x}$

$= \dfrac{\sqrt{4} \times \sqrt{x - 1/4}}{2x}$ ✓WORKING.

$= \dfrac{2\sqrt{x-1/4}}{2x}$

$\sin\theta = \dfrac{\sqrt{x - 1/4}}{x}$

ANSWERED QUITE WELL. NO MAJOR PROBS.

c)

(i) AFTER t HRS ...

$\left.\begin{array}{l}\text{MARY'S POS'N: } 25 - 4t \\ \text{JOHN'S POS'N: } 6t\end{array}\right\}$ ✓ USE $D = S \times T$

TOTAL DIST. APART ⇒ USE PYTHAG.

$d^2 = (25-4t)^2 + (6t)^2$ ✓WORKING.

$= 625 - 200t + 16t^2 + 36t^2$

$= 52t^2 - 200t + 625$

STUDENTS HAD TROUBLE DETERMINING THE EXPRESSION FOR MARY'S POS'N & USING PYTHAG.

(ii) $L = 52t^2 - 200t + 625$

$$\frac{dL}{dt} = 104t - 200 \quad \checkmark$$

MAX/MIN EXISTS AT $\frac{dL}{dt} = 0$

$$0 = 104t - 200$$

$$t = 1\tfrac{12}{13} \text{ OR } \frac{25}{13} \quad \checkmark$$ (DON'T ROUND OFF HERE!)

CHECK IF MAX/MIN!

$$\frac{d^2L}{dt^2} = 104, \quad \frac{d^2L}{dt^2} > 0$$ ∴ CONCAVE UP ∴ MIN ✓

MANY STUDENTS FAILED TO DETERMINE IF THE STATIONARY PT. WAS A MAX OR MIN.

OTHERS DID NOT SHOW THAT THEY UNDERSTOOD THE RESULT OF A POSITIVE SECOND DERIVATIVE... BE THOROUGH IN YOUR ANSWERS!

(iii) $t = \frac{25}{13} \Rightarrow L = 432.694\ldots$

REM: $d^2 = L$

$$\therefore d = 20.8\ldots \text{ km}$$
$$= 21 \text{ km} \quad \checkmark$$

"NEAREST KM!"