Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2008

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1,2	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and series	3,4,5	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	6,7,8	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	9, 10	

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**	**%**
Marks	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/12**	**/120**	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

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Question 1 **Start a new booklet**

(a) Evaluate $\dfrac{2.4\times\sqrt{30}}{24.9}$ correct to 3 significant figures. **1**

(b) Solve $x^2-3=3x+1$ **2**

(c) Express $\dfrac{5}{3-2\sqrt{3}}$ with a rational denominator. **2**

(d) Solve and graph on the number line $|3x-1|<8$. **3**

(e) A patient in hospital is fed intravenously (into the vein) 3.6 litres of fluid per 24 hours. If there are 15 drops of fluid per mL, find how many drops per minute the patient receives. **2**

(f) Simplify $\dfrac{2}{x(x-3)}-\dfrac{1}{x}$ **2**

Question 2 **Start a new booklet**

The line L has equation $x+2y=5$ and P is the point $(2,4)$.

(i) On a number plane, mark the origin O, the point P and draw the line L. **1**

(ii) Find the midpoint M, of the interval OP. **1**

(iii) Show M lies on the line L. **1**

(iv) Find the gradients of the line OP and the line L. **2**

(v) Show the line L is the perpendicular bisector of the interval OP. **2**

(vi) Line L meets the $x-$axis at Q. Find the co-ordinates of Q. **1**

(vii) A line is drawn through O parallel to PQ and it meets line L in R.
Find the equation of ~~QR~~.OR. **2**

(viii) Explain why PQOR is a rhombus. **2**

Question 3 **Start a new booklet**

(a)

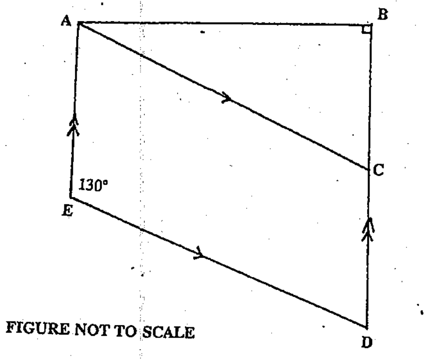


FIGURE NOT TO SCALE

In the diagram $AE \parallel BD$ and $AC \parallel ED$, $\angle AED = 130^0$ and $\angle ABC = 90^0$.

(i) Copy this diagram onto your answer sheet.

(ii) Find the size of $\angle BAC$ giving reasons. **2**

(b) Differentiate

(i) xe^{3x} **2**

(ii) $\dfrac{2x^4-3}{x^2}$ **2**

(c) (i) Find the primitive function of $\dfrac{1}{3x^2}$ **1**

(ii) Find exactly in simplest form $\int_2^3 \dfrac{x}{x^2-1}$ **2**

(d) Find the range of values of k if the equation $4x^2 - kx + 1 = 0$ has no real roots. **3**

Question 4 **Start a new booklet**

(a) If α and β are the roots of the equation $(3x-2)^2 + 4 = 0$

Find **(i)** $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $3\alpha^2 + 3\beta^2$ **2**

(b) An arithmetic progression has a first term 1 and last term 14. The sum of the series is 90.

(i) Find the number of terms in the series. **2**

(ii) Show that the common difference is $\frac{13}{11}$. **2**

(c) Two dice are rolled. The score for the roll is given by the difference between the numbers on the uppermost faces (e.g. if the numbers are 2 and 6, the score is 4).

Find the probability that the score will be

(i) 0 **2**

(ii) At least 3. **2**

Question 5 **Start a new booklet**

(a) If $\log_x 128 = \frac{7}{3}$, find x. 1

(b) **(i)** Sketch the graph of $y = 5\cos\frac{x}{2}$ for $-360^0 \le x \le 360^0$. 2

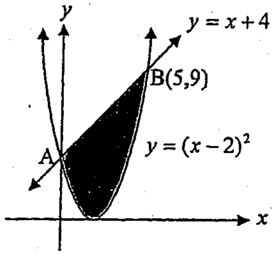
(ii) Mark clearly on your graph the point or points where

$5\cos\frac{x}{2} = -1$. 1

(iii) Calculate the value(s) of x which satisfy the equation

$5\cos\frac{x}{2} = -1$. Express your answer(s) to the nearest minute. 2

(c)



NOT TO SCALE

The graphs of $y = (x-2)^2$ and $y = x+4$ intersect at the point A and the point B(5,9).

(i) Show that the point A lies on the y-axis. 2

(ii) Write down the two inequalities whose intersection describes the shaded area shown in the diagram above. 1

(iii) Find the area of the shaded regions bounded by the graphs of $y = (x-2)^2$ and $y = x+4$. 3

Question 6 **Start a new booklet**

(a) For the curve $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 12$,

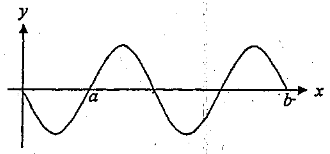
(i) Find any turning points and determine their nature. 6

(ii) Find any points of inflexion. 2

(iii) Sketch the curve clearly labelling points of intersection with the axes and the features you have found in (i) and (ii). 1

(iv) For what value of x is the curve concave upwards? 1

(b)



The graph of $y = 2\cos(2x + \frac{\pi}{2})$ is shown over two complete cycles.

(i) Find the value of b. 1

(ii) Given that $\int_0^a 2\cos(2x+\frac{\pi}{2})dx = -8$, find $\int_0^b 2\cos(2x+\frac{\pi}{2})\,dx$ without using calculus. 1

Question 7 **Start a new booklet**

(a) For the curve $y^2 - 2y - 6x = 0$ find

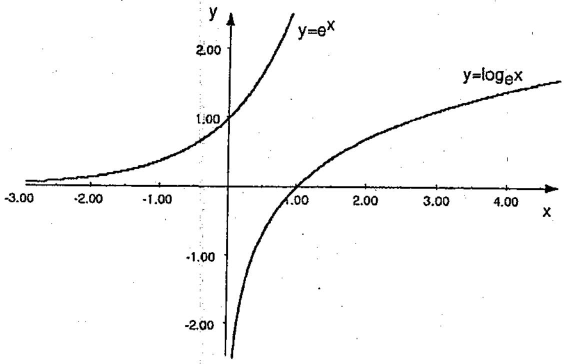
(i) the co-ordinates of the focus 3

(ii) the equation of the directrix. 1

(b) Prove that the line $y = 2x + c$ cuts the curve $y = x^2 + 6x + 7$ at two distinct points if $c > 3$. 2

(c) Evaluate $\sum_{r=1}^{\infty} 3^{-r}$ 2

(d) The graphs show the two functions $y = e^x$ and $y = \log_e x$.



(i) With reference to the graph above, explain how the two graphs $y = e^x$ and $y = \log_e x$ are related to each other. 1

(ii) Show that the equation of the tangent drawn at $x = 2$ on the graph of $y = \log_e x$ is given by the equation

$x - 2y - 2 + \log 4 = 0$ 2

(iii) find the acute angle that the tangent in **(ii)** makes with the x - axis, to the nearest degree. 1

Question 8 **Start a new booklet**

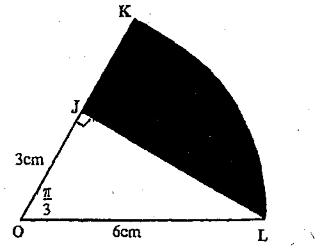


FIGURE NOT TO SCALE

(a) In the diagram KL is an arc of a circle with centre 0 and radius 6cm. OJ =3cm, $\angle$KOL=$\frac{\pi}{3}$ and JL $\perp$ OK.

Calculate the perimeter of the shaded region JKL. Give your answer correct to 1 decimal place. 4

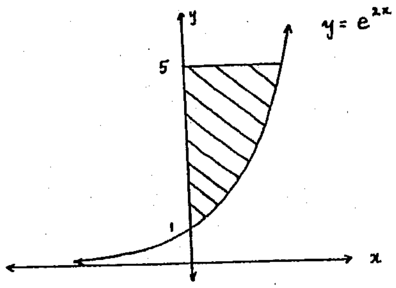
(b) **(i)** Copy and complete the table below for $f(x) = \left(\log_e \sqrt{x}\right)^2$, calculating each value correct to 3 decimal places. 1

x	1	2	3	4	5
$f(x)$	0	0.120			

(ii) Using Simpson's Rule with 5 function values, show that

$\int_1^5 \left(\log_e \sqrt{x}\right)^2 dx \doteqdot 1.22$ 1

(c)



The diagram above shows the region bounded by the curve $y = e^{2x}$, the y-axis and the line $y = 5$.

(i) Show that $x = \log_e \sqrt{y}$ 1

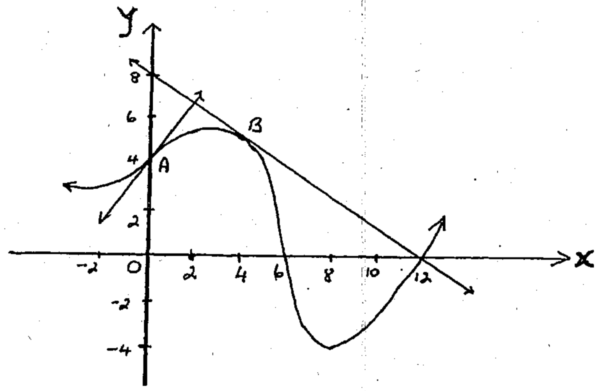
(ii) The shaded area is rotated about the y-axis. Write down the integral equal to the volume formed. 1

(iii) Evaluate the volume of this solid of revolution using the approximation in **Part (b) (ii)** above, leaving your answer correct to 2 significant figures. 1

(d) A function $y = f(x)$ has $\frac{d^2y}{dx^2} = 6x - 2$ and a stationary point at $(3,0)$. Find $f(x)$. 3

Question 9 Start a new booklet

(a)



The above is a graph of the function $y = f(x)$. Tangents are drawn at A(0,4) and B(4,5). Use the graph to evaluate:

(i) $f(6)$

(ii) $f'(4)$

(iii) $f'(8)$

(iv) $f''(0)$ 4

(b) A school softball team has a probability of 0.8 of losing or drawing any match and a probability of 0.2 of winning any match.

(i) Find the probability of the team winning at least one of the three consecutive matches. 2

(ii) What is the least number of consecutive matches the team must play to be 90% certain it will win at least one match? 2

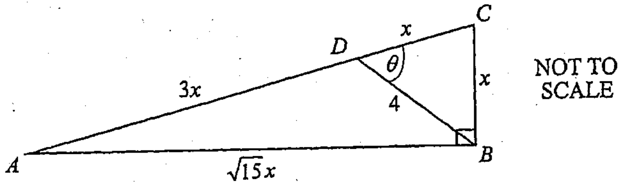
(c) Maxamillian's daughter was born on the 1st January. On that day he opened a trust account by depositing \$250. Each year, on her birthday, he deposited \$250 into this trust fund. He continued to do this up to and including her 17th birthday. When she turned eighteen, he collected the total amount including interest from this account and presented to her.
This account paid an interest of 6% p.a. compounded every six months.

(i) Show the initial deposit amounted to approximately \$724.57 after 18 years. 1

(ii) How much did Maximillian give his daughter on her eighteenth birthday? 3

Question 10 **Start a new booklet**

(a)



In the diagram, ABC is a right angled triangle where $AB=\sqrt{15}\,x$ cm and $BC= x$ cm. The point D lies on AC and CD = BC = x cm, AD = $3x$ cm and BD = 4 cm. Let $< BDC = \theta$.

(i) Use the cosine rule to show that $\cos\theta = \frac{2}{x}$. 1

(ii) Use the sine rule in triangle BCD to show that $\sin\theta = \frac{\sqrt{15}\,x}{16}$. 2

(iii) Hence show that $15x^4 - 256x^2 + 1024 = 0$. 2

(iv) Explain why one of the solutions to the equation in part (iii), namely $x = 2.53$ (to 2 decimal places), could not be the value of x indicated in the diagram above. 1

(b) ABCDE is a pentagon of fixed perimeter P cm. Its shape is such that ABE is an equilateral triangle and BCDE is a rectangle. If the length of AB is x cm :

(i) Show that the length BC is $\frac{\mathrm{P}-3x}{2}$ cm. **1**

(ii) Show that the area of the pentagon is given by

$A=\frac{1}{4}[2\mathrm{P}x-(6-\sqrt{3})x^2]$ **2**

(iii) Find the value of $\frac{\mathrm{P}}{x}$ for which the area of the pentagon is a maximum. **3**

END OF EXAMINATION

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2UNIT TRIAL HSC – 2008 – SOLUTIONS

Question 1

a) $\frac{2.4 \times \sqrt{30}}{24.9} = 0.527925..$

$= 0.528$ (3 sig fig) ✓

- mostly well done, but a significant number of students wrote 0.53... clearly not understanding significance.

b) $x^2 - 3 = 3x + 1$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ ✓

$x = 4, \; x = -1$ ✓

- mostly well done.

c) $\frac{5}{3-2\sqrt{3}} \times \frac{3+2\sqrt{3}}{3+2\sqrt{3}}$ ✓

$= \frac{15 + 10\sqrt{3}}{9 - 12}$

$= \frac{15 + 10\sqrt{3}}{-3}$

$= -5 - \frac{10}{3}\sqrt{3}$ ✓

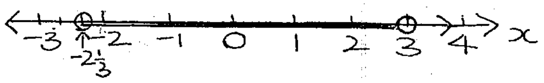
- mostly well done
- many made errors with signs!

d) $|3x - 1| < 8$

$3x - 1 < 8, \quad -3x + 1 < 8$

$3x < 9 \qquad -3x < 7$ ✓

$x < 3$ ✓ $\qquad x > -2\frac{1}{3}$



✓

- mostly well done
- those who didn't do well did not have or do the negative case properly.

e) Rate $= 3.6$ L / 24 h

$= 3600$ mL/1440 min

$= 2.5$ mL/min ✓

$= 2.5 \times 15$ drops/min

$= 37.5$ drops/min ✓

- usually well done
- check reasonableness of answer – a torrent of 135000 drips/min is quite unreasonable!!

f) $\frac{2}{x(x-3)} - \frac{1}{x}$

$= \frac{2 - (x-3)}{x(x-3)}$ ✓

$= \frac{5 - x}{x(x-3)}$ ✓

- often had an extra x, which caused problems cancelling (ie $\frac{2x - x(x-3)}{x^2(x-3)}$)
- Signs!! $(x-5)$ or $(x-1)$ was con

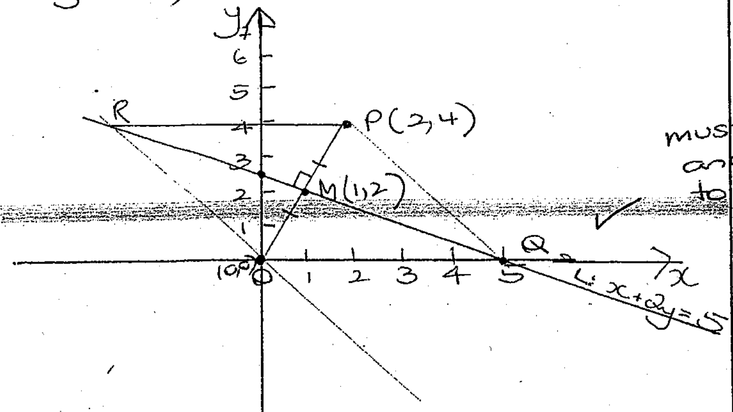
Question 2.

line L: $x + 2y = 5$, $P(2,4)$

$x = 0, \; y = 2\frac{1}{2}$

$y = 0, \; x = 5$

i)



✓

The graphing was poorly done. Give the intercepts, use a ruler and indicate interval size on axes

must have O, P, and line L have to be shown.

ii) $M\left(\frac{0+2}{2}, \frac{0+4}{2}\right)$

$= M(1, 2)$ ✓

iii) $x + 2y = 5$

L.H.S $= 1 + 2 \times 2$

$= 5$

$=$ R.H.S. ✓

∴ M lies on $x + 2y = 5$

iv) $m_{OP} = \frac{4-0}{2-0}$

$=$ [illegible] ✓

$x + 2y = 5$

$2y = -x + 5$

$y = -\frac{1}{2}x + \frac{5}{2}$

$m \quad = -\frac{1}{}$ ✓

Q2 cont'd

v) $m_{OP} = 2$

$\perp m = -\frac{1}{2}$ $M(1,2)$ ✓

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 1)$

$2y - 4 = -x + 1$ ✓

$x + 2y - 5 = 0$ } which is line L

OR since L passes through ✓ the midpt. of OP and $m_L \times m_{OP} = -1$ i.e. $-\frac{1}{2} \times 2 = -1$ ✓ then line L is the perpendicular bisector of OP.

many students did not explain why L is also the bisector of OP ∴ lost one mark

vi) $Q(5,0)$ ✓

vii) $m_{PQ} = \frac{0-4}{5-2}$

$= \frac{-4}{3}$

$\therefore m_{OR} = \frac{-4}{3}$ ✓

$m = -\frac{4}{3}$, $O(0,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x - 0)$

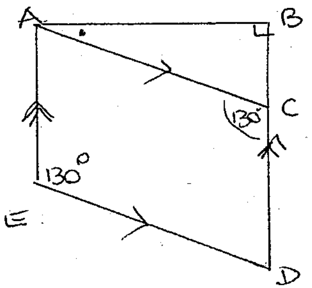
$y = -\frac{4}{3}x$ ✓

ie $4x + 3y = 0$

viii) the diagonals bisect each other at right angles. ✓✓ (or other suitable reason)

Students need to learn the tests for all types of quadrilaterals

Question 3



$\angle DCA = 130°$ (opp. ∠'s of ∥grm =) ✓

$\angle BAC + 90 = 130$ (ext. ∠ △ABC)

$\angle BAC = 40°$ ✓

reasons must be given

poor reasoning with many students

Q3 cont'd

(b) i) Let $y = \overset{u}{x}\,\overset{v}{e^{3x}}$

$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$

$= e^{3x} + x \cdot 3e^{3x}$ ✓

$= e^{3x}(1 + 3x)$ ✓

ii) let $y = \frac{2x^4 - 3}{x^2}$

$= 2x^2 - \frac{3}{x^2}$

$= 2x^2 - 3x^{-2}$ ✓

$y' = 4x + 6x^{-3}$

$= 4x + \frac{6}{x^3}$ ✓

* or use the quotient rule.

many did not simplify

(c) i) $\int \frac{1}{3x^2}\,dx = \frac{1}{3}\int x^{-2}\,dx$

$= -\frac{1}{3}x^{-1} + c$

$= -\frac{1}{3x} + c$ ✓

ii) $\int_2^3 \frac{x}{x^2-1}\,dx = \frac{1}{2}\left[\ln(x^2-1)\right]_2^3$ ✓

$= \frac{1}{2}[\ln 8 - \ln 3]$

$= \frac{1}{2}\ln\frac{8}{3}$ ✓

(d) $4x^2 - kx + 1 = 0$

$a = 4,\ b = -k\ \ c = 1$

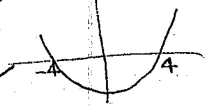
No real roots $\Rightarrow \Delta < 0$ ✓

$b^2 - 4ac < 0$

$k^2 - 16 < 0$ ✓

$(k-4)(k+4) < 0$

$-4 < k < 4$ ✓



many had a good idea i.e. $\Delta < 0$ but could not solve properly

Question 4

(a) $(3x-2)^2 + 4 = 0$

$9x^2 - 12x + 4 + 4 = 0$

$9x^2 - 12x + 8 = 0$

$a = 9 \quad b = -12, \quad c = 8$

i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{12}{9}$

$= \frac{4}{3}$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{8}{9}$ ✓

iii) $3\alpha^2 + 3\beta^2 = 3(\alpha^2+\beta^2)$

$= 3\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ ✓

$= 3 \times \left[\frac{16}{9} - \frac{16}{9}\right]$

$= 0$ ✓

(b) AP: $a = 1 \quad l = 14 \quad S_{14} = 90$

i) $S_n = \frac{n}{2}(a+l)$

$90 = \frac{n}{2}(1+14)$ ✓

$180 = 15n$

$n = 12$ ✓

12 terms in the series.

ii) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$90 = 6\left[2 + (12-1)d\right]$ ✓

$90 = 6(2 + 11d)$

$15 = 2 + 11d$ ✓

$11d = 13$

$d = \frac{13}{11}$ as req'd.

c)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

✓

i) $P(0) = \frac{6}{36}$

$= \frac{1}{6}$ ✓

ii) P(score at least 3)

$= \frac{12}{36}$ ✓

$= \frac{1}{3}$ ✓

Some student had difficulty in simplifying the equation into $ax^2+bx+c=0$.

Mostly well done.

Some students did not rewrite $3\alpha^2 + 3\beta^2$ correctly

Mostly well done

Many used T_n expression to correctly determine d.

* a diagram need no be shown, but some working has to be shown.

Lack of table or working for some students

Question 5

(a) $\log_x 128 = \frac{7}{3}$

$x^{\frac{7}{3}} = 128$

$x = 128^{\frac{3}{7}}$ ✓

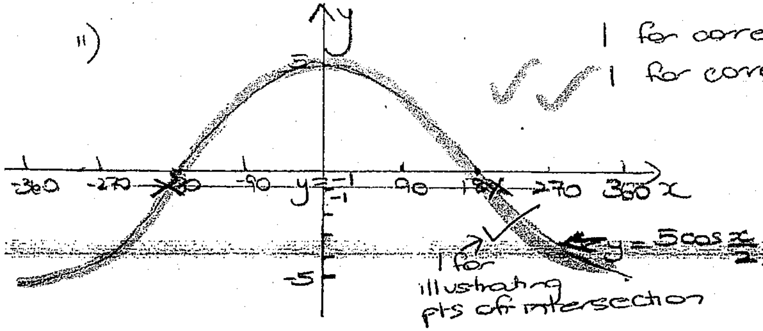
$x = 8$

(b) i) $y = 5\cos\frac{x}{2}$

$A = 5 \quad P = \frac{2\pi}{\frac{1}{2}}$

$= 4\pi \; (180°)$

ii)



1 for correct shape ✓

1 for correct Amplitude/Range. ✓

1 for illustrating pts of intersection ✓

iii) $5\cos\frac{x}{2} = -1$

$\cos\frac{x}{2} = -\frac{1}{5}$ ✓

$\frac{x}{2} = 101°32', -101°32'$

$x = 203°4', -203°4'$ ✓

(c) $y = x + 4$

$y = (x-2)^2$

$(x-2)^2 = x + 4$

$x^2 - 4x + 4 = x + 4$

$x^2 - 5x = 0$

$x(x-5) = 0$ ✓

$x = 0, \quad x = 5$

when $x = 0$, $y = 4 \Rightarrow (0,4)$

when $x = 5$, $y = 9 \Rightarrow (5,9)$ ✓

A is (0,4) which lies on

(a)

*Many 24 students coul not do. This using the log. defn

(b)

* Graph poor drawn in ma instances

- axes must labelled
- divisions o axes need t be shown

* y=-1 was o not in a rele position compared to y=-5

*$\cos\frac{x}{2} = -\frac{1}{5}$ is not equiva to cosx = -

* need to observe gra & realise the was a ± valu

(c) i) some stud didn't actuall show that A lies on the y-axis MUST explain that the x co is zero.

Some used the substitution method, but d not sub x=0 both eq'ns to

Q5 (cont'd)

ii) $y < x+4 \cap y > (x-2)^2$ ✓

iii) $A = \int_0^5 (x+4)\,dx - \int_0^5 (x^2-4x+4)\,dx$ ✓

$= \int_0^5 (x+\cancel{4} - x^2 + 4x - \cancel{4})\,dx$

$= \int_0^5 5x - x^2\,dx$

$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$ ✓

$= \left(\frac{125}{2} - \frac{125}{3}\right) - 0$ ✓

$A = 20\frac{5}{6}\,u^2$

(ii) accepted ← $y < x+4 \cap y \geq (x-2)^2$ also.

iii) generally well done but there were still some careless error with the actual integration and/or substituting

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Question 6.

(a) $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 12$

$f'(x) = x^2 - 2x - 8$

$f''(x) = 2x - 2$

i) for statny pts $f'(x) = 0$ ✓

$\underline{\text{ie}}$ $x^2 - 2x - 8 = 0$.

$(x-4)(x+2) = 0$

$x = 4,\ x = -2$ ✓

when $x = 4$, $y = -14\frac{2}{3}$ $\Rightarrow (4, -14\frac{2}{3})$ ✓

when $x = -2$ $y = 21\frac{1}{3}$ $\Rightarrow (-2, 21\frac{1}{3})$ ✓

$x = 4$: $f''(4) > 0 \Rightarrow$ $\underline{\text{min}^m}$ at $(4, -14\frac{2}{3})$ ✓

$x = -2$: $f''(-2) < 0 \Rightarrow$ $\underline{\text{max}^m}$ at $(-3, 21\frac{1}{3})$ ✓

ii) for possible inflexions $f''(x) = 0$

$\underline{\text{ie}}$ $2x - 2 = 0$

$2x = 2$

$x = 1$ ✓

test $x = 1$

x	1^-	1	1^+
$f''(x)$	$-$	0	$+$

∴ concavity changes.

when $x = 1$

$y = \frac{1}{3} - 1 - 8 + 12$

$= 3\frac{1}{3}$

∴ inflexion at $(1, 3\frac{1}{3})$ ✓

Many students did not state why $f'(x)=$ others did not let it equal zero but still found 2 x values fr an expression

} y values

The test for max/min was often not shown

} test for max/min.

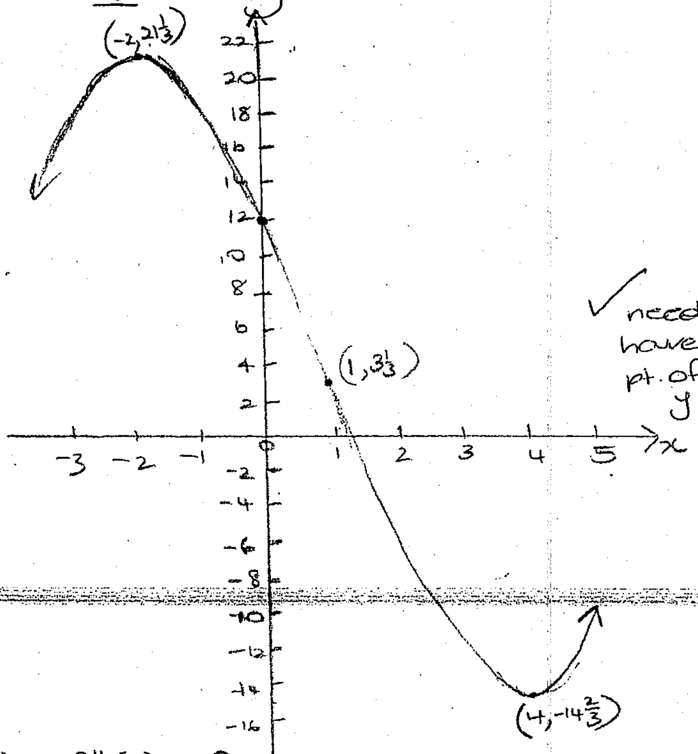
The test for the change in concavity was often ignored

*1 only if did not test for change in concavity.

Question 6 (cont'd)

* cuts y axis ⇒ $x = 0$

$\underline{\text{ie}}$ at $y = 12$



✓ need to have max^m/min^m/ pt. of inflexion + y int. labelled.

iv) $f''(x) > 0$

$\underline{\text{ie } x > 1}$ ✓

(b) i) $P = \frac{2\pi}{2}$

$= \pi$

2 cycles

∴ $b = 2\pi$ ✓

ii) $\int_0^b 2\cos\left(2x + \frac{\pi}{2}\right)dx$

$= -8 + 8 + -8 + 8$

$= 0$ ✓

Sketch was mostly well done but lack of detail in some cases.

students included $x = 1$, others could not use their graph to answer this.

Some students did not recognise that there was 2 cycles for the curve.

Some people did not realise that -8 was appropriate & tried to use absolute value.

Question 7

(a) $y^2 - 2y - 6x = 0.$

$y^2 - 2y = 6x$

$y^2 - 2y + 1 = 6x + 1$ ✓

$(y-1)^2 = 6\left(x + \frac{1}{6}\right)$

$V\left(-\frac{1}{6}, 1\right)$ ✓ ⊂

$4a = 6$

$a = \frac{3}{2}$

i) $F\left(\frac{4}{3}, 1\right)$ ✓

ii) $x = -1\frac{2}{3}$ ✓

- students got the completing the square wrong, adding 4 instead of 1.

- students did not realise that the parabola was in the form $x = y^2$ not $y = x^2$.

(b) $y = 2x + c$

$y = x^2 + 6x + 7$

$\therefore x^2 + 6x + 7 = 2x + c$

$x^2 + 4x + 7 - c = 0.$ ✓

$a = 1 \quad b = 4 \quad c = 7 - c.$ ✓

2 distinct roots $\Rightarrow \Delta > 0$

ie $b^2 - 4ac > 0$

$16 - 4(7 - c) > 0$ ✓

$16 - 28 + 4c > 0$

$4c > 12$

$c > 3$ as req'd.

- students tried to sub in a value, eg. let $c = 4$, and then show 2 distinct roots.

(c) $\sum_{r=1}^{\infty} 3^{-r} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

$a = \frac{1}{3}, \; r = \frac{1}{3}$ ✓

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{3}}{\frac{2}{3}}$

$= \frac{1}{3} \times \frac{3}{2}$

$= \frac{1}{2}$ ✓

- some students let $r = 3$ or at times $a = 3$

Question 7 (cont'd)

(d) i) $y = e^x$ and $y = \log_e x$ are reflections of each other in the line $y = x$ ✓

students used ~~the~~ words such as opposite & reciprocal instead of inverse.

ii) $y = \log_e x$

$y' = \frac{1}{x}$

at $x = 2$: $y' = \frac{1}{2}$ ✓

done well

when $x = 2$ $y = \log_e 2$

$y - y_1 = m(x - x_1)$

$y - \log_e 2 = \frac{1}{2}(x - 2)$

$2y - 2\log_e 2 = x - 2$

$x - 2y - 2 + 2\log_e 2 = 0$

$x - 2y - 2 + \log_e 2^2 = 0$ ✓

$x - 2y - 2 + \log_e 4 = 0$ (as req'd)

iii) $m = \tan\theta$

$\therefore \tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2}$

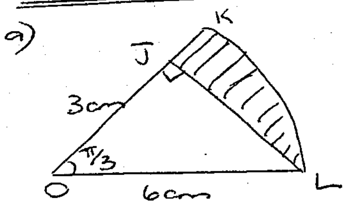
$\theta = 26^\circ 33' 54''$ ✓

$\theta = 27^\circ$ (nearest degree)

done well

Question 8:

a)



$6^2 = JL^2 + 3^2$

$36 = JL^2 + 9$

$JL^2 = 27$

$JL = \sqrt{27}$

$\underline{JL = 3\sqrt{3}}$ ✓

$\underline{JK = 3 \text{ cm}}$ ✓

$l = r\theta$

$= \frac{\pi}{3} \times 6$

$= 2\pi$ ✓

$\therefore KL = 2\pi$

Perimeter $= 3\sqrt{3} + 3 + 2\pi$

$= 14.5$ cm (1dp) ✓

b) i) $f(x) = (\log_e \sqrt{x})^2$

x	1	2	3	4	5
$f(x)$	0	0.125	0.302	0.480	0.648

✓

$\int_1^5 (\log_e \sqrt{x})^2 dx$

$= \frac{h}{3}\left\{ f(x_1) + f(x_5) + 2f(x_3) + 4f(x_2) + 4f(x_4) \right\}$

$= \frac{1}{3}\left\{ 0 + 0.648 + 2 \times 0.302 + 4 \times 0.125 + 4 \times 0.48 \right\}$ ✓

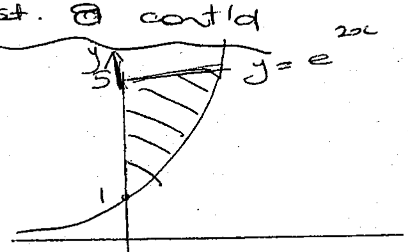
$\doteqdot 1.22$ (as req'd.)

care
eg. $\sqrt{36-9} \neq 5$

perimeter
not area

Quest. 8 cont'd

c)



(i) $y = e^{2x}$

$\log_e y = \log_e e^{2x}$

$\log_e y = 2x$

$x = \frac{1}{2} \log_e y$

$= \log_e y^{\frac{1}{2}}$

$\underline{x = \log_e \sqrt{y}}$ as req'd ✓

ii) $V = \pi \int_1^5 (\log_e \sqrt{y})^2 \, dy$ ✓

iii) $V = \pi \left[(1.22) \right]$ ✓

$\underline{= 3.8 \, u^3}$ (2 sig fig)

(d) $\frac{d^2y}{dx^2} = 6x - 2$

$\frac{dy}{dx} = 3x^2 - 2x + c$ ✓

$\frac{dy}{dx} = 0, x = 3$

$0 = 3(3)^2 - 2(3) + c$

$= 27 - 6 + c$

$c = -21$

$\therefore \frac{dy}{dx} = 3x^2 - 2x - 21$ ✓

$y = x^3 - x^2 - 21x + k$

$x = 3, y = 0$: $0 = 27 - 9 - 63 + k$

$= -45 + k$ ✓

$k = 45$

$\therefore \underline{y = x^3 - x^2 - 21x + 45}$

more steps
required in
many solutions

missed
connection
with this Q
and Q b)ii)

many careless
errors
+

many did not
find last
constant

Question 9

(a) i) $f(6) = 0$ ✓

ii) $(0,8)\ (12,0)$

$m = \frac{-8}{12}$

$\therefore f'(4) = -\frac{2}{3}$ ✓

iii) $f'(8) = 0$. ✓

iv) $f''(0) = 0$. ✓

b) $P(w) = 0.2 \quad P(\tilde{w}) = 0.8$

$P(\text{Win at least one}) = 1 - P(\text{win none})$ ✓ Done well.

$= 1 - (0.8)^3$

$= 1 - 0.512$

$= 0.488$ ✓

$\frac{61}{125}$

ii) $P(\text{winning at least one in } n \text{ matches}) = 1 - (0.8)^n$

Now $1 - (0.8)^n = 0.9$ ✓

$0.8^n = 0.1$

$\log_e 0.8^n = \log_e 0.1$

Done well.

$n = \frac{\log_e 0.1}{\log_e 0.8}$

$= 10.3\ldots$ ✓

$\therefore$ 11 matches need to be played.

(c) $P = \$250 \quad R = 0.06/\text{pa}$

$n = 18$ years $= 36\ \frac{1}{2}$ yrs $\quad R = 0.03 / 6$ months

i) $A_1 = P(1+R)^n$

Done well.

$= 250(1.03)^{36}$ ✓

$A_1 = \$724.57$

$\therefore$ initial deposit amounts to \$724.57.

Q9 (cont'd)

ii)

A_1 amounts to: $25(1.03)^{36}$

A_2 amounts to: $250(1.03)^{34}$

A_3 amounts to: $250(1.03)^{32}$

⋮

A_{16} amounts to: $250(1.03)^{6}$

A_{17} amounts to: $250(1.03)^{4}$

A_{18} amounts to: $250(1.03)^{2}$

- students got r and a incorrec
- Also did not ge the powers corre ✓

$$\text{Total amount} = 250\left(\underbrace{1.03^2 + 1.03^4 + \ldots + 1.03^{36}}\right)$$

G.S. $a = 1.03^2 \quad r = 1.03^2, n = 18$ ✓

$S_n = a(r^n - 1)$

$= \frac{1.03^2(1.03^{36} - 1)}{1.03^2 - 1}$

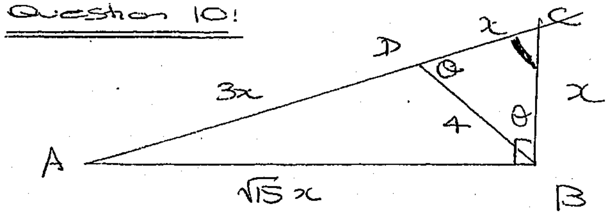
$\text{Total} = 250 \times S_{18}$

$= 250 \times 33.06869\ldots$

$= 8267.173556$ ✓

Total $= \$8267.17$ (nearest ¢)

Question 10!



1) In $\triangle DCB$.

$\cos\theta = \frac{x^2 + 4^2 - x^2}{2 \times 4 \times x}$

$= \frac{16}{8x}$ ✓

$\cos\theta = \frac{2}{x}$ (as req'd)

generally well done

Question 10 (cont'd)

ii) In $\triangle ABC$:

$$\sin C = \frac{\sqrt{15}x}{4x} = \frac{\sqrt{15}}{4}$$ ✓

Now in $\triangle BCD$

$$\frac{\sin\theta}{x} = \frac{\frac{\sqrt{15}}{4}}{4}$$

$$\sin\theta = \frac{\frac{\sqrt{15}}{4}x}{4} = \frac{\sqrt{15}x}{16}$$ ✓

iii) $\sin^2\theta + \cos^2\theta = 1$

$$\frac{15x^2}{256} + \frac{4}{x^2} = 1$$ ✓

$$\frac{15x^4}{256} + 4 = x^2$$

$$15x^4 + 1024 = 256x^2$$ ✓

$$15x^4 - 256x^2 + 1024 = 0$$

(as req'd)

iv) $\sin\angle C = \frac{\sqrt{15}}{4}$

$= 79°31'$

if $x = 2.53$

$$\cos\theta = \frac{2}{x} = \frac{2}{2.53}$$

$\therefore \theta = 37°46'$

Now in $\triangle BCD$

$37°46' \times 2 + 79°31'$ ✓

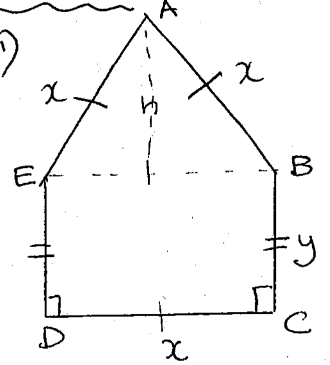
$\neq 180°$.

$\therefore x$ cannot equal 2.53.

- this preperatory link was often missed.
- otherwise well done
- asked to form this equation, not solve it! Many students wasted a lot of time here because they didnt READ the question
- not well done - many could not interpret the requirements correctly
- many statements (with no supporting evidence) gained no marks.

Q10 cont'd

(b) i)



Let $AB = x$ cm

$BC = y$ cm.

Perimeter (P) $= 3x + 2y$

$\therefore 2y = P - 3x$

$y = \frac{P-3x}{2}$ cm as req'd

demonstrate this ✓

- usually well done

ii) $h^2 + \left(\frac{1}{2}x\right)^2 = x^2$

$$h^2 + \frac{1}{4}x^2 = x^2$$

$$h^2 = \frac{3}{4}x^2$$

$$h = \frac{\sqrt{3}}{2}x$$

Area = Area $\triangle$ + Area rect.

$= \frac{1}{2}bh + xy$

$= \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x + x\left(\frac{P-3x}{2}\right)$ ✓

$= \frac{\sqrt{3}}{4}x^2 + \frac{Px - 3x^2}{2}$

$= \frac{\sqrt{3}x^2 + 2Px - 6x^2}{4}$ ✓

$= \frac{1}{4}\left[2Px - x^2(6 - \sqrt{3})\right]$

$A = \frac{1}{4}\left[2Px - (6-\sqrt{3})x^2\right]$ as req'd

- $A_T = \frac{1}{2}x.x.\sin 60$ also used well for Area $\triangle$.
- usually well done

iii) $A' = \frac{1}{2}P - \frac{1}{2}(6-\sqrt{3})x$

$A'' = -\frac{1}{2}(6-\sqrt{3}) < 0 \Rightarrow \max^m$ ✓

for statny values $A' = 0$

ie $\frac{1}{2}P - \frac{1}{2}(6-\sqrt{3})x = 0$ ✓

$\cancel{\frac{1}{2}}P = \cancel{\frac{1}{2}}(6-\sqrt{3})x$

$x = \frac{P}{6-\sqrt{3}}$

$\therefore \frac{P}{x} = 6 - \sqrt{3}$.

the value of $\frac{P}{x}$ that makes the area a ma is $(6-\sqrt{3})$ cm. ✓

- many differentiation issues!
- signs (as always!)
- $0 = \frac{P}{2} - 3x + \frac{\sqrt{3}}{2}x$ often became $\frac{P}{2} = -3x + \frac{\sqrt{3}}{2}x$ resulting in $\frac{P}{x} = \sqrt{3} - 6$!
- Check signs!!!
- many did not demonstra

Question 9 (continued)

(b) (i) Sketch the graphs of $y = \cos x$ and $y = \frac{1}{2}\tan x$ from $x = \frac{-\pi}{2}$ to $x = \frac{\pi}{2}$ on the same set of axes. **1**

(ii) By solving the equation $\cos x = \frac{1}{2}\tan x$ find the point of intersection of the two graphs that lies between $x = 0$ and $x = \frac{\pi}{2}$. **2**

Question 10 (12 marks)

(a) Prove that the limiting sum of the series $1 + \sin^2 x + \sin^4 x + \sin^6 x + ...$ is equal to $1 + \tan^2 x$ where $\tan x$ is defined. **3**

(b) Fred and Wilma take out a home loan of $400 000 to be repaid over 20 years at an interest rate of 6% per annum compounding monthly. They repay the loan in instalments of $\$P$ at the end of each month *after* the monthly interest has been calculated.

(i) Show that the amount left to be repaid after 3 months (just after Fred and Wilma have paid their third instalment) is given by **2**

$$\$400\,000 \times 1.005^3 - P(1 + 1.005 + 1.005^2)$$

(ii) Given that the home loan is completely repaid in 20 years find the value of P. **3**

(iii) Fred and Wilma decide to pay off the loan at $4000 per month instead. After how many months will the loan be repaid in this case? **4**