SOLUTIONS

CRANBROOK SCHOOL

2013

YEAR 12

TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16

Total marks - 82

Section I

7 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

75 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

7 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is $\dfrac{1+\sqrt{5}}{7-2\sqrt{5}}$ as a fraction with a rational denominator?

(A) $\dfrac{12+9\sqrt{5}}{29}$

(B) $\dfrac{17+9\sqrt{5}}{29}$

(C) $\dfrac{12+9\sqrt{5}}{69}$

(D) $\dfrac{17+9\sqrt{5}}{69}$

4 How many solutions are there to the equation $\cos x = x$ for $0 \le x \le 2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3

5 What values of x is the curve $f(x)=x^3+x^2$ concave down?

(A) $x<-\dfrac{1}{3}$

(B) $x>-\dfrac{1}{3}$

(C) $x<-3$

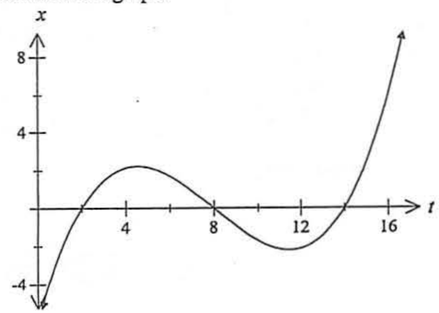
(D) $x>3$

7 What is the value of $\sum_{r=1}^{50}(2r-4)$?

(A) 96
(B) 2350
(C) 2450
(D) 4700

8 What values of k does the quadratic equation $x^2-4kx+4k=0$ have no real roots?
(A) $0<k<4$
(B) $0>k>4$
(C) $-1<k<0$
(D) $0<k<1$

9 The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph.



When was the particle at rest?
(A) $t=4.5$ and $t=11.5$
(B) $t=0$
(C) $t=2$, $t=8$ and $t=14$
(D) $t=1.5$ and $t=8$

10 What is solution to the equation $\cos\theta=-\frac{\sqrt{3}}{2}$ for $0^\circ\le\theta\le360^\circ$?

(A) $\theta=30^\circ$ or 330°
(B) $\theta=60^\circ$ or 300°
(C) $\theta=150^\circ$ or 210°
(D) $\theta=120^\circ$ or 240°

Section II

90 marks
Attempt Questions 11 - 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet.

All necessary working should be shown in every question.

Question 11 (13 marks) **START A NEW BOOKLET**

a) Write 5,140,000 in scientific notation 1

b) Differentiate $y=2x^3(3-x^2)$ with respect to x 2

c) Solve $|4x-1|<3$ 2

d) Factorise $3x^2-5x-2$ 2

e) Find integers a and b such that $(5-\sqrt{3})^2=a+b\sqrt{3}$? 2

f) Find $\int \frac{x}{x^2+1}\,dx$ 2

h) Simplify $\log_{10}20A-\log_{10}2A$ 2

Question 12 (15 marks) **START A NEW BOOKLET**

a) Show that $\dfrac{cosec\theta\, sec\theta}{\tan\theta}=cosec^2\theta$ **2**

b) The quadratic equation $3x^2-4x+1=0$ has roots α and β.
Find:

i) $\alpha\beta+(\alpha+\beta)$ **2**

ii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ **2**

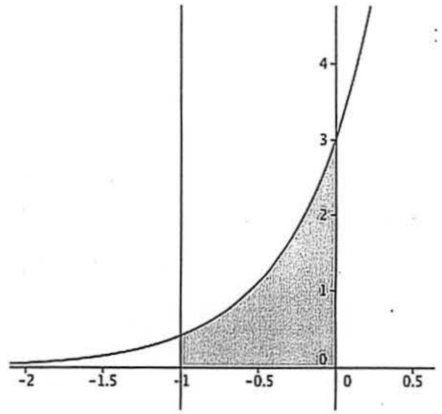
c) Differentiate with respect to x, and factorise where possible:

i) $\dfrac{2}{\sqrt{3-x^2}}$ **2**

ii) $\dfrac{\ln x}{e^x}$ **2**

d) A geometric series has a 3rd term of 16, and a 6th term of 128.
Find the first term and the common ratio. **2**

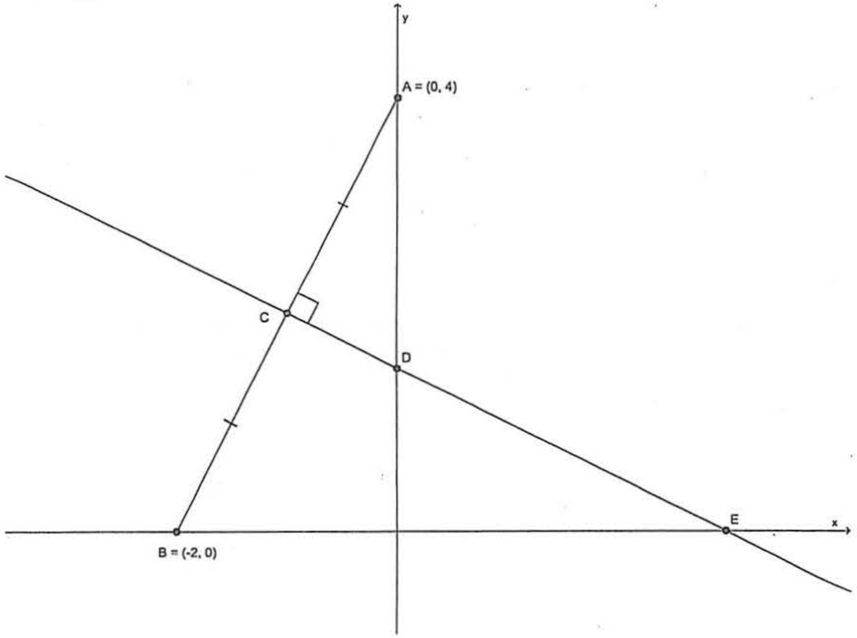
e) The diagram below shows the region bounded by the x-*axis*, the y-*axis*, $y=3e^{2x}$ $x=-1$.



Find the volume of the solid formed, correct to 2 decimal places, if the shaded region is rotated about the x-*axis*. **3**

Question 13 (15 marks) **START A NEW BOOKLET**

a) The diagram shows the points $A\ (0,4)$ and $B\ (-2,0)$. Point C is the midpoint of AB. Line CE is drawn perpendicular to AB and crosses the y −axis at D and the x −axis at E.



i) Find the coordinates of C. **1**

ii) Show that CE has equation $x + 2y - 3 = 0$. **3**

iii) Find the coordinates of E. **1**

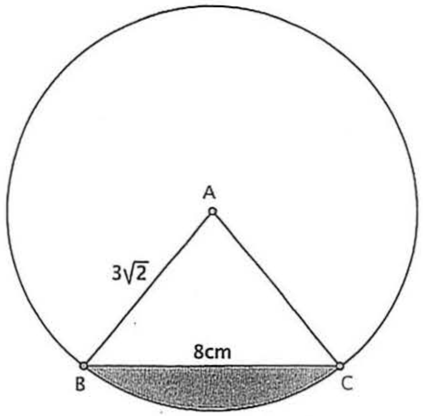
iv) Show that the distance AE is 5 units. **1**

v) Find the equation of the circle centred at E with radius AE. **1**

vi) Show that point B lies on the circle found in part v). **1**

b) Use Simpsons rule with five function values to approximate $\int_{-3}^{1} 3^x \, dx$ **3**

c) The following diagram shows a circle with a centre A, radius $3\sqrt{2}\ cm$, and a chord of length $8cm$.

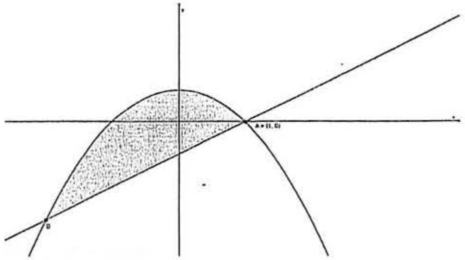


i) Use the cosine rule to find $\angle BAC$ in radians, correct to 2 decimal places. **3**

ii) Hence find, correct to the nearest cm^2, the area of the minor segment cut off by the chord. **1**

Question 14 (8 marks) **START A NEW BOOKLET**

b) The curve $y = 1 - x^2$ and the straight line $y = x - 1$ are shown in the diagram below. They intersect at Point A and Point B. Point A has the coordinates (1, 0).



i) Show that B has the coordinates (-2, -3) **1**

ii) Find the area of the shaded region. **2**

c) After an initial advertising campaign for a new product was ceased, the sales of that product declined at a rate proportional to the current monthly sales S at any time, i.e. $\frac{dS}{dt} = -kS$, where time is in months.

i) Prove that $S = S_0 e^{-kt}$ satisfies the equation $\frac{dS}{dt} = -kS$, where k, S_0 are constants. **1**

ii) If sales drop by 20% in four months, find the exact value of k. **2**

iii) If this trend continued, in what time would monthly sales be half the original sales? **2**

Question 15 (15 marks) **START A NEW BOOKLET**

a) Boxes are stacked in layers, where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether.

i) Write down the number of boxes in the bottom layer. **2**

ii) Show that there are $\frac{1}{2}n(n+11)$ boxes altogether **1**

b) A parabola below has equation $x^2 = 12y$. One point on the parabola is $A(6, 3)$.

i) Find the equation of the focal chord through A **2**

ii) Find the coordinates of the point B, the other endpoint of the focal chord through A. **1**

iii) Find the equation of the tangent to the parabola at A. **3**

c) Wendy has set up her superannuation fund, and has accumulated \$134,000. However, due to an accident she is no longer able to work and make any further contribution to the fund. Wendy is leaving the money in the superannuation fund to accumulate interest at 8% p.a. compounded annually. However she needs to withdraw \$24,000 at the end of each year for normal living expenses.

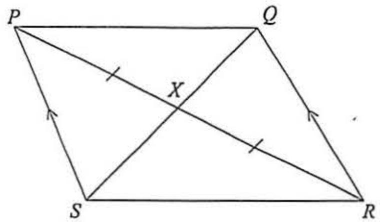
i) Show that at the end of the first year she has \$120,720 left in the superannuation fund **1**

ii) Show that an expression for the amount she has in the fund after n years is given by $A_n = 300000 - 166000(1.08)^n$ **3**

iii) Hence find how many years the fund will last before there is no money in it. **2**

Question 16 (9 marks) **START A NEW BOOKLET**

a)



In the diagram above, $PX = XR$ and PS is parallel to QR

i) Prove that $\Delta PXS \equiv \Delta RXQ$ **3**

ii) Hence show that $PQ = RS$ **1**

b) A piece of wire of length 50 cm is to be cut into two sections, one of which is used to form a square, the other of which is used to form a circle. The length of the edge of the square is x cm.

i) Show that the radius of the circle is given by $r = \dfrac{25-2x}{\pi}$ **1**

ii) Find the area of the circle **1**

iii) Find the total area of the square and the circle **1**

iv) Find the value for x for which the total area of the square and the circle is a minimum. **2**

End of paper

STANDARD INTEGRALS

$\int x^n\,dx$	$= \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n<0$
$\int \frac{1}{x}\,dx$	$= \ln x,\ x>0$
$\int e^{ax}\,dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax\,dx$	$= \frac{1}{a}\sin ax,\ a \neq 0$
$\int \sin ax\,dx$	$= -\frac{1}{a}\cos ax,\ a \neq 0$
$\int \sec^2 ax\,dx$	$= \frac{1}{a}\tan ax,\ a \neq 0$
$\int \sec ax \tan ax\,dx$	$= \frac{1}{a}\sec ax,\ a \neq 0$
$\int \frac{1}{a^2+x^2}\,dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}}\,dx$	$= \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$
$\int \frac{1}{\sqrt{x^2-a^2}}\,dx$	$= \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$
$\int \frac{1}{\sqrt{x^2+a^2}}\,dx$	$= \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x,\ x>0$

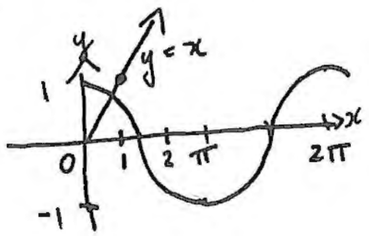
2UNIT TRIAL CRANBROOK 2013

MULTIPLE CHOICE

1/ $\dfrac{(1+\sqrt{5})}{(7-2\sqrt{5})} \times \dfrac{(7+2\sqrt{5})}{(7+2\sqrt{5})}$

$= \dfrac{7+9\sqrt{5}+10}{49-20} = \dfrac{17+9\sqrt{5}}{29}$

(B)

4/ 

Sketch $y = \cos x$ and $y = x$

Remember $\pi \doteqdot 3.1$

$\therefore$ use this scale to draw $y = x$ THEN 1 PT OF INT.

(B)

5/ $f'(x) = 3x^2 + 2x$

$f''(x) = 6x + 2$

c c down $\therefore f''(x) < 0$

$6x + 2 < 0 \quad x < -\frac{1}{3}$

(A)

7/ $\displaystyle\sum_{r=1}^{50} (2r-4) = -2 + 0 + 2 + \ldots$

$a = T_1 = -2 \quad l = T_{50} = 96$

$S_{50} = \frac{n}{2}(a+l)$

$= 25(-2+96)$

$= 2350$

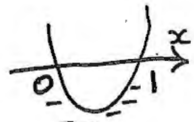
(B)

8/ No real roots $\Delta < 0$

$(-4k)^2 - 16k < 0$

$16k^2 - 16k < 0$

$16k(k-1) < 0$

DRAW IT 

Remember curve is less than zero. where y-Values are less than zero ie when $0 < k < 1$ (D)

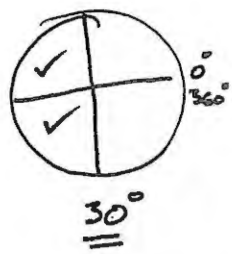
9/ AT REST $\rightarrow V = 0$

stationary points on the displacement curve will be zeros of velocity curve ie $t = 4.5,\ 11.5$

(A)

10/ $\cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = 180° - 30°,$
$180° + 30°$
$= 150°, 210°$



(C)

11.A) 5,140,000

5.14×10^6 ✓

↑

Number between 1 and 10

Solutions

12 2u

B) $y = 2x^3(3-x^2)$

$u = 2x^3 \quad v = 3-x^2$

$u' = 6x^2 \quad v' = -2x$ ✓

$\frac{dy}{dx} = -4x^4 + 6x^2(3-x^2)$

$= 18x^2 - 10x^4$ ✓ (2)

c) $|4x-1| < 3$

Case 1: Positive

$4x-1 < 3$

$4x < 4$

$x < 1$ ✓

Case 2: Negative

$-(4x-1) < 3$

$4x-1 > -3$

$4x > -2$

$x > -\frac{1}{2}$

$-\frac{1}{2} < x < 1$ ✓ (2)

D) $3x^2 - 5x - 2$

$= (3x+1)(x-2)$ (2)

E) $(5-\sqrt{3})^2 = (5-\sqrt{3})(5-\sqrt{3})$

$= 25 - 10\sqrt{3} + 3$

$= 28 - 10\sqrt{3}$ ✓

$a = 28$

$b = -10$ ✓ (2)

f) $\int \frac{x}{x^2+1}\, dx$

$= \frac{1}{2} \ln(x^2+1) + C$ ✓ ✓ (2)

H) $\log_{10} 20A - \log_{10} 2A$

$= \log_{10}\left(\frac{20A}{2A}\right)$ ✓

Cancel A's !!

$= \log_{10} 10$

$= 1$ ✓ (2)

Q12

a) $LHS = \dfrac{\operatorname{cosec}\theta \sec\theta}{\tan\theta}$

$= \dfrac{\operatorname{cosec}\theta \times \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ ✓

$= \operatorname{cosec}\theta \times \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta}$

$= \operatorname{cosec}\theta \;\operatorname{cosec}\theta$

$= \operatorname{cosec}^2\theta = RHS$ ✓

always check RHS ...
Notice on RHS we have $\operatorname{cosec}^2\theta$
So leave the $\operatorname{cosec}\theta$

Reduce all other terms to $\sin\theta, \cos\theta$

Fraction theory needs refining for a number of boys.

b) $\alpha+\beta = -\frac{b}{a} = --\frac{4}{3} = \frac{4}{3}$ $\quad \alpha\beta = \frac{c}{a} = \frac{1}{3}$

i) $\alpha\beta + \alpha+\beta = \frac{4}{3}+\frac{1}{3} = \frac{5}{3}$ ✓✓

ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \dfrac{\frac{4}{3}}{\frac{1}{3}}$

$= \frac{4}{\cancel{3}} \times \frac{\cancel{3}}{1}$

$= 4$ ✓✓

Again basic fraction division and addition - common denominators need to be understood.

c) i) $\dfrac{2}{\sqrt{3-x^2}} = 2(3-x^2)^{-\frac{1}{2}}$ ← CHAIN RULE QUESTION!!

$f'(x) = -\frac{1}{\cancel{2}} \times \cancel{2}(3-x^2)^{-\frac{3}{2}} \times -2x$

$= \dfrac{2x}{(3-x^2)^{3/2}}$ ✓✓

NOT A QUOTIENT rule question
While it will still work IF DONE CAREFULLY it takes more time & most who did this were NOT careful ☹

ii) $\dfrac{d}{dx}\left(\dfrac{\ln x}{e^x}\right) = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{e^x \times \frac{1}{x} - (\ln x)e^x}{e^{2x}}$

$= \dfrac{\cancel{e^x}\left(\frac{1}{x} - \ln x\right)}{e^{2x}} = \dfrac{1 - x\ln x}{xe^x}$ ✓✓

(ii) is a quotient rule Q.
NOW SHOULD NOT leave as double decker fraction!!

d) $T_3 = ar^2 = 16$ ①
$T_6 = ar^5 = 128$ ②

$\dfrac{ar^5}{ar^2} = \dfrac{128}{16}$
$r^3 = 8$
$\therefore r = 2$ ✓

subst into ①
$ar^2 = 16$
$a(4) = 16$
$a = 4$ ✓

e) $V = \pi\int_{-1}^{0} (3e^{2x})^2$ $\qquad 3^2 = 9$!!!!

$= \pi\int_{-1}^{0} 9e^{4x}\,dx$

$= 9\pi\left[\frac{e^{4x}}{4}\right]_{-1}^{0}$

$= \frac{9\pi}{4}\left[e^0 - e^{-4}\right]$

$\therefore$ Volume $\doteqdot 6.94\ u^3$

RTQ if it says 2DP. DO IT! ☺

Q13 i) Midpoint $C = \left(\frac{0+-2}{2}, \frac{4+0}{2}\right)$
$= (-1, 2)$ ✓

(ii) $m_{AB} = \frac{4}{2} = 2$
$\therefore m_{CE} = -\frac{1}{2}$

$y-2 = -\frac{1}{2}(x+1)$
$2y-4 = -x-1$
$x+2y-3=0$ ✓✓✓

If asked to show $x+2y-3=0$ SHOW THIS IN THIS FORMAT
Some were sloppy ☹

(iii) Let $y=0$ in $x+2y-3=0$
$x=3$ $\quad\therefore E$ is $(3,0)$ ✓

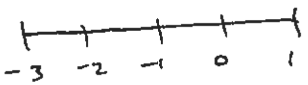
iv) $AE = \sqrt{9+16} = 5$ ✓

v) $(x-a)^2+(y-b)^2 = r^2$
$(x-3)^2 + y^2 = 25$ ✓

Some students need to learn this!! (a,b) is the CENTRE, r = radius

vi) JUST SUBSTITUTE! ✓

$\left.\begin{array}{l} x=-2 \\ y=0 \end{array}\right\}$ $(-2-3)^2+0 = 25$
$LHS = 25 = RHS$
$\therefore B$ lies on circle!

b)  Clearly $h = 1$

OR $\frac{b-a}{h} = \frac{1--3}{4} = 1$

$\therefore \int_{-3}^{1} 3^x \, dx$

$= \frac{h}{3}\left(y_0 + y_n + 4(y_1 + y_3) + 2(y_2)\right)$ use $\underline{\underline{y}}$ values

$= \frac{1}{3}\left(\frac{1}{27} + 3 + 4\left(\frac{1}{9}+1\right) + 2(3^{-1})\right)$

$= 2.27$ (2dp) ✓✓✓

Learn the procedure & TAKE CARE!

c(i) Learn the rule $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos A = \frac{(3\sqrt{2})^2 + (3\sqrt{2})^2 - 8^2}{2 \times 3\sqrt{2} \times 3\sqrt{2}}$

$= -\frac{28}{36}$

$\therefore A = 2.46$ (2dp) RTQ

ii) $A_{segment} = \frac{1}{2} r^2 (\theta - \sin\theta)$.

$= 16\text{ cm}^2$ (using 2.46)

$= 17\text{ cm}^2$ using exact value

both were given marks.

QUESTION 14 - 8 Marks

(b) $y = 1 - x^2 \qquad y = x - 1$

(i) $1 - x^2 = x - 1$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\therefore x = -2, 1$

when $x = -2$, $y = (-2) - 1$
$= -3$

$\therefore B(-2, -3)$

(ii) $A = \int_{-2}^{1} (1 - x^2) - (x - 1)\, dx$

$= \int_{-2}^{1} 2 - x - x^2\, dx$

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^{1}$

$= \left(2(1) - \frac{(1)^2}{2} - \frac{(1)^3}{3} \right) - \left(2(-2) - \frac{(-2)^2}{2} - \frac{(-2)^3}{3} \right) = 4\frac{1}{2} \text{ units}^2$

(c)

(i) $S = S_0 e^{-kt}$

$\frac{dS}{dt} = -k \cdot S_0 e^{-kt}$

$\therefore \frac{dS}{dt} = -k \cdot S$

as required.

(ii) $t = 0$, $S = 100\% = S_0 = 1$

$t = 4$, $S = 80\% = 0.8$

$\therefore 0.8 = 1 \cdot e^{-k \cdot 4}$

$0.8 = e^{-4k}$

$\log_e 0.8 = -4k$

$\therefore k = \frac{\ln 0.8}{-4}$

(iii) $S = 0.5$

$\therefore 0.5 = 1e^{-kt}$

$\therefore \log_e 0.5 = -kt$

$\therefore t = \frac{\ln 0.5}{-k}$

$\therefore t = \frac{\ln 0.5}{-\left(\frac{\ln 0.8}{-4}\right)}$

$\therefore t = 12.43$ months (2.d.p.)

QUESTION 15

a. NOTE THAT THIS IS AN AP: EACH LAYER INCREASING BY 1.

i. $T_n = ?$ $\quad T_n = a + (n-1)d$ ✓

$n = n$ $\quad = 6 + (n-1) \times 1.$

$a = 6$ $\quad = 5 + n$ ✓

$d = 1$

- MANY DID NOT COMPLETELY SIMPLIFY... ATTENTION TO DETAIL, PLEASE!

ii. $S_n = ?$ $\quad S_n = n/2 \left[2a + (n-1).d\right]$

$n = n$

$a = 6$ $\quad = n/2 \left(12 + n - 1\right)$

$d = 1$ $\quad S_n = n/2 \left(n + 11\right)$ ✓

- NO PROBLEMS.

b.

i. NOTE: $x^2 = 4ay \rightarrow x^2 = 12y$

$\therefore a = 3$

$\therefore$ FOCUS $= (0, 3)$ ✓

USING $(6,3)$ & $(0,3)$:

$$\frac{y-3}{x-0} = \frac{3-3}{6-0}$$ ...2 PT. FORMULA.

$$\frac{y-3}{x} = 0$$

$$y = 3$$ ✓

OR NOTE THAT A LINE THRU $(0,3)$ & $(6,3)$ MUST BE $y = 3$ ✓✓ ☺

- THERE WAS MINIMAL EXPLANATION/WORKING FOR THIS QUESTION.
- SHOW THE MARKER HOW YOU GOT FROM $a = 3 \Rightarrow y = 3$. FOR ALL THEY CAN TELL, YOU HAVE ASSUMED FOCUS = EQ'N OF A LINE...
- IF YOU USE DIAGRAMS IN YOUR REASONING, KEEP THEM NEAT & LARGE.

ii. AS PARABOLA IS SYMMETRICAL ABOUT y-AXIS, $\therefore$ B WOULD BE $(-6, 3)$ ✓

- NO PROBS!

iii. $x^2 = 12y$

$y = \frac{x^2}{12}$ RE-ARRANGE

$y' = \frac{2x}{12} = \frac{x}{6}$ ✓ DIFFERENTIATE

AT A $(6,3)$ ... $y' = 6/6 = 1$ ✓

$y - 3 = 1(x - 6)$ ... PT/GRAD. FORMULA.

$y = x - 3$

OR

$x - y - 3 = 0$ ✓

- ANSWERED WELL.
- A MINOR ANNOYANCE WAS NOT RECOGNISING CORRECT GENERAL FORM.

c.

i. $A_1 = \$134\,000 \times \text{INTEREST} - \text{WITHDRAWAL}$
$= \$134\,000 \times 1.08 - 24\,000$
$= \$120\,720$ ✓

ii. $A_1 = A_0 \times 1.08 - 24\,000$

$A_2 = A_1 \times 1.08 - 24\,000$
$= (A_0 \times 1.08 - 24\,000) \times 1.08 - 24\,000$
$= A_0 \times 1.08^2 - 24\,000 \cdot 1.08 - 24\,000$
$= A_0 \times 1.08^2 - 24\,000\,(1 + 1.08)$

$A_3 = A_2 \times 1.08 - 24\,000$
$= \left(A_0 \times 1.08^2 - 24\,000\,(1 + 1.08)\right) \times 1.08 - 24000$
$= A_0 \times 1.08^3 - 24\,000\,(1 + 1.08 + 1.08^2)$

$\therefore A_n = A_0 \times 1.08^n - 24000\,\underbrace{(1 + 1.08 + \dots 1.08^{n-1})}$

$\$134\,000$

G.P.
$n = n$
$a = 1$
$r = 1.08$

$S_n = \frac{1\,(1.08^n - 1)}{0.08}$

$A_n = 134\,000 \times 1.08^n - \frac{24\,000\,(1.08^n - 1)}{0.08}$ ✓

EXPAND

$= 134\,000 \times 1.08^n - 300\,000\,(1.08^n - 1)$
$= 134\,000 \times 1.08^n - 300\,000 \times 1.08^n + 300\,000$
$A_n = 300\,000 - 166\,000 \times 1.08^n$ ✓

- GENERALLY ANSWERED WELL.
- SOME STUDENTS JUMPED FROM $A_1 \rightarrow A_n$. YOU NEED TO "BUILD" THE EQ'N MORE.

PTO ↗

- QUITE A FEW STUDENTS FOUND VALUES FOR $n = 1, 2, 3$ FOR BOTH THE SERIES-BASED EXPRESSION AND THE EXPRESSION PROVIDED. AS THEY FOUND BOTH TO BE EQUIVALENT, THEY TOOK THAT AS A PROOF.
- THIS IS NOT MATHEMATICALLY SOUND AS THERE WAS ONLY A LIMITED SET OF VALUES TESTED

iii. NO \$ LEFT $\Rightarrow A_n = 0$

$0 = 300\,000 - 166\,000\,(1.08)^n$

$166\,000\,(1.08)^n = 300\,000$

$1.08^n = \frac{300}{166}$

USE LOGS

$\log_{1.08}\left(\frac{300}{166}\right) = n$ ✓

CHANGE OF BASE

$\frac{\log_{10}\left(\frac{300}{166}\right)}{\log_{10}(1.08)} = n$

$\therefore n = 7.689\ldots$ ✓ YEARS
(7.7, 8, etc... were also accep

- ANSWERED WELL. ONLY A FEW MINOR ALGEBRAIC ERRORS.

Give proper reasons!

Q16.) i) In $\triangle PXS$ and $\triangle RXQ$

$PX = XR$ Given

$\angle SPX = \angle QRX$ (Alternate angles on parallel lines are equal) ✓

$\angle PXS = \angle QXR$ (Vertically opposite angles equal) ✓

$\therefore \triangle PXS \equiv \triangle RXQ$ (AAS) ✓

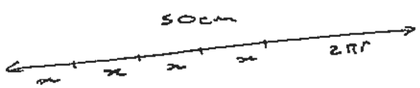
ii) $PS = QR$ corresponding sides of congruent $\triangle$s.

As $PS = QR$ and $PS \parallel QR$, $PSRQ$ is a parallelogram. Equal and parallel opposite sides.

$\therefore PQ = SR$ Opposite sides of a parallelogram.

Note: * You could also prove congruency of $\triangle PXQ$ and SXR.
or
* Diagonals bisect each other, $\therefore$ Parallelogram

6) i)



$4x + 2\pi r = 50$

$r = \dfrac{50-4x}{2\pi}$

$\boxed{r = \dfrac{25-2x}{\pi}}$

ii) $A = \pi r^2$

$= \pi\left(\dfrac{25-2x}{\pi}\right)^2$

$= \dfrac{\pi(25-2x)^2}{\pi^2}$

Square the π!

$A = \dfrac{(25-2x)^2}{\pi}$

or

$A = \dfrac{625 - 100x + 4x^2}{\pi}$

Careful here

iii) $A_T = x^2 + \dfrac{625 - 100x + 4x^2}{\pi}$

$\dfrac{dA_T}{dx} = 2x - \dfrac{100}{\pi} + \dfrac{8x}{\pi}$

let $\dfrac{dA_T}{dx} = 0$ stat pt.

$0 = 2x + \dfrac{8x}{\pi} - \dfrac{100}{\pi}$

$-8x - 2\pi x = -100$

$2\pi x + 8x = 100$

$x(2\pi + 8) = 100$

$x = \dfrac{100}{2\pi + 8}$

$= \dfrac{50}{\pi + 4}$

$x = 7.0\text{cm}$ (1dp) ✓

Check min:

$\dfrac{d^2A_T}{dx^2} = 2 + \dfrac{8}{\pi}$

> 0 ✓

$\therefore$ min