Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2007

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

Time allowed: 3 hours
(Plus 5 minutes reading time)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1,2,3	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, probability and series	4,5,6	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	7,8,9	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new page

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1: (12 marks) MARKS
Use a SEPARATE writing booklet

a) Factorise completely $ab - a - bx + x$ 2

b) Simplify $|2| + |-5|$ 1

c) Find integers a and b such that

$$\frac{1}{\sqrt{3}+2} = a\sqrt{3} + b$$ 2

d) Find the value of $\cos\frac{\pi}{8}$, correct to 3 decimal places. 2

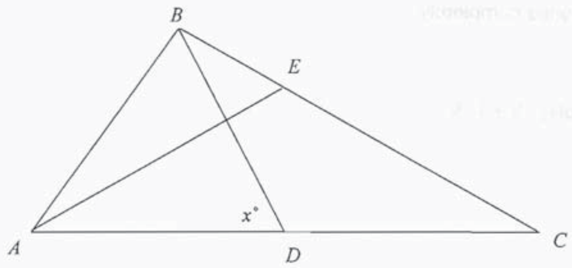
e) Solve $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0^\circ \le \theta \le 360^\circ$ 2

f) Graph on a number line the values of x for which $|x-2| \ge 1$ 2

g) Simplify $\frac{2^{n+1} - 2^{n}}{2^{2n+1} - 2^{2n}}$ 1

Question 2: (12 marks) MARKS
Use a SEPARATE writing booklet

a)



Triangle ABC is a right angled triangle with $\angle ABC = 90^\circ$. D is a point on AC such that $AB = BD = DC$. E lies on BC such that AE bisects $\angle BAD$. Let $\angle ADB = x^\circ$.

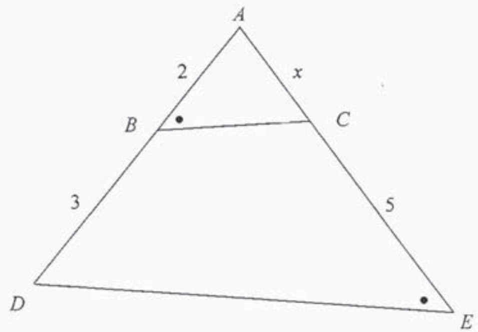
Copy the diagram into your booklet showing this information.

i) Show that $\angle DBC = (2x - 90)^\circ$ 2

ii) Hence find the value of x. 1

iii) Show that triangle AEC is isosceles 2

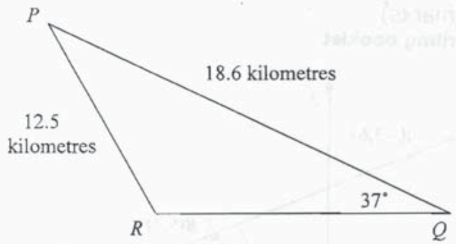
b)



In the diagram above, $\angle ABC = \angle AED$, $AB = 2$, $BD = 3$, $CE = 5$ and $AC = x$. Copy the diagram into your booklet

i) Prove that triangle ABC is similar to triangle AED 3

ii) Hence find the value of x 2

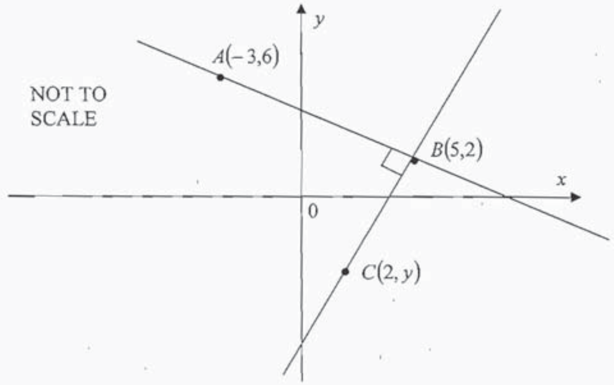
c) 

In the diagram above, $PQ = 18.6$ kilometres, $PR = 12.5$ kilometres and $\angle PQR = 37^\circ$. $\angle PRQ$ is obtuse. Find the size of $\angle PRQ$ correct to the nearest minute. 2

Question 3: (12 marks)

MARKS

Use a SEPARATE writing booklet



The diagram shows the origin 0 and the points $A(-3,6)$, $B(5,2)$ and $C(2,y)$.

The lines AB and BC are perpendicular.

Copy or trace this diagram onto your writing sheet.

a) Show that A and B lie on the line $x + 2y = 9$ 2

b) Show that the length of AB is $4\sqrt{5}$ units. 1

c) Find the perpendicular distance from 0 to AB. 1

d) Find the area of triangle $A0B$. 1

e) Show that C has coordinates $(2,-4)$. 2

f) Does the line AC pass through the origin? Explain. 2

g) The point D is not shown on the diagram. The point D lies on the x axis and $ABCD$ is a rectangle. Find the coordinates of D. Note: D is **not** the point of intersection of line AB extended to meet the x axis. 2

h) On your diagram, shade the region satisfying the inequality $x + 2y \geq 9$. 1

Question 4: (12 marks) MARKS

Use a SEPARATE writing booklet

a)

i) On the same set of axes, sketch the graphs of $y=\sqrt{4-x^2}$ and $y=|x+2|$. 2

ii) Hence, shade the region bounded by $y=\sqrt{4-x^2}$ and $y=|x+2|$ 1

iii) Using the diagram or otherwise, explain why

$$\int_{-2}^{0}\sqrt{4-x^2}-(x+2)dx=\pi-2$$

b) From seven cards showing numbers

1, 1, 2, 3, 3, 3, 3

two cards are chosen at random and without replacement. What is the probability that

i) both cards show a 1 1

ii) the sum of the two numbers on the cards is greater than 4? 2

c)

The spinner shown above is used in a game. Once spun, it is equally likely to stop at any one of the letters **A**, **E I**, **O** or **U**.

i) If the spinner is spun twice, find the probability that it stops on the same letter twice. 2

ii) How many times must the spinner be spun for it to be 99% certain that it will stop on the letter **E** at least once? 2

Question 5: (12 marks) MARKS

Use a SEPARATE writing booklet

a) Solve $x^2+2x+1=4$. 2

b) A given parabola has a focus with coordinates $(2,-1)$ and a directrix with equation $y=3$. Find the equation of the parabola and state its focal length. 3

c) Find the exact solutions of $2\sin^2\theta-\sin\theta-1=0$ for $0\le\theta\le2\pi$. 4

d) The quadratic equation $x^2+mx+n=0$ has one root that is twice the other. Find the value of $\frac{m^2}{n}$. 3

Question 6: (12 marks)

Use a SEPARATE writing booklet

MARKS

a) The first three terms of an arithmetic series are 6+10+14+.....

Calculate the number of terms needed to give a sum of 390. 4

b) The geometric series $a + ar + ar^2 + ...$ has a second term of $\frac{1}{4}$ and has a limiting sum of 1.

i) Show that $a = 1 - r$ 1

ii) Solve a pair of simultaneous equations to find r 2

c) The sum of the first 8 terms of a geometric series is 17 times the sum of its first 4 terms. Find the common ratio. 3

d)

i) Write down the discriminant of $2x^2 - 3x + k$ 1

ii) For what values of k does $2x^2 - 3x + k$ have an equal real roots? 1

Question 7: (12 marks)

Use a SEPARATE writing booklet

MARKS

a) Differentiate with respect to x:

i) $\sqrt{x}$ (leave answer as surd) 1

ii) $x^3 e^{-3x}$ (answer in factored form) 2

iii) $\frac{\tan x}{2x+1}$ 2

b) Find $\int \frac{e^{2x}}{e^{2x}+4}\,dx$ 2

c) Evaluate $\int_0^{\frac{\pi}{4}} \left(\frac{1}{2}x + \cos 2x\right)dx$ 2

d) Find the equation of the normal to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$. 3

Question 8: (12 marks) MARKS

Use a SEPARATE writing booklet

a) The table shows the values of a function $f(x)$ for five values of x. 2

x	2	2.5	3	3.5	4
$f(x)$	3.7	1.2	9.8	4.1	2.7

Use Simpson's Rule with these five values to find an approximation to

$\int_2^4 f(x)dx$ (Answer to 2 decimal places)

b) Consider the curve given by $y = 6x^2 - x^3$

i) Find the coordinates of the two stationary points 2

ii) Determine the nature of the stationary points 2

iii) Show that there exists a point of inflexion when $x = 2$ 1

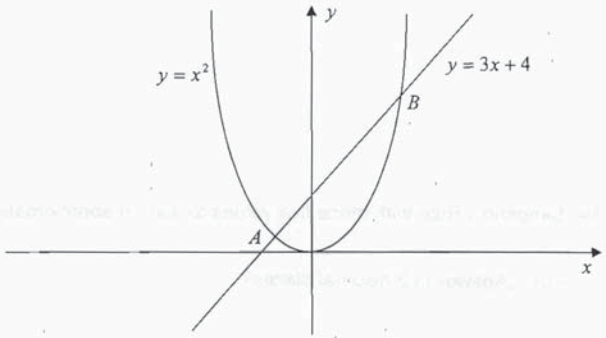
iv) Sketch the curve for the domain $-2 \le x \le 6$ 2

c) Find the area bounded by the curve $y = \sqrt{x+2}$, the x-axis and the line $x = 7$. 3

Question 9: (12 -marks) MARKS

Use a SEPARATE writing booklet

a)



i) The curve $y = x^2$ and the line $y = 3x + 4$ intersect at the points A and B as shown in the diagram above. 2

Find the x coordinates of the points A and B.

ii) Find the area bounded by the curve $y = x^2$ and the line $y = 3x + 4$. 3

b) A cylindrical container closed at both ends is made from a sheet of thin plastic. The surface area of the cylinder is 600π *centimetres*^{2}.

i) Show that the height h of the cylinder is given by the expression:

$h = \frac{300}{r} - r$, where r is the radius. 2

ii) Find an expression for the volume V in terms of r. 2

iii) Find the height of the container if the volume is to be a maximum. 3

Question 10: (12 marks) MARKS

Use a SEPARATE writing booklet

a) Sketch $y = 4\cos 3x$ for $0 \le x \le 2\pi$ 2

b) Find the sum of the series

$$\log x + \log x^2 + \log x^3 + \ldots\ldots + \log x^{10}$$ 2

c) Bronwyn borrows \$500 000 from a finance company to buy a house. She pays interest at 6% per annum, calculated quarterly on the balance still owing. The loan is to be repaid at the end of 20 years with equal quarterly repayments of \$P.

Let A_n equal the amount owing after the n^{th} repayment.

i) Show that after the first quarterly repayment of \$P Bronwyn owes an amount equivalent to A_1=\$507 500 - \$P 1

ii) Find an expression for the amount still owing after 3 repayments of \$P. 1

iii) Find the value of \$P to the nearest cent. 3

d) Find the volume generated when the curve $y = \sqrt{\cot x}$ is rotated about the x-axis between $x = \frac{\pi}{3}$ and $x = \frac{\pi}{4}$. Leave your answer in exact form. 3

FORT STREET HS

2007 TRIAL HSC SOLUTIONS

Question 1

(a) $ab - a - bx + x$

$= a(b-1) - x(b-1)$ ✓

$= (b-1)(a-x)$ ✓

(b) $|2| + |-5| = 2 + 5$

$= 7$ ✓

(c) $\frac{1}{\sqrt{3}+2} = \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$ ✓

$= \frac{\sqrt{3}-2}{3-4}$

$= -(\sqrt{3}-2)$

$= -\sqrt{3}+2$

which is in the form ✓ $a\sqrt{3}+b$ where $a=-1$ and $b=2$

(d) $\cos\frac{\pi}{8} = 0.9238795\ldots$ ✓

$= 0.924$ ✓

(correct to 3 dp)

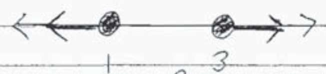
(e) $\theta = 180° - 30°$ or $360° - 30°$ ✓

$= 150°$ or $330°$ ✓

f) $x - 2 \geq 1$ $\quad -x + 2 \geq 1$

$x \geq 3$ $\quad -x \geq -1$

OR $x \leq +1$ ✓



✓

(g) $\frac{2^{n+1} - 2^n}{2^{2n+1} - 2^{2n}}$

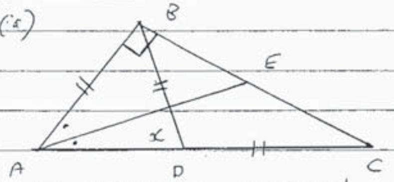
$= \frac{2^n(2-1)}{2^{2n}(2-1)}$

$= 2^{n-2n}$

$= 2^{-n}$ ✓

Question 2

(a)



i) $x = \angle ADB = \angle DAB$ (equal sides isos △) ✓

∴ $\angle ABD = 180° - 2x°$ (angle sum △) ✓

∴ $\angle DBC = 90° - (180 - 2x)°$

$= (2x - 90)°$ ✓

ii) $\angle DBC = \angle BCD = (2x - 90)°$

(Equal sides isos △)

∴ $x = 2(2x - 90)$ (ext ∠)

$x = 4x - 180$

$-3x = -180$

$x = 60$ ✓

iii) $\angle BAD = 60°$ (see i)

$\angle EAD = 30°$ (AE bisector) ✓

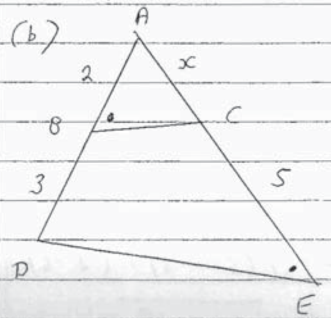
$\angle DBC = \angle DCB$

$= (2x - 90)°$

$= 30°$ ✓

∴ △AEC isosceles

(b)



i) In △s ABC, AED

∠A is common ✓

$\angle ABC = \angle AED$ (data) ✓

∴ △ABC ||| △AED

(equiangular) ✓

ii) $\frac{AB}{AE} = \frac{AC}{AD}$

(Corresponding sides in sim △s)

$\frac{2}{x+5} = \frac{x}{5}$ ✓

$x^2 + 5x - 10 = 0$

$x = \frac{-5 \pm \sqrt{25+40}}{2}$

As $x > 0$

$x = \frac{-5+\sqrt{65}}{2}$ ✓

c) $\frac{\sin R}{18.6} = \frac{\sin 37}{12.5}$ ✓

$\sin R = 0.895\ldots$

$\angle PRQ = 180° - 63°34'$

$= 116°26'$ ✓

Question 3

a) $x + 2y = 9$

Point A $(-3, 6)$

test by substitution

$-3 + 2 \times 6 = 9$ ✓

Point B $(5, 2)$

test by substitution

$5 + 2 \times 2 = 9$ ✓

b) $AB = \sqrt{(5--3)^2 + (2-6)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= \sqrt{16} \times \sqrt{5}$ ✓

$= 4\sqrt{5}$ units

c) $\frac{|1\times 0 + 2\times 0 - 9|}{\sqrt{1^2+2^2}}$

$= \frac{9}{\sqrt{5}}$ units ✓

d) Area $= \frac{1}{2} \times 4\sqrt{5} \times \frac{9}{\sqrt{5}}$

$= 18$ units^2 ✓

e) grad AB $= \frac{2-6}{5+3} = -\frac{1}{2}$

grad of BC is thus 2 ✓

Since the product of the gradients of perpendicular lines is -1

grad BC $= \frac{y-2}{2-5}$

$= \frac{y-2}{-3}$

Solve $\frac{y-2}{-3} = 2$

$y = -6 + 2$

$= -4$ ✓

∴ C has co-ords $(2, -4)$

f) gradient of AO $= \frac{6}{-3}$

$= -2$ ✓

gradient of OC $= \frac{-4}{2}$

$= -2$

∴ A, O, C are collinear ✓

∴ AC passes through O

g) Let D have co-ordinates $(x, 0)$ gradient of AD × gradient of AB $= -1$

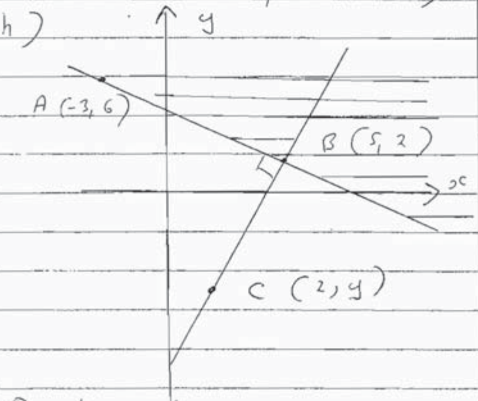
$\frac{6}{-3-x} \times -\frac{1}{2} = -1$ ✓

$-6 = 6 + 2x$

$x = -6$ ✓

∴ D is the point $(-6, 0)$

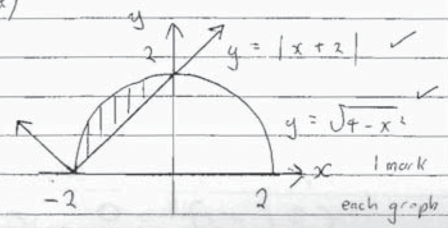
h)



Question 4

a)

i)



ii) shaded region ✓

iii) $\int_{-2}^{0} \left[\sqrt{4-x^2} - (x+2)\right] dx$

= Area of shaded region

$= \frac{1}{4}$ Area of circle − Area of Δ

$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times 2 \times 2$ ✓ ✓

$= \frac{1}{4}\pi \times 2^2 - 2$

$= \pi - 2$

b)

i) $P(\text{both } 1) = \frac{2}{7} \times \frac{1}{6}$

$= \frac{1}{21}$ ✓

ii) $P(\text{sum greater than } 4)$

$= P(2+3) + P(3+2) + P(3+3)$

$= \frac{1}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{1}{6} + \frac{4}{7} \times \frac{3}{6}$ ✓

$= \frac{4}{42} + \frac{4}{42} + \frac{12}{42}$

$= \frac{20}{42} = \frac{10}{21}$ ✓

c)

i) $P(\text{any letter twice})$

$= 5 \times P(AA)$

$= 5 \times \frac{1}{5} \times \frac{1}{5}$ ✓

$= \frac{1}{5}$ ✓

ii) $P(\bar{E}) = 1 - \frac{1}{5} = \frac{4}{5}$

Solve $1 - \left(\frac{4}{5}\right)^n = \frac{99}{100}$

$1 - 0.8^n = 0.99$ ✓

$0.8^n = 0.01$

$n = \frac{\log_e 0.01}{\log_e 0.8}$

$= 20.63$

$= 21$ (n is an integer)

Question 5

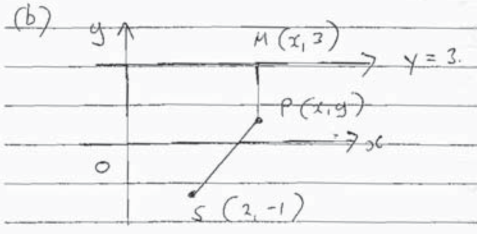
(a) $x^2 + 2x + 1 = 4$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$ ✓

$x = -3$ or 1 ✓

(b)



$SP^2 = PM^2$ for parabola

$(x-2)^2 + (y-(-1))^2 = (3-y^2)$ ✓

$(x-2)^2 + y^2 + 2y + 1 = 9 - 6y + y^2$

$(x-2)^2 = -8y + 8$ ✓

$(x-2)^2 = -8(y-1)$

is of the form

$(x-h)^2 = -4a(y-k)$

∴ Focal length is 2 ✓

(c) $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$ ✓

$\sin\theta = -\frac{1}{2}$ or $\sin\theta = 1$ ✓

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$ ✓ (1 or 2 solns) ✓✓ (3 solns)

(d) $x^2 + mx + n = 0$

Let the roots be α and 2α

The sum of the roots is

$3\alpha = -m$ ✓

The products of the roots is

$2\alpha^2 = n$ ✓

$\frac{m^2}{n} = \frac{9\alpha^2}{2\alpha^2}$

$= \frac{9}{2}$ ✓

Question 6

(a) $6 + 10 + 14 + \ldots$

This is an arithmetic progression with $a = 6$ and $d = 4$ ✓

$S_n = \frac{n}{2}[2a + (n-1)d]$

$390 = \frac{n}{2}[12 + (n-1)4]$

$780 = 12n + 4n^2 - 4n$ ✓

$4n^2 + 8n - 780 = 0$

$n^2 + 2n - 195 = 0$

$(n+15)(n-13) = 0$ ✓

$n = 13$ or -15

∴ 13 terms required for the sum to be 390 ✓

(b) i) $\frac{a}{1-r} = 1$

$a = 1 - r$ (1) ✓

ii) $T_2 = ar = \frac{1}{4}$ (2)

Substituting (1) into (2)

$(1-r)r = \frac{1}{4}$ ✓

$r - r^2 = \frac{1}{4}$

$4r^2 - 4r + 1 = 0$

$(2r-1)^2 = 0$ ✓

$r = \frac{1}{2}$

(c.) $S_8 = 17 S_4$

$\frac{a(r^8 - 1)}{r - 1} = \frac{17a(r^4 - 1)}{r - 1}$

$r^8 - 1 = 17r^4 - 17$

$r^8 - 17r^4 + 16 = 0$

$(r^4 - 1)(r^4 - 16) = 0$

$r = \pm 1 \quad r = \pm 2$

(d) i) $\Delta = b^2 - 4ac$

$= 9 - 8k$ ✓

ii) $9 - 8k \le 0$

$k = \frac{9}{8}$ ✓

Question 7

(a) (i) $\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx} x^{\frac{1}{2}}$

$= \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}}$ ✓

(ii) $\frac{d}{dx}(x^3 e^{-3x})$

$= x^3 - 3e^{-3x} + 3x^2 e^{-3x}$ ✓

$= 3x^2 e^{-3x}(1 - x)$ ✓

(iii) $\frac{d}{dx}\left(\frac{\tan x}{2x + 1}\right)$ ✓

$= \frac{(2x + 1)\sec^2 x - \tan x \cdot 2}{(2x + 1)^2}$

$= \frac{2x\sec^2 x + \sec^2 x - 2\tan x}{(2x + 1)^2}$

✓ ✓

(b) $\int \frac{e^{2x}}{e^{2x} + 4}\, dx = \frac{1}{2}\ln(e^{2x} + 4) + c$

(c) $\int_0^{\frac{\pi}{4}} \frac{1}{2}x + \cos 2x \, dx$

$= \left[\frac{x^2}{4} + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$ ✓

$= \frac{\frac{\pi^2}{16}}{4} + \frac{1}{2}$

$= \frac{\pi^2 + 32}{64}$ ✓

(d) $y = x \sin x$

$\frac{dy}{dx} = x\cos x + \sin x$

when $x = \frac{\pi}{2}$ grad tan $= 1$

grad normal $= -1$ ✓

∴ the equation of normal at $(\frac{\pi}{2}, \frac{\pi}{2})$ is

$y - \frac{\pi}{2} = -1(x - \frac{\pi}{2})$ ✓

$y - \frac{\pi}{2} = -x + \frac{\pi}{2}$

$x + y - \pi = 0$ ✓

Question 8

a) $\int_2^4 f(x)\,dx \doteqdot \frac{\frac{1}{2}}{3}\left[f(2) + 4\{f(2.5) + f(3.5)\} + 2f(3) + f(4)\right]$

$= \frac{1}{6}(3.7 + 2.7 + 4 \times (1.2 + 4.1) + 2 \times 9.8)$ ✓

$= \frac{1}{6}(6.4 + 4 \times 5.3 + 19.6)$

$= 7.87$ (2 dec pl's) ✓

b) $y = 6x^2 - x^3$

(i) Stationary points occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x - 3x^2 = 0$

$3x(4 - x) = 0$ ✓

Stationary points occur when $x = 0$ and $x = 4$

Stationary points are $(0, 0)$ $(4, 32)$ ✓

(ii) $\frac{d^2y}{dx^2} = 12 - 6x$

when $x = 0$ $\frac{d^2y}{dx^2} = 12 > 0$

∴ min turning pt at $x = 0$ ✓

when $x = 4$ $\frac{d^2y}{dx^2} = -12 < 0$ ✓

∴ max turning pt at $x = 4$

minimum turning point $(0, 0)$

maximum turning point $(4, 32)$

(iii) Point of inflexion when $\frac{d^2y}{dx^2} = 0$ and concavity changes

$\frac{d^2y}{dx^2} = 12 - 6x = 0$

$6x = 12$ ✓

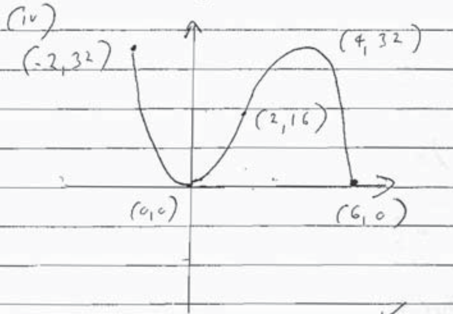
$x = 2$

x	< 2	2	> 2
$\frac{d^2y}{dx^2}$	+	0	−

∴ Change in concavity and point of inflexion at $x = 2$

Point of inflexion is $(2, 16)$

(iv)



1 mk correct shape ✓

1 mk endpoints ✓

(c) Area $= \int_{-2}^{7} (x + 2)^{\frac{1}{2}} dx$ ✓

$= \left[\frac{2}{3}(x + 2)^{\frac{3}{2}}\right]_{-2}^{7}$ ✓

$= \frac{2}{3} \cdot 27$

$= 18$ units^2 ✓

Question 9

a) i) Solving simultaneously to find the points of intersection between $y = x^2$ and $y = 3x + 4$

$x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$ ✓

$(x + 1)(x - 4) = 0$

$x = -1, \quad x = 4$ ✓

at A $x = -1$ and at B $x = 4$

(ii) Area $= \int_{-1}^{4}(3x+4)\,dx - \int_{-1}^{4} x^2\,dx$ ✓

$= \left[\frac{3x^2}{2} + 4x - \frac{x^3}{3}\right]_{-1}^{4}$ ✓

$= \left(\frac{3\times4^2}{2} + 4\times4 - \frac{4^3}{3}\right) - \left(\frac{3\times(-1)^2}{2} + 4\times-1 - \frac{-1^3}{3}\right)$

$= 18\frac{2}{3} + 2\frac{1}{6}$

$= 20\frac{5}{6}$ units^2 ✓

b) i) $SA = 2\pi r^2 + 2\pi r h$

$600\pi = 2\pi r^2 + 2\pi r h$ ✓

$r^2 + rh = 300$

$h = \frac{300 - r^2}{r}$

$= \frac{300}{r} - r$ ✓

ii) $V = \pi r^2 h$

$= \pi r^2\left(\frac{300}{r} - r\right)$ ✓

$= \pi(300r - r^3)$ ✓

iii) $\frac{dV}{dr} = \pi(300 - 3r^2) = 0$

$r = 10$ ✓

$\frac{d^2V}{dr^2} = -6\pi r < 0$ ✓

$\therefore$ Max V at $r = 10$

$h = \frac{300}{10} - 10$

$= 20$ cm ✓

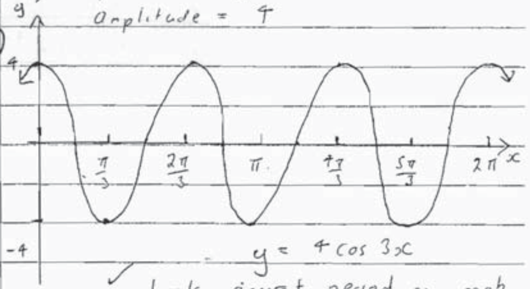
when V is max

c)

Question 10

a) period $= \frac{2\pi}{3}$

Amplitude $= 4$



✓ 1 mk correct period on graph

✓ 1 mk correct amplitude on graph

b) arithmetic progression

$a = \log x \quad d = \log x$ ✓

$S_{10} = \frac{10}{2}\left[2\log x + 9\log x\right]$ ✓

$= 55\log x$

c)

i) $A_1 = 500\,000(1.015) - \$P$

$= \$507\,500 - \$P$ ✓

ii) $A_2 = (500\,000(1.015) - \$P)1.015 - \$P$

$A_3 = (500000(1.015)^2 - \$P(1 + 1.015))1.015 - \$P$

$= 500000(1.015)^3 - \$P(1 + 1.015 + 1.015^2)$ ✓

(iii) 20 years = 80 repayments

$A_{80} = \$500\,000(1.015)^{80} - \$P(1 + 1.015 + 1.015^2 + \ldots + 1.015^{79})$

$\$500\,000(1.015)^{80} = \$P(1 + 1.015 + 1.015^2 + \ldots + 1.015^{79})$ ✓

$\$P = \frac{500000(1.015)^{80}}{1 + 1.015 + 1.015^2 + \ldots + 1.015^{79}}$

$= \frac{500000(1.015)^{80}}{\frac{(1.015^{80} - 1)}{0.015}}$

$= \frac{500000(1.015)^{80} \times 0.015}{1.015^{80} - 1}$ ✓

$= \$10\,774.16$ (nearest cent) ✓

(d) $V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} y^2\,dx = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot x\,dx$

$= \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos x}{\sin x}\,dx$ ✓

$= \pi\left[\ln(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ ✓

$= \pi\left[\ln\left(\sin\frac{\pi}{3}\right) - \ln\left(\sin\frac{\pi}{4}\right)\right]$

$= \pi\left[\ln\frac{\sqrt{3}}{2} - \ln\frac{1}{\sqrt{2}}\right]$

$= \pi\ln\frac{\sqrt{6}}{2}$ units^3 ✓