FORT STREET HIGH SCHOOL

Name: ______________________

Teacher: ______________________

Class: ______________________

2015

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11, 12, 13
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	14, 15
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	16

Total Marks 100

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section
Section II 90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section I

10 marks

Attempt Questions 1-10

Circle the correct response in your Question Book.

1. The domain of the function $f(x)=\dfrac{1}{\sqrt{4x^2-1}}$ is:

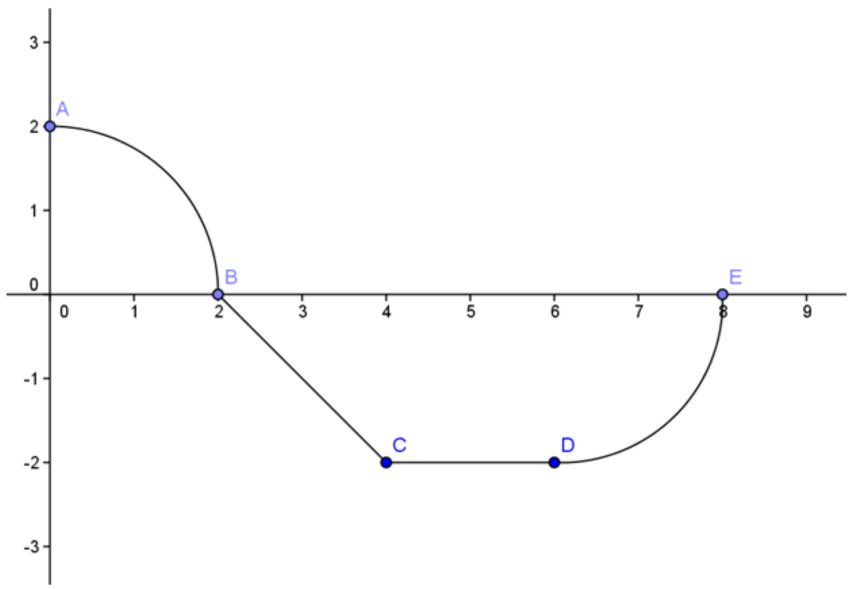
   (A) $-\dfrac{1}{2}<x<\dfrac{1}{2}$

   (B) $x<-\dfrac{1}{2}\, and\; x>\dfrac{1}{2}$

   (C) $x\le-\dfrac{1}{2}\, and\; x>\dfrac{1}{2}$

   (D) $-\dfrac{1}{2}\le x<\dfrac{1}{2}$

Use the following diagram to answer questions 2 and 3
(note: the curved sections between A and B, D and E are parts of a circle).



2. Evaluate $\displaystyle\int_0^8 f(x)dx$

   (A) 12

   (B) 6

   (C) -6

   (D) 0

3. For what values of x satisfying $0 < x < 8$ is the function f NOT differentiable?

(A) $x = 2 \, and \, x = 4$

(B) $x = 2 \, and \, x = 6$

(C) $x = 6 \, and \, x = 8$

(D) $x = 0 \, and \, x = 4$

4. What is the derivative of $x\cos x$ with respect to x?

(A) $-\sin x$

(B) $-x\sin x$

(C) $x\sin x - \cos x$

(D) $-x\sin x + \cos x$

5. What is the value of $\int_0^1 e^{2x} + 1 dx$?

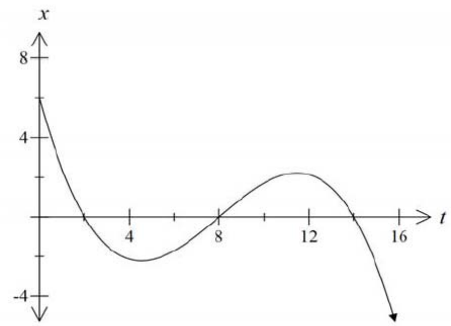
(A) $\frac{1}{2}e^2$

(B) $\frac{1}{2}(e^2 + 1)$

(C) e^2

(D) $e^2 + 1$

6. The displacement, x metres, from the origin of a particle moving in a straight line at any time (t seconds) is shown in the graph



When was the particle at rest?

(A) t = 4.5 and t = 11.5

(B) t = 0

(C) t = 2, t = 8 and t = 14

(D) t = 1.5 and t = 8

7. Which one of the following statements is true for the equation $7x^2 - 5x + 2 = 0$?

(A) No real roots

(B) One real root

(C) Two real distinct roots

(D) Three real roots

8. A particle moves along the x-axis with acceleration $3t - 2$ u/s^2. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds?

(A) 37.5 units to the right

(B) 37.5 units to the left

(C) 51.5 units to the right

(D) 51.5 units to the left

9. Find the maximum value of $y = -x^2 + x + 11$

(A) 11.25

(B) 0.5

(C) 12.25

(D) 1

10. Integrate $7x - \frac{2}{x^2}$

(A) $\frac{7x^2}{2} + \frac{2}{x} + c$

(B) $\frac{7x^2}{2} - \frac{2}{x} + c$

(C) $\frac{7x^2}{2} + \frac{2}{x^3} + c$

(D) $\frac{7x^2}{2} - \frac{2}{x^3} + c$

Question 11 (15 marks)

(a) Find $\log_5 12$ correct to 3 significant figures **2**

(b) Factorise fully $3x^2+5x-2$ **2**

(c) Express $\dfrac{5}{\sqrt{3}+2}$ in the from $a\sqrt{3}+b$ **2**

(d) Solve $|2x-1|<3$ **2**

(e) If $f(x)=2\sin 3x$ find the exact value of $f'\left(\dfrac{\pi}{18}\right)$ **2**

(f) Find $\displaystyle\int \frac{x}{2}-e^{2x}dx$ **2**

(g)

i. Sketch, on the same graph, the curves $y=x^2-4x+3$ and $y=x+3$.Show the x and y intercepts in the sketch. **2**

ii. Hence shade the region defined by $\begin{Bmatrix} y<x^2-4x+3 \\ y\le x+3 \end{Bmatrix}$ simultaneously. **1**

Question 12 (15 marks)

(a) Paula has 10 playing cards in her hand, consisting of 5 different pairs of cards. She shuffles the cards and places two of them face up on the table.

i. Find the probability that these two cards are not a pair **1**

ii. A third card is placed on the table. Find the probability that this card forms a pair with one of the two cards already on the table. **1**

(b) A person invests \$800 at the beginning of each year in a superannuation fund. Compound interest is paid at 10% per annum on the investment. The first \$800 is to be invested at the beginning of 2016 and the last is to be invested at the beginning of 2045. Calculate to the nearest dollar:

i. The amount to which the 2016 investment will have grown by the beginning of 2046 **2**

ii. The amount to which the total investment will have grown by the beginning of 2046 **2**

(c) Solve $2\cos^2 x - 1 = 0$ for $-\pi \le x \le \pi$ **3**

(d) Find the equation of the tangent to $y = 2\sin 2x - 3$ at the point where $x = 0$ **3**

(e) Find the value of x if $\sum_{n=0}^{\infty} \frac{9}{x^{n+1}} = 18$ **3**

Question 13 (15 marks)

(a) Show that $\dfrac{\tan\theta}{\sec\theta-1}-\dfrac{\tan\theta}{\sec\theta+1}=2\cot\theta$ **2**

(b) Maureen is raising money for charity by jumping on a Pogo Stick. Her challenge is to jump between two points, A and B, 20 times. On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps. On her third attempt she takes 51 jumps. She continues this pattern for all 20 attempts.

i. How many jumps did she make on her 20^{th} attempt? **2**

ii. How many jumps did she make altogether? **2**

(c) For the parabola $y=\dfrac{1}{8}x^2-x+3$

i. Find the co-ordinates of the vertex and focus **2**

ii. Find the equation of the normal to the parabola at the point where $x=-4$. Write your answer in general form. **3**

iii. Find the point on the parabola at which the tangent is parallel to $y=3x+1$ **2**

(d) A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective.

Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective. Give your answer in exact form. **2**

Question 14 (15 marks)

(a)

i. Copy and complete the function box in your writing booklet for the function $y = \frac{4}{x}$. Answer to 2 d.p. **1**

x	1	1.25	1.5	1.75	2
y					

ii. Use the table in part i. and two applications of Simpson's Rule to find an approximation for the area bounded by the curve $y = \frac{4}{x}$, the x-axis and the lines $x = 1$ and $x = 2$. Answer to 2 d.p. **2**

(b) Consider the curve $y = x^3 - 12x^2 + 36x$

i. Find the x and y intercepts **2**

ii. Find any stationary points and determine their nature **3**

iii. Hence, sketch for $-1 \le x \le 7$, showing all features found in parts i. and ii. **2**

(c) Find the area bounded by the line $y = 3x + 1$ and the parabola with equation $y = 2x^2 + 4x - 5$ **3**

(d) Ten kilograms of sugar is placed in a container of water and begins to dissolve. After t hours the amount A kg of undissolved sugar is given by

$$A = 10e^{-kt}$$

i. Evaluate k, given that A = 3.2 when t = 4 **1**

ii. After how many hours (to the nearest hour) does 1 kg of sugar remain undissolved? **1**

Question 15 (15 marks)

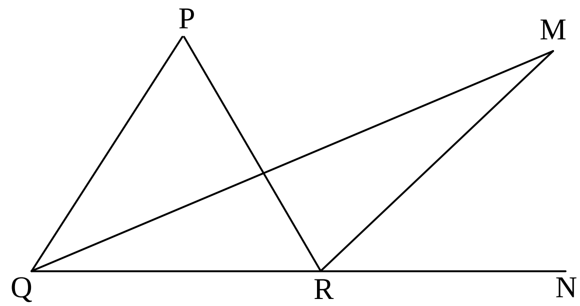


Figure not to scale

(a) PQR is an isosceles triangle in which PQ = PR and $\angle QPR = 104°$. QR is produced to N. QM bisects $\angle PQR$ and RM bisects $\angle PRN$.

i. In your writing booklet draw a neat sketch and mark on it all the given information. **1**

ii. Find the size of $\angle PQR$. Give reasons for your answer. **2**

iii. Find the size of $\angle QMR$. Give reasons for your answer. **3**

(b) What is the exact volume of the solid of revolution formed by rotating the curve $y = \sec x$ about the x-axis, for $0 \le x \le \dfrac{\pi}{3}$? **3**

(c) A particle moves in a straight line so that its velocity v in metres per second at time t is given by $v = 4 - 2t$.

At time $t = 0$, the particle is at $x = 1$.

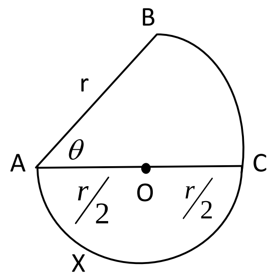
i. Find the displacement x of the particle as a function of t. **2**

ii. When is the particle at rest and what is its acceleration at that time? **2**

iii. Find the distance the particle travels in the first 4 seconds. **2**

Question 16 (15 marks)

(a) A cam is formed with cross-section as shown in the figure



The cross-section consists of a semi-circle AXC centre O and radius $\frac{r}{2}$ and a sector ABC of radius r, centre A and angle θ.

i. Show that the perimeter of the cam ABCX in terms of r and θ is given by $P = r\left(\frac{\pi}{2} + \theta + 1\right)$ **1**

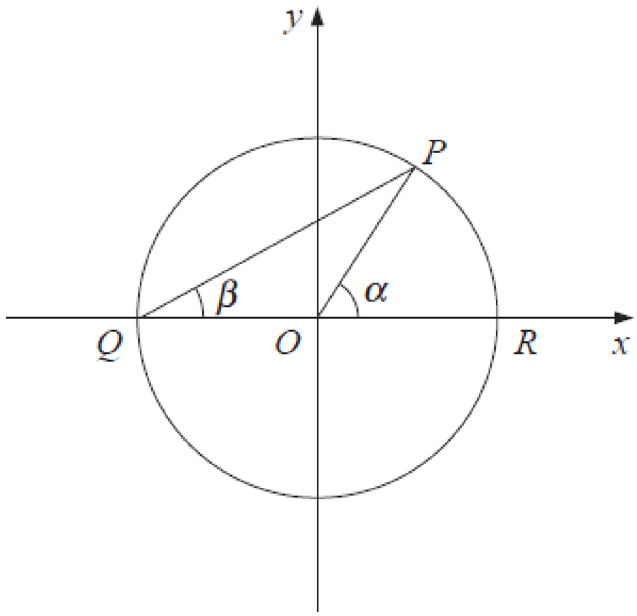
ii. If the area of the cross-section of the cam is 1 square unit, show that the perimeter P is given by **2**

$$P = \frac{2}{r} + r(1 + \frac{\pi}{4})$$

iii. Show that the least perimeter occurs when $r^2 = \frac{8}{\pi + 4}$ and calculate the value of θ to the nearest degree. **3**

Question 16 (continued)

(b) In the diagram, Q is the point (-1,0), R is the point (1,0), and P is another point on the circle with centre O and radius 1. Let $\angle POR = \alpha$ and $\angle PQR = \beta$, and let $\tan\beta = m$.



i. Explain why ΔOPQ is isosceles, and hence deduce that $\alpha = 2\beta$ **1**

ii. Find the equation of the line PQ **2**

iii. Show that x coordinates of P and Q are solutions of the equation **2**

$$(1+m^2)x^2 + 2m^2x + m^2 - 1 = 0$$

iv. Using this equation, find the coordinates of P in terms of m **2**

v. Hence deduce that $\tan 2\beta = \dfrac{2\tan\beta}{1-\tan^2\beta}$ **2**

2015 Fort Street High Mathematics Trial – Solutions

Section I: Multiple Choice

1	$4x^2 - 1 > 0$ $4x^2 > 1$ $x^2 > \frac{1}{4}$ $\therefore x < -\frac{1}{2}\, and\ x > \frac{1}{2}$	1 Mark: B
2	$\frac{1}{2} \times 2 \times 2 + 2 \times 2 = 6$ Since it is below the axes $\int_0^8 f(x)dx = -6$	1 Mark: C
3	Function is not smooth and continuous at $x = 2\, and\ x = 4$	1 Mark: A
4	$-x\sin x + \cos x$	1 Mark: D
5	$\int_0^1 e^{2x} + 1dx = \left[\frac{1}{2}e^{2x} + x\right]_0^1$ $= \left(\frac{1}{2}e^2 + 1\right) - \left(\frac{1}{2} + 0\right)$ $= \frac{1}{2}e^2 + \frac{1}{2}$ $= \frac{1}{2}(e^2 + 1)$	1 Mark: B
6	Particle comes to rest at v = 0, which is a stationary point	1 Mark: A
7	$\Delta = 25 - 4 \times 7 \times 2$ $= 25 - 56$ < 0 $\therefore$ no real roots	1 Mark: A
8	$\ddot{x} = 3t - 2$ $\dot{x} = \frac{3}{2}t^2 - 2t + c$ $t = 0, \dot{x} = 2 \therefore c = 2$ $\dot{x} = \frac{3}{2}t^2 - 2t + 2$ $x = \frac{1}{2}t^3 - t^2 + 2t + k$ $t = 0, x = 4 \therefore k = 4$ $x = \frac{1}{2}t^3 - t^2 + 2t + 4$ $At\quad t = 5$ $x = \frac{1}{2} \times 125 - 25 + 10 + 4 = 51.5$	1 Mark: C

9	$x=-\frac{b}{2a}$ $=-\frac{1}{2\times -1}$ $=\frac{1}{2}$ $At\ x=\frac{1}{2}, y=-\left(\frac{1}{2}\right)^2+\frac{1}{2}+11=11.25$	1 Mark: A
10	$\int 7x-2x^{-2}=\frac{7}{2}x^2+2x^{-1}+c$ $=\frac{7}{2}x^2+\frac{2}{x}+c$	1 Mark: A

Section II:

Q11

(a)

$$\log_5 12 = \frac{\ln 12}{\ln 5} \quad ❶$$
$$= 1.543959...$$
$$\approx 1.54 (to\, 3\, sig.\, figs.) \quad ❶$$

- Many answered to 3 decimal places

(b)

$$3x^2 + 5x - 2 = 3x^2 + 6x - x - 2$$
$$= 3x(x+2) - 1(x+2) \quad ❶$$
$$= (x+2)(3x-1) \quad ❶$$

- Well done

(c)

$$\frac{5}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{5\sqrt{3}-10}{3-4} \quad ❶$$
$$= 10 - 5\sqrt{3}$$
$$= -5\sqrt{3} + 10 \quad ❶$$

- Well done

(d)

$$|2x-1| < 3$$
$$2x - 1 < 3$$
$$2x < 4$$
$$x < 2 \quad ❶$$
$$or$$
$$2x - 1 > -3$$
$$2x > -2$$
$$x > -1 \quad ❶$$
$$\therefore -1 < x < 2$$

- Well done

(e)

$$f(x) = 2\sin 3x$$
$$f'(x) = 6\cos 3x \quad ❶$$
$$f'\left(\frac{\pi}{18}\right) = 6\cos\frac{3\pi}{18}$$
$$= 6\cos\frac{\pi}{6}$$
$$= 6 \times \frac{\sqrt{3}}{2}$$
$$= 3\sqrt{3} \quad ❶$$

- Many wrote $\cos\frac{\pi}{6} = \frac{1}{2}$. Students need to learn the exact ratios for sin, cos and tan.

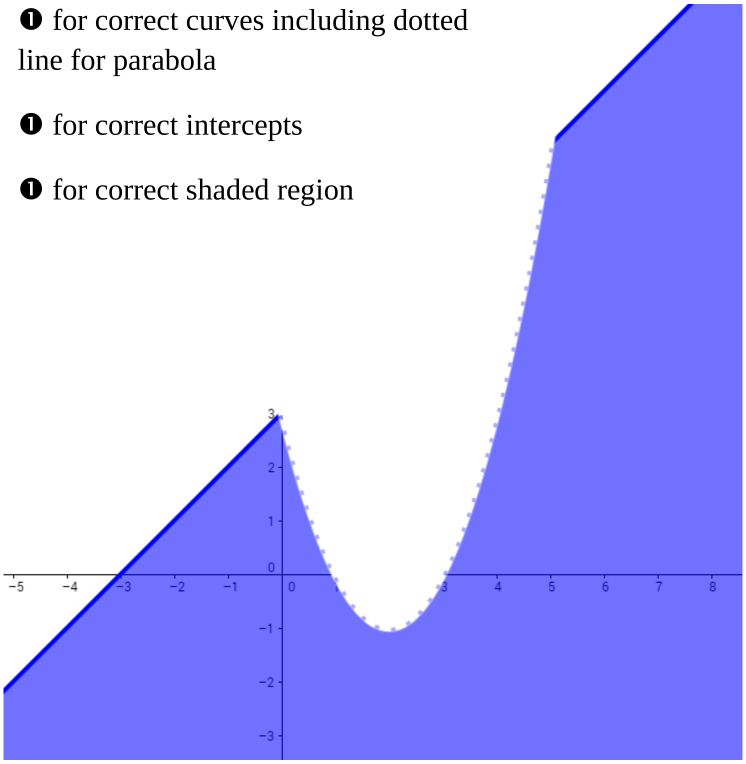
(f)

$$\int \frac{x}{2} - e^{2x}\,dx = \frac{x^2}{4} - \frac{1}{2}e^{2x} + c \quad ❶ \textit{ for each component}$$

❶ for correct curves including dotted line for parabola

❶ for correct intercepts

❶ for correct shaded region



- Many forgot the dashed line for the parabola. Read questions carefully.

Q12

(a)

i.

$10 \times 9 = 90$ (sample space)

$10 \times 8 = 80$ (for 2nd card not matching)

$\therefore P(not\, a\, pair) = \frac{80}{90} = \frac{8}{9}$ ❶

ii.

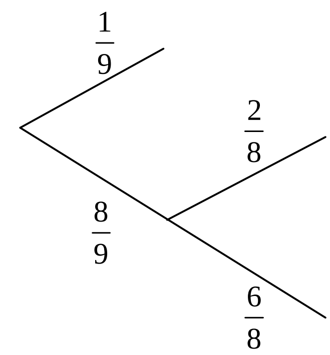
$10 \times 8 \times 2 = 160$

$10 \times 9 \times 8 = 720$

$\therefore P(3rd\ a\ pair) = \frac{160}{720} = \frac{2}{9}$ ❶

Comments

- This was poorly answered
- Also,



$\therefore \frac{8}{9} \times \frac{2}{8} = \frac{2}{9}$

(b)

i.

$\$800(1.1)^{30} = \$13,959.52...$ ❶

$= \$13,960$ ❶

ii.

$800(1.1)^{30} + 800(1.1)^{29} + ... + 800(1.1)^{1}$

$= 800((1.1)^{30} + (1.1)^{29} + ... + (1.1))$

GP with a = 1.1, n =30, r =1.1

$S_{30} = \frac{1.1(1.1^{30} - 1.1)}{1.1 - 1}$

$= \frac{1.1(1.1^{30} - 1)}{0.1}$

$= 11(1.1^{30} - 1)$ ❶

$\therefore 800(11(1.1^{30} - 1)) = 144,754.7399... \approx \$144,755$ ❶

(c)

$2\cos^2 x - 1 = 0$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm \frac{1}{\sqrt{2}}$

$For\ -\pi \le x \le \pi \quad x = \frac{3\pi}{4}, \frac{\pi}{4}, \frac{-\pi}{4}, \frac{-3\pi}{4}$

❶ *values* ❶ *signs* ❶ *solutions*

- Many students did not put the $\pm$ signs, hence, did not get all the values

(d)

$y = 2\sin 2x - 3$

$At\ x = 0 \quad y = 2\sin 0 - 3 = -3$ ❶

$y' = 4\cos 2x$

$At\ x = 0 \quad y' = 4$ ❶

$y - y_1 = m(x - x_1)$

$y - -3 = 4(x - 0)$

$y + 3 = 4x$

$y = 4x - 3$ ❶

- This part was answered well

(e)

$$\sum_{n=0}^{\infty} \frac{9}{x^{n+1}}$$

$i.e.\ \frac{9}{x} + \frac{9}{x^2} + \frac{9}{x^3} + ... = 18$ ❶

$LHS\ is\ S_\infty\ where\ r = \frac{1}{x}\ and\ a = \frac{9}{x}$

$$18 = \left(\frac{\frac{9}{x}}{1 - \frac{1}{x}} \right)$$ ❶

$18\left(1 - \frac{1}{x}\right) = \frac{9}{x}$

$18(x - 1) = 9$

$18x - 18 = 9$

$27 = 18x$

$x = \frac{3}{2}$ ❶

- Some students used AP to solve this.
- Poorly answered

Q13

(a)

i. $\dfrac{\tan\theta}{\sec\theta-1}-\dfrac{\tan\theta}{\sec\theta+1}=2\cot\theta$

$$LHS=\frac{\tan\theta}{\sec\theta-1}-\frac{\tan\theta}{\sec\theta+1}$$
$$=\frac{\tan\theta(\sec\theta+1)-\tan\theta(\sec\theta-1)}{\sec^2\theta-1} \text{ ❶}$$
$$=\frac{\tan\theta\sec\theta+\tan\theta-\tan\theta\sec\theta+\tan\theta}{\tan^2\theta}$$
$$=\frac{2\tan\theta}{\tan^2\theta} \text{ ❶}$$
$$=2\cot\theta$$
$$=RHS$$

(b)

i.

$AP: a=45\ , d=3$

$T_n=a+(n-1)d$

$T_1=45\ \therefore\ a=45$

$T_2=48$

$T_3=51\ \therefore\ d=3$ ❶ *for correct a and d*

$\therefore T_{20}=45+(20-1)\times3=102$ *jumps* ❶

ii.

$$S_{20}=\frac{n}{2}(2a+(n-1)d)$$
$$=\frac{20}{2}(2\times45+3\times19) \text{ ❶}$$
$$=1470 \text{ jumps ❶}$$

(c)

i.

$$8y=x^2-8x+24$$
$$=x^2-8x+(-4)^2-(-4)^2+24$$
$$=(x-4)^2+8$$
$$8y-8=(x-4)^2$$
$$4(2)(y-1)=(x-4)^2 \text{ ❶}$$

$\therefore$ a = 2, vertex is (4, 1) and Focus is (4, 5) ❶

Comments

- Generally well done but setting out needs attention
- Generally well done
- Some used $x=-\dfrac{b}{2a}$ to find vertex but then couldn't find focus, therefore no mark awarded

ii.

$$y' = \frac{1}{4}x - 1$$
$$y' = -1 - 1 = -2$$
$$\therefore m_{normal} = \frac{1}{2} \quad ❶$$
$$At\ x = -4, y = \frac{1}{8}(-4)^2 - (-4) + 3 = 9 \quad ❶\ i.e.\left(-4, 9\right)$$
$$y - 9 = \frac{1}{2}(x - -4)$$
$$2y - 18 = (x + 4)$$
$$\therefore x - 2y + 22 = 0 \quad is\,the\,normal \quad ❶$$

- Generally well done
- NOTE: mark was not deducted for incorrect general form (but maybe it will in HSC so take care.)

iii.

$$m = 3\,and\ y' = \frac{1}{4}x - 1$$
$$equating\ the\,two$$
$$3 = \frac{1}{4}x - 1 \quad ❶$$
$$\therefore x = 16$$
$$y = \frac{1}{8}(16)^2 - 16 + 3$$
$$\therefore (16, 19) \quad ❶$$

- Overall well done

(d)

6% batteries 4% bulbs ->defective
94% batteries 96% bulbs ->good

P(Both battery and bulb good)

$$= 94\% \times 96\% \quad ❶$$
$$= \frac{9024}{10000}$$
$$= \frac{564}{625} \quad ❶$$

- Generally well done. Some made the assumption of $1 - 0.06 \times 0.04$

Q14

Comments

(a)

i.

x	1	1.25	1.5	1.75	2
y	4	3.2	2.67	2.29	2

ii.

$$A \approx \frac{h}{3}\{(y_0 + y_n) + 4(y_1 + y_3) + 2(y_2)\}$$

$$\approx \frac{0.25}{3}\{(4+2) + 4(3.2+2.29) + 2\times 2.67\}$$ ➊

$$\approx 2.78$$ ➊

- Common to misapply Simpson's Rule. Many students did not realise that 5 function values were needed for 2 applications. Also many misquoted the rule, sometimes 2 mistakes were made.

(b)

i.

$$y = x^3 - 12x^2 + 36x$$

$$y = x(x^2 - 12x + 36)$$

$$= x(x-6)^2$$ ➊

$$\therefore\ y = 0\ \&\ x = 0, x = 6 \text{ are the intercepts}$$ ➊

- Well done apart from careless errors.

ii.

$$y' = 3x^2 - 24x + 36$$

$$y' = 0$$

$$0 = 3x^2 - 24x + 36$$

$$= x^2 - 8x + 12$$

$$= (x-6)(x-2)$$

$$\therefore\ x = 2, 6 \text{ are possible turning points}$$ ➊

$$y'' = 6x - 24$$ ➊

$$at\ x = 2, y'' = 12 - 24 < 0$$

$$\therefore \max$$

$$at\ x = 6, y'' = 36 - 24 > 0$$

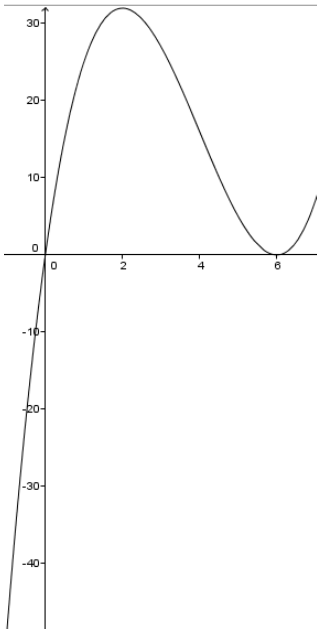
$$\therefore \min$$

$$x = 2, y = 32$$

$$x = 6, y = 0$$

$$\therefore (2,32)\ is \max \& (6,0)\ is \min$$ ➊

iii.



❶ for correct domain
❶ for shape, labelling of intercepts & max/min points

(c) $y = 3x + 1, y = 2x^2 + 4x - 5$

$$3x + 1 = 2x^2 + 4x - 5$$
$$0 = 2x^2 + x - 6$$
$$= 2x^2 + 4x - 3x - 6$$
$$= 2x(x+2) - 3(x+2)$$
$$= (x+2)(2x-3)$$
$$\therefore x = -2, x = 1\frac{1}{2}$$ ❶

$$A = \int_{-2}^{\frac{3}{2}} (3x+1) - (2x^2 + 4x - 5)dx$$ ❶

$$= \int_{-2}^{\frac{3}{2}} -2x^2 - x + 6\,dx$$

$$= \left[-\frac{2}{3}x^3 - \frac{1}{2}x^2 + 6x\right]_{-2}^{\frac{3}{2}}$$

$$= -\frac{2}{3}\left(\frac{3}{2}\right)^3 - \frac{1}{2}\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right) - \left[-\frac{2}{3}(-2)^3 - \frac{1}{2}(-2)^2 + 6(-2)\right]$$

$$= \frac{343}{24} units^2$$ ❶

- Some students made errors in the simultaneous equations and had the wrong endpoints for the integration. Careless errors in evaluating the integral common.

(d)

i.

$$A = 10e^{-kt}$$
$$t = 4, A = 3.2$$

$$\begin{aligned} 3.2 &= 10e^{-4k} \\ 0.32 &= e^{-4k} \\ \ln(0.32) &= -4k \\ k &= -\frac{1}{4}\ln(0.32) \\ &\approx 0.2848585708 \\ &\approx 0.285 \text{ ❶} \end{aligned}$$

ii.

$$\begin{aligned} 1 &= 10e^{-0.285t} \\ \frac{1}{10} &= e^{-0.285t} \\ \ln\frac{1}{10} &= -0.285t \\ t &= -\frac{1}{0.285}\ln(0.1) \\ &= 8.07942... \\ &\approx 8\,hrs \text{ ❶} \end{aligned}$$

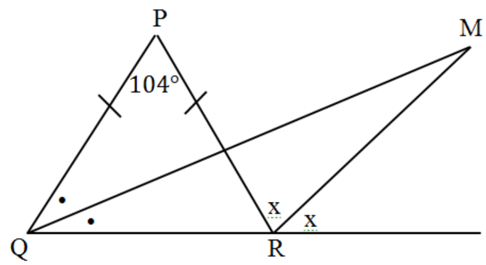
- Well done apart from careless errors.

Q15

Comments

(a)

i.



- Students need to draw/copy diagrams larger.
- Mostly answered well with reasons given for statements.

ii.

$$\angle PQR = \frac{1}{2}(180 - 104)$$
$$= 38° \ (\angle \, sum \, of \ isosceles \, \Delta)$$

❶ for answer and ❶ for reason

iii.

$$\therefore \angle PRQ = 38° (base \, \angle 's \, of \ isosceles \, \Delta) \text{ ❶}$$
$$\therefore \angle PRN = 180 - 38 = 142° (straight \, line) \text{ ❶}$$
$$Hence \ \angle MRN = \frac{1}{2} \times 142° = 71° (RM \ bi\sec ts - given)$$
$$\therefore \angle MRQ = 71 + 38 = 109° (adjacent \, \angle 's)$$
$$\therefore \angle QMR = 180 - 109 - 19 = 52° (\angle \, sum \, of \ \Delta) \text{ ❶}$$

(b) $y = \sec x$

- Students need to show substitution step.

$$V = \pi \int_0^{\frac{\pi}{3}} \sec^2 x \quad dx$$
$$= \pi \left[\tan x\right]_0^{\pi/3} \text{ ❶}$$
$$= \pi\left(\sqrt{3} - 0\right) \text{ ❶}$$
$$= \sqrt{3}\pi \, units^3 \text{ ❶}$$

(c)

i.

- Remember to include c. Some students forgot/did not find the value of constant.

$$\frac{dx}{dt} = 4 - 2t$$
$$x = \int 4 - 2t \ dt$$
$$= 4t - t^2 + c \text{ ❶}$$
$$At \ x = 1, \ t = 0 \ \therefore \ c = 1$$
$$\therefore \ x = -t^2 + 4t + 1 \text{ ❶}$$

ii.

$$\frac{dx}{dt}=0$$
$$0=4-2t$$
$$\therefore t=2$$ ❶
$$\frac{d^2x}{dt^2}=-2$$ ❶

iii.

$$t=0, x=1$$
$$t=2, x=1+8-4=5 \therefore 4 units\,travelled$$ ❶
$$t=4, x=1+4\times 4-(4)^2=1 \therefore 4 units\,travelled$$
$$\therefore travlled\ a\ total\ of\ 8 units$$ ❶

Comments

- Mostly well done

- Many students misinterpreted the question: finding distance travelled “in the fourth second” instead of “in the first 4 seconds”.

Q16

(a)

i.

$$P = arcAXB + arcBC + AB$$
$$= \frac{1}{2}2\pi\left(\frac{r}{2}\right) + r\theta + r \text{ ❷}$$
$$= r\left(\frac{\pi}{2} + \theta + 1\right)$$

ii.

$$A = Semicircle + \sec tor$$
$$= \frac{1}{2}\pi\left(\frac{r}{2}\right)^2 + \frac{1}{2}r^2\theta$$
$$= \frac{\pi}{8}r^2 + \frac{1}{2}r^2\theta$$

$now\, A = 1, so$

$$1 = r^2\left(\frac{\pi}{8} + \frac{\theta}{2}\right)$$
$$\frac{1}{r^2} = \frac{\pi}{8} + \frac{\theta}{2}$$
$$\frac{\theta}{2} = \frac{1}{r^2} - \frac{\pi}{8}$$
$$\theta = \frac{2}{r^2} - \frac{\pi}{4} \text{ ❶}$$

$Substituting\ for\, \theta\, in\, P = r\left(\frac{\pi}{2} + \theta + 1\right) gives$

$$P = r\left(\frac{\pi}{2} + \frac{2}{r^2} - \frac{\pi}{4} + 1\right)$$
$$= r\frac{\pi}{2} + \frac{2}{r} - \frac{\pi r}{4} + r \text{ ❶}$$
$$= \frac{2}{r} + r\frac{\pi}{4} + r$$
$$= \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right) as\, required$$

Comments

- Still very poor at attempting "Show" Questions
- Explanation of each component was poor, particularly the semicircle AXB
- $\frac{1}{2}r^2\theta$ for area of sector very poorly known. Need to learn formulae correctly.
- In (i) and (ii): many used $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{r}{2}\right)^2$ which is clearly incorrect. Should be $\frac{1}{2}\pi R^2 = \frac{1}{2}\pi\left(\frac{r}{2}\right)^2$ (i.e. use a different pronumeral)
- Several students "got lost" and did not make θ the subject. Need to focus on what you're trying to do. Show P formula, which has no θ, means θ has to be eliminated somehow. This gives your working a **direction**.

iii.

$$For\ stationary\ pt, \frac{dP}{dr} = 0$$

$$\frac{dP}{dr} = -\frac{2}{r^2} + 1 + \frac{\pi}{4}; \quad \frac{dP}{dr} = 0; \quad \frac{d^2P}{dr^2} = 4r^{-3} > 0$$

$$\therefore \min\ (as\, r > 0)$$

$$0 = -\frac{2}{r^2} + 1 + \frac{\pi}{4}$$ ❶

$$\frac{2}{r^2} = \frac{4+\pi}{4}$$

$$\frac{r^2}{2} = \frac{4}{4+\pi}$$

$$r^2 = \frac{8}{4+\pi}$$

Then

$$from\ part\,(ii), \theta = \frac{2}{r^2} - \frac{\pi}{4}$$

$$= \frac{2}{\left(\frac{8}{\pi+4}\right)} - \frac{\pi}{4}$$

$$= \frac{4+\pi}{4} - \frac{\pi}{4}$$

$$= 1 + \frac{\pi}{4} - \frac{\pi}{4}$$ ❶

$$= 1\ radian$$

$$= 1 \times \frac{180}{\pi}$$

$$\approx 57°$$ ❶

- Very poor at explaining what was happening – often a lot of very disconnected mathematical statements where logic flow was difficult or impossible to follow (or indeed was not present).

Comments

(b)

i. OP is a radius, as is OQ

$\therefore \Delta POQ\, is\, isosceles$

$\therefore \angle P = \beta\, (base\, \angle 's\ of\ isosceles\, \Delta)$

$\therefore \alpha = 2\beta\, (ext. \angle\ sum\, of\ opposite\, \text{int}. \angle 's)$

- Reasons often missing (no marks given in these cases)

ii. $PQ\, has\ gradient\ m = \tan \beta$

$Q(-1, 0)$ ❶

$\therefore y - 0 = m(x - -1)$

$y = mx + m$ ❶

- Many students used $\tan \beta$ instead of *m* [look at the question overall- there's *m* all through the equations – use it instead of $\tan \beta$, which comes in only in the last part]

iii.

$P\,and\,Q\,are\,on\,x^2+y^2=1\,and\;y=mx+m$ ❶

So, solving simultaneously:

$x^2+m^2(x+m)^2=1$

$x^2+m^2x^2+2m^2x+m^2=1$

$x^2(1+m^2)+2m^2x+m^2-1=0$

Which is the given equation, so P and Q satisfy it. ❶

Q is (-1, 0) => $x=-1$

$$\begin{aligned}LHS &= (1+m^2)\left(-1\right)^2+2m^2\left(-1\right)+m^2-1\\ &= 1+m^2-2m^2+m^2-1\\ &= 0\\ &= RHS\end{aligned}$$

- Some managed to show for Q by substituting $x=-1$, which gained 1 mark

iv.

$$\begin{aligned}x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ &= \frac{-2m^2\pm\sqrt{4m^4-4(1+m^2)(m^2-1)}}{2(1+m^2)}\\ &= \frac{-2m^2\pm\sqrt{4m^4-4(m^4-1)}}{2(1+m^2)}\\ &= \frac{-2m^2\pm\sqrt{4}}{2(1+m^2)}\\ &= \frac{-2m^2\pm 2}{2(1+m^2)}\\ &= \frac{-2m^2-2}{2(1+m^2)};\frac{-2m^2+2}{2(1+m^2)}\\ &= -1;\frac{-(m^2-1)}{(m^2+1)} \quad ❶\end{aligned}$$

i.e. P has x value $\frac{1-m^2}{1+m^2}$

$$\begin{aligned}y &= mx+m\\ &= m\left(\frac{1-m^2}{m^2+1}\right)+m\\ &= m\left(\frac{-m^2+1+m^2+1}{m^2+1}\right)\\ &= \frac{2m}{m^2+1} \quad ❶\end{aligned}$$

$\therefore P\,is\left(\frac{1-m^2}{m^2+1},\frac{2m}{m^2+1}\right)$

Comments

- This was the commonly used method.
- Simpler and more elegant attempts used root relationships (see Alternate Method), which came out easily.
- Those who use this method often did not resolve adequately for the x value of P, nor identify Q's x value (so P must be the other)
- y value often not found
- Much of the algebra work involving algebraic fractions was extremely poorly done
- Point specifically asked for so give P's coordinates

Alternate Method:

Roots are -1 and x_p ,so

$$x_p - 1 = \frac{-b}{a}$$
$$= \frac{-2m^2}{1+m^2}$$
$$x_p = 1 - \frac{2m^2}{1+m^2}$$
$$= \frac{1+m^2-2m^2}{1+m^2}$$
$$= \frac{1-m^2}{1+m^2}$$
$$y = mx + m$$
$$= m\left(\frac{1-m^2}{1+m^2}\right) + m$$
$$= \frac{m\left(1-m^2\right) + m\left(1+m^2\right)}{1+m^2}$$
$$= \frac{2m}{1+m^2}$$
$$\therefore P\, is \left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$$

v.

$m_{OP} = \tan\alpha$ ❶

But also

$$m_{OP} = \frac{y_P - y_0}{x_P - x_0}$$
$$= \frac{\frac{2m}{m^2+1} - 0}{\frac{1-m^2}{m^2+1} - 0}$$
$$= \frac{2m}{m^2+1} \times \frac{m^2+1}{1-m^2}$$
$$= \frac{2m}{1-m^2}$$ ❶

$$So \tan\alpha = \frac{2m}{1-m^2}$$
$$But\; m = \tan\beta \; and \; \alpha = 2\beta$$
$$Hence \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$$

- Many extension 1 candidates tried to state the double angle formula here without linking to the question- this was awarded no marks. They were amongst many who struggled to see the relationship between gradient definition and $m = \tan\theta$ definition from trigonometry.