Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2006

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

Time allowed: 3 hourS
(Plus 5 minutes reading time)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, probability and series	4, 6, 8	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	3, 5, 7	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	9, 10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new page

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks) (Start a new page)

a) Given that $v = \frac{1}{\sqrt{t^2+1}}$ find the value of v when $t = 2.04 \times 10^3$, giving your answer in scientific notation correct to 2 significant figures. **2**

b) Find those values of x which satisfy the inequality $|5-2x| \geq 3$. **2**

c) Given $\log_6 2 = 0.3869$ and $\log_6 3 = 0.6131$ find

i) $\log_6(1.5)$ **1**

ii) $\log_6 24$ **1**

d) Simplify: $\frac{a^2-a}{a} \times \frac{a^2+2a+1}{a^2-1}$ **2**

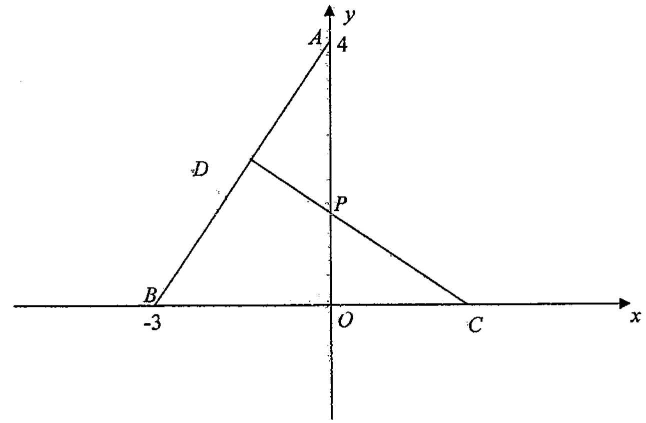
e) If $f(x) = \begin{cases} -2 & x \leq -3 \\ x+1 & -3 < x < 1. \\ x^2 & x \geq 1 \end{cases}$

i) Find the value of $f(-3)+f(0)-f(2)$ **1**

ii) Write down an expression for $f(a^2+1)$ **1**

f) Find x if $\log_{16} 2 = \log_x 3$ **2**

Question 2: (12 marks) (Start a new page)



In the diagram AB=BC and CD is perpendicular to AB.

CD intersects the y axis at P.

Copy the diagram onto your answer sheet.

a) Find the length of AB **1**

b) Hence show the co-ordinates of C are (2,0) **1**

c) Show the equation of CD is 3x+4y=6. **3**

d) Show that the co-ordinates of P are $(0, 1\frac{1}{2})$. **1**

e) Use Pythageras' theorem on ΔPOC to show the length of CP is $2\frac{1}{2}$ units. **1**

f) Prove Δ ADP is congruent to Δ COP. **3**

g) Hence calculate the area of the quadrilateral DPOB. **2**

Question 3: (12 marks) (Start a new page)

a) Differentiate

i) $y = x\sqrt{x}$ **1**

ii) $y = \dfrac{e^x + 2x}{x}$ **2**

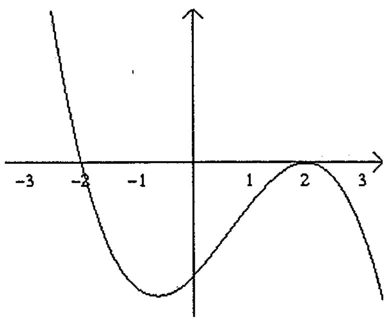
iii) $y = (3-4x)^5$ **1**

b) Find

i) $\int e^{\frac{x}{2}} dx$ **1**

ii) $\int_1^2 \left(2x - \dfrac{1}{x^2}\right) dx$ **2**

c) The figure below shows the graph of $y = f'(x)$ where $f'(x)$ is the derivative of a function $f(x)$. The domain of $f(x)$ and $f'(x)$ is $-3 \le x \le 3$



Copy the diagram onto your answer sheet.

i) By considering the graph of $y = f'(x)$, explain why the graph of $y = f(x)$ has two, and only two stationary points. **1**

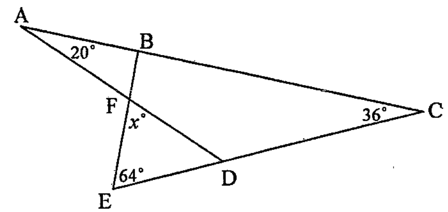
ii) For what values of x does $y = f(x)$ have a relative maximum? Justify your answer. **2**

iii) Given that $f(-3) = 0$ sketch a possible graph of $y = f(x)$ on the same axes that you drew the graph $y = f'(x)$. **2**

Question 4: (12 marks) (Start a new page)

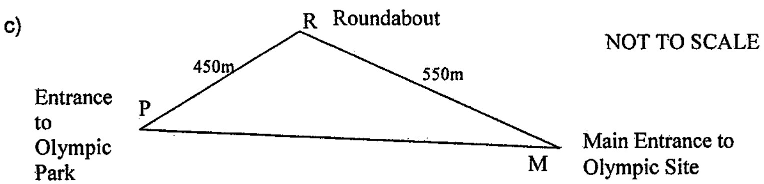
a) Evaluate $\sum_{n=5}^{14} 17 - 2n$. **2**

b)



Find the value of x, giving reasons. **2**

c)



The diagram shows the route MPR, followed by a shuttle bus from the main entrance to the Sydney Olympic Site (M), via a roundabout (R), to the entrance to Olympic Park (P), where the principal stadiums are sited. R is 550 metres at a bearing of 325° from M, and P is 450 metres at a bearing of 250° from R. It is proposed that an overhead cable car be built directly from M to P.

i) Copy the diagram onto your page and show that $\angle MRP$ measures $105°$. **1**

ii) Calculate the distance MP, covered by the cable car (to the nearest m). **2**

d) Let α and β be the roots of the equation $x^2 - 5x + 2 = 0$. Find the values of

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

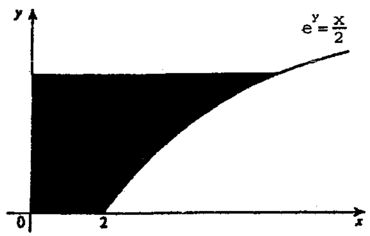
iii) $(\alpha+1)(\beta+1)$ **1**

e) Solve $2\sin\theta = -1$ for $0° \le \theta° \le 360°$ **2**

Question 5: (12 marks) (Start a new page)

a) The curve $y = ax^3 + \frac{b}{x^2}$ cuts the x-axis at $x = 1$, and the gradient of the tangent at this point is 2. Find the values of a and b. **3**

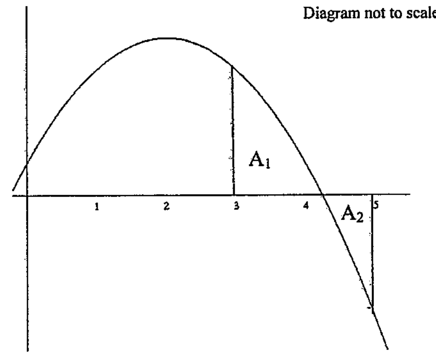
b)



The shaded region shown in the diagram is bounded by the x and y axes, the curve $e^y = \frac{x}{2}$ and the line $y = 1$

A solid is formed by rotating the shaded region about the y axis. Show that the volume of the solid is given by $4\pi \int_0^1 e^{2y} dy$ and find the volume correct to 2 decimal places. **3**

c) The graph of $y = 1 + 4x - x^2$ over the domain $0 \le x \le 5$ is shown below



i) What is the range of y over this domain? **1**

ii) Evaluate $\int_3^5 1 + 4x - x^2 \, dx$ and hence determine which area A_1 or A_2 is greater? Give reasons. **3**

d) Show that $\frac{\sec^2\theta}{\tan^2\theta} = \frac{1}{\sin^2\theta}$. **2**

(Q5 continued over)

Question 6: (12 marks) (Start a new page)

a) For what values of m does the quadratic equation

$$(5m-3)x^2-4mx+m+1=0$$

have one real root. **3**

b) Find the coordinates of the vertex and the focus and the equation of the directrix of the parabola $y^2-2y-15=4x$. **3**

c) A caterer organises parties for groups of up to 200. She calculates the cost price of a party by allowing $22 per head for the first 10 guests, $21 per head for the next 10 guests, and so on, allowing one dollar less per head for each subsequent group of 10 guests or part thereof.

i) Show that the cost price, in dollars, for each guest in the n^{th} group of 10 guest, or part thereof, is given by

$$T_n=23-n$$

where T_n is the n^{th} term of an arithmetic series. **1**

ii) Find the increase in the cost price of a party if 4 more persons are added to a guest list of 85. **1**

iii) Determine the cost price of a party attended by 115 people. **2**

iv) If the caterer wishes to make a 25% profit on the cost price, calculate the average charge per head (to the nearest 5 cents) for a party of 115 guests. **2**

Question 7: (12 marks) (Start a new page)

a) A defence system of a city has both ground-to-air missiles and anti-aircraft guns. The missiles have a 0.85 chance of hitting an attacking plane while the anti-aircraft guns have a 0.2 chance of hitting it.

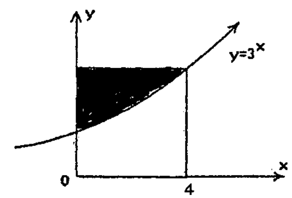
i) A single plane attacks the city. Find the probability that it will not be shot down by either the missile or the gun. **2**

ii) If two planes attack the city, what is the probability at least one is shot down? **2**

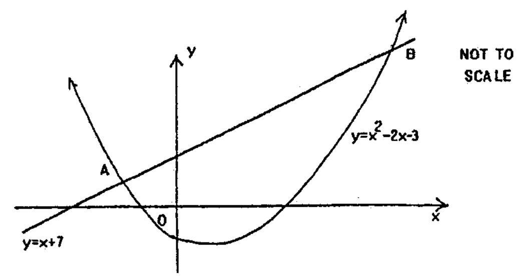
b)

i) Use the Trapezoidal Rule with 2 strips (i.e. 3 function value), to find an approximate value for $\int_0^4 3^x\,dx$. **2**

ii) Hence find an approximate value for the shaded area in the diagram below. **2**



c) The diagram shows the graphs $y=x^2-2x-3$ and $y=x+7$. The graphs intersect at the points A and B.



i) Find the coordinates of A and B. **2**

ii) Find the area enclosed by $y=x^2-2x-3$ and $y=x+7$. **2**

Question 8: (12 marks) (start a new page)

a) A PIN number for a security code consists of a 3 digit number where each number can be any of the numbers (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

i) Find the probability that a PIN number entered at random will break the code. **1**

ii) A thief knows that the middle number is 4 or 6. What is the probability that she will break the code (entering one PIN number at random)? **2**

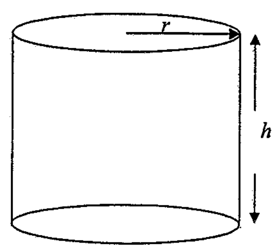
b) A geometric series has a second term of 6 and the ratio of the seventh term to the sixth is 3.

i) Find the common ratio r. **1**

ii) What is the first term a? **1**

iii) Calculate the sum of the first 12 terms. **1**

c) With the breaking of the drought in the Narrabri area, Farmer Jo wants to install a new water tank. It is to be in the shape of a closed cylinder with radius r metres and height h metres as shown in the diagram. The surface area of metal to be used in the tank's construction is 30 square metres.



i) Find an expression for h in terms of r. **2**

ii) Show that the volume V m^3 of the tank is given by the formula $V = 15r - \pi r^3$. **1**

iii) Find the radius (to the nearest cm) if the volume of the tank is to be maximised. **3**

Question 9: (12 marks) (Start a new page)

a) Don planted a Grevillia hedge. The plants were 20cm tall when he planted them. After 1 year they were 1m tall. The next year they grew 60% of the previous years growth to a height of 148cm. They continued to grow 60% of the previous year's growth each year until they reached their maximum height.

i) What was the height of each Grevillia after 3 years? **1**

ii) Calculate the maximum height of the Grevillia hedge. **2**

b) Consider the curve given by the function $f(x) = x^2 + \frac{1}{x^2}$.

i) State the domain of the function. **1**

ii) Determine if the function is odd or even.

iii) Describe the behaviour of the curve as x gets very close to zero. **1**

iv) Find the coordinates of the stationary points and determine their nature. **4**

v) Explain why the curve is concave up for all values of x in its domain. **1**

vi) Sketch the curve for $-2 \le x \le 2$. **2**

Question 10: (12 marks) (Start a new page)

a) The population of a certain town at the beginning of the year 2000 was 100 000. The population increases (due to births and new arrivals) by 12% each year, but also decreases (due to departures and deaths) by E people each year.

i) Prove that the population at the beginning of 2001 was $112000 - E$. 1

ii) Show that the population after 2 years is $125440 - 2.12E$. 1

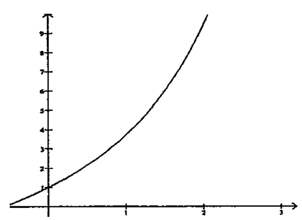
iii) If the population at the beginning of 2006 is 140000, calculate the decrease E due to departures and deaths each year (to the nearest whole person). 4

b)

i) Show that $\frac{d(xe^x)}{dx} = e^x + xe^x$. 1

ii) Hence, by integrating both sides of the equation in (i), show that $\int_0^2 xe^x\,dx = e^2 + 1$. 2

iii) The region that lies between the $x-axis$ and the curve $y = e^x + x$ as shown in the diagram is rotated about the $x-axis$ between $x = 0$ and $x = 2$ to form a solid.



Find the volume of this solid (leave your answer in exact form). 3

END OF EXAMINATION

2006 TRIAL HSC (2 UNIT)

Question 1.

a) $V = \frac{1}{\sqrt{t^2+1}}$

$= \frac{1}{\sqrt{(2.04\times10^3)^2+1}}$

$= 2.40292\ldots\times10^{-7}$

$= 2.4\times10^{-7}$ (2 sig fig)

b) $|5-2x| \geq 3$

$5-2x \geq 3$, $5-2x \leq -3$

$-2x \geq -2$, $-2x \leq -8$

$x \leq 1$, $x \geq 4$

c) i) $\log_6(1.5) = \log_6\left(\frac{3}{2}\right)$

$= \log_6 3 - \log_6 2$

$= 0.6131 - 0.3869$

$= 0.2262$

ii) $\log_6 24 = \log_6(6\times4)$

$= \log_6 6 + \log_6 4$

$= 1 + 2\log_6 2$

$= 1 + 2\times0.3869$

$= 1.7738$

OR $\log_6 24 = \log_6 3 + \log_6 8$

$= \log_6 3 + \log_6 2^3$

$= 0.6131 + 3\times0.3869$

$= 1.7738$.

d) $\frac{a^2-a}{a}\times\frac{a^2+2a+1}{a^2-1} = \frac{\cancel{a}(a-1)}{\cancel{a}}\times\frac{(a+1)^2}{(a-1)(a+1)}$

$= a+1$.

Q1 e) i) $f(3) + f(0) - f(2)$

$= -2 + 1 - 4$

$= -5$ ✓

ii) $f(a^2+1) = (a^2+1)^2$

$= a^4 + 2a^2 + 1$ ✓

f) $\log_{16} 2 = \log_x 3$

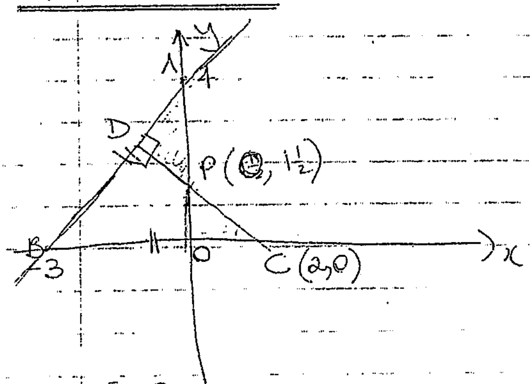
$\frac{1}{4} = \log_x 3$

$x^{\frac{1}{4}} = 3$ ✓

$x = 3^4$

$x = 81$ ✓

QUESTION 2



a) $AB = 5$ units (by Pythagoras in ΔABO) ✓

b) $BC = 5$ units ($AB = BC$)

since B is $(-3,0)$ then $-3+5=2$ ✓

$\therefore C(2,0)$

c) $A(0,4)$ $B(-3,0)$ $C(2,0)$

$m_{AB} = \frac{-4}{-3}$

$= \frac{4}{3}$

$\therefore m_{CD} = -\frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{3}{4}(x-2)$ ✓

$4y = -3x + 6$

eqn CD: $3x + 4y = 6$ (as req'd) ✓

d) $3x + 4y = 6$

cuts y axis $\Rightarrow x = 0$

$4y = 6$ ✓

$y = \frac{3}{2} \Rightarrow \therefore P(0, 1\frac{1}{2})$

e) $CP^2 = 1{\cdot}5^2 + 2^2$

$= 2{\cdot}25 + 4$

$= 6{\cdot}25$ ✓

$CP = 2\frac{1}{2}$ units

f) In Δ's ADP & COP

i) $\angle ADP = \angle POC = 90^\circ$ (given)

ii) $\angle DPA = \angle OPC$ (vert. opp.) ✓

iii) $AP = PC = 2\frac{1}{2}$ ($CP = 2\frac{1}{2}$ proved above, $AP = 4 - 1\frac{1}{2} = 2\frac{1}{2}$) ✓

$\therefore \Delta ADP \equiv \Delta COP$ (AAS) ✓

g) area quad DPOB $= \Delta APO - \{\Delta ADP / \Delta POC\}$ ✓

$= \frac{1}{2} \times 3 \times 4 - \frac{1}{2} \times 2 \times 1\frac{1}{2}$

$= 6 - 1\frac{1}{2}$ ✓

$= 4\frac{1}{2}\ u^2$.

QUESTION 3

a) i) $y = x\sqrt{x}$

$= x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}$ ✓ (either form)

ii) $y = \frac{e^x + 2x}{x}$ $\quad \frac{u}{v}$

$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \frac{x(e^x+2) - (e^x+2x)}{x^2}$ ✓

$$\frac{dy}{dx} = \frac{xe^x + 2x - e^x - 2x}{x^2}$$

$$= \frac{xe^x - e^x}{x^2}$$

$$= \frac{e^x(x-1)}{x^2}$$

✓ (either form)

iii) $y = (3-4x)^5$

$$\frac{dy}{dx} = 5(3-4x)^4 \times -4$$

$$= -20(3-4x)^4$$ ✓

b) i) $\int e^{\frac{x}{2}}\,dx$

$$= \frac{e^{\frac{x}{2}}}{\frac{1}{2}} + c$$

$$= 2e^{\frac{x}{2}} + c$$ ✓

ii) $\int_1^2 \left(2x - \frac{1}{x^2}\right) dx$

$$= \int_1^2 \left(2x - x^{-2}\right) dx$$

$$= \left[x^2 + x^{-1}\right]_1^2$$ ✓

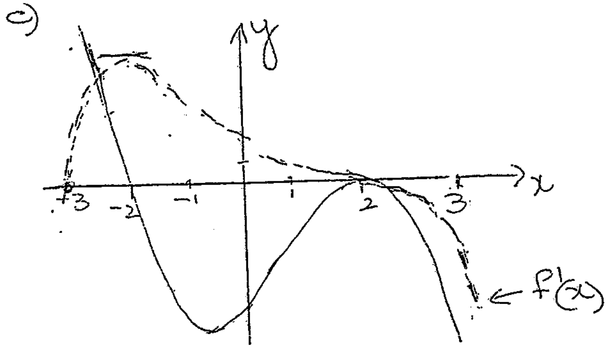
$$= \left[x^2 + \frac{1}{x}\right]_1^2$$

$$= \left(4 + \frac{1}{2}\right) - (1+1)$$

$$= 2\tfrac{1}{2}$$ ✓

Q3

c)

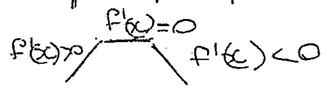


i) $y = f(x)$ has only 2 statny pts at $x = -2$ and $x = 2$ as $f'(x)$ only cuts the x axis twice. (ie $f'(x) = 0$ twice) ✓

ii) $x = -2$. ✓

x	-2^-	-2	-2^+
$f'(x)$	+ve	0	-ve

✓



$\therefore$ a max^m turning pt. at $x = 2$.

iii) $f(-3) = 0$

$\therefore x = -3, y = 0$.

$x = 2$

x	2^-	2	2^+
$f'(x)$	-ve	0	-ve

✓ + 1 for graph

or 2 for showing features

Q4. a) $\sum_{n=5}^{14} 17-2n$

$= 7+5+3+\ldots+-11$

$a=7,\ d=-2,\ n=10$

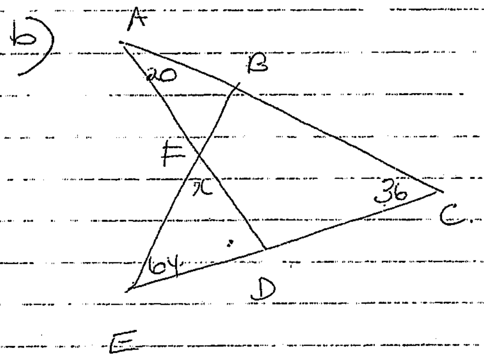
$S_n = \frac{n}{2}(a+l)$

$S_{10} = 5(7-11)$

$= 5 \times -4$

$= -20.$ ✓

b)

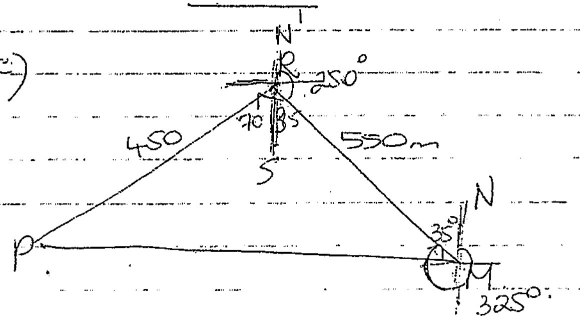


$\angle ADE = 56$ (ext. L $\triangle$ ACD) ✓

$x+64+56=180$ (L sum $\triangle$ EFD)

$\underline{x=60}$ ✓

c)



$\angle SRP = 250-180$

$= 70°$

$\angle SRM = 35°$ (alt. L's on // lines)

$\angle MRP = 35+70$ ✓

Q4

c) $MP^2 = 450^2 + 550^2 - 2 \times 450 \times 550 \times \cos 105°$ ✓

$= 633115.4273$

$MP = 795.6855$

$MP = 796$ m (nearest m) ✓

d) $x^2 - 5x + 2 = 0$

$a=1 \quad b=-5 \quad c=2$

i) $\alpha+\beta = \frac{-b}{a}$ ✓

$= 5$

ii) $\alpha\beta = \frac{c}{a}$ ✓

$= 2$

iii) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= 2 + 5 + 1$

$= 8.$ ✓

e) $2\sin\theta = -1 \qquad 0° \le \theta \le 360°$

$\sin\theta = -\frac{1}{2}$

acute $\theta = 30°$

S | A
T | C

$\theta = 210°, 330°.$ ✓ ✓

QUESTION 5:

a) $y = ax^3 + \frac{b}{x^2}$ ← bx^{-2}

cuts x axis $\Rightarrow y = 0 \Rightarrow (1, 0)$
and $f'(1) = 2$

$0 = a + b$ (1) ✓

$\frac{dy}{dx} = 3ax^2 - 2bx^{-3}$
$= 3ax^2 - \frac{2b}{x^3}$

when $x = 1$, $\frac{dy}{dx} = 2$

$2 = 3a - 2b$ (2) ✓

from (1) $a = -b$
$\therefore 2 = -3b - 2b$
$-5b = 2$
$b = -\frac{2}{5}$ and
$\therefore a = \frac{2}{5}$ ✓

b) $e^y = \frac{x}{2}$ $\quad x = 2e^y$
$x^2 = 4e^{2y}$

$V = \pi \int x^2\, dy$
$V = \pi \int_0^1 4e^{2y}\, dy$ ✓ 1 for showing this.

$V = \frac{4\pi}{2}\left[e^{2y}\right]_0^1$
$= 2\pi\left[e^2 - 1\right]$ ✓
$V = 40.14\ u^3$ (2dp) ✓

Q5

c) i) $y = 1 + 4x - x^2$

$x = 2$ $y = 1 + 8 - 4$
$= 5$

$x = 5$ $y = 1 + 20 - 25$
$= -4$

R: $-4 \le y \le 5$ ✓

ii) $\int_3^5 1 + 4x - x^2\, dx$
$= \left[x + 2x^2 - \frac{x^3}{3}\right]_3^5$

$= \left(5 + 50 - \frac{125}{3}\right) - \left(3 + 18 - \frac{27}{3}\right)$ ✓
$= 1\frac{1}{3}$

$\therefore A_1$ is greater than A_2 ✓ as the overall area is positive. ✓

a) $\frac{\sec^2\theta}{\tan^2\theta} = \frac{1}{\sin^2\theta}$

L.H.S. $= \frac{1}{\cos^2\theta} \div \tan^2\theta$
$= \frac{1}{\cos^2\theta} \div \frac{\sin^2\theta}{\cos^2\theta}$ ✓
$= \frac{1}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{\sin^2\theta}$ ✓
$= \frac{1}{\ldots}$

QUESTION 6.

a) $(5m-3)x^2 - 4mx + m + 1 = 0.$

$a = 5m-3 \quad b = -4m \quad c = m+1$

for one real root (equal roots?) $\Rightarrow \Delta = 0.$

$b^2 - 4ac = 0$

$(-4m)^2 - 4(5m-3)(m+1) = 0$ ✓

$16m^2 - 4(5m^2 + 2m - 3) = 0$

$16m^2 - 20m^2 - 8m + 12 = 0.$

$-4m^2 - 8m + 12 = 0$

$-4(m^2 + 2m - 3) = 0.$ ✓

$(m - 1)(m + 3) = 0$

$\therefore \underline{m = 1}, \ \underline{m = -3}$ for one real root. ✓

b) $y^2 - 2y - 15 = 4x$

$y^2 - 2y + 1 = 4x + 15 + 1$

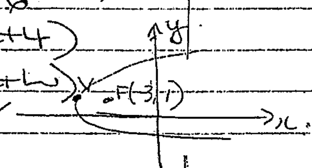
$(y-1)^2 = 4x + 16$

$(y-1)^2 = 4(x+4)$

cf $(y-k)^2 = 4(x+h)$ ✓

vertex $(-4, 1)$ ✓

$a = 1$



$F(-3, 1)$ ✓

directrix : $x = -5.$ ✓

c) $a = 22, \ d = -1 \quad T_n = n^{th}$ multiple of 10

$T_n = a + (n-1)d$

$= 22 + (n-1) \times -1$

$= 22 - n + 1$

$T_n = 23 - n$ ✓

$T_9 = 23 - 9$

$= 14$ (\$14 / head for 9th group of 10)

Extra cost $= 4 \times \$14$

$= \$56$ ✓

Q6 c) iii) 111^{th} person $\rightarrow 120^{th}$ person

$= 12^{th}$ group of 10

$T_{12} = 23 - 12$

$= \$11$ / head

101st person to 110^{th} person

$= 11^{th}$ grp of 10

$T_{11} = 23 - 11$

$= \$12$ / head

$\text{Cost} = 10 \times [\underbrace{\$22 + \$21 + \$20 + \ldots + \$12}] + \$11 \times 5$ ✓

$= 10 \times \frac{n}{2}(a + l) + 55$

$= 10 \times \frac{11}{2}(22 + 12) + 55$

$= 55 \times 34 + 55$

$= \$1925$ ✓

OR find cost for 12 whole groups − 5 people

$\text{Cost} = \frac{n}{2}(a+l)$

$= \frac{12}{2}(220 + 11 \times 10) - 5 \times 11$

$= \$1925$

iv) Cost + 25% profit $= 1.25 \times \$1925$

$= \$2406.25$ ✓

Av. charge / head $= \dfrac{\$2406.25}{115}$ ✓

$= \$20.95$

QUESTION 7

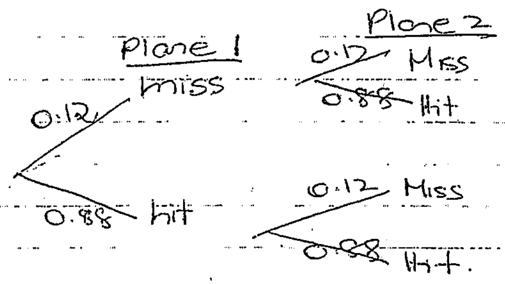
a) $P\left(\begin{smallmatrix}\text{hit by}\\ \text{Missile}\end{smallmatrix}\right) = P(M) = 0{\cdot}85 \quad P(\tilde{M}) = 0{\cdot}15$

$P\left(\begin{smallmatrix}\text{hit by}\\ \text{gun}\end{smallmatrix}\right) = P(G) = 0{\cdot}2 \quad P(\tilde{G}) = 0{\cdot}8$

i) $P\left(\begin{smallmatrix}\text{plane}\\ \text{missed by both}\end{smallmatrix}\right) = 0{\cdot}15 \times 0{\cdot}8$

$= 0{\cdot}12$

ii)



$P(\text{at least 1 hit}) = 1 - P(MM)$

$= 1 - (0{\cdot}12)^2$

$= 0{\cdot}9856$

b) i) $\int_0^4 3^x\,dx \doteqdot \frac{h}{2}\left[f(x_1) + f(x_3) + 2f(x_2)\right]$

$= \frac{2}{2}\left[f(0) + f(4) + 2f(2)\right]$ $\quad h = \frac{b-a}{n} = \frac{4-0}{2} = 2$

$= \left[3^0 + 3^4 + 2 \times 3^2\right]$

$= 100$

ii) Shaded region $\doteqdot$ area rectangle $- 100$

$= 81 \times 4 - 100$

shaded area $\doteqdot 224\ u^2$

Q7

c) i) $y = x^2 - 2x - 3$ ①

$y = x + 7$ ②

$\therefore x^2 - 2x - 3 = x + 7$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\therefore x = 5, \quad x = -2.$

when $x = 5$, $y = 12 \Rightarrow (5, 12)$

when $x = -2$, $y = 5 \Rightarrow (-2, 5)$

$\therefore A(-2, 5) \quad B(5, 12)$

ii) $A = \int_{-2}^{5} (x+7) - (x^2 - 2x - 3)\,dx$

$= \int_{-2}^{5} -x^2 + 3x + 10\,dx$

$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 10x\right]_{-2}^{5}$

$= \left(-\frac{125}{3} + \frac{75}{2} + 50\right) - \left(\frac{8}{3} + 6 - 20\right)$

$= 57\frac{1}{6}\ u^2$

QUESTION 8.

a) i) $P\left(\begin{smallmatrix}\text{break}\\\text{code}\end{smallmatrix}\right) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

$= \frac{1}{1000}$ ✓

4 or 6

ii) $P(\text{breakcode}) = \frac{1}{10} \times \frac{1}{2} \times \frac{1}{10}$ ✓

$= \frac{1}{200}$ ✓

b) GS: $T_2 = 6$ $\quad \frac{T_7}{T_6} = 3$

i) $\frac{\cancel{ar^6}}{\cancel{ar^5}} = 3$

$r = 3$ ✓

ii) $T_2 = ar = 6$

$a \times 3 = 6$

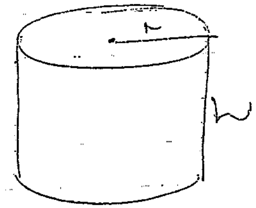
$a = 2$ ✓

iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{12} = \frac{\cancel{2}(3^{12} - 1)}{\cancel{2}}$

$S_{12} = 531\ 440$ ✓

=

c) 

i) $SA = 2\pi r^2 + 2\pi r h$

$30 = 2\pi r^2 + 2\pi r h$ ✓

$15 = \pi r(r + h)$

$\frac{15}{\pi r} = r + h$

$\boxed{h = \frac{15}{\pi r} - r}$ ✓

ii) $V = \pi r^2 h$

$= \pi r^2 \left(\frac{15}{\pi r} - r\right)$ must be shown!

$V = 15r - \pi r^3$ ✓

iii) $\frac{dV}{dr} = 15 - 3\pi r^2$

$\frac{d^2V}{dr^2} = -6\pi r$

for ~~stat~~ stat^y ~~pts~~ values $\frac{dV}{dr} = 0$.

✓

$15 - 3\pi r^2 = 0$.

$3\pi r^2 = 15$

$\pi r^2 = 5$

$r = \pm\sqrt{\frac{5}{\pi}}$

$r = \sqrt{\frac{5}{\pi}}$ $(r > 0)$ ✓

test $r = \sqrt{\frac{5}{\pi}}$, $\frac{d^2V}{dr^2} < 0 \Rightarrow \max^m$

$\therefore$ for $\max^m$ vol, $r = \sqrt{\frac{5}{\pi}}$

$= 1.2615\ldots$ m ✓

$r = 12.6$ cm (nearest cm)

QUESTION 9:

a) 20cm Now; 1m Yr 1; 4yr 48cm; Yr 3 176.8

1st years growth

$a = 80\text{cm}$ $\quad r = 0.6$, $n = 3$

$T_3 = ar^2$

$= 80 \times 0.36$

$= 176.8$ cm. ✓

OR

$S_3 = \frac{a(1-r^n)}{1-r}$

$= \frac{80\text{cm}(1-0.6^3)}{0.4}$

$= 156.8$

Height $= 20 + 156.8$

$= 176.8$ cm.

ii) "Growth Max" $= \frac{a}{1-r}$

$= \frac{80}{1-0.6}$

$= 200$ ✓

$\therefore$ Max height $= 200 + 20$

$= 220$ cm. ✓

b) $f(x) = x^2 + \frac{1}{x^2}$

i) domain: all reals, $x \neq 0$

ii) $f(-x) = (-x)^2 + \frac{1}{(-x)^2}$

$= x^2 + \frac{1}{x^2}$

$\therefore f(x) = f(-x)$

$\therefore$ even function

✓ must have domain + even fn

iii) as $x \to 0$ $\quad f(x) \to \infty$ ✓

iv) $f(x) = x^2 + \frac{1}{x^2}$

$= x^2 + x^{-2}$

$f'(x) = 2x - 2x^{-3}$

$f''(x) = 2 + 6x^{-4}$

$= 2 + \frac{6}{x^4}$

for stat'y values $f'(x) = 0$

$2x - \frac{2}{x^3} = 0$. ✓

$2x^4 - 2 = 0$

$2(x^4 - 1) = 0$

$x^4 = 1$

$x = \pm 1$ ✓

if $x = 1$, $y = 2$

$x = -1$, $y = 2$

$f''(1) > 0 \Rightarrow$ min^m at $(1, 2)$

$f''(-1) > 0 \Rightarrow$ min^m at $(-1, 2)$

1 for testing max/min

1 for y values.

v) for concave up $f''(x) > 0$.

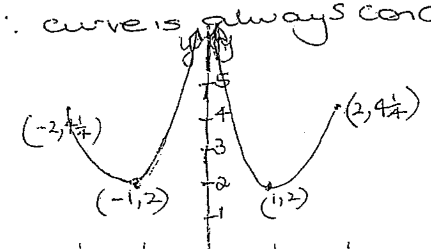
$2 + \frac{6}{x^4} > 0$.

x^4 is always +ve

$2 +$ ve > 0 ✓

$\therefore$ curve is always concave up.

vi)



1 for stat'y pts + end pts marked

1 for shape / axes labelled + asymptote.

QUESTION 10

a) $P_{2000} = 100\,000$

i) $P_{2001} = 100\,000 + 12\% \times 100\,000 - E$ ✓

$= 112\,000 - E$

ii) $P_{2002} = P_{2001} + 12\% \times P_{2001} - E$

$= P_{2001}(1.12) - E$

$= [112000 - E]\,1.12 - E$ ✓

$= 125440 - 1.12E - E$

$= 125440 - 2.12E$

$= 125440 - E(1 + 1.12)$

iii) $P_{2006} = 100\,000 \times 1.12^6 - E(\underbrace{1 + 1.12 + \ldots + 1.12^5}_{\text{G.S. } a=1,\ r=1.12,\ n=6})$ ✓

$140\,000 = 100000 \times 1.12^6 - E\,(S_6)$

$= 100000 \times 1.12^6 - E\left[\dfrac{1(1.12^6 - 1)}{0.12}\right]$ ✓

$E\left(\dfrac{1.12^6 - 1}{0.12}\right) = 100\,000 \times 1.12^6 - 140000$ ✓

$E = 7070.97$

$E = 7071$ (to nearest whole reason) ✓

b) i) $\dfrac{d}{dx}(xe^x) = v\cdot\dfrac{du}{dx} + u\cdot\dfrac{dv}{dx}$

$= e^x \cdot 1 + x\cdot e^x$ ✓

$= e^x + xe^x$

Q10

b) ii) $\int \dfrac{d}{dx} xe^x = \int e^x + xe^x\, dx$

$xe^x = \int e^x dx + \int xe^x dx$

$= e^x + \int xe^x dx$

$\therefore \int xe^x dx = (xe^x - e^x)\, dx$ ✓

Now $\int_0^2 xe^x dx = \left[xe^x - e^x\right]_0^2$

$= [(2e^2 - e^2) - (0 - 1)]$ ✓

$= e^2 + 1$ (as req'd)

$V = \pi \int y^2 dx$

$= \pi \int_0^2 (e^x + x)^2 dx$

$= \pi \int_0^2 e^{2x} + 2xe^x + x^2 dx$ ✓

$= \pi\left[\int_0^2 (e^{2x} + x^2)\, dx + \int_0^2 2xe^x dx\right]$

$= \pi\left\{\left[\dfrac{e^{2x}}{2} + \dfrac{x^3}{3}\right]_0^2 + \underbrace{2(e^2+1)}_{\text{from (ii)}}\right\}$ ✓

$= \pi\left[\left(\dfrac{e^4}{2} + \dfrac{8}{3}\right) - \left(\dfrac{1}{2} + 0\right) + 2(e^2+1)\right]$

$= \pi\left[\dfrac{e^4}{2} + 2e^2 + 4\tfrac{1}{6}\right] u^3$ ✓