Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2009

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4: TRIAL HSC

Mathematics

TIME ALLOWED: 3 HOURS
(PLUS 5 MINUTES READING TIME)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1,2,6	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and logarithms	5,7,	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	3,4,10	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	8,9	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

2009 TRIAL

QUESTION 1 (12 marks) Start a NEW booklet. **Marks**

(a) Factorise $16x^2 - 25$ **1**

(b) Find the value of $17^{-0.5}$ to two decimal places **2**

(c) Convert $\frac{4\pi}{5}$ radians to degrees **1**

(d) Simplify $\frac{x}{2}+\frac{3x-1}{3}$ **2**

(e) Evaluate $\int_1^2 4x+7\,dx$ **2**

(f) Express $0.\dot{2}\dot{3}$ as a fraction. Show working. **2**

(g) Solve $5-3x<9$ **2**

QUESTION 2 (12 marks) Start a NEW booklet. **Marks**

(a) For the points A (3,2) and B (-5,-5),

(i) Find gradient between A and B **1**

(ii) Find midpoint of A and B **1**

(iii) Find distance between A and B. Answer as a surd. **1**

(iv) Show that the equation of the line l through A and B is $7x-8y-5=0$ **2**

(v) Show that the point C (-3,4) does not lie on the line l **1**

(vi) Find the perpendicular distance from the line l to (-3,4) **2**

(b) Find the equation of the line through (2,3) and the point of intersection of $x+2y-3=0$ and $2x+3y-7=0$ **4**

QUESTION 3 (12 marks) Start a NEW booklet.

(a) Differentiate (i) $(x^2-1)^{11}$ **1**

(ii) $\tan(3x)$ **1**

(b) Find the equation of the tangent to the curve $y = xe^x$ at the point (1,e) **4**

(c) Differentiate $y=\frac{\sin x}{1+\cos x}$ **3**

and hence show that $\frac{dy}{dx}=\frac{1}{1+\cos x}$

(d) The curve $y=3x+\frac{a}{x^2}$ has a turning point at $x=3$. **3**

Find the constant a

QUESTION 4 (12 marks) Start a NEW booklet. **Marks**

(a) Find the primitives (i.e. indefinite integrals) of:

(i) e^{2x} **1**

(ii) $\sin 6x$ **1**

(b) Evaluate

(i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx$ **2**

(ii) $\int_{9}^{13} \frac{dx}{x-7}$ **2**

(c) The following gives values of $f(x) = x\log x$

x	1	2	3	4	5
$f(x)$	0	1.39	3.30	5.55	8.05

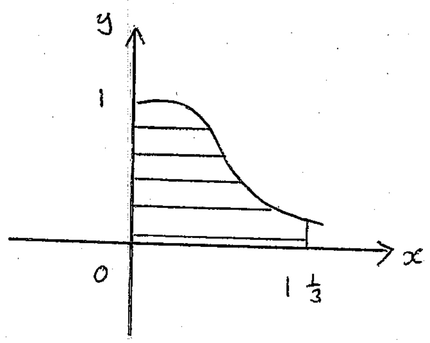
Use Simpson's rule with these five values to find an approximation to two decimals places of **3**

$$\int_1^5 x\log x\,dx$$

(d) Find the area between the curve $y = \frac{1}{(1+3x)^2}$, **3**

the x-axis and the ordinates $x = 0$ and $x = 1\frac{1}{3}$ as

shown in the sketch below.



QUESTION 5 (12 marks) Start a NEW booklet. **Marks**

(a) (i) The co-ordinates of P are (2,1). Show that P lies on both the parabolas $4y = x^2$ and $4y = (x-4)^2$. Show that P is the only point of intersection of the two curves. **3**

(ii) Find the equation of the tangent at P to the parabola $4y = (x-4)^2$. **2**

(iii) Find the co-ordinates of the other point Q at which this tangent intersects the parabola $4y = x^2$ **3**

(b) The roots of $2x^2 - 3x - 7 = 0$ are α and β. Find:-

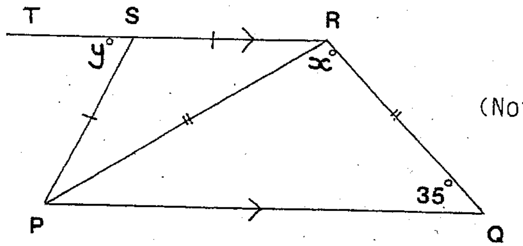
(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

QUESTION 6 (12 marks) Start a NEW booklet. **Marks**

(a)



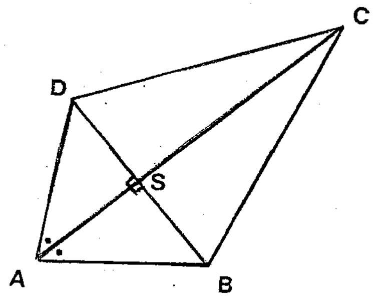
(Not to scale)

The diagram (not to scale) shows a quadrilateral PQRS, in which $PQ \parallel SR$, $PS = SR$, and $PR = RQ$. Also, T is a point on *RS* produced. Draw a neat sketch of this diagram in your answer book.

(i) Given that $\angle RQP = 35^0$, and $\angle PRQ = x^0$, find *x*, giving reasons. **3**

(ii) If also $\angle TSP = y^0$, find y, giving reasons **3**

(b)



(Not to scale)

In the diagram (not to scale), ABCD is a quadrilateral. The diagonals *AC, BD* interest at right angles, and $\angle DAS = \angle BAS$. Draw a neat sketch of the above diagram in your answer book.

(i) Explaining the reason for each step, use congruent triangles to prove that DA=AB. **3**

(ii) Hence prove that DC=CB **3**

QUESTION 7 (12 marks) Start a NEW booklet **Marks**

(a) Two ordinary dice, with the numbers 1 to 6 on their faces are thrown. What is the probability that:-

(i) they both show 6? **1**

(ii) they show a 1 and a 6? **1**

(iii) at least one of them shows a 1? **2**

(iv) they show a total of six? **1**

(b) On a destroyer there are two lines of defence against aircraft attack. These are a surface-to-air missile and a 15mm rapid-firing gun. The probability of success in hitting an attacking aircraft with each line of defence is respectively 0.9 and 0.8. Find the probability of hitting an attacking aircraft before it penetrates both defences. **3**

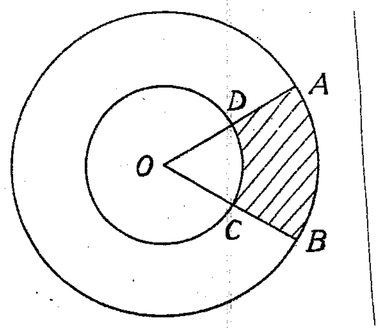
(c) Given $\log_2 3 = 1.58496$, find, correct to two decimal places:-

(i) $\log_2 9$ **2**

(ii) $\log_2 12$ **2**

QUESTION 8 (12 marks) Start a NEW booklet | **Marks**

(a)



The diagram shows two concentric circles centre 0 and radii 20 cm and 10 cm respectively. ODA and OCB are straight lines and the angle between OA and OB is 60^0.

Find, correct to 3 significant figures:-

(i) the perimeter of the shaded region ABCD **2**

(ii) the area of the shaded region ABCD **2**

(b) From a point O the point P bears 120^0 from North and is 12.3 km away. The point Q is 15.2 km South West of O.

(i) Mark the relative positions of O, P, Q on a sketch. **1**

(ii) What is the size of $\angle POQ$? **1**

(iii) Calculate the distance PQ in kilometres (rounded off correct to one decimal place). **2**

(c) The area under the curve $y=\sqrt{9-x^2}, -3 \le x \le 3,$ is rotated about the x - axis. Find the volume of the solid of revolution thus obtained. Name the solid. **4**

QUESTION 9 (12 marks) Start a NEW booklet. | **Marks**

(a) The first three terms of an arithmetic series are 50, 43, 36.

(i) Write down a formula for the nth term. **1**

(ii) If the last term of the series is -27, how many terms are there in the series? **1**

(iii) Find the sum of the series. **2**

(b) A loan of $1000 is to be repaid by equal annual instalments, repayments commencing at the end of the first year of the loan. Interest , at the rate of 10 per cent, is calculated each year on the balance before each repayment, and added to that balance.

If the annual instalment is P dollars, prove that:

(i) the amount owing at the beginning of the second year of the loan is (1100 - P) dollars. **2**

(ii) the amount owing at the beginning of the third year of the loan is (1210 - 2.1P) dollars **2**

(iii) if the loan (including interest charges) is exactly repaid at the end of n years, then **4**

$$P=\frac{100(1.1)^n}{(1.1)^n-1}$$

QUESTION 10 (12 marks) Start a NEW booklet. **Marks**

(a) A function *f(x)* is defined by the rule

$$f(x) = 9x(x-2)^2$$

in the domain $-1 \le x \le 3$.

(i) Draw a sketch of the graph of $y = f(x)$, showing clearly the turning points, the intercepts with x and y axes, and the values at the end-points of the domain. **6**

(ii) What is the range of $f(x)$? **1**

(b) A cylindrical can is to hold a volume of 600cm^3.

(i) Show that the can's surface area can be expressed in terms of radius r as:- **1**

$$SA = \frac{1200}{r} + 2\pi r^2$$

(ii) Find the radius r and height h for the minimum surface area to hold a volume of 600cm^3. (Answer to 2 decimal places.) **4**

(For a cylinder $V = \pi r^2 h$, $SA = 2\pi rh + 2\pi r^2$)

END OF EXAMINATION

FORT STREET HIGH SCHOOL
TRIAL HSC 2009
MATHEMATICS 2U
SOLUTIONS

QUESTION ONE

(a) $16x^2 - 25 = (4x-5)(4x+5)$ ✓

(b) $17^{-0.5} = 0.242535625\ldots\ldots$ ✓

$= 0.24$ (to 2dp) ✓

(c) $\frac{4\pi^c}{5} = \frac{4}{5} \times 180^\circ$

$= 144^\circ \quad (\pi^c = 180^\circ)$ ✓

(d) $\frac{x}{2} + \frac{3x-1}{3}$

$= \frac{3x}{6} + \frac{2(3x-1)}{6}$ ✓

$= \frac{3x + 6x - 2}{6}$

$= \frac{9x-2}{6}$ ✓

(e) $\int_1^2 (4x+7)\,dx$

$= \left[2x^2 + 7x\right]_1^2$ ✓

$= (8+14) - (2+7)$

$= 13$ ✓

Although a very basic integration, students had some difficulty.

(f) Let $x = 0.2323\ldots$ ✓ (1)

$100x = 23.2323\ldots$ (2)

$99x = 23$ (2) − (1)

$x = \frac{23}{99}$ ✓

OR $0.\dot{2}\dot{3} = 0.23 + 0.0023 + 0.000023 \ldots$

∴ Infinite sum of a geometric progression

where $a = 0.23 \quad r = 0.01$

$S = \frac{a}{1-r} = \frac{0.23}{1-0.01}$ ✓

$= \frac{0.23}{0.99} = \frac{23}{99}$

ie $0.\dot{2}\dot{3} = \frac{23}{99}$ ✓

Usually well done, however, some students did not know how to proceed.

(g) $5 - 3x < 9$

$-3x < 9$ ✓

$x > -\frac{4}{3}$ ✓

QUESTION TWO

(a) (i) grad $AB = \frac{2-(-5)}{3-(-5)} = \frac{7}{8}$ ✓

(ii) midpoint $AB = \left(\frac{3+(-5)}{2}, \frac{2+(-5)}{2}\right)$

$= \left(-1, -\frac{3}{2}\right)$ ✓

(iii) distance $AB = \sqrt{(3-(-5))^2 + (2-(-5))^2}$

$= \sqrt{8^2+7^2}$

$= \sqrt{113}$ ✓

(iv) line thru' $(3, 2)$ with gradient $\frac{7}{8}$

$y - 2 = \frac{7}{8}(x-3)$ ✓

$8y - 16 = 7x - 21$ ✓

$7x - 8y - 5 = 0$

(v) Substitute $(-3, 4)$ into

$7x - 8y - 5 = 0$ ✓

$7 \times -3 - 8 \times 4 - 5 = -58 \neq 0$

$\therefore (-3, 4)$ does not lie on line ℓ.

(vi)

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|7 \times -3 + -8 \times 4 - 5|}{\sqrt{7^2+8^2}}$ ✓

$= \frac{|-58|}{\sqrt{113}}$

$= \frac{58}{13}$ ✓

Some students had difficulty with substitution into the formula.

(b) using "k" method.

required line is

$x + 2y - 3 + k(2x + 3y - 7) = 0$ ✓

substitute $(2, 3)$

$2 + 6 - 3 + k(4 + 9 - 7) = 0$

$5 + 6k = 0$

$k = -\frac{5}{6}$ ✓

substitute $k = -\frac{5}{6}$

$x + 2y - 3 - \frac{5}{6}(2x + 3y - 7) = 0$ ✓

$6x + 12y - 18 - 10x - 15y + 35 = 0$

$-4x - 3y + 17 = 0$

$\underline{4x + 3y - 17 = 0}$ ✓

OR

$x + 2y - 3 = 0$ (1)

$2x + 3y - 7 = 0$ (2)

$2x + 4y - 6 = 0$ (1) $\times 2 =$ (3)

(3) − (2) $y + 1 = 0$

$y = -1$

Substitute $y = -1$ in (1)

$x - 2 - 3 = 0$ ✓

$x = 5$

gradient from $(2, 3)$ to $(5, -1)$

$= \frac{3-(-1)}{2-5} = -\frac{4}{3}$ ✓

required equation

$y - 3 = -\frac{4}{3}(x - 2)$ ✓

$3y - 9 = -4x + 8$ ✓

$\underline{4x + 3y - 17 = 0}$

Most students preferred to use the solution involving simultaneous equations.

corrections please.

QUESTION THREE

(a) (i) $\frac{d}{dx}(x^2-1)^{11}$

$= 11(x^2-1)^{10} \times 2x$

$= 22x(x^2-1)^{10}$ ✓

Some students forgot to write the power of 10.

(ii) $\frac{d}{dx} \tan(3x)$

$= 3\sec^2(3x)$ ✓

Some wrote x instead of $3x$

(b) (i) $y = xe^x$

$\frac{dy}{dx} = xe^x + e^x$ ✓

Some did not learn Product rule to differentiate $y = uv$

at $x = 1$ $\frac{dy}{dx} = e + e$

$= 2e$ ✓

equation of tangent through $(1, e)$ with gradient $= 2e$ is

ok

$y - e = 2e(x-1)$ ✓

$y - e = 2ex - 2e$

$y = 2ex - e$ ✓

(c) $y = \frac{\sin x}{1+\cos x}$

$\frac{dy}{dx} = \frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$ ✓

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \frac{1+\cos x}{(1+\cos x)^2}$ $(\sin^2 x + \cos^2 x = 1)$ ✓

$= \frac{1}{1+\cos x}$ ✓

Some had problems differentiating $y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(d) $y = 3x + \frac{a}{x^2}$

$= 3x + ax^{-2}$

$\frac{dy}{dx} = 3 - \frac{2a}{x^3}$

Some had proble[ms] with the derivat[ive] of $y = x^{-2}$, [...] $\frac{dy}{dx} = x^{-3}$ an[d] NOT x^{-1}

At the turning point when $x = 3$

$\frac{dy}{dx} = 0$

$3 - \frac{2a}{3^3} = 0$

$3 = \frac{2a}{27}$

$2a = 3 \times 27$

$a = \frac{81}{2}$

$= 40\frac{1}{2}$

OK if they wer[e] correct in $\frac{dy}{dx}$

QUESTION FOUR

(a) (i) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$ ✓

① Answer + c

[Lose 1 mark if "+c" omitted in either (i) or (ii)]

(ii) $\int \sin 6x\,dx = \frac{1}{6}\left(-\cos 6x\right) + c$

$= -\frac{1}{6}\cos 6x + c$ ✓

① Answer + c

[some mistakes with $\frac{1}{2}, \frac{1}{6}$ or ⊖]

(b) (i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx = \left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓

1. Integration

[Some students wrote ⊖sin x.]

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ ✓

1. Subst. to get answer

②

(ii) $\int_{9}^{13} \frac{dx}{x-7} = \left[\log_e (x-7)\right]_9^{13}$ ✓

1. Integration

[Zero marks if failed to recognise "Log Integral".]

$= \log_e 6 - \log_e 2$

[Some students were unable to simplify ln 6 − ln 2, making errors in evaluation.]

$= \log_e \frac{6}{2}$

$= \log_e 3$ ✓

$(= 1.0986\ldots)$

1. Simplified answer

② [Allow decimal if rounded correctly]

(c) $\int_1^5 x \log x\,dx \doteq \frac{3-1}{6}\left[f(1) + 4f(2) + f(3)\right]$

$+ \frac{5-3}{6}\left[f(3) + 4f(4) + f(5)\right]$

[Students using $\frac{h}{3}$ were more successful than those using $\frac{b-a}{6}$.]

$= \frac{1}{3}\left[f(1) + f(5) + 2f(3) + 4\left(f(2) + f(4)\right)\right]$ ✓

1. Using correct h in a correct version of Simpson's Rule.

$= \frac{1}{3}\left(0 + 8.05 + 2(3.30) + 4(1.39 + 5.55)\right)$ ✓

1. Substituting y-values into correct formula.

$= 14.1366\ldots$ ✓

1. Answer

$= 14.14$ (to 2 dec pls)

③ [allow other rounding]

or use $h = 1$ (by inspection, or $\frac{b-a}{n} = \frac{5-1}{4} = 1$)

in $\frac{h}{3}\left[y_0 + 4(y_1 + y_3) + 2(y_2) + y_4\right]$ etc.

[No carry-on mark for evaluating after incorrect formula.]

[Some students mixed up "odds" and "evens" or failed to get ...]

(d) $A = \int_0^{\frac{4}{3}} \frac{1}{(1+3x)^2}\,dx$ ✓

1. Setting up a correct definite integral expression

[N.B. Many students use poor integral notation e.g. missing out "dx", although no mark was deducted for this (Generous!)]

$= \int_0^{\frac{4}{3}} (1+3x)^{-2}\,dx$

$= \left[\frac{(1+3x)^{-1}}{-1 \times 3}\right]_0^{\frac{4}{3}}$ ✓

1. Correct Integration step.

[Very poorly done. Many students attempted a log integral or expanded $\frac{1}{(1+3x)^2} = \frac{1}{1+6x+9x^2}$. Many students missed the factor of 3 in the denominator.]

$= -\frac{1}{3}\left[\frac{1}{1+3x}\right]_0^{\frac{4}{3}}$

$= -\frac{1}{3}\left(\frac{1}{1+4} - \frac{1}{1+0}\right)$

$= -\frac{1}{3}\left(\frac{1}{5} - 1\right)$

$= -\frac{1}{3} \times -\frac{4}{5}$

$= \frac{4}{15}$ units² ✓

1. Substitution to get answer.

[Many students thought incorrectly that $F(0) = \frac{1}{1+0} = 0$ This is wrong!]

③

any corrections please.

QUESTION FIVE

(a) (i) Substituting coordinates of P into

$4y = x^2$

$4 \times 1 = 2^2$

$\therefore$ P lies on $4y = x^2$

Substituting coordinates of P into

$4y = (x-4)^2$

$4 \times 1 = (2-4)^2$

$4 = (-2)^2$

$\therefore$ P lies on $4y = (x-4)^2$ ✓

Solving simultaneously

$4y = x^2$ (1)

$4y = (x-4)^2$ (2)

Substituting (1) into (2)

$x^2 = (x-4)^2$

$x^2 - (x-4)^2 = 0$ ✓

$x^2 - (x^2 - 8x + 16) = 0$

$8x - 16 = 0$

$8x = 16$

$x = 2$

$\therefore y = 1$ ✓

Hence P is the only point of intersection of the two curves

Some students did not show that P satisfied both equations.

Some students did not state that P only satisfied but implied it.

Most students tried to solve 2 equations simultaneously.

Mainly well done.

(ii) $4y = (x-4)^2$

$y = \frac{1}{4}(x-4)^2$

$\frac{dy}{dx} = \frac{1}{2}(x-4)$ ✓

$\therefore m = \frac{1}{2} \times -2 = -1$ at $x = 2$

Some students found the differentiation difficult.

Equation of tangent at $(2,1)$ with gradient $= -1$ is

$y - 1 = -1(x-2)$

$y - 1 = -x + 2$

$y = 3 - x$ or $x + y - 3 = 0$ ✓

Incorrect general form by some stud

(iii) Solving simultaneously

$4y = x^2$ (1) $\quad y = 3 - x$ (2)

Substituting (2) into (1)

$4(3-x) = x^2$ ✓

$12 - 4x = x^2$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\therefore x = -6$ or 2 ✓

at $x = -6$ $\quad y = 3 - (-6)$

$= 9$

$\therefore$ Q has co-ordinates $(-6, 9)$ ✓

Co-ordinates for Q we incorrect because of incorrect equation o tangent.

(b) (i) $\alpha + \beta = \frac{3}{2}$ ✓

Well done

(ii) $\alpha\beta = -\frac{7}{2}$ ✓

Well done.

(iii) $(\alpha + \beta)^2 - 2\alpha\beta$ ✓

$= \left(\frac{3}{2}\right)^2 - 2 \times -\frac{7}{2}$

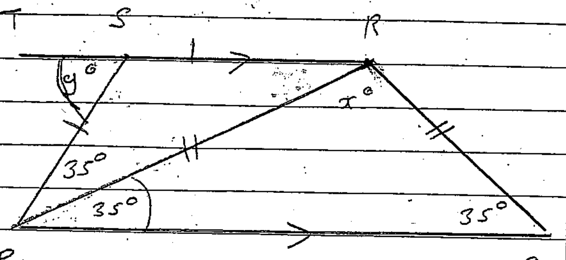
$= \frac{9}{4} + 7$

$= \frac{37}{4} = 9\frac{1}{4}$ ✓

Many incorrectly expanded $(\alpha+\beta)^2$ and said $\alpha^2+\beta^2 = (\alpha+\beta$

QUESTION SIX

(c)



(i) $\angle RPQ = \angle RQP = 35^\circ$

($\triangle RPQ$ is isosceles) ✓

$\angle PRQ = 180^\circ - 2 \times 35^\circ$

(angle sum $\triangle RPQ$) ✓

$= 110^\circ$

$\therefore x = 110^\circ$ ✓

(ii) $\angle SRP = \angle RPQ = 35^\circ$

(alternate angles $SR \parallel PQ$) ✓

$\angle SPR = \angle SRP = 35^\circ$

($\triangle SPR$ is isosceles) ✓

$\angle TSP = \angle SRP + \angle SPR$

(exterior angle)

$= 35^\circ + 35^\circ$

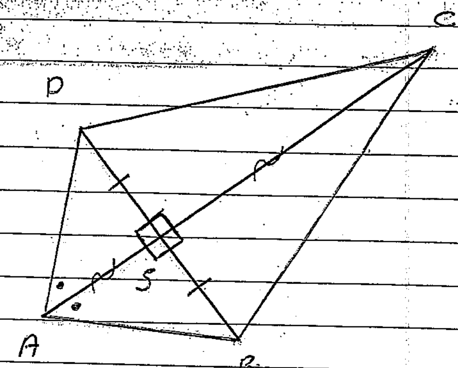
$= 70^\circ$

$\therefore y = 70^\circ$ ✓

* some students ignored the request to make a neat sketch for both parts. (this makes it hard to see exactly what each student is referring to)

a) generally well done although setting out and reasoning need to be refined.

(b)



(i) In $\triangle$'s ASD and ASB

AS is a common side

$\angle DAS = \angle BAS$ (given)

$\angle DSA = \angle BSA = 90^\circ$ (diagonals intersect at rt. $\angle$'s given) ✓

$\therefore \triangle ASD \equiv \triangle ASB$ (AAS)

$\therefore DA = AB$ (corres sides in congt $\triangle$s) ✓

and $DS = BS$

(ii) In $\triangle$'s DCS and BCS

CS is a common side ✓

$DS = BS$ (proved above)

$\angle CSD = \angle CSB$ (diagonals intersect at rt. angles)

$\triangle DCS \equiv \triangle BCS$ (SAS)

$\therefore DC = CB$ (corres sides in congt $\triangle$'s) ✓

* Marks may well be deducted in the H.S.C if they are presented in the same manner as they were submitted in the trial

Some general knowledge of setting out congruence proofs although in many cases it was sloppy!

- Reasoning need to be succinct and correct.
- need to start off with In $\triangle$'s
- state tests correctly eg SAS not ASS (no such congruence test of AAA)
- notation for is congruent to is $\equiv$

ii) some students stated that ABCD was a kite $\therefore$ DC = BC.

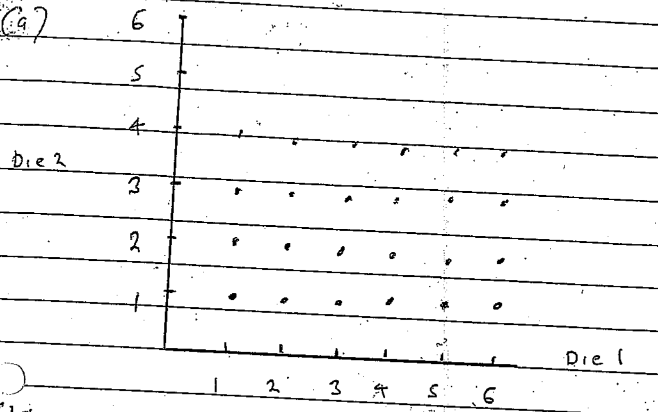
But the defn of a kite is two pairs of adj. sides equal and diagonals intersect at rt. $\angle$'s and it is the 2nd pair of adj. sides we are trying to prove equal!

* some students used $\triangle$'s ADC & ABC which works as well

Comments (13)

QUESTION SEVEN

(a)



Using a graphical representation to determine the number of favourable outcomes in each case:

(i) 1 favourable outcome (6, 6)
P (both show 6) = $\frac{1}{36}$ since n(S) = 36 ✓

(ii) 2 favourable outcomes (1, 6) and (6, 1)
P (show a 1 and a 6) = $\frac{2}{36} = \frac{1}{18}$ ✓

(iii) 11 favourable outcomes (1, 1) (1, 2) (2, 1) (1, 3) (3, 1) (1, 4) (4, 1) (1, 5) (5, 1) (1, 6) (6, 1) ✓
P (at least one 1) = $\frac{11}{36}$ ✓

(iv) 5 favourable outcomes (1, 5) (2, 4) (3, 3) (4, 2) and (5, 1)
P (total 6) = $\frac{5}{36}$ ✓

* Part (a) was fairly well done.
- Some students just wrote answers withou[t] working out or drawi[ng] a diagram.

(14)

(b) P (hitting attacking aircraft)
= 1 − P (missing aircraft with both defences) ✓
= 1 − (P (missile missing) × P(gun missing))
= 1 − 0.1 × 0.2 ✓
= 1 − 0.02
= 0.98 ✓

* Some students solved this as 0.9 × 0.8 = 0.7[2] because they di[d] not make a tree diagram.

(c) (i) $\log_2 9 = \log_2 3^2$
$= 2 \log_2 3$ ✓
$= 2 \times 1.584962$
$= 3.16992$
$= 3.17$ (to 2 dec pls) ✓

(ii) $\log_2 12 = \log_2 (4 \times 3)$
$= \log_2 4 + \log_2 3$ ✓
$= \log_2 2^2 + 1.584962$
$= 2 \log_2 2 + 1.584962$
$= 2 + 1.584962$
$= 3.584962$
$= 3.58$ (to 2 dec pl) ✓

* mostly well do[ne]. Some students tried to solve th[is] by change of base. this was not appropriate because of the wording in the question.

Also some simp[le] mistakes eg. $\log_2 3^2 = \log_2 \ldots \times \log \ldots$

QUESTION EIGHT

(a) (i) Let $r_1 = OD$ and $r_2 = OA$

Perimeter $= AD + BC + r_1\theta + r_2\theta$

$= 10 + 10 + \frac{10\pi}{3} + \frac{20\pi}{3}$ ✓

$= 20 + 10\pi$

$= 51.4$ cm (to 3 sig figs) ✓

Many students did not provide answers to three significant figures.

(ii) Area $= \frac{1}{2} r_2^2\theta - \frac{1}{2} r_1^2\theta$

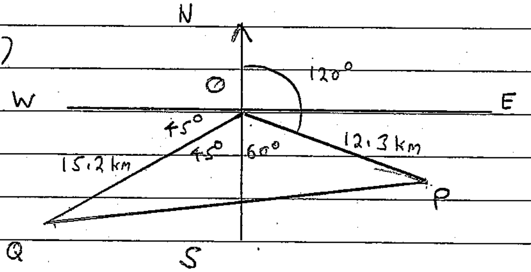
$= \frac{\theta}{2}(r_2^2 - r_1^2)$

$= \frac{\pi}{6}(400 - 100)$ ✓

$= \frac{300\pi}{6} = 50\pi$

$= 157$ cm^2 (to 3 sig figs) ✓

(b) (i)



✓

* Many students had difficulty doing diagram.

* Some students had a plus sign instead of a minus sign in cosine rule.

(ii) $\angle POQ = 105°$ (from sketch) ✓

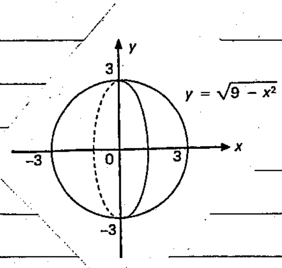
(iii) By the cosine rule in $\triangle QOP$

$PQ^2 = OQ^2 + OP^2 - 2 \times OQ \times OP \times \cos\angle POQ$

$= (15.2)^2 + (12.3)^2 - 2 \times 15.2 \times 12.3 \times \cos 105°$

$PQ = 21.9$ km to 1 dec pl

(c)



$V = \pi \int_{-3}^{3} y^2\,dx$

$= \pi \int_{-3}^{3} \left(\sqrt{9 - x^2}\right)^2 dx$ ✓

$= 2\pi \int_0^3 (9 - x^2)\,dx$

$= 2\pi \left[9x - \frac{x^3}{3}\right]_0^3$ ✓

$= 2\pi\left[\left(27 - \frac{27}{3}\right) - (0 - 0)\right]$

$= 2\pi \times 18$

$= 36\pi$ units^3 ✓

The shape of the solid is a sphere ✓

OR Since the shape of the volume is a sphere ✓

$V = \frac{4}{3}\pi r^3$ ✓ $r = 3$

$= \frac{4}{3}\pi 3^3$

$= 4 \times \pi \times 9$ ✓

$= 36\pi$ units^3 ✓

Many students forgot to write down the shape produced or identified it as a hemisphere or circle.

Part (a) generally well done.
Part (b) was very poorly done. Many students did not set out their ~~algebra and reasoning~~ clearly enough to "PROVE THAT..."

QUESTION NINE

(a) (i) $t_n = a + (n-1)d$

$a = 50 \quad d = -7$

$t_n = 50 + (n-1)(-7)$ ✓

$= 50 - 7n + 7$

$= 57 - 7n$

① Using formula correctly.

(ii) $57 - 7n = -27$

$-7n = -84$

$n = 12$ ✓

There are 12 terms in the series

① Answer.

(iii) $S_n = \frac{n}{2}(a + L)$

[or can use $S_n = \frac{n}{2}[2a + (n-1)d]$]

$n = 12 \quad a = 50 \quad L = -27$

$S_{12} = \frac{12}{2}(50 - 27)$ ✓

$= 6 \times 23$

$= 138$ ✓

1. Using formula correctly.
1. Answer.

②

(b) (i) Amount owing at the beginning of the second year

$A_1 = 1000\left(1 + \frac{10}{100}\right) - P$ ✓

$= 1000(1.1) - P$ ✓

$= 1100 - P$

IMPORTANT. If a Qn says "Prove that..." or "Show that..." you must set out algebra and reasoning clearly.

1. Getting ~~from interest rate~~ $\longrightarrow \times(1.1)$
1. Showing $1000(1.1) - P \longrightarrow$ answer.

②

(ii) Amount owing at the beginning of the third year.

$A_2 = \left(1000(1.1) - P\right)1.1 - P$ ✓

or can use $A_2 = A_1(1.1) - P$

$= 1000(1.1)^2 - P\,1.1 - P$

$= 1000(1.1)^2 - P(1 + 1.1)$

$= 1210 - 2.1P$ ✓

Observation: "Beginning of 2^{nd} Yr" means immediately after repayment at end of 1^{st} Yr.

1. Showing $A_2 = A_1(1.1) - P$.
1. Simplifying $\longrightarrow$ answer.

②

OR $(1100 - P)(1.1) - P$ ✓

$= 1100(1.1) - (1.1)P - P$

$= 1100(1.1) - P(1 + 1.1)$

$= 1210 - 2.1P$ ✓

(iii) Continuing from part (ii)

Amount owing after 3 years

$A_3 = \left[1000(1.1)^2 - P(1 + 1.1)\right](1.1) - P$

$= 1000(1.1)^3 - P(1 + 1.1 + 1.1^2)$ ✓

⋮ continuing the pattern

Amount owing after n years

$= 1000(1.1)^n - P(1 + 1.1 + 1.1^2 + \ldots 1.1^{n-1})$ ✓

$1 + 1.1 + (1.1)^2 + \ldots + (1.1)^{n-1}$ is a geometric series with $a = 1 \quad r = 1.1$

If after n years the loan is repaid

$1000 \times (1.1)^n - P\left(\frac{(1.1)^n - 1}{1.1 - 1}\right) = 0$ ✓

$1000 \times (1.1)^n - 10P\left[(1.1)^n - 1\right] = 0$

$10P\left[(1.1)^n - 1\right] = 1000 \times (1.1)^n$ ✓

$P = \frac{1000 \times (1.1)^n}{10\left[(1.1)^n - 1\right]}$

$= \frac{100 \times (1.1)^n}{(1.1)^n - 1}$

N.B. Part (b)(iii) was clearly understood by many students but very poorly set out to "Prove that...". For full marks you needed:

1. Expression for A_3 or equ... to show pattern leading to A...

Not enough just to write down $A_n = \ldots$

1. Correct expression for A_n or equivalent.

Some students made errors such as omitting P or writi... $-P(1 + 1.1 + 1.1^2 + \ldots + 1.1^{n-2}$ This is wrong for end of n^{th} ye...

1. Identifying and using a Geometric Series correctly

Note: best to state clearly "Geom. Series with n terms, $a = 1$, $r = 1.1$, so $S_n = \frac{a(r^n - 1)}{r - 1}$".

Few students showed this clearly but mark was given for a part of this information.

1. Using $A_n = 0$ and simplifying $\longrightarrow$ answer.

④

QUESTION TEN

(a) (i) $f(x) = 9x(x-2)^2$

$= 9x(x^2 - 4x + 4)$

$= 9x^3 - 36x^2 + 36x$

$f'(x) = 27x^2 - 72x + 36$ ✓ OK.

$f''(x) = 54x - 72$ ✓

stationary points occur when $f'(x) = 0$

$27x^2 - 72x + 36 = 0$

$9(3x^2 - 8x + 4) = 0$

$9(3x-2)(x-2) = 0$

$x = \frac{2}{3}$ or 2 ✓ ok

$f\left(\frac{2}{3}\right) = 9\left(\frac{2}{3}\right)\left(\frac{2}{3} - 2\right)^2$

$= 9 \times \frac{2}{3} \times \left(\frac{-4}{3}\right)^2 = \frac{32}{3}$

$f''\left(\frac{2}{3}\right) < 0$ ✓

$\therefore$ max turning pt at $\left(\frac{2}{3}, \frac{32}{3}\right)$

students lost 1 mark for failing to investigate the max and MIN turning points using the second derivatives.

$f(2) = 18(2-2)^2 = 0$

$f''(2) > 0$

$\therefore$ min turning pt at $(2, 0)$ ✓

at $x = 0$ $y = 0$

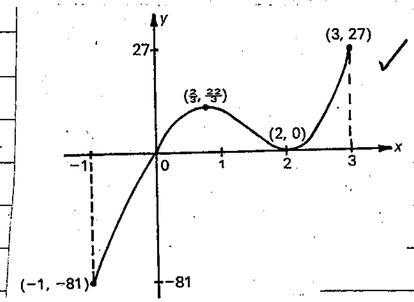
at $y = 0$ $x = 0$ $x = 2$

Values at the end-points of domain

$f(-1) = -9 \times (-3)^2 = -81$

$f(3) = 27 \times 1^2 = 27$

(i) continued



✓

Some students lost 1 mark for not sketching the curve nicely and not showing critical points on the graph.

(ii) Range $-81 \le f(x) \le 27$ ✓ ok

(b)(i) $V = \pi r^2 h = 600$

$h = \frac{600}{\pi r^2}$

$SA = 2\pi r h + 2\pi r^2$

$= 2 \times \pi \times r \times \frac{600}{\pi r^2} + 2\pi r^2$ ✓ ok

$= \frac{1200}{r} + 2\pi r^2$

(ii) $\frac{dSA}{dr} = -1200r^{-2} + 4\pi r$ ✓

$= \frac{-1200}{r^2} + \frac{4\pi r^3}{r^2}$

good as SA was given if students knew the $\frac{dy}{dx}$ of $\frac{1}{x}$ is $\frac{-1}{x^2}$

stationary point occurs when $\frac{dSA}{dr} = 0$

$4\pi r^3 - 1200 = 0$

$r^3 = \frac{1200}{4\pi}$

$r = \sqrt[3]{\frac{300}{\pi}}$

$= 4.57$ cm ✓

Some had problems with re-arranging the formula and finding the cube root of r.

$\frac{d^2SA}{dr^2} = \frac{2400}{r^3} + 4\pi > 0$ at $r = 4.57$

$\therefore$ min SA at $r = 4.57$ cm ✓

$h = \frac{600}{\pi \times 4.57^2} = 9.14$ cm ✓

Students lost 1 mark if the value of r was not stated as min by not investigating $\frac{d^2SA}{dr^2}$