FORT STREET HIGH SCHOOL

YEAR 12

TRIAL HSC 2004

MATHEMATICS

Time allowed: 3 hours
Plus Reading Time: 5 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions
- The marks allocated for each question are indicated.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Each new question is to be started on a **new page.**
- If required additional paper may be obtained from the Examination Supervisor on request.

Name: Feiyi Zhang Class: 12A

Teacher: Mr Bayas

Question No	1	2	3	4	5	6	7	8	9	10	Total
Marks	8 /12	12 /12	7 /12	10 /12	6 /12	10 /12	10 /12	5 /12	10 /12	6 /12	84/120 ~~84~~

Question 1 (12 marks) Marks:

a) Simplify $\frac{1}{2}\sqrt{48}-\sqrt{12}+\sqrt{147}$ 2

b) Factorise x^2-4y^2-x-2y 2

c) Solve the equation $\frac{x-6}{3}-\frac{x-1}{2}=1$ 2

d) Solve $\tan x^0 = 1$ for $0^0 \le x^0 \le 360^0$ 1

$(x-1)(x-1)$
$= x^2 - x - x$
$x^2 - x - x + 1$
$(x-1)(x+1)$
$-\frac{1}{3}$

e) Use the table of standard integrals to evaluate $\int \frac{1}{\sqrt{x^2+16}}dx$ 2

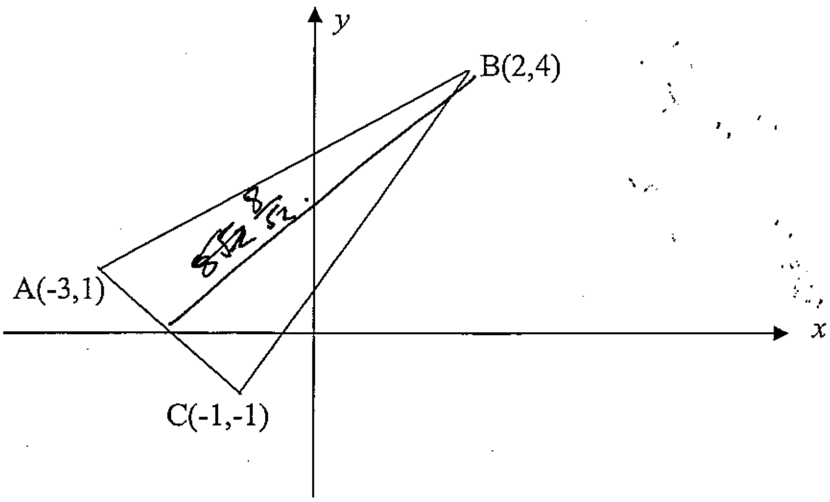
f) A man earns \$91,500 in 1998 and invests 15% of his earnings in an account earning 10% p.a. compounded annually. How much interest has he accrued at the end of 5 years. 3

91 500 1998

x^2-4y^2-x-2y.
$x^2-x \quad -4y^2-2y$.
x^2-x
$x(x-1) - 2y(2y-1)$

Question 2: (12 marks) **Marks:**

a)



A B

In the diagram the co-ordinates of A, B and C respectively are (-3,1), (2,4) and (-1,-1).

C

i) Show $\triangle ABC$ is isosceles. **2**

ii) Find the equation of AC. **2**

iii) Find the perpendicular distance from B to the line AC. **2**

iv) Find the area of $\triangle ABC$. **2**

b) A parabola has equation

$$(x-1)^2 = 12(y+2)$$

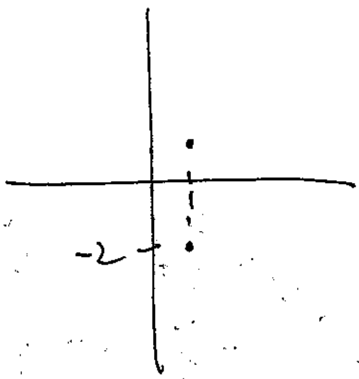
i) What is the vertex? **1**

ii) What is the focal length? **1**

iii) Find the co-ordinates of the focus. **1**

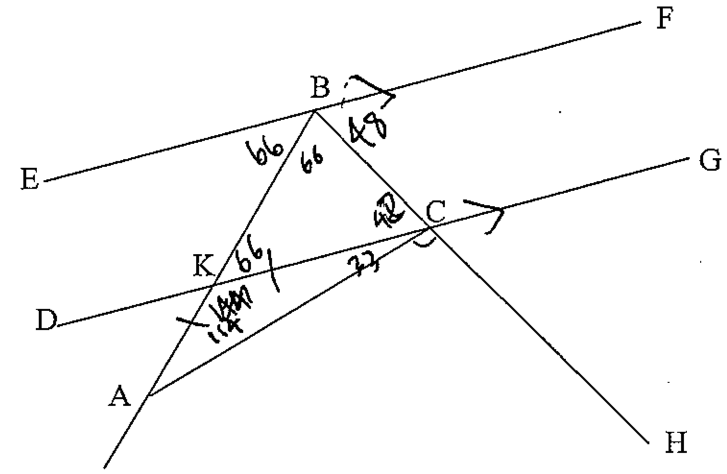
iv) Find the equation of the directrix. **1**

$\frac{5}{3} \times \frac{5}{3} = \frac{25}{9}$



Question 3: (12 marks) **Marks**

a)

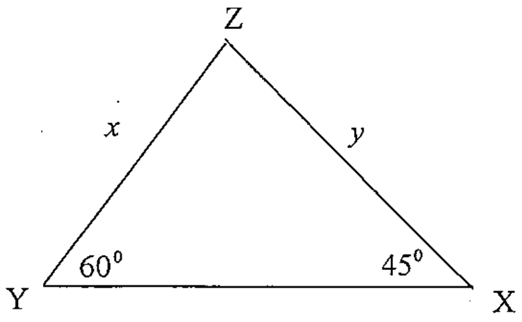


For the diagram above EF II DG, KC=KA
$\angle EBA = 66^0$, $\angle FBC = 48^0$.

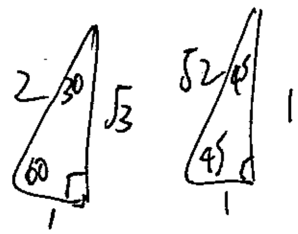
i) Copy the diagram showing the information given 1

ii) Find the size of $\angle ACH$ giving reasons for your answer 3

b)



NOT TO SCALE



3

In the above diagram $\angle YXZ = 45^0$, $\angle XYZ = 60^0$. Find the exact value of the ratio $\dfrac{x}{y}$.

c) Jillian walks 1.2 km from point S on a bearing of 178^0 to point T, then turns due east and walks a further 1.6 km to point V

i) Draw a diagram representing this information, showing the size of $\angle$STV.

ii) Calculate her distance (to the nearest 10^{th} of a kilometre) and bearing (to nearest degee) from S. 1 4

$\frac{2}{\sqrt{2}} \times \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{2}\times\sqrt{3}} = \frac{2}{\sqrt{6}}$ $2^{\frac{1}{2}} + 3^{\frac{1}{2}} =$ $\frac{\sqrt{3}y}{x}$ $\frac{2x}{\sqrt{2}} \times x^2$ $\frac{2x}{\sqrt{2}} \times x = 2x$ $\frac{2x}{\sqrt{2}} \times \frac{1}{2x}$ $\frac{2}{5}$

Question 4: (12 marks) **Marks:**

a) Find the value of e^{π} correct to 3 significant figures **1**

b) Differentiate the following functions

i) $(2x+1)^8$ **2**

ii) $\dfrac{x}{\log_e x}$ **2**

c) Find the following integrals

i) $\int e^{3x+1}dx$ **2**

ii) $\int \dfrac{5\cancel{dx}\,1}{5x-2}$ **2**

iii) If $\displaystyle\int_0^{\ln 2} \frac{e^x}{e^x+1}\, dx = \ln a$, find the value of a. **3**

Marks:

Question 5: (12 marks)

a) The first three terms of an arithmetic series are 25 + 19 + 13 + ……

i) Find the 20th term 1

ii) How many terms will it take for the sum of the terms to become negative 2

b) If the sum of n terms of a series is given by the formula $S_n = 3n^2$ find an expression for the n th term and show that the series is arithmetic 3

c) For the series $2e^{-1} + 4e^{-2} + 8e^{-3} + \cdots$

i) Find S_n 2

ii) Find S_∞ 2

iii) Find an expression for $S_\infty - S_n$ 2

$3n^2 = \frac{n}{2}[2a+(n-1)d]$.

~~$3n = \frac{n}{2}(a+l)$~~

$6n^2 = n(2a+(n-1)d$

~~$6n = na+nl$.~~

$6n = 2a+(n-1)d$

$6n = 2a+dn-d$.

$T_n = a+(n-1)d$.

$2e^{-1}+2$

$4e$

$S_n = \frac{n}{2}[a+(n-1)d]$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_n = \frac{a(r^n-1)}{r-1}$

$T_n = a+(n-1)d$

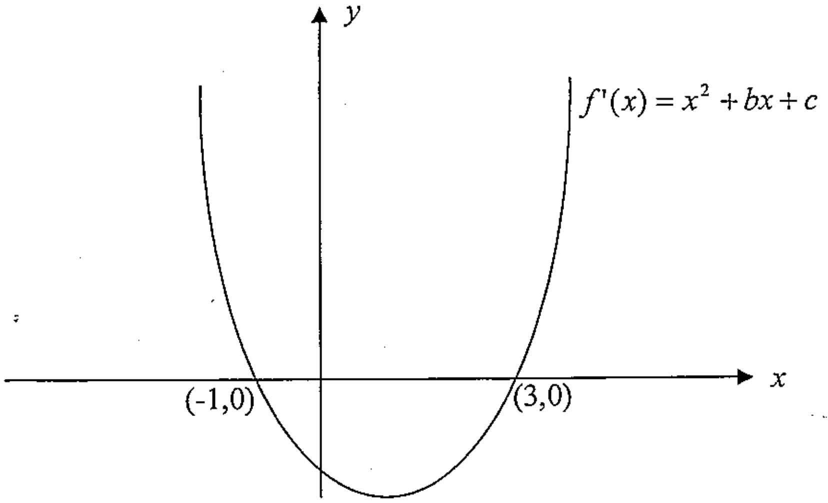
$\left(\frac{2}{e^1}\right)^2 = \frac{4}{e^2}$ $(4e^{-2})(2e^{-1})$ $2e^{-1}\left(\frac{1}{1-1}\right.$

$= 8e^{-3}$

Marks:

Question 6: (12 marks)

a)



$f'(x) = x^2 + bx + c.$
$f(x) = \frac{x^3}{3} + \frac{bx^2}{2} + cx + C.$
at x $(-1,0)$
$0 \; y = \frac{-1^3}{3} + \frac{b(-1)^2}{2} + c(-1$

The gradient function $f'(x) = x^2 + bx + c$ has been sketched in the above diagram

i) Show that $b = -2$ and $c = -3$ **2**

ii) State the values of x for which $y = f(x)$ has turning points and determine the nature of the turning points of $y = f(x)$ **2**

iii) If $y = f(x)$ passes through the point (0,1) find $f(x)$. **2**

iv) Show that the point of inflection of $y = f(x)$ occurs at $\left(1, -2\frac{2}{3}\right)$. **2**

b)

i) A box of 12 golf balls contains 8 white and 4 yellow balls. A boy selects 2 golf balls at random. What is the probability they are different colours? **2**

ii) Another box of 12 golf balls contains 6 white, 4 yellow and 2 pink balls. A boy selects 2 golf balls at random. What is the probability they are different colours? **2**

$y = x^2$
$y' = 2x$

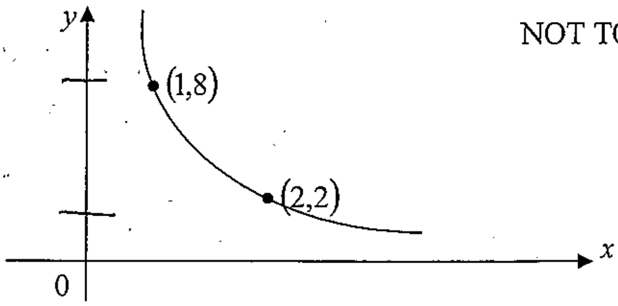
$y = \frac{1}{3}x^3$
$y' = 1x^2$

$-1x^2 / x$

Marks:

Question 7: (12 marks)

a)



NOT TO SCALE

$\pi \int \frac{8}{y}$ × $\frac{1}{8}$

The diagram shows the graph of $y = \frac{8}{x^2}$ (i.e. $x^2 = \frac{8}{y}$) for $x > 0$.

The arc of the graph between (1, 8) and (2, 2) is rotated about the y axis. Find the volume of the solid formed (in exact form). **3**

b) Consider the function $y = 5xe^{-x}$.

i) Copy and complete the following table, giving the value to 2 decimal places. **1**

x	0	1	2	3	4
y	0	1.84	1.35		0.37

ii) Find $\int_0^4 5xe^{-x}dx$ using Simpson's Rule with 5 function values. **2**

iii) Find $\frac{dy}{dx}$. **2**

iv) Find the value of x for which $\frac{dy}{dx} = 0$. **1**

v) Show that $\frac{d^2y}{dx^2} = 5e^{-x}(x-2)$. **2**

vi) What is the x coordinate of the point of inflection on the graph of $y = 5xe^{-x}$? **1**

$y = 2x^2$
$4 = 4x$

Question 8: (12 marks) **Marks:**

a) For what values of k will the roots of the quadratic equation **3**

$$kx^2 - 2(k+1)x + 4 = 0 \text{ be real?}$$

b) Given that one root of the quadratic equation $3x^2 + bx + c = 0$ is three times the other root prove that $b^2 - 16c = 0$. **3**

c)

i) From a packet of mixed seed it was estimated that the probability of any seed planted yielded a white carnation was 0.02.
If n seeds are planted, write down expressions for

(α) The probability of no white carnations **1**

(β) The probability of at least one white carnation. **2**

ii) How many seeds must be planted for you to be at least 98% certain of obtaining a white carnation. **3**

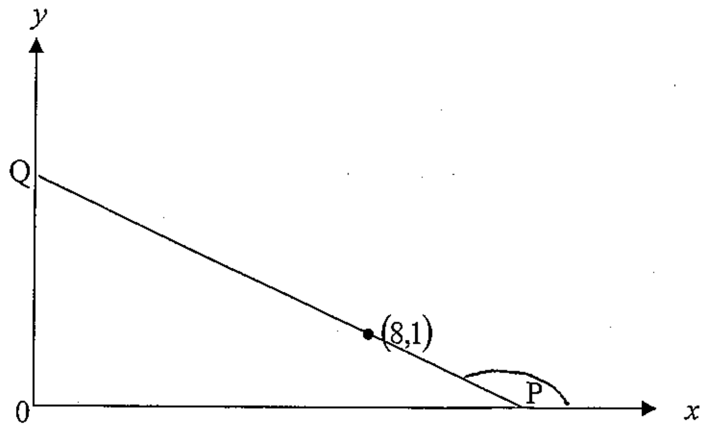
$\cancel{4(k-2}\ x$

$(-2k-2)(-2k-2)$

$= 4k^2 + 4k + 4k + 4$

$= 4k^2 + 8k + 4.$

Question 9: **(12 marks)** **Marks:**



$-m^{-1}$

$= 1m^{-2}$

A line is drawn through the point (8,1) to cut the positive x axis at P and the positive y axis at Q. The gradient of *PQ* is *m*.

a) Find the equation of *PQ* in terms of *m*. **1**

b) Show that the coordinates of *P* are $\left(\frac{8m-1}{m},0\right)$. **1**

c) Find the coordinates of *Q*. **2**

d) Show that the area of $\triangle OPQ$ is $\frac{1}{2}\left(16-64m-\frac{1}{m}\right)$. **2**

e) Find the value of *m* for which this area is a minimum, noting that *m* is negative. **4**

f) Show that the minimum area is 16 unit^2. **2**

$\frac{1}{2m^2}$

$\frac{1}{2}m^{-2}$

-1

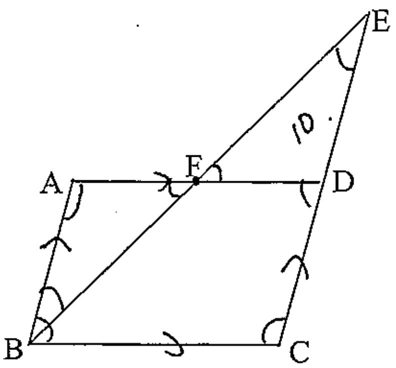
$\frac{8m-1}{m} \times 2 = \frac{2(8m-1)}{m}$

$= \frac{16m-2}{m}.$

Marks:

Question 10: (12 marks)

a)



In the diagram above ABCD is a parallelogram and point F is the midpoint of AD.

i) Show that triangles AFB and FED are congruent. **2**

ii) Show that triangles EFD and EBC are similar. **2**

iii) Hence find the area of ABCD given that the area of ΔEFD is 10 square units. **2**

b)

i) Sally has just turned 18 years old and wishes to purchase a \$25,000 car by her 25th birthday. She visits the Big Bank and discovers that she can earn interest at the rate of 0.65% per month. Assuming that it will take her exactly one month to make all the necessary financial arrangements how much must she deposit each month to achieve her goal? **3**

ii) How much less would she need to deposit monthly if her parents gave her \$3000 for her 18th birthday to use as an initial deposit? **3**

END OF EXAMINATION

9 years.
96 months
− one months
= 95 months

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

TRIAL HSC SOLUTIONS

Mathematics - 2004

QUESTION ONE	COMMENTS.

a) $\frac{1}{2}\sqrt{48} - \sqrt{12} + \sqrt{147} = \frac{1}{2} \times 4\sqrt{3} - 2\sqrt{3} + 7\sqrt{3}$ ✓

$= 7\sqrt{3}$ ✓

b) $x^2 - 4y^2 - x - 2y = (x-2y)(x+2y) - (x+2y)$ ✓

$= (x+2y)(x-2y-1)$ ✓

c) $\frac{x-6}{3} - \frac{x-1}{2} = 1$

$2(x-6) - 3(x-1) = 6$ ✓

$-x - 9 = 6$

$x = -15$ ✓

d) $\tan x^\circ = 1$

$x = 45^\circ$ or 225° ✓

e) $\int \frac{1}{\sqrt{x^2+16}}\,dx = \ln\left(x + \sqrt{x^2+16}\right)$ ✓✓

f) Amount invested = \$13725 ✓

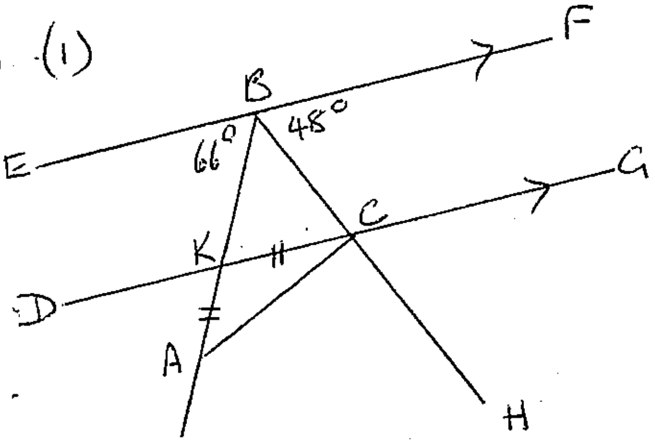
Interest = $\$13725 \times 1.1^5 - 13725$ ✓

= \$8379.25 ✓

QUESTION TWO	COMMENTS
a) (i) $AB = \sqrt{3^2+5^2} = \sqrt{34}$ ✓	a) i) Well Done
$BC = \sqrt{5^2+3^2} = \sqrt{34}$ ✓	
$\therefore \triangle ABC$ is isosceles	
(ii) Equation of AC is $\dfrac{y-1}{x+3} = \dfrac{-1-1}{-1+3}$ ✓	ii) Well Done
$2y-2 = -2x-6$	
$2x+2y+4 = 0$	
$x+y+2 = 0$ ✓	
(iii) $d = \dfrac{\lvert 2+4+2 \rvert}{\sqrt{1^2+1^2}}$ ✓	iii) many students need to learn the 'perpendicular distance' formula & to remember eqn. of line being used must be in general form so that the correct values for 'a', 'b', 'c' are substituted
$= \dfrac{8}{\sqrt{2}}$ ✓ $(4\sqrt{2})$	
(iv) Area $= \frac{1}{2} \times \sqrt{2^2+2^2} \times \dfrac{8}{\sqrt{2}}$ ✓	iv) Well Done
$= \frac{1}{2} \times 2\sqrt{2} \times \dfrac{8}{\sqrt{2}}$	
$= 8$ units ✓	
b) (i) Vertex $= (1,-2)$ ✓	b) Well Done.
(ii) Focal length $= 3$ ✓	
(iii) Focus is $(1,1)$ ✓	
(iv) Directrix is $y = -5$. ✓	

QUESTION THREE	COMMENTS

a) (i)



students made KC = AC.

otherwise good.

(ii) $\angle BKC = 66^\circ$ - alternate angles

$\therefore \angle CKA = 114^\circ$ - straight angle

$\angle KCA = \frac{1}{2} \times 66^\circ$ - isosceles Δ

$\angle BCK = 48^\circ$ - alternate angles

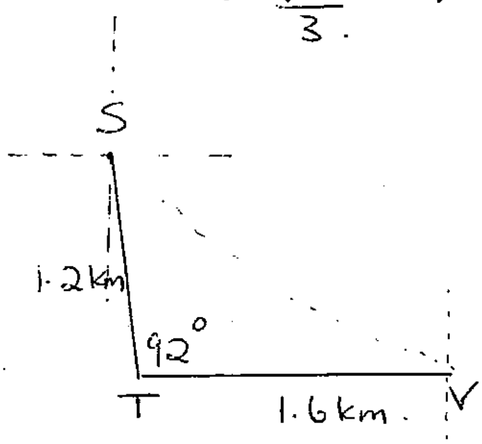
$\therefore \angle ACH = 180^\circ - (33 + 48)$
$= 99^\circ$.

b) $\dfrac{x}{\sin 45^\circ} = \dfrac{y}{\sin 60^\circ}$

$\dfrac{x}{y} = \dfrac{\sin 45^\circ}{\sin 60^\circ}$
$= \dfrac{1}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}}$
$= \dfrac{2}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$
$= \dfrac{\sqrt{6}}{3}$.

well done

c) (i)



• many students drew V. west of T.

QUESTION THREE (contd)

(ii) $d^2 = 1.2^2 + 1.6^2 - 2 \times 1.2 \times 1.6 \cos 92^\circ$ ✓ students forgot to square root at the end.

$d = 2.0$ km to nearest 0.1 km ✓

$$\frac{\sin \angle SVT}{1.2} = \frac{\sin 92^\circ}{d}$$

$$\sin \angle SVT = \frac{1.2 \sin 92^\circ}{d}$$ ✓

$\therefore \angle SVT = 36^\circ$ to nearest degree.

$\angle TSV = 52^\circ$

$\therefore$ Bearing from S $= 178 - 52$

$= 126^\circ T$. ✓

QUESTION 4	COMMENTS
a) $e^{\pi} = 23 \cdot 1$ ✓	
b) (i) $\frac{d}{dx}(2x+1)^8 = 8(2x+1)^7 \times 2$ ✓ $= 16(2x+1)^7$ ✓	correct notation rarely used 3 methods 1) this method ←
(ii) $\frac{d}{dx} \frac{x}{\log_e x} = \frac{\log_e x - x \times \frac{1}{x}}{(\log_e x)^2}$ ✓ $= \frac{\log_e x - 1}{(\log_e x)^2}$ ✓	or 2) let $y =$ $\therefore \frac{dy}{dx} =$ or 3) let $f(x) =$ $\therefore f'(x) =$ ← $(\log_e x)^2 \neq 2\log_e$
c) (i) $\int e^{3x+1}\,dx = \frac{1}{3} e^{3x+1} + C$ ✓✓	* constant often left out.
(ii) $\int \frac{dx}{5x-2} = \frac{1}{5}\ln(5x-2) + C$ ✓✓	
(iii) $\int_0^{\ln 2} \frac{e^x}{e^x+1}\,dx = \ln(e^x+1)\Big]_0^{\ln 2}$ ✓ $= \ln(3) - \ln 2$ ✓ $= \ln 1 \cdot 5.$ $\therefore a = 1 \cdot 5.$ ✓	*some used a calculator rather than recognise $e^{\ln 2} = 2$ nor $\ln 3 - \ln 2 = \ln(\frac{3}{2}) = 1 \cdot 5$.

QUESTION 5

a) i) $T_{20} = 25 + 19 \times -6$

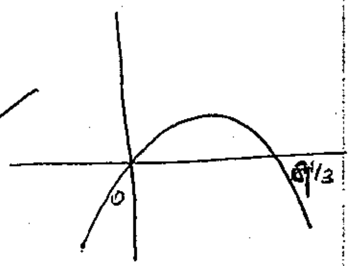
① $= -89$

(ii) $S_n = \frac{n}{2}(50 - 6(n-1))$

② $= 25n - 3n^2 + 3n$

If $28n - 3n^2 < 0$

$n(28 - 3n) < 0$



$\therefore n > 9\frac{1}{3}$

$\therefore$ 10 terms are needed for negative sum.

b) $T_n = S_n - S_{n-1}$

③ $= 3n^2 - 3((n-1)^2)$

$= 3n^2 - 3[n^2 - 2n + 1]$

$= 6n - 3$

Series is $3, 9, 15, \ldots$

ie AP with $d = 6$.

c) $S_n = \dfrac{2e^{-1}\left((2e^{-1})^n - 1\right)}{2e^{-1} - 1}$

② $= \dfrac{\frac{2}{e}\left(\frac{2^n}{e^n} - 1\right)}{\frac{2}{e} - 1}$

$= \dfrac{\frac{2}{e}\left(\frac{2^n}{e^n} - 1\right)}{\frac{2 - e}{e}}$

$(2e^{-1})^n$ not $2e^{-n}$.

COMMENTS

Some - incorrect n value, others missed -6

Many used T_n not S_n..

Few could work out S_1, S_2 etc

Some didn't tidy up T_n ie

?? $T_n = 3 + (n-1)6$

$r > 1$ missed

1 for substitution

1 for tidy up

QUESTION 5 | COMMENTS

$$= \frac{2}{2-e}\left[\left(\frac{2}{e}\right)^n - 1\right] \checkmark$$

$$S_\infty = \frac{2/e}{1 - 2/e} \checkmark$$

2

$$= \frac{2}{e} \times \frac{e}{e-2}$$

$$= \frac{2}{e-2} \checkmark$$

Some approx. on calculator.

$$S_\infty - S_n = \frac{2}{e-2} - \frac{2}{2-e}\left[\left(\frac{2}{e}\right)^n - 1\right] \checkmark$$

2

$$= \frac{2}{e-2}\left[1 + \left(\frac{2}{e}\right)^n - 1\right]$$

$$= \frac{2}{e-2}\left(\frac{2}{e}\right)^n \checkmark$$

1 for subtraction of S_n & S_∞ from working

1 for correct answer.

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r<1$$

$$= \frac{2e^{-1}\left[1 - (2e^{-1})^n\right]}{1 - 2e^{-1}}$$

$$= \frac{\frac{2}{e}\left[1 - 2^n e^{-n}\right]}{1 - \frac{2}{e}}$$

$$= \frac{2}{e}\left[1 - 2^n e^{-n}\right] \times \frac{e}{e-2} = \frac{2}{e-2}\left[1 - \left(\frac{2}{e}\right)^n\right]$$

QUESTION 6	COMMENTS

a) (i) $f'(x) = x^2 + bx + c$

$f'(3) = 9 + 3b + c = 0$ ①

$f'(-1) = 1 - b + c = 0$ ②

① − ② ⇒ $8 + 4b = 0$

$b = -2$

$1 + 2 + c = 0$

$c = -3$

(ii) $f(x)$ has turning points when

$f'(x) = 0$ ie $x = -1$ and 3.

At $x = -1$ max. turning pt.

$x = 3$ min. turning pt

(iii) $f(x) = \int x^2 - 2x - 3$

$= \frac{x^3}{3} - x^2 - 3x + c$

If passes through $(0, 1)$ then

$1 = c$

$\therefore \boxed{y = \frac{x^3}{3} - x^2 - 3x + 1}$

(iv) $f''(x) = 2x - 2$

For point of inflection

$f''(x) = 0$ and changes sign

$\therefore x = 1$. $x = 1 - \varepsilon < 0$: $x = 1 + \varepsilon > 0$

$\therefore$ Pt of inflection at $(1, -2\frac{2}{3})$

One mark solving $f''(x) = 2x - 2 = 0$
One mark finding $f(1)$ or testing for change in concavity

Q6 contd	COMMENTS

(i) $P(\text{both different}) = P(WY)$ or $P(YW)$

$\frac{32}{132} + \frac{32}{132} = \frac{64}{132}$ $\quad = \frac{8}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{8}{11}$ ✓

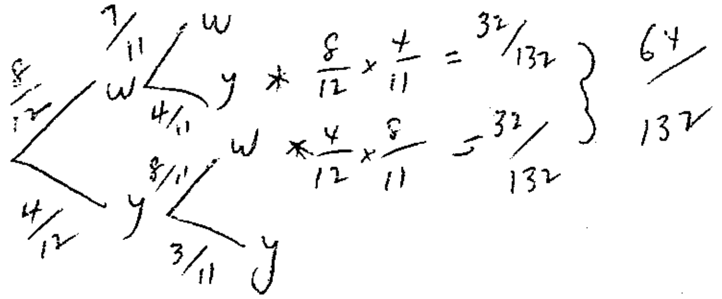
$= \frac{16}{33}$ ✓

(ii) $P(\text{both different}) = P(W\bar{W}) + P(Y\bar{Y}) + P(P\bar{P})$

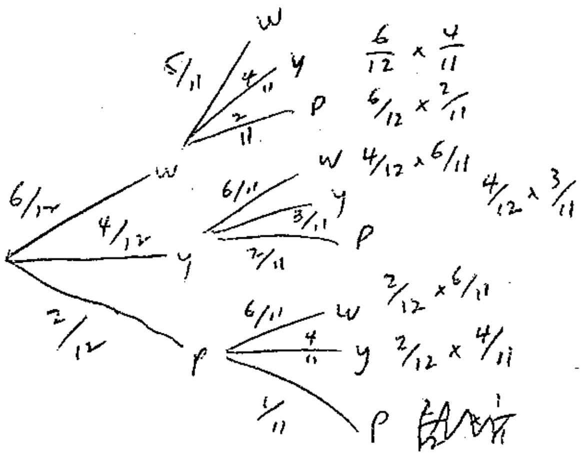
$= \frac{6}{12} \times \frac{6}{11} + \frac{4}{12} \times \frac{8}{11} + \frac{2}{12} \times \frac{10}{11}$ ✓

$= \frac{2}{3}$ ✓

i)



ii)



QUESTION 7.	COMMENTS
a) $V = \pi \int_2^8 x^2 \, dy$ $= \pi \int_2^8 \frac{8}{y} \, dy$ ✓ $= \pi \left[8 \ln y \right]_2^8$ ✓ $= \pi (8 \ln 8 - 8 \ln 2)$ ✓ $= 8\pi \ln 4$. cubic units	Well done. although many used wrong limits.
b) (i) Correct value $= 0.75$. ✓	
(ii) $\int_0^4 5xe^{-x} = \frac{2}{6}(0 + 4 \times 1.84 + 1.35) + \frac{2}{6}(1.35 + 4 \times 0.75 + 0.37)$ ✓ $= 4.48 \, u^2$. ✓	
(iii) $\frac{dy}{dx} = -5xe^{-x} + e^{-x} \times 5$ ✓ $= 5e^{-x}(1-x)$ ✓	
(iv) $\frac{dy}{dx} = 0$ if $x = 1$. ✓	
(v) $\frac{d^2y}{dx^2} = 5e^{-x} \times -1 + (1-x) \times -5e^{-x}$ ✓ $= -5e^{-x} - 5e^{-x}(1-x)$ $= 5e^{-x}(-1 - 1 + x)$ ✓ $= 5e^{-x}(x-2)$. ✓	
(vi) For point of inflection $x = 2$	

QUESTION 8 | Comments

a) For real roots $\Delta \geq 0$.

$\Delta = \left[-2(k+1)\right]^2 - 4 \times k \times 4$ ✓

$= 4(k^2+2k+1) - 16k$.

$= 4k^2 - 8k + 4$ ✓

$= 4(k^2-2k+1) = 4(k-1)^2$ ✓

≥ 0. for all values of k.

b) Roots are α, 3α

$4\alpha = \frac{-b}{3}$ $\quad \therefore \alpha = \frac{-b}{12}$

$3\alpha^2 = \frac{c}{3}$. ✓

$3\left(\frac{-b}{12}\right)^2 = \frac{c}{3}$. ✓

$\frac{9b^2}{144} = c$.

$9b^2 = 144c$.

$b^2 - 16c = 0$. ✓

c) (i)(α) P(no white carnations) $= (0.98)^n$ ✓

(β) P(at least 1 white) $= 1 - P$ (no whites) ✓

$= 1 - 0.98^n$ ✓

(ii) $1 - 0.98^n \geq 0.98$ ✓

$-0.98^n \geq -0.02$.

$0.98^n \leq 0.02$

$n \ln 0.98 \leq \ln 0.02$ ✓

$n \geq \frac{\ln 0.02}{\ln 0.98}$ $\quad . n \geq 194$ ✓

QUESTION 19	Comments.

a) Equation of PQ is

(i) $y - 1 = m(x-8)$

$y = mx - 8m + 1.$

(ii) At P $y=0.$

$0 = mx - 8m + 1$

$mx = 8m - 1$

$x = \dfrac{8m-1}{m}$

$\therefore P = \left(\dfrac{8m-1}{m}, 0\right).$

students forgot to express these as points.

(iii) At Q, $x = 0.$

$\therefore y = 1 - 8m.$

$Q = (0, 1-8m).$

(iv) Area $\Delta OPQ = \dfrac{1}{2} \times \left(\dfrac{8m-1}{m}\right) \times (1-8m)$

$= \dfrac{1}{2}\left(\dfrac{8m - 64m^2 - 1 + 8m}{m}\right)$

well done.

$= \dfrac{1}{2}\left(\dfrac{16m - 64m^2 - 1}{m}\right)$

$= \dfrac{1}{2}\left(16 - 64m - \dfrac{1}{m}\right)$

(v) $\dfrac{dA}{dm} = \dfrac{1}{2}\left(-64 + \dfrac{1}{m^2}\right)$

$= 0$ if $m = \pm\dfrac{1}{8}$

$m < 0$ since line has negative slope

* when differentiating $\dfrac{1}{2m^2}$ many did this $(2m)^{-2}$

$-2(2m)^{-3}$?

$$\frac{d^2A}{dm^2} = -\frac{1}{m^3}$$

> 0 if $m = -\frac{1}{8}$ $\therefore$ Minimum area at $m = -\frac{1}{8}$.

(vi) Min. Area $= \frac{1}{2}\left(16 - 64 \times -\frac{1}{8} - \frac{1}{-1/8}\right)$

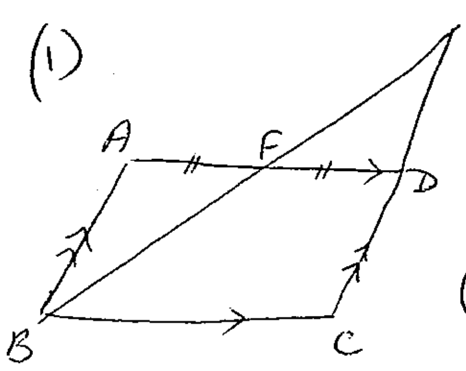
$= \frac{1}{2}\left(16 + 8 + 8\right)$

$= 16 \text{ units}^2$.

v. good.

QUESTION 10	COMMENT

a) (i)



In ΔAFB & ΔFED

(i) $AF = FD$ given

(ii) $\angle AFB = \angle EFD$ vertically opposite

(iii) $\angle ABF = \angle FED$ alternate angles $AB \parallel CE$.

$\therefore \Delta AFB \equiv \Delta FED$ AAS Test.

(ii) In ΔEFD : ΔEBC

i) $\angle E$ is common

(ii) $\angle EFD = \angle EBC$ - corresponding angles

$\therefore \Delta EFD \ ||| \ \Delta EBC$ since equiangular

(iii) If area $\Delta EFD = 10u^2$ then

area $\Delta EBC = 4 \times 10 u^2$ since ratio of sides is 1 : 2

$\therefore$ Since $\Delta AFB \equiv \Delta FED$ then area ABCD is $40 u^2$.

b) $25000 = M\left[(1.0065)^{83} + (1.0065)^{82} + (1.0065)^{81} + \ldots + \ldots \cdot (1.0065)^{1}\right]$.

$$= m\left(1.0065\frac{\left[(1.0065)^{83} - 1\right]}{0.0065}\right)$$

$\therefore m = \$226.71$ to nearest cent.

QUESTION 10 (contd) | COMMENT.

(ii) $25000 = 3000(1.0065)^{83} + m\left(\frac{1.0065\left[(1.0065)^{83}-1\right]}{(.0065)}\right)$

$m = \$180.13$ to nearest cent.

∴ Sally would save \$46.58 per month.