Name: ______________________________

Teacher: ____________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2013

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11, 12
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	13, 14
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	15, 16

Total Marks 100

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section
Section II 90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SECTION 1

Multiple choice questions : Answer on the answer sheet provided.

1 For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

2 The table below shows the values of a function $f(x) = \sqrt{25 - x^2}$ for six values of x.

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	4.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25 - x^2}\,dx$ using trapezoidal rule using these six function values?

(A) 10.74

(B) 12.65

(C) 18.98

(D) 37.96

3 The semi-circle $y = \sqrt{9 - x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

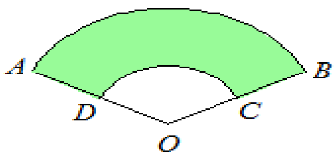
(A) $V = \pi \int_0^3 (9 - x^2)dx$

(B) $V = 2\pi \int_0^3 (9 - x^2)dx$

(C) $V = \pi \int_0^3 (9 - y^2)dy$

(D) $V = 2\pi \int_0^3 (9 - y^2)dy$

4 A car windscreen wiper traces out the area *ABCD* where *AB* and *CD* are arcs of circles with a centre *O* and radii 40 cm and 20 cm respectively. Angle *AOB* measures 120º.



Not to scale

What is the area *of ABCD?*

(A) 419 cm^2

(B) 1257 cm^2

(C) 1676 cm^2

(D) 2095 cm^2

5 What is the correct expression for the integral $\int \cos\frac{x}{3}\,dx$?

(A) $-3\sin\frac{x}{3}+c$

(B) $-3\cos\frac{x}{3}+c$

(C) $3\sin\frac{x}{3}+c$

(D) $3\cos\frac{x}{3}+c$

6 What is the derivative of $(1+\log_e x)^4$?

(A) $4(1+\log_e x)^3$

(B) $\dfrac{(1+\log_e x)^5}{5}$

(C) $\dfrac{4(1+\log_e x)^3}{x}$

(D) $\dfrac{(1+\log_e x)^5}{5x}$

7 What is the value of $\sum_{r=1}^{40}(3r-7)$?

(A) 109

(B) 2180

(C) 2260

(D) 2380

8 What points on the curve $y = x^3 - 4x^2 + 2x$ have a tangent parallel to the $2x + y = 3$?

(A) $(-\frac{2}{3}, -\frac{92}{27})$ and $(-2, -28)$

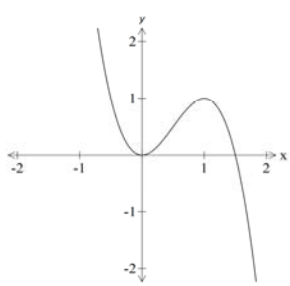
(B) $(-\frac{2}{3}, -\frac{92}{27})$ and $(2, -4)$

(C) $(\frac{2}{3}, -\frac{4}{27})$ and $(-2, -28)$

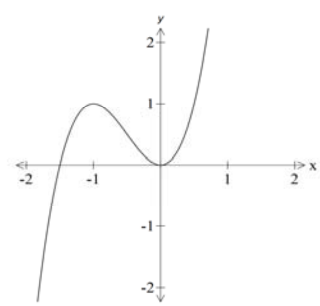
(D) $(\frac{2}{3}, -\frac{4}{27})$ and $(2, -4)$

9 Which of the following is the graph of $f(x) = 2x^3 - 3x^2$?

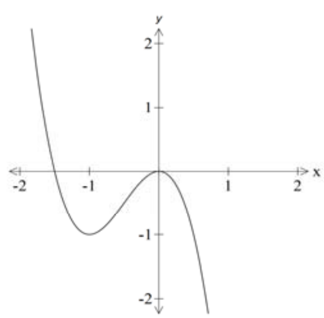
(A)



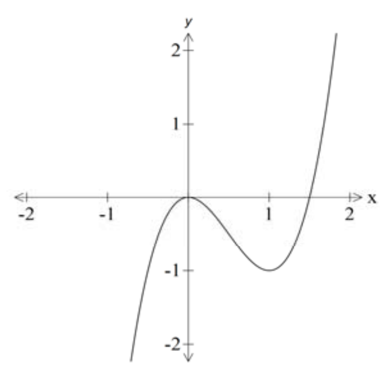
(B)



(C)

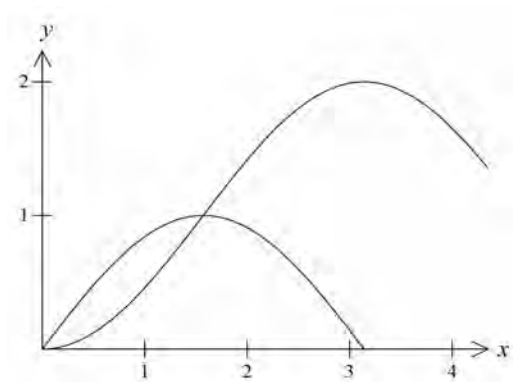


(D)



10 The diagram below shows the graph of $y = \sin x$ and $y = 1 - \cos x$.

These graphs intersect at $(0,0)$ and $(\frac{\pi}{2}, 1)$.



What is the value of the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \le x \le \pi$?

(A) 2

(B) $2 + \pi$

(C) $2 - \pi$

(D) π

END OF SECTION 1

<u>SECTION II</u> <u>All necessary working must be shown</u>

<u>Question 11</u> **(15 marks)** **Start a new answer booklet** **Marks**

(a) Harry lives in Homebush and is starting a new job in the city. He needs to catch a train to get to work. His new boss says that he cannot be late on the first two days of his new job or he will lose it. The probability that his train will arrive on time is **0.96**

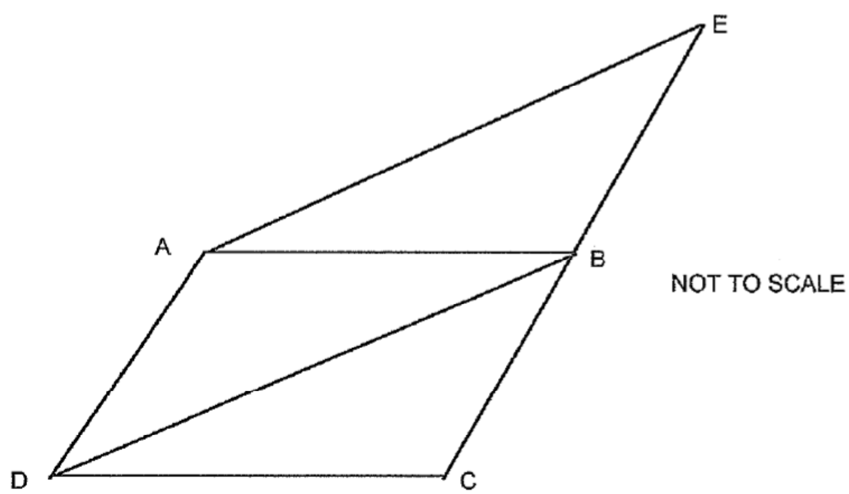
(i) What is the probability that Harry's train is late on the first day? 1

(ii) What is the probability of the train being late on the first two days? 1

(iii) What is the probability of Harry keeping his job? 1

(iv) What is the probability that Harry arrives late on exactly one of the first three days of his new job? (do not round off your answer). 1

(b) ABCD is a rhombus. CB is produced to E such that CB = BE.
Copy the diagram onto your worksheet.



(i) Prove that $\Delta ABE \equiv \Delta DCB$. 3

(ii) Hence explain why AE is parallel to DB. 1

(iii) State, giving reasons, what type of quadrilateral is AEBD 1

(c) (i) Find the range of values of k such that the following simultaneous equations have two solutions. 4

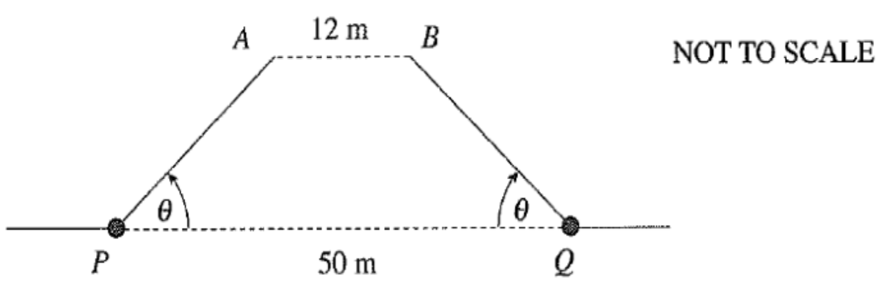
$$y = x + k$$
$$2x^2 + y^2 = 6$$

(ii) Find the value of n if the roots of the equation, 2

$$4x^2 - 20x + n = 0,$$ differ by 2.

<u>Question 12</u> (15 marks) **Start a new answer booklet**

(a)



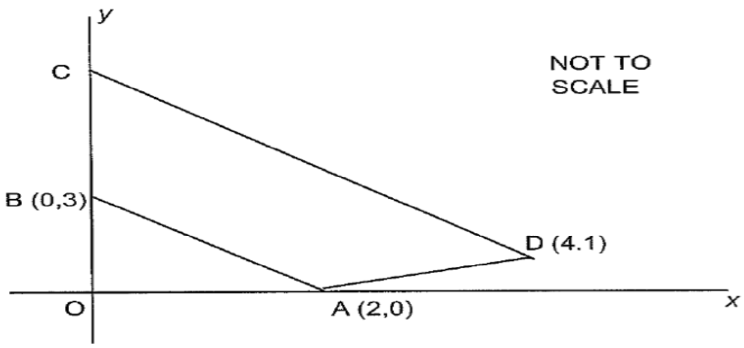
Copy this diagram onto your worksheet.

The figure shows the side view of a bridge opened to let boats pass underneath. When the equal arms of the bridge *PA* and *QB* are lowered, they meet exactly to form the straight roadway *PQ*, which is 50 metres long. When the arms *PA* are *QB* are raised through an angle θ as shown, the `corridor ' *AB* is 12 metres wide.

Calculate the size of angle θ to the nearest degrees.

3

(b)



In the diagram above, the coordinates of A, B and D are (2, 0), (0, 3) and (4, 1) respectively. Point C lies on the y-axis such that AB is parallel to DC.

Copy the above diagram onto your worksheet.

(i) What type of quadrilateral is ABCD? 1

(ii) Write down the gradient of AB. 1

(iii) Show that the equation of DC is $3x + 2y - 14 = 0$. 2

(iv) Find the coordinates of point C. 1

(v) Show that the length of AB = $\sqrt{13}$ units. 1

(vi) Find the length of CD in exact form. 1

(vii) Find the shortest distance from A to CD in exact form. 1

(viii) Hence or otherwise, find the area of quadrilateral ABCD. 2

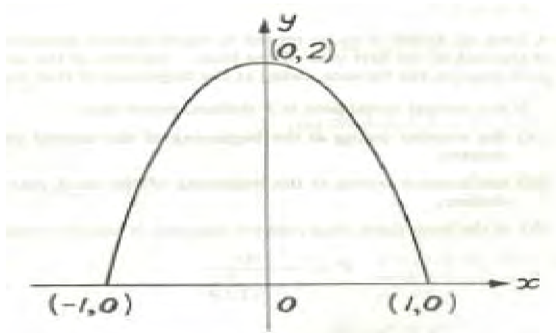
(c) For the curve $y = \ln(x - 2)$

(i) Write down its domain. 1

(ii) Sketch the curve. 1

<u>Question 13</u> **(15 marks)** **Start a new answer booklet**

(a) An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of an arc of either a cosine curve or a parabola, as illustrated on axes below.



NOT TO SCALE

(i) If the arch is made in the shape of the curve **3**

$$y = 2cos\frac{\pi}{2}\,x$$

Find the area of the window. (Answer in terms of π)

(ii) If the arch is made the shape of an arc of a parabola, **2**

Show that the equation of the parabola is $y = 2 - 2x^2$

(iii) Hence, find the area of the window made in the shape of an arc. **2**

(b) Consider the curve given by $y = x^3 - 6x^2 + 9x + 4$.

(i) Find the coordinates of the stationary points and determine their nature. **4**

(ii) Find the coordinates of any points of inflexion. **2**

(iii) Sketch the curve, showing all of the above information. **2**

Question 14 **(15 marks)** **Start a new answer booklet**

(a) (i) Sketch a graph of the function $y = \ln x$ for the domain $1 \leq x \leq 3$.
Shade the region which is above the $x-$axis and enclosed by the function
$y = \ln x$ and $y = \ln 3$. 1

(ii) Show that when the region is rotated about the $y-$ axis, 4
the volume generated is $V = 4\pi\ units^3$.

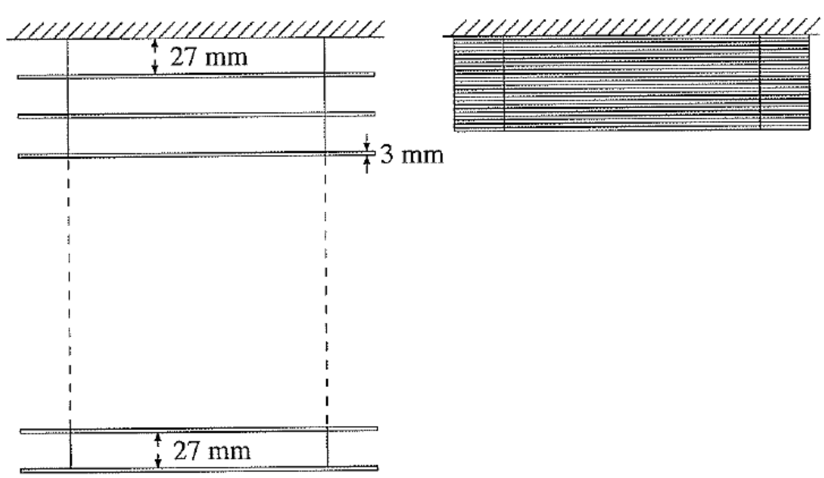
(b) A rural water dam is to be emptied by means of a control valve. The valve operates so that the volume of the water, V litres, remaining in the dam varies with time, t minutes, according to the equation

$\frac{dV}{dt} = -bt$, where b is a constant.

(i) Initially the dam contains 250 000 litres of water. 2
Show that after t minutes $V = 250\,000 - \frac{1}{2}bt^2$.

(ii) If $b = 0.431$, at what rate to the nearest litres will the dam be emptying
when $V = 85\,000$ litres? 2

(c)



A venetian blind consists of twenty-five slats, each 3mm thick. When the blind is down, the gap between the top slat and the top of the blind is 27mm and the gap between the adjacent slats is also 27mm, as shown in the first diagram.

(i) Show that when the blind is raised, the bottom slat rises 675mm. 1

(ii) How far does the next slat rise? 1

(iii) Explain briefly why the distances of the slats rise form an arithmetic sequence. 2

(iv) Find the sum of all the distances that the slats rise when the blind is raised. 2

Question 15 **(15 marks)** **Start a new answer booklet**

(a) A particle is moving in a straight line, starting from the origin. At times t seconds the particle has a displacement of x metres from the origin and a velocity $m\,s^{-1}$. The displacement is given by $x = 2t - 3\,log_e\,(t + 1)$.

(i) Find an expression for v. 1

(ii) Find the initial velocity. 1

(iii) Find when the particle comes to rest. 2

(iv) Find the distance travelled by the particle in the first three seconds. (Answer to four decimal places). 3

(b) The population P of mosquitoes in a laundry is growing exponentially according to the equation $P = 50e^{kt}$, where t, is the time in days after the insects are first counted. After four days the population has doubled.

(i) Show that the constant is $k = \frac{1}{4}\ln 2$. 2

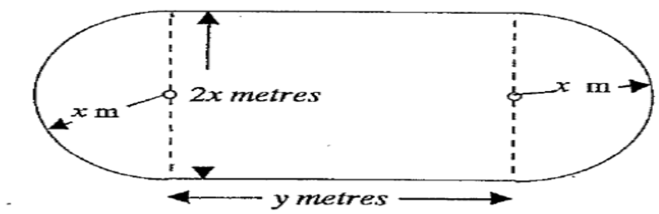
(ii) How many mosquitoes will there be after 10 days? 2

(iii) At what rate is the population increasing after 10 days? 2

(iv) How long will it take to the nearest number of days for the number of mosquitoes to be 1000? 2

<u>Question 16</u> **(15 marks)** **Start a new answer booklet**

(a) A high school plans to construct a new athletics track. The track will be rectangular with semi-circular ends. The perimeter of the track must be 400 metres.
Let the length of the straight be y metres and the width of the field be $2x$ metres.



(i) Show that $y = 200 - \pi x$. 1

(ii) If A represents the area of the athletics field, show that 2
$A = 400x - \pi x^2$.

(iii) Show that $x = \frac{200}{\pi}$, when the enclosed area of the athletics track 2
is maximum.

(iv) Hence find the other dimension? 1

(v) Calculate the maximum area of the new athletics track. (Answer in terms of π). 2

(b) Alex borrowed $60 000 to buy a small business. He was charged 6% per annum on the balance owing and he repaid the loan plus interest in equal monthly repayment over 5 years.

(i) Show that Alex owed $ (60 300 – M) immediately after making his first monthly repayment of $\$M$.

1

(ii) Show that Alex owed \$ [$60000\ (1.005)^3 - M\ (1.005^2 + 1.005 + 1\ )$]

1

immediately after he made three monthly repayments.

(iii) Calculate his monthly repayment, $\$M$ to the nearest five cents.

2

(iv) Calculate the total amount of interest paid.

1

(c) Show that

2

$$sin^2(225\,°)cosec\ (315°) = -\frac{1}{\sqrt{2}}$$

END OF EXAMINATION

Multiple choice questions : Answer on the answer sheet provided.

1 What values of x is the curve $f(x)=2x^3+x^2$ concave down?

(A) $x<-\frac{1}{6}$

(B) $x>-\frac{1}{6}$

(C) $x<-6$

(D) $x>6$

2 The table below shows the values of a function $f(x)=\sqrt{25-x^2}$ for six values of x.

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	4.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25-x^2}\,dx$ using trapezoidal rule with these six values?

(A) 10.74
(B) 12.65
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3 The semi-circle $y=\sqrt{9-x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

(A) $V=\pi\int_0^3 (9-x^2)\,dx$

(B) $V=2\pi\int_0^3 (9-x^2)\,dx$

(C) $V=\pi\int_0^3 (9-y^2)\,dy$

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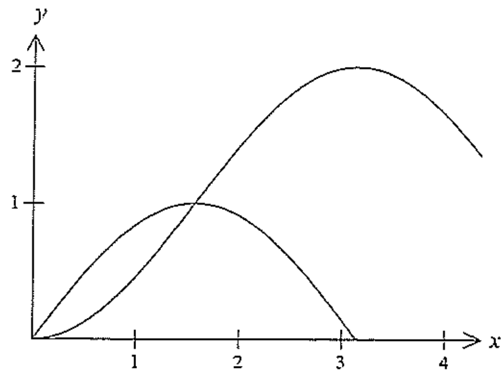
These graphs intersect at $(0,0)$ and $(\frac{\pi}{2}, 1)$.



What is the value of the area between $y = \sin x$ and $y = 1 - \cos x$ over the domain $0 \leq x \leq \pi$?

(A) 2

(B) $2 + \pi$

(C) $2 - \pi$

(D) π

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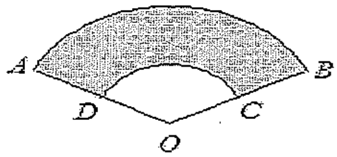

Not to scale

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6 What is the derivative of $(1+\log_e x)^4$?

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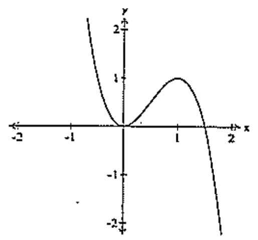
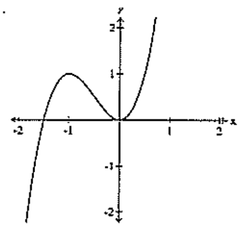
9 Which of the following is the graph of $f(x) = 2x^3 - 3x^2$?

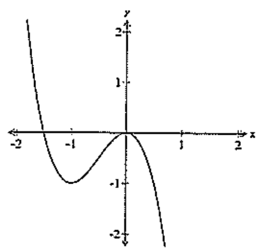
(A)



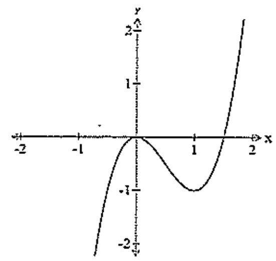
(B)



(C)



(D)



MATHEMATICS : 2013 TRIAL HSC SOLUTIONS

Question 11

Mostly well done

a (i) $P(L) = 0.04$ ✓

ii) $P(LL) = (0.04)^2$
$= 0.0016$ ✓

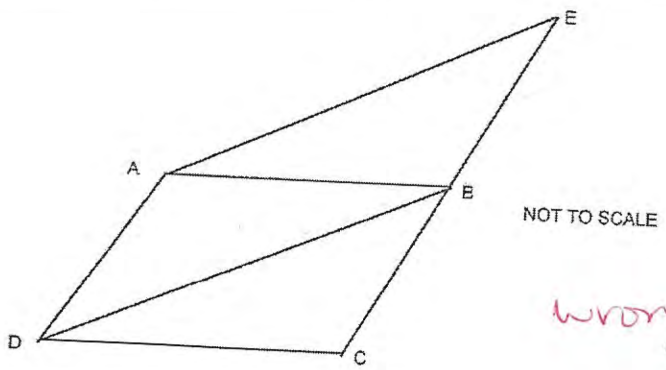
iii) $P(\bar{L}\bar{L}) = 1 - 0.0016$
$= 0.9984$ ✓

also. $0.96 \times 0.96 = 0.9216$

iv) $P(1/3) = 3(0.04 \times 0.096 \times 0.96)$
$= 0.110592$ ✓

also $0.96 \times 0.96 \times 0.04$
$= 0.036864$

b



wrong assumptions

1. $\angle EAB = \angle BDC$ alternate angles

2. students proved $\Delta DAB \equiv \Delta BCD$

(i) In Δ's ABE, ΔDCB

$AB = CD$ } sides of
$AD = BC$ } rhombus ✓
$BE = CD$ (given)

$\angle ABE = \angle DCE$ (alternate $\angle$'s $AB \parallel DC$) ✓

$\therefore \Delta ABE \equiv \Delta DCB$ (SAS)
✓

ii) AE ∥ DB because

$\angle AEB = \angle DBC$

(Corresponding $\angle$'s in congruent Δ's)

well done

iii) Since $\Delta DBC \equiv \Delta AEC$

$DB = AE$

$\therefore$ AEBD is parallelogram because opposite sides are equal and parallel.

C i) $y = x + k$ — ①

$2x^2 + y^2 = 6$ — ②

students made mistakes with the expansion of $(x+k)^2$

Sub ① into ②

$2x^2 + (x+K)^2 = 6$

$2x^2 + x^2 + 2xK + K^2 = 6$

$3x^2 + 2xK + (K^2 - 6) = 0$

2 solutions when $\Delta > 0$

$(2K)^2 - 4(3)(K^2 - 6) > 0$

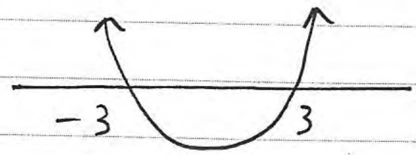
$4K^2 - 12K^2 + 72 > 0$

$-8K^2 + 72 > 0$

2

→

$K^2 - 9 < 0$ ✓



$-3 < x < 3$ ✓

$\therefore$ There are two sets of solutions when $-3 < K < 3$

ii) $4x^2 - 20x + n = 0$

let the roots be α and $\alpha + 2$

Sum: $2\alpha + 2 = -\frac{b}{a}$

$2\alpha + 2 = 5$

$2\alpha = 3$

$\alpha = \frac{3}{2}$ ✓

mostly well done.

Product: $\alpha(\alpha + 2) = \frac{c}{a}$

$\alpha^2 + 2\alpha = \frac{n}{4}$

$\left(\frac{3}{2}\right)^2 + 2\left(\frac{3}{2}\right) = \frac{n}{4}$

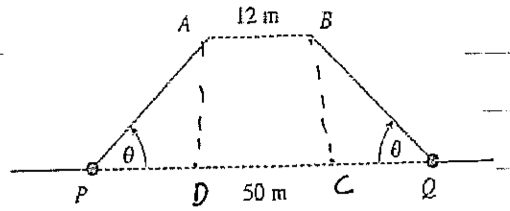
$\frac{9}{4} + 3 = \frac{n}{4}$

$n = 9 + 12$

$= 21$ ✓

3

Question 12



a) $PD = QC$

$= \frac{50-12}{2}$

$= 19$ ✓

$PA = QB$ (given)

$= \frac{50}{2}$

$= 25$ ✓

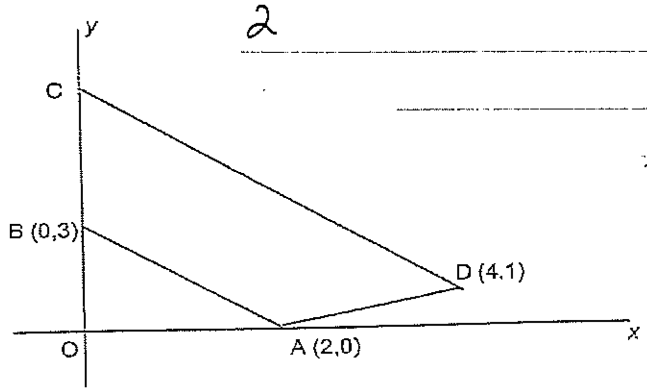
$\cos\theta = \frac{19}{25}$

$\theta = 41^\circ$ ✓

b i) ABCD is a Trapezium ✓
(AB ∥ DC)

ii) $m_{AB} = \frac{\text{rise}}{\text{run}}$

$= -\frac{3}{2}$ ✓



Generally well done - some students did not show working for sides PD + PA but awarded the mark anyway.

④

iii)	$M_{DC} = M_{AB} = \frac{-3}{2}$	
	Equation of line DC:	
	$y - 1 = \frac{-3}{2}(x - 4)$ ✓	Need to show enough working for the 2 marks.
	$2y - 2 = -3x + 12$ ✓	this line not counted as it was a 'show' question.
	$3x + 2y - 14 = 0$ as required	
iv)	when $x = 0$	
	$3(0) + 2y = 14$	
	$y = 7$	
	$\therefore$ C is $(0, 7)$ ✓	occasional error here
v	$AB = \sqrt{(\Delta x)^2 + (\Delta y)^2}$	
	$= \sqrt{9 + 4}$ ✓	'Carry on' errors awarded provided a new error was not made
	$= \sqrt{13}$ u	
vi)	$CD = \sqrt{(\Delta y)^2 + (\Delta x)^2}$	
	$= \sqrt{6^2 + 4^2}$	
	$= \sqrt{52}$ u ✓	

vii) $d = \dfrac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|3\times 2+0-14|}{\sqrt{9+4}}$

$= \dfrac{8}{\sqrt{13}}$ ✓ u or $\dfrac{8\sqrt{13}}{13}$ u.

A common error was $|6-14| = 6$!

viii) Area $= \left(\dfrac{a+b}{2}\right)h$ ✓

$= \dfrac{(\sqrt{13}+\sqrt{52})}{2} \times \dfrac{8}{\sqrt{13}}$

$= 12 \text{ units}^2$ ✓

Some students did not know formula! fatal error (−2 marks)

c i) $y = \ln(x-2)$

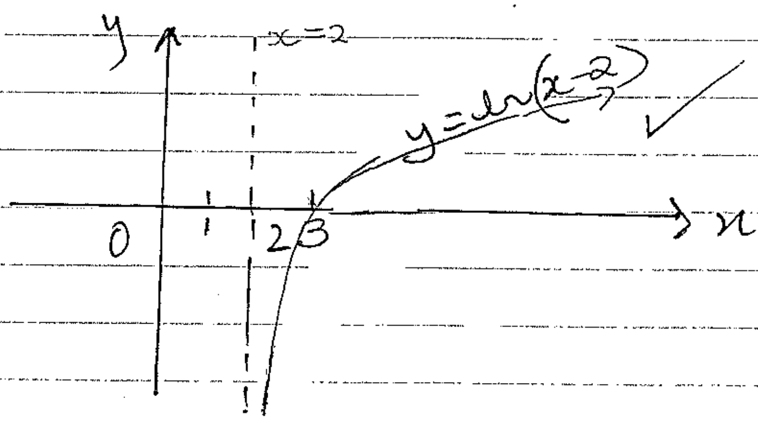
D: $x - 2 > 0$

$x > 2$ ✓

ii) $\ln(x-2) = 0$

$x - 2 = e^0$

$x = 3$



USE A RULER FOR AXES! Must have x + y axes labelled. Sketches must be of a suitable size. Relative scale is req'd on x axis.

6

Label asymptote with eqn.

Question 13

a i) $$\text{Area} = \int_{-1}^{1} 2\cos\frac{\pi}{2}x\,dx$$ ✓

$$= 2\int_{0}^{1} 2\cos\frac{\pi}{2}x\,dx$$

$$= 4\left[\frac{2}{\pi}\sin\frac{\pi}{2}x\right]_0^1$$ ✓

$$= \frac{8}{\pi}(1-0)$$

$$= \frac{8}{\pi}\ m^2$$ ✓

Many students failed to simplify $\frac{8}{\pi}\sin\frac{\pi}{2}$ to $\frac{8}{\pi}$ presumably because they are not yet proficient with working in radians.
Also the rule $\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)+c$ was poorly applied with students unable to write $\frac{1}{\pi/2}$ as $\frac{2}{\pi}$.

ii) $y = ax^2 + bx + c$

when $x = 0$, $y = 2$

$\therefore y = ax^2 + bx + 2$

$\therefore c = 2$ ✓

axis of symmetry:

$$x = \frac{-b}{2a}$$

$$= 0$$

$\therefore b = 0$

$y = ax^2 + 2$

$x = 1$, $y = 0$

$a + 2 = 0$

$a = -2$ ✓

$\therefore y = -2x^2 + 2$

Major problem was using what you were required to prove as part of the proof. i.e. As the question was, "Show that the equation of the parabola is $y = 2 - 2x^2$", it is incorrect to use $y = 2 - 2x^2$ as part of the proof.
The correct technique is to start with one of the general forms of a parabola like $y = ax^2 + bx + c$ or $(x-h)^2 = -4ay$ or $y = k(x-1)(x+1)$ and use the characteristics of the graphs to establish the equation 7 as $y = 2 - 2x^2$.

iii) $A = \int_{-1}^{1} 2 - 2x^2 \, dx$

$= 2\left[2x - \frac{2}{3}x^3\right]_0^1$ ✓

$= 2\left(2 - \frac{2}{3}\right)$

$= \frac{8}{3} \text{ m}^2$ or $\left(2\frac{2}{3} \text{ m}^2\right)$ ✓

Very well done

b $y = x^3 - 6x^2 + 9x + 4$

i) $\frac{dy}{dx} = 3x^2 - 12x + 9$

S.P when $\frac{dy}{dx} = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$ ✓

$\therefore f(1) = 8$

$f(3) = 4$

$\therefore$ Stationary points are

$(1, 8)$ and $(3, 4)$ ✓

This question was generally well done.

Students solutions to b part ii could be improved by stating "Possible points of inflexion occur when $\frac{d^2y}{dx^2} = 0$ which leads to $x = 2$ and then confirming the existence of the Point of Inflexion by showing a change of concavity with a table of the 2nd derivative.

The nature:

$f''(x) = 6x - 12$

$f''(1) = -6$

$f''(3) = 6$

$\therefore f''(1) < 0$ ✓

So $(1, 8)$ is a maximum

$f''(3) > 0$ ✓

$\therefore (3, 4)$ is a minimum

ii) P.O.I when $f''(x) = 0$

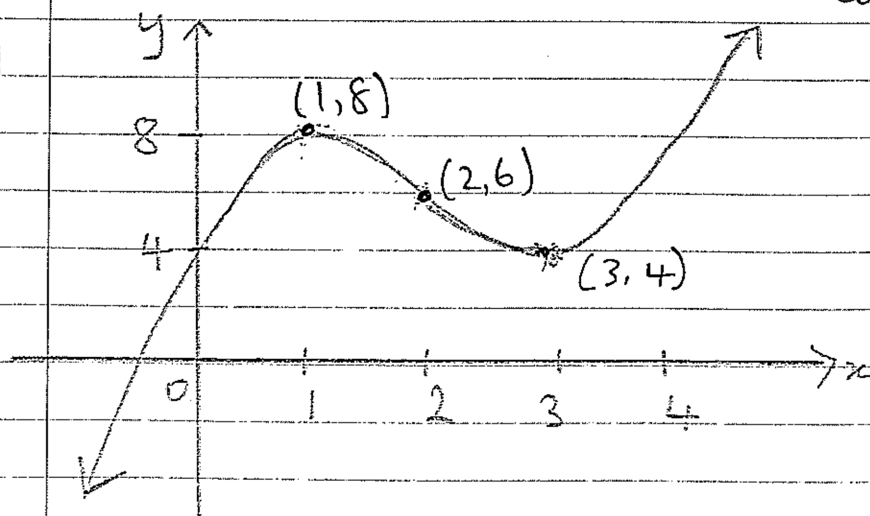
$6x - 12 = 0$

$x = 2$ ✓

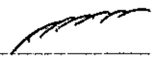
$f''(2) = 6$ ✓

$\therefore$ P.O.I is $(2, 6)$ Also when discussing the concavity.

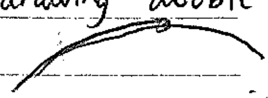
iii)



Students should draw neat half page sketches, use a pencil and avoid feathering



and drawing double lines



If you make a mistake erase it and draw it again.

✓ shape

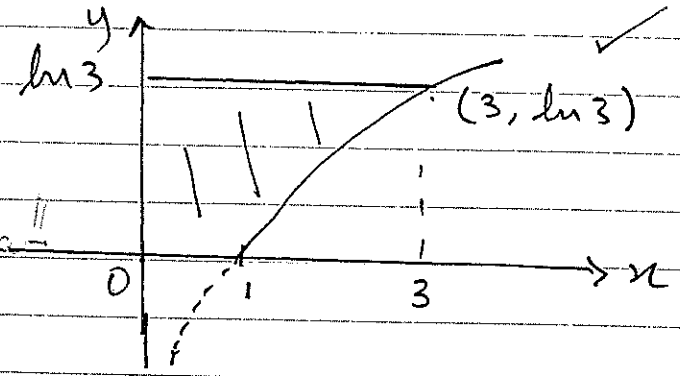
✓ y intercept + P.O.I + T. Points.

9

Question 14

a i) $y = \ln x$

1 for line $y=\ln x$, $y=\ln 3$ and shading the correct area



- many could not graph $y=\ln x$, or made it look like a straight line
- many could not shade between the given lines (which included the x-axis).

$$y = \ln x$$
$$x = e^y$$

ii) $$V = \pi \int_0^{\ln 3} x^2 \, dy$$

- set up generally well done

1 x^2 expression
$$= \pi \int_0^{\ln x} e^{2y} \, dy$$

- integration generally well done

1 integration and limits correctly substituted
$$= \pi \left[\frac{e^{2y}}{2} \right]_0^{\ln 3}$$

$$= \frac{\pi}{2} \left[e^{2\ln 3} - e^0 \right]$$

- many could not, or left out, resolving the powers.

1 resolving powers
$$= \frac{\pi}{2} \left[e^{\ln 9} - 1 \right]$$

1 resolution to answer
$$= \frac{\pi}{2} (9-1)$$

- generally well done

$$= 8\pi \text{ units}^3$$

b i) $\frac{dV}{dt} = -bt$

$V = -\int bt\, dt$

① integration $V = -\frac{bt^2}{2} + c$ ✓

$t = 0$

$V = 250\,000$

① initial conditions used correctly $\therefore C = 250\,000$ ✓

$\therefore V = 250\,000 - \frac{b}{2}t^2$

• generally well done, but a significant number of students could not do the process of using the initial conditions to show how the eqn was derived. (ie. $t=0$, $V=250000$)

ii) $85000 = 25000 - \frac{0.431}{2}t^2$

$t = \sqrt{\frac{330\,000}{0.431}} = 765661.2529$

① finds correct 't' value for given V.

$= 875$ minutes ✓

$\frac{dV}{dt} = -bt$

$= -0.431 \times 875$

① finds and interprets rate correctly $= -377$ litres/min ✓

emptying at 377 l/min.

• Many tried to interpret this as a growth/decay problem.

• too many students decided this was a rate!

• question does not ask for a value of $\frac{dV}{dt}$!! (too many students stopped at this point and did not interpret their result).

c i) No. of slats = 25

① shows/interprets working correctly

The bottom slat rises $= 27 \times 25$

$= 675$ mm

ii) 2^{nd} last slat $= 27 \times 24$

① correct.

$= 648$ mm

iii) 1^{st} slat moves up 27 mm

2^{nd} " " 54 mm

3^{rd} " " 81 mm

① establishes pattern.

$\therefore\ 27 + 54 + 81 + \dots + 675$

① test for AP correct

$d_1 = 54 - 27 = 27$

$d_2 = 81 - 54 = 27$

Since $d_1 = d_2$, it is an arithmetic sequence

iv) $S_n = \frac{n}{2}(a + l)$

$n = 25$, $a = 27$, $l = 675$

① set up correct

$= \frac{25}{2}(27 + 675)$

① answer correct $= 8775$ mm

Alternate $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{25}{2}(2.27 + (25-1).27)$

$= \frac{25}{2} . 702$

$= 8775$ mm

also $a = 675$ $d = -27$

$S_n = \frac{25}{2}(2.675 + (25-1)\times -27)$

$= \frac{25}{2}(702)$

$= 8775$

(i) and (ii):

- various methods, but generally well done.
- as the question (in pt (ii)) asks you to show/explain this is an AP, you cannot use Tn formula - you haven't shown its an AP yet!

(iii) many did not develop the sequence, thus making it difficult to demonstrate the AP.

- this definition was rarely used (because of previous point).
- an explanation of constant difference was awarded one mark.

- this was often incorrect when sign of d was confused. If $a = 675$ then $d = -27$, not 27.
- otherwise generally well done.

Question 15

a i) $V = \frac{dx}{dt}$ $\qquad x = 2t - 3\log_e(t+1)$

$= 2 - \frac{3}{t+1}$ ✓

ii) when $t = 0$

$V = 2 - 3$

$= -1$ m/s ✓

(the particle is moving to the left)

iii) rest: $V = 0$

$2 - \frac{3}{t+1} = 0$

$\frac{3}{t+1} = 2$ ✓

$2t + 2 = 3$

$t = \frac{1}{2}$ sec ✓

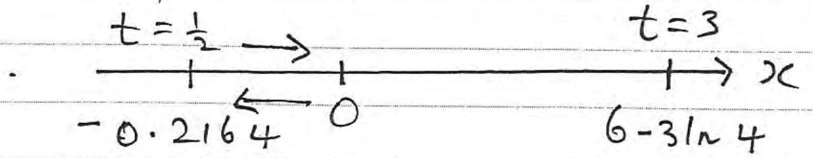
iv) Distance travelled:

$t = \frac{1}{2}$

$x = 2(\frac{1}{2}) - 3\ln\frac{3}{2}$

$= -0.2164$ m ✓

$t = 3$, $x = 6 - 3\ln 4$ ✓



The distance travelled is the sum of two parts. Evaluate these as well as the distance.

$\therefore$ Total distance $= 2(0.2164) + (6 - 3\ln 4)$

$= 2.2734$ m

13

b i) $P = 50e^{Kt}$

$100 = 50e^{4K}$ ✓ better if they derive it, as otherwi[se] they invariably find a way to get to it, albeit not always correctly.

$$\left.\begin{aligned} e^{4K} &= 2 \\ 4K &= \ln 2 \end{aligned}\right\} ✓$$

$$K = \frac{1}{4}\ln 2$$

ii) $P = 50e^{10 \times \frac{\ln 2}{4}}$ ✓

$= 283$ mosquitoes ✓

iii) $\frac{dP}{dt} = KP$ The rate $\left(\frac{dP}{dt}\right)$ is proportional to the population (P). What is the constant of proportionality, and what is $\frac{dP}{dt}$ when $t=10$ s.?

$= \frac{1}{4}\ln 2 \times 50e^{10 \times \frac{1}{4}\ln 2}$

$= 49$ mosquitoes / day ✓

iv) $50e^{Kt} = 1000$

$e^{Kt} = 20$

$Kt = \ln 20$ ✓

$t = \ln 20 \div \frac{1}{4}\ln 2$

$\simeq 17$ days ✓

Question 16

a i) $P = 2y + 2\pi r$

$2y + 2\pi x = 400$

$2y = 400 - 2\pi x$ ✓ well done

$y = 200 - \pi x$

ii) $A = 2xy + \pi r^2$

$= 2x(200 - \pi x) + \pi r^2$ ✓ well done.

$= 400x - 2\pi x^2 + \pi r^2$ ✓

$= 400x - \pi x^2$ as required

iii) $\frac{dA}{dx} = 400 - 2\pi x$

When maximum:

$400 - 2\pi x = 0$

$2\pi x = 400$ ✓

$x = \frac{200}{\pi}$

To show it is maximum:

x	$\frac{199}{\pi}$	$\frac{200}{\pi}$	$\frac{201}{\pi}$
$\frac{dA}{dx}$	2	0	-2
	+	0	−

✓

$\therefore$ The area is maximum when $x = \frac{200}{\pi}$

most students failed to show why ~~this~~ the area is maximum when $x = \frac{200}{\pi}$.

15

iv) $P = 400\ m$

$y = 200 - \pi x$

$= 200 - \pi\left(\frac{200}{\pi}\right)$

$= 0$ ✓

$\therefore$ The area is maximum when $y = 0$ (a circle)

v $\text{Area} = 2xy + \pi r^2 \quad (y=0)$

$= \pi\left(\frac{200}{\pi}\right)^2$ ✓

$= \frac{40000}{\pi}\ m^2$ ✓

b i) $P = \$60\,000$, $R = 1.005$

$n = 60$ repayments.

$A_1 = PR - M$

$= \$60000 \times 1.005 - M$ ✓

$= \$60300 - M$

ii) $A_2 = (PR - M)R - M$

$= PR^2 - MR - M$

$A_3 = PR^3 - MR^2 - MR - M$

$= PR^3 - M(R^2 + R + 1)$ ✓

$= \$60000(1.005)^3 - M(1.005^2 + 1.005 + 1)$

as required.

Well done.

16

iii) $A_{60} = 0$

$$0 = PR^{60} - m(\underbrace{R^{59} + R^{58} + \dots + R + 1}_{GP})$$

$$= PR^{60} - m\left(\frac{R^{60}-1}{R-1}\right)$$

$$m\left(\frac{R^{60}-1}{R-1}\right) = PR^{60}$$

$$m = \frac{PR^{60}(R-1)}{(R^{60}-1)}$$

$$= \frac{\$60\,000(1\cdot005)^{60}(1\cdot005-1)}{(1\cdot005^{60}-1)}$$

$$= \frac{\$60\,000(1\cdot005)^{60}(0\cdot005)}{(1\cdot005^{60}-1)}$$

$= \$1,159\cdot95$ /month — well done.

iv) Interest $= (m \times 60) - \$60\,000$
$= \$9,597$

c $\sin 225° = -\sin 45°$
$= -\frac{1}{\sqrt{2}}$

$\operatorname{cosec} 315° = \frac{1}{\sin 315°} = \frac{1}{-\frac{1}{\sqrt{2}}}$ — well done.
$= -\sqrt{2}$

$\therefore \sin^2 225 \operatorname{cosec} 315° = \frac{1}{2} \times -\sqrt{2}$ (17)

$= \frac{-\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$ as required.

END