FORT STREET HIGH SCHOOL

Name: ____________________________

Teacher: ____________________________

Class: ____________________________

2017

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
H1	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4	Constructs arguments to prove and justify results, manipulates algebraic expressions involving logarithmic and exponential functions, expresses practical problems in mathematical terms based on simple given models	12, 13
H5, H6, H7	Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems, uses the derivative to determine the features of the graph of a function, uses the features of a graph to deduce information about the derivative	11, 14, 15
H8, H9	Uses techniques of integration to calculate areas and volumes, synthesises mathematical solutions to harder problems and communicates them in appropriate form	16

Total Marks 100

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section I

10 marks

Attempt questions 1-10

Allow about 15 minutes for this section

Answer each question by circling the letter for the correct alternative on this sheet.

---

1 If $\sqrt{48}+\sqrt{3}=\sqrt{x}$, what is the value of x?

(A) $\sqrt{51}$

(B) $5\sqrt{3}$

(C) 75

(D) 3

2 Find the equation of a line with a y intercept of $(0,-6)$ and parallel to the line joining $(1,2)$ to the origin.

(A) $y=-2x+6$

(B) $y=-2x-6$

(C) $y=2x+6$

(D) $y=2x-6$

3 $27-m^3$ factorised fully is.

(A) $(9-m)(9+m)$

(B) $(3-m)(9+3m+m^2)$

(C) $(3+m)(9-3m+m^2)$

(D) $(9-m^2)(9+m)$

4 Find $\int \sec^2 2x\,dx$

(A) $\frac{1}{2}\tan x+C$

(B) $\tan x+C$

(C) $2\sec 2x+C$

(D) $\frac{1}{2}\tan 2x+C$

5 Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$.

(A) $y = 2x - 2e^2$

(B) $y = 2e^2 x - 3e^2$

(C) $y = 2e^2 x - e^2$

(D) $y = e^2 x - 3e^2$

6 Evaluate $\sum_{p=2}^{5} p^2$

(A) 29

(B) 50

(C) 54

(D) 100

7 What is the greatest value of the function $y = 4 - 2\cos x$?

(A) 2

(B) 4

(C) 6

(D) 8

8 Find the value of x if $\log_{10}(2x+4) = 1 + \log_{10} x$.

(A) $x = \frac{1}{2}$

(B) $x = 1$

(C) $x = 2$

(D) $x = 5$

9 Two-digit numbers are formed from the digits 2, 3, 4, 6 with no repetition of digits allowed. A two-digit number is then selected at random. What is the probability that the number is even?

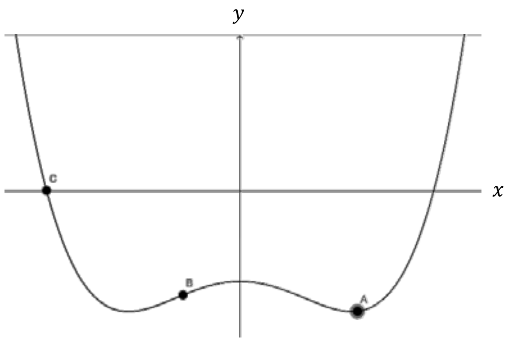
(A) $\frac{1}{12}$

(B) $\frac{1}{8}$

(C) $\frac{3}{4}$

(D) $\frac{5}{12}$

10 The diagram below shows a sketch of the curve $y = 3x^4 - 2x^2 - 1$. The curve has a local minimum at A, a point of inflexion at B, and cuts the x axis at C. What are the coordinates of C?



NOT TO SCALE

(A) $\left(-\frac{1}{3}, 0\right)$

(B) $\left(-\sqrt{3}, 0\right)$

(C) $\left(-\frac{1}{\sqrt{3}}, 0\right)$

(D) $\left(-1, 0\right)$

Section II

90 marks

Attempt questions 11-16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a *new* booklet.

(a) Express $\dfrac{\sqrt{3}+1}{5-\sqrt{2}}$ with a rational denominator **2**

(b) Express $\dfrac{3}{2x-3}-\dfrac{2}{2x+3}$ as a single fraction in its simplest form **2**

(c) Write $4.2\dot{5}\dot{3}$ as a mixed fraction **2**

(d) Consider the parabola with equation $y^2-8y=12x-28$

(i) Show that the equation can be written as $(y-4)^2=12(x-1)$ **2**

(ii) Find the coordinates of the focus. **1**

(e) Differentiate

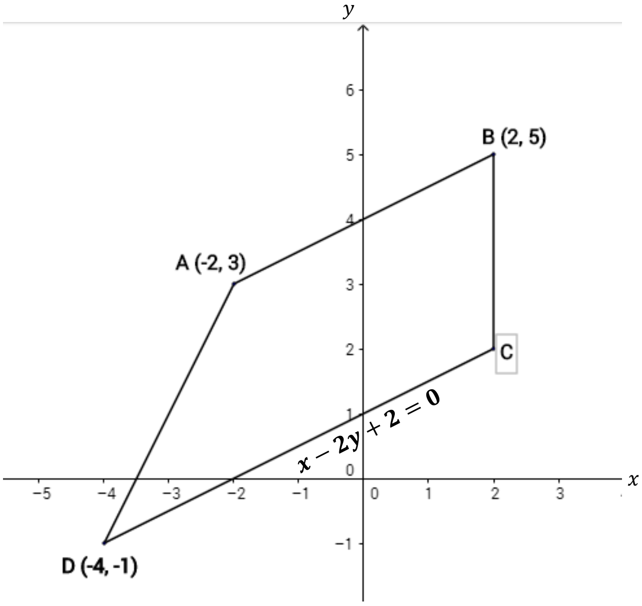
(i) $(x^2-1)^8$ **2**

(ii) $\cos\left(\dfrac{x}{3}\right)$ **1**

(f) Find the exact value of $\displaystyle\int_1^3 \frac{2x^2}{x^3+1}\,dx$ **3**

Question 12 (15 marks) Start a *new* booklet.

(a) In the trapezium $ABCD$, coordinates of the points A, B and D are $(-2, 3)$, $(2, 5)$ and $(-4, -1)$, respectively. The equation of the line DC is $x - 2y + 2 = 0$. Copy or trace the diagram into your booklet.



NOT TO SCALE

(i) The line BC is parallel to the y axis. Find the coordinates of C. **2**

(ii) Show that AB is parallel to CD **1**

(iii) Find the perpendicular distance from A to the line DC. **2**

(iv) Find the length of AB. **2**

(v) Find the area of the trapezium $ABCD$ **3**

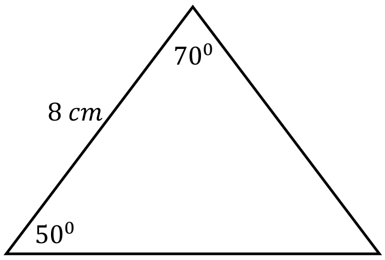
(b) Solve $\tan^2\theta = 3$ in the domain $0 \le \theta \le 2\pi$ **2**

(c) (i) On the same set of axes, sketch the graphs of $y = x^2 - 4$ and $y = x + 1$ **2**

(ii) Shade the region defined by $y \ge x^2 - 4$ and $y \le x + 1$ **1**

Question 13 (15 marks) Start a *new* booklet.

(a) Calculate, correct to 2 decimal places, the longest side of the triangle. **2**



(b) Show that $y = -3x^2 + x - 4$ is negative definite. **3**

(c) (i) On the same set of axes sketch the curve $y = e^{-\frac{1}{2}x}$ and the line $x = 2$. **2**

(ii) The curve cuts the y axis at P. Find the gradient of the tangent to the curve $y = e^{-\frac{1}{2}x}$ at P. **2**

(iii) Show that the equation of the normal to the curve $y = e^{-\frac{1}{2}x}$ at P is $y = 2x + 1$ **2**

(iv) Shade the area enclosed by the curve, the normal and the line $x = 2$. Show that it is equal to $4 + 2e^{-1}$. **4**

Question 14 (15 marks) Start a *new* booklet.

(a) (i) Find the set of values of x for which $|3x-1|<2$. **3**

(ii) Show the solution set on a number line. **1**

(b) State the domain and range of the function $y=\sqrt{x^2-4}$ **2**

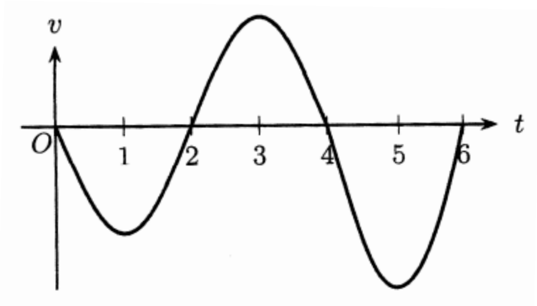
(c) The probability of Emily winning a particular game of chance is 30%.

(i) If she is to play 3 games, what is the probability she will win at least one game? **1**

(ii) What is the probability she will win at least one game if she is to play 5 games? **1**

(iii) What is the least number of games she must play before the probability of her winning at least one game is 99%? **2**

(d) The graph below shows the velocity, v m/s, of a particle moving along a straight line for 6 seconds. The particle starts from rest from a fixed point O $(0,0)$ at time $t=0$.



(i) At what times is the acceleration equal to zero? **2**

(ii) At what time is the displacement of the particle equal to zero? **1**

(iii) At what times does the particle change direction? **1**

(iv) At what times does the particle have the greatest acceleration? **1**

Question 15 (15 marks) Start a *new* booklet.

(a) When k is added to each of the numbers 12, 9 and 7, respectively, the resulting set forms a geometric series.

(i) Write the geometric series in terms of k. **1**

(ii) Write an expression for the common ratio in terms of k **1**

(iii) Find the value of k **2**

(iv) Determine whether a limiting sum exists for this geometric series. If so calculate its value. **2**

(b) A country borrowed \$500 million on January 1 2013 from the International Monetary Fund to build infrastructure. The country agreed to pay \$1.5 million at the end of each month after a compounded interest of 0.6% per month has been added to the debt.

(i) Show that the amount owed by the country on March 1 2013 is **2**

$$A = 5\times10^{8}\times1.006^{2} - 1.5\times10^{6}\left(1+1.006\right)$$

(ii) On November 30 2013 the country decided that their monthly payment for December would be \$2 585 562.38. Show that their debt on January 1 2014 will be \$517 520 479.60. **4**

(iii) The new government elected in the country decided to further increase their monthly payment to \$5 million beginning from the payment of January 2014 until the debt is paid. How long will it take for the debt to be paid off? Give your answer in years and months. **3**

Question 16 (15 marks) Start a *new* booklet.

(a) A chemical substance being made in a laboratory will be airlifted by helicopter to a hospital immediately after it is completed. The substance decomposes and the amount M in kilograms present at any time t hours is given by $M = Ae^{-kt}$ where A and k are constants.

If $\frac{1}{4}$ of the mass of this substance is present after 4 hours and 4 kg of this substance will reach the hospital in 6 hours.

(i) Find the exact value of k **2**

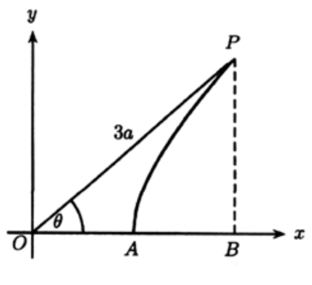
(ii) Find the value of A, the original mass of the chemical substance **2**

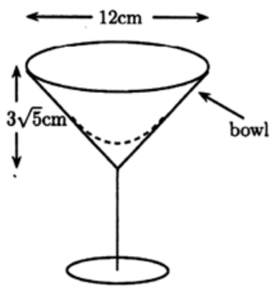
(b) Given the function $y = \dfrac{10}{3+2\sin x}$ in the domain $0 \le x \le 2\pi$.

(i) Find all stationary points and determine their nature. **3**

(ii) Using 5 function values evaluate by Simpson's Rule $\int_0^{2\pi} \dfrac{10}{3+2\sin x}\,dx$ **2**

(c) A part of the graph of $y = \sqrt{x^2 - a^2}$ in the first quadrant intersects the x axis at A. P is the point on the curve such that $OP = 3a$ and $\angle POA = \theta$. B is the foot of the perpendicular from P to the x axis. The region bounded by AB, PB and the curve is rotated about the x axis to obtain the volume of liquid that can be placed in the conical bowl of a brass cup.





(i) Show that the volume of the liquid that can be placed in the conical bowl is **3**

$$V = \frac{a^3\pi}{3}\left(27\cos^3\theta - 9\cos\theta + 2\right)$$

(ii) Given that the height OB of the conical bowl is $3\sqrt{5}$ cm and the diameter of its rim is 12 cm, find the exact volume of liquid that the cup will hold when full. **3**

End of paper

2017 Fort Street High Mathematics Trial – Solutions

Section I: Multiple Choice

1	$=\sqrt{48}+\sqrt{3}$ $=4\sqrt{3}+\sqrt{3}$ $=5\sqrt{3}$ $=\sqrt{75}$ $\therefore x=75$	1 Mark: C
2	Gradient $=2$ y intercept -6 $y=2x-6$	1 Mark: D
3	$3^3-m^3=(3-m)(9+3m+m^2)$	1 Mark: B
4	$\int \sec^2 2x\ dx=\frac{1}{2}\tan 2x+C$	1 Mark: D
5	$y'=2e^{2x}$ at $x=1$ $\quad y'=2e^2$ $\qquad y=e^2$ $y=2e^2x+b$ $e^2=2e^2(1)+b$ $b=-e^2$ $\therefore y=2e^2x-e^2$	1 Mark: C
6	$4+9+16+25=54$	1 Mark: C
7	Highest value occurs when $x=\pi$ $y=4-2\cos\pi$ $=4-2(-1)$ $=4+2$ $=6$	1 Mark: C
8	$\log_{10}(2x+4)-\log_{10}x=1$ $\log_{10}\left(\frac{2x+4}{x}\right)=1$ $10^1=\frac{2x+4}{x}$ $10x=2x+4$ $8x=4$ $\therefore x=\frac{1}{2}$	1 Mark: A
9	$\begin{array}{c\|c\|c\|c\|c} & 2 & 3 & 4 & 6 \\ \hline 2 & \text{X} & & \checkmark & \checkmark \\ 3 & \checkmark & \text{X} & \checkmark & \checkmark \\ 4 & \checkmark & & \text{X} & \checkmark \\ 6 & \checkmark & & \checkmark & \text{X} \end{array}$ Total number of outcomes $=12$ Favourable outcomes $=9$ P(Even) $=\frac{9}{12}$ $=\frac{3}{4}$	1 Mark: C
10	y coordinate of C is 0 Factorise $3x^4-2x^2-1=0$ $(3x^2+1)(x^2-1)=0$ $3x^2=-1$ (no solution) $x^2=1$ $x=\pm1$ Since C lies on the negative x axis then $x=-1$	1 Mark: D

Section II:

Q11

(a)

$$= \frac{\sqrt{3}+1}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} \quad ❶$$

$$= \frac{5\sqrt{3}+\sqrt{6}+5+\sqrt{2}}{25-2}$$

$$= \frac{5\sqrt{3}+\sqrt{6}+5+\sqrt{2}}{23} \text{ or } \frac{\left(5+\sqrt{2}\right)\left(\sqrt{3}+1\right)}{23} \quad ❶$$

(b)

$$= \frac{6x+9-2(2x-3)}{4x^2-9}$$

$$= \frac{6x+9-4x+6}{4x^2-9} \quad ❶$$

$$= \frac{2x+15}{4x^2-9} \quad ❶$$

(c)

$$\text{let } x = 4.2\dot{5}\dot{3}$$

$$10x = 42.\dot{5}\dot{3}$$

$$1000x = 4253.\dot{5}\dot{3}$$

$$1000x - 10x = 4253.\dot{5}\dot{3} - 42.\dot{5}\dot{3} \quad ❶$$

$$990x = 4211$$

$$x = \frac{4211}{990}$$

$$x = 4\frac{251}{990} \quad ❶$$

(d) (i)

Completing the square:

$$y^2 - 8y + \left(-\frac{8}{2}\right)^2 = 12x - 28 + \left(-\frac{8}{2}\right)^2 \quad ❶$$

$$(y-4)^2 = 12x - 28 + 16$$

$$(y-4)^2 = 12x - 12$$

$$(y-4)^2 = 12(x-1) \quad \text{as req'd} \quad ❶$$

(ii)

$$x \text{ coordinate } = 1+3$$

$$= 4$$

$$y \text{ coordinate } = 4$$

$$\therefore \; focus = (4,4) \quad ❶$$

Comments

(a) Generally done well.

(b) Some students transferred information from the question incorrectly – many wrote $2(2x+3)$ instead of $2(2x-3)$

(c) Could also be answered using a geometric series

Some students left their answer as an improper fraction

A few people lost a mark because they didn't show working out

(d) (i) Generally done well

(ii) Student wrote coordinates of the vertex instead of the focus others confused the x and y coordinates when finding the focus)

(e) (i)

let $u = x^2 - 1$

$$y = u^8 \qquad u = x^2 - 1$$

$$\frac{dy}{du} = 8u^7 \qquad \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 8\left(x^2 - 1\right)^7 \times 2x \quad ❶$$

$$\frac{dy}{dx} = 16x\left(x^2 - 1\right)^7 \quad ❶$$

(ii)

$$y = \cos\left(\frac{x}{3}\right)$$

$$\frac{dy}{dx} = -\frac{1}{3}\sin\left(\frac{x}{3}\right) \quad ❶$$

(f)

$$\int_1^3 \frac{2x^2}{x^3 + 1} = \frac{2}{3}\left[\ln\left(x^3 + 1\right)\right]_1^3 \quad ❶$$

$$= \frac{2}{3}\left[\ln(28) - \ln(2)\right] \quad ❶$$

$$= \frac{2}{3}\left[\ln\frac{28}{2}\right]$$

$$= \frac{2}{3}\ln(14) \quad ❶$$

Comments

(e) (i) Generally done well. A few careless errors made along the way

(ii) Done well

(f) Some students didn't recognise that the primitive was a log function and used creative methods to integrate

A few integrated the function as $\ln x^3$ instead of $\ln(x^3 + 1)$

There were a few careless errors – students squared instead of cubing the x term after integrating

Q12	**Comments**
(a) (i) x coordinate of C = x coordinate of B; $x = 2$ ❶; y coordinate of C: $2 - 2y + 2 = 0$, $y = 2$ ❶; Coordinates of C are $(2, 2)$	(a) (i) Well done
(ii) Gradient AB $= \frac{5-3}{2+2} = \frac{1}{2}$; Gradient DC $= \frac{-1-2}{-4-2} = \frac{1}{2}$; $\therefore$ Gradient AB = Gradient DC ❶	(ii) Well done
(iii) d $= \frac{\lvert ax+by+c \rvert}{\sqrt{a^2+b^2}} = \frac{\lvert x-2y+2 \rvert}{\sqrt{1^2+(-2)^2}} = \frac{\lvert -2-2(3)+2 \rvert}{\sqrt{5}}$ ❶ $= \frac{6}{\sqrt{5}}$ ❶	(iii) Well done although a few students wrote a decimal approximation instead of an exact value which caused errors in v)
(iv) AB $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2+2)^2+(5-3)^2}$ ❶ $= \sqrt{16+4} = 2\sqrt{5}$ ❶	(iv) Well done.

(v)

Length of DC

$$= \sqrt{(2+4)^2 + (2+1)^2}$$ ❶

$$= \sqrt{36+4}$$

$$= 3\sqrt{5}$$

Area of ABDC

$$= \frac{1}{2}h(a+b)$$

$$= \frac{1}{2} \times \frac{6}{\sqrt{5}} \times \left(2\sqrt{5} + 3\sqrt{5}\right)$$ ❶

$$= \frac{3}{\sqrt{5}} \times 5\sqrt{5}$$

$$= 15\,\text{units}^2$$ ❶

(b)

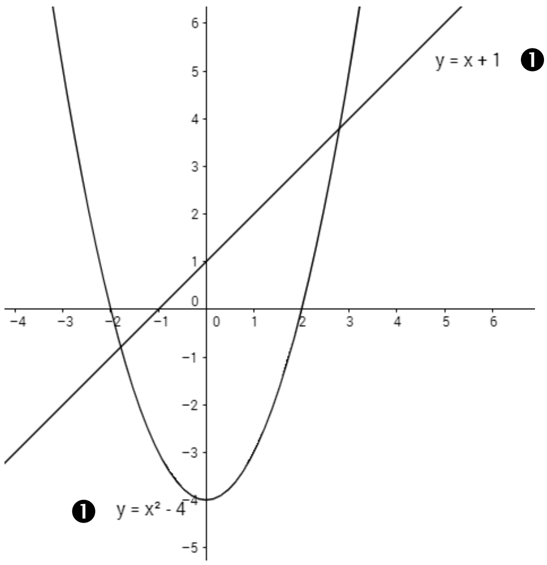
$$\tan^2\theta = 3$$

$$\tan\theta = \pm\sqrt{3}$$ ❶

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$ ❶

(c)

(a)



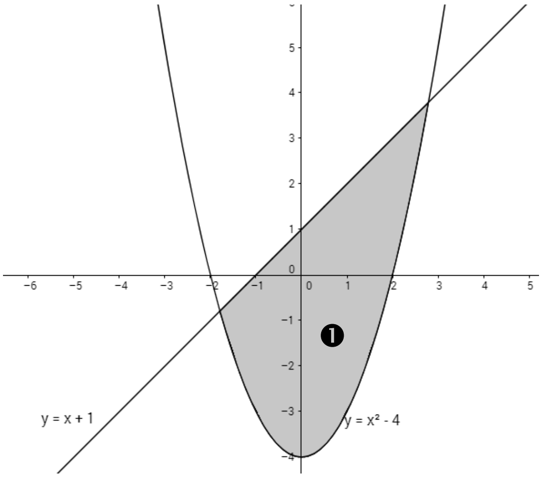
Comments

(v) Well done, although some students did not substitute correctly into the area of a trapezium formula.

(b) As a result of leaving out the negative square root, many students gave only two answers.

(c) Well done, but a few students lost marks for drawing untidy lines and curves or for failing to write x and y-intercepts.

(b)

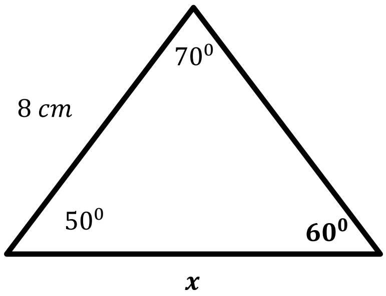


Comments

Mostly well done but a few students did not draw enough of the line and curve and were unable to show the full extent of the region.

Q13

(a)



Missing angle $= 60^0$

$$\frac{a}{Sin\,A} = \frac{b}{Sin\,B}$$

$$\frac{8}{Sin\,60^0} = \frac{x}{Sin\,70^0} \qquad ❶$$

$$x = \frac{8}{Sin\,60^0} \times Sin\,70^0$$

$$x = 8.68 \qquad ❶$$

(b)

$$\begin{aligned}\Delta \quad &= (1)^2 - 4\times(-3)\times(-4)\\ &= -47\\ &< 0 \ \therefore \text{ no real roots} \qquad ❶\end{aligned}$$

Since $a < 0$ (i.e. leading coefficient <0)
then the function is concave down,
hence $f(x) < 0$ for all real x. ❶

Since $\Delta < 0$ and $f(x) < 0$ for all real x
then function is negative definite ❶

OR

$$\begin{aligned} f(x) \quad &= -3\left(x^2 - \frac{x}{3} + \left(\frac{1}{6}\right)^2 + \frac{4}{3} - \left(\frac{1}{6}\right)^2\right)\\ &= -3\left(\left(x - \frac{1}{6}\right)^2 + \frac{47}{36}\right)\\ &< 0 \qquad \text{since } \left(x - \frac{1}{6}\right)^2 \geq 0 \text{ for all real } x\end{aligned}$$

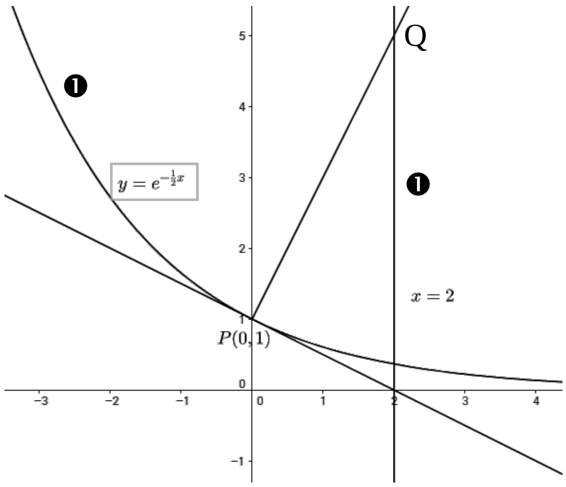
Comments

(a) Well done

(b) Many students forgot to state a=-3<0.

(c)

i.



(c) (i) Well done

ii.

$$\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x} \quad ❶$$

Coordinates of P are (0,1) ❶ (correct coordinates)

$$\frac{dy}{dx} = -\frac{1}{2}e^{0}$$

Therefore gradient of tangent is $-\dfrac{1}{2}$

(ii) Well done

iii.

$$\text{Gradient of normal} = -\frac{1}{\text{Gradient of tangent}}$$

$$= 2 \quad ❶$$

Equation of normal:

$$y - 1 = 2(x - 0) \quad ❶$$

$$\therefore y = 2x + 1 \quad \text{as req'd}$$

(iii) Well done although a few students lost a mark for not showing every step.

iv.

$$A = \int_0^2 2x + 1\, dx - \int_0^2 e^{-\frac{1}{2}x}\, dx \quad ❶$$

$$= \left[x^2 + x\right]_0^2 - \left[-2e^{-\frac{1}{2}x}\right]_0^2 \quad ❶$$

$$= \left[\left(2^2 + 2\right) - 0\right] - \left[-2e^{-1} - \left(-2e^{0}\right)\right] \quad ❶$$

$$= 6 + 2e^{-1} - 2$$

$$= 4 + 2e^{-1} \quad \text{as req'd} \quad ❶$$

(iv) Well done but many students over complicated their solution by dividing the region up into triangles and trapeziums. These students seem not to understand the process of finding the area between two curves as a subtraction of integrals.

Alternative Method

Comments

At $x = 2$,

$$y = 2\times 2+1$$

$$= 5 \qquad \text{i.e. } Q(2,5)$$

$$\text{Area of Trapezium} = \frac{2}{2}(1+5)$$

$$= 6$$

$$\text{Area Shaded} = \text{Area of Trapezium} - \int_0^2 e^{-\frac{1}{2}x}\ dx$$

$$= 6 - \left[-2e^{-\frac{1}{2}x}\right]_0^2$$

$$= 6 + 2e^{-1} - 2e^{0}$$

$$= 4 + 2e^{-1} \quad \text{as req'd}$$

Q14

(a)

i.

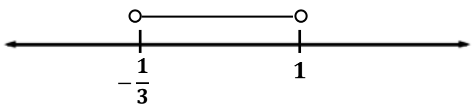
$3x-1<2$ and $3x-1>-2$

$3x<3$ $\quad$ $3x>-1$

$x<1$ ❶ $\quad$ $x>-\frac{1}{3}$ ❶

Therefore, $-\frac{1}{3}<x<1$ ❶

ii.



❶

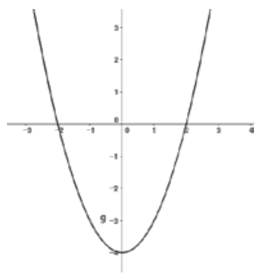
(b)

Domain :

$x^2-4\geq 0$

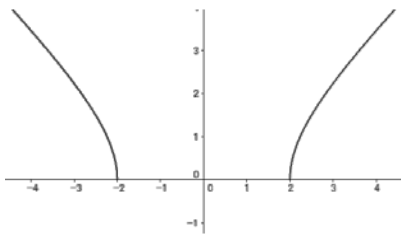
$(x-2)(x+2)\geq 0$

$\therefore\ x\leq -2$ and $x\geq 2$ ❶



Range :

$y\geq 0$ ❶



(c)

i. $1-(0.7)^3=0.657$ ❶

ii. $1-(0.7)^5=0.83193$ ❶

iii.

$1-(0.7)^n=0.99$ ❶

$0.7^n=0.01$

$n\log(0.7)=\log(0.01)$

$n=13$ ❶

Comments

(a) (i) Writing both values of x with 1 expression

(ii) Shade in the dots and shade below the line

Drawing lines past the dots

(b) Give only one inequality of x.
Swapped direction of inequalities

Confused about a range and domain

(c) Rounding down the value of n instead of up. E.g. 12.98, rounding to 12

Cannot use log to solve this problem

Cannot use log to evaluate

(d)

i. In a velocity time graph acceleration is zero at the stationary points. Therefore, accleration is zero at $1, 3$ and 5. ❷ *all* 3 *correct*
❶ 1 *error*
⓿ 2 *errors*

ii. Displacement is zero when area above the t axis and below the curve is equal to the area below the t axis and above the curve. From the graph, the area between $t = 0$ and $t = 2$ is below the axis and is equal to the area between $t = 2$ and $t = 4$ which is above the axis.

Therefore, the displacement = 0 when $t = 4$ ❶

iii. The particle changes direction at $t = 2$ and $t = 4$ seconds as the velocity changes from negative to positive and then from positive to negative. ❶

iv. The gradient of the graph represents acceleration. As the steepest gradient occurs at $t = 6$ the greatest acceleration occurs at $t = 6$ seconds. ❶

Comments

(d) (i) Adding t=0 and 6; cannot read stationary points from a velocity time graph

(ii) Adding t=0 and t=6 with 4 Cannot see that the changes in direction only occurs a t=4

(iii) Didn't understand and see t as odd numbers. Adding t=0 & 6

(iv) Well done but many students over complicated their solution by dividing the region up into triangles and trapeziums. These students seem not to understand the process of finding the area between two curves as a subtraction of integrals

Cannot see t=6 to be the greatest acceleration; adding t=0, 2 and 4

Q15

(a)

i. $12+k, 9+k, 7+k$........ ❶

ii. $r=\dfrac{9+k}{12+k}$ ❶

iii. $$\frac{9+k}{12+k}=\frac{7+k}{9+k}$$ ❶
$$(9+k)^2=(7+k)(12+k)$$
$$81+18k+k^2=84+19k+k^2$$
$$k=-3$$ ❶

iv.
$$r=\frac{9-3}{12-3}$$
$$=\frac{6}{9}$$
since $|r|<1$ then a limiting sum exists ❶
$$S_\infty=\frac{a}{1-r}$$
$$=\frac{9}{1-\frac{6}{9}}$$
$$S_\infty=27$$ ❶

(b)

i. Amount owed on Feb 1 2013
$A_1=5\times10^8\times1.006-1.5\times10^6$ ❶

Amount owed on Mar 1 2013
$$A_2=A_1\times1.006-1.5\times10^6$$
$$=\left(5\times10^8\times1.006-1.5\times10^6\right)\times1.006-1.5\times10^6$$
$$=5\times10^8\times1.006^2-1.5\times10^6\times1.006-1.5\times10^6$$
$$=5\times10^8\times1.006^2-1.5\times10^6\left(1+1.006\right)\text{ as req'd}$$ ❶

ii.

Amount owed on Nov 30 2013 :
$$A_3=5\times10^8\times1.006^3-1.5\times10^6\left(1+1.006+1.006^2\right)$$
$$\vdots \qquad\qquad \vdots$$
$$A_{11}=5\times10^8\times1.006^{11}-1.5\times10^6\times\left(1+1.006+1.006^2+......+1.006^{10}\right)$$ ❶
$$=5\times10^8\times1.006^{11}-1.5\times10^6\times S_{11}$$
where $S_{11}=\dfrac{a\left(r^n-1\right)}{r-1}\Rightarrow a=1,\ r=1.006$ ❶

Comments

i) Well done

ii) Some students do not know that $r=\dfrac{T_2}{T_1}=\dfrac{T_3}{T_2}=etc$

iii) Well done.

iv) The question asked you to determine whether the limiting sum exists. For this, students needed to state why the limiting sum exists. Calculating the limiting sum was well done.

b) i) Overall, this SHOW question was well done.

ii) This is a SHOW question. This means all steps needed to be explained. Students lost marks if they went straight to using S_n formula without establishing the pattern, giving the related series.

Comments

hence,

$$A_{11} = 5\times10^8 \times 1.006^{11} - 1.5\times10^6 \times \frac{1\left(1.006^{11}-1\right)}{1.006-1}$$

$$A_{11} = \$517\ 004\ 017.80 \quad ❶$$

Amount owed on Jan 1 2014 :

$$A_{12} = 517\ 004\ 017.80 \times 1.006^1 - 258\ 5562.38$$
$$= \$517\ 520\ 479.60 \text{ as req'd} \quad ❶$$

iii.

$$A_{13} = 517\ 520\ 479.60 \times 1.006^1 - 5\times10^6$$
$$A_{14} = A_{13} \times 1.006 - 5\times10^6$$
$$= 517\ 520\ 479.60 \times 1.006^2 - 5\times10^6 \left(1.006+1\right)$$
$$\vdots \qquad \vdots$$
$$A_n = 517\ 520\ 479.60 \times 1.006^n - 5\times10^6 \times \left(1+1.006+1.006^2+.....+1.006^{n-1}\right)$$
$$= 517\ 520\ 479.60 \times 1.006^n - 5\times10^6 \times S_n$$

When the loan is paid off $A_n = 0$, hence

$$0 = 517\ 520\ 479.6 \times 1.006^n - 5\times10^6 \times \frac{1\left(1.006^n-1\right)}{1.006-1} \quad ❶$$

$$0 = 3\,105\,122.88 \times 1.006^n - 5\times10^6 \times 1.006^n + 5\times10^6$$

$$0 = 1.006^n \left(3\,105\,122.88 - 5\times10^6\right) + 5\times10^6$$

$$1.006^n = \frac{5\times10^6}{1\,894\,877.12}$$
$$= 2.63869...$$
$$n = \frac{\log\left(2.63869...\right)}{\log\left(1.006\right)}$$
$$n = 162.198...$$
$$\therefore n = 163 \text{ months} \quad ❶$$

Total time to pay off the loan:

$$= 163 + 12 \ (2013)$$
$$= 175 \text{ months}$$
$$= 14 \text{ years } 7 \text{ months} \quad ❶$$

iii) Many students understood that logarithms had to be used but forgot to include the year 2013.

Q16

(a)

i.

$M = Ae^{-kt}$ when $t=0,\ M = M_0$

$t=4,\ M=\frac{1}{4}M_0$

$t=6,\ M=4$

$M_0 = Ae^0$

$\therefore A = M_0$ ❶

$\frac{1}{4}M_0 = M_0e^{-4k}$

$\frac{1}{4} = e^{-4k}$

$\therefore k = \frac{1}{2}\ln(2)\ OR\ \frac{1}{4}\ln(4)$ ❶

ii.

$M = M_0e^{-\frac{\ln(2)}{2}t}$

$4 = M_0e^{-\frac{\ln(2)}{2}\times 6}$ ❶

$M_0 = \frac{4}{e^{-3\ln(2)}}$

$\therefore M_0 = 32kg$ ❶

(b)

i.

$y = 10(3+2\sin x)^{-1}$

$\frac{dy}{dx} = \frac{-20\cos x}{(3+2\sin x)^2}$

Stationary point exists when $\frac{dy}{dx}=0$

Hence,

$\frac{-20\cos x}{(3+2\sin x)^2}=0$

$\cos x = 0$

$\therefore$ Stationary points exist at $x=\frac{\pi}{2}$ and $\frac{3\pi}{2}$ ❶

Comments

(a) (i) No initial value considered

Don’t know how to evaluate e and ln functions

(ii) Don’t know how to use $\frac{dy}{dx}=0$ to find stationary points

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\frac{dy}{dx}$	$-\frac{20}{9}$	0	$\frac{20}{9}$	0	$-\frac{20}{9}$
	-ve	0	+ve	0	-ve

❶

$\therefore \frac{\pi}{2}$ is a minimum and $\frac{3\pi}{2}$ is a maximum ❶

Also accept answers in decimals

ii.

$h=\frac{\pi}{2}$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x)$	$\frac{10}{3}$	2	$\frac{10}{3}$	10	$\frac{10}{3}$

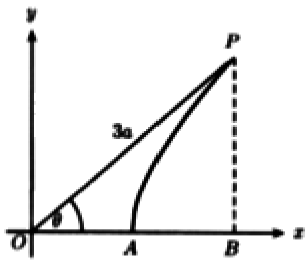
$$A = \frac{h}{3}\left(\frac{10}{3}+\frac{10}{3}+4\left(2+10\right)+2\times\frac{10}{3}\right)$$ ❶

$$= \frac{\pi}{6}\left(\frac{20}{3}+48+\frac{20}{3}\right)$$

$\therefore A = \frac{92\pi}{9}$ units^2 or 32.114(3 dp) ❶

(c)

i.



Calculate limits of the integral:

x coordinate of A:

$$y = 0$$
$$0 = \sqrt{x^2-a^2}$$
$$\therefore x = a$$

x coordinate of B:

$$\cos\theta = \frac{OB}{3a}$$

$$OB = 3a\cos\theta$$
$$\therefore x = 3a\cos\theta$$

❶ for correct x coordinate for A & B

Comments

Do not know how to use the table to check min and max points

ii. Confused with formula, swapped 4 and 2 in multiplying

Comments

Cannot find the x coordinates of A and B

$$V = \pi \int y^2 dx \qquad y = \sqrt{x^2 - a^2}$$
$$y^2 = x^2 - a^2$$

$$V = \pi \int_{a}^{3a\cos\theta} x^2 - a^2 dx$$

$$= \pi \left[\frac{x^3}{3} - a^2 x \right]_{a}^{3a\cos\theta} \quad ❶$$

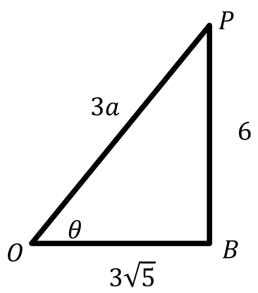
$$= \pi \left[\left(\frac{(3a\cos\theta)^3}{3} - 3a^3\cos\theta \right) - \left(\frac{a^3}{3} - a^3 \right) \right]$$

$$= \pi \left(\frac{27a^3\cos\theta - 9a^3\cos\theta + 2a^3}{3} \right)$$

$$= \frac{a^3\pi}{3}\left(27^3\cos\theta - 9\cos\theta + 2\right) \text{ as req'd} \quad ❶$$

iii.

$$PB = \frac{1}{2} \text{the diameter}$$
$$\therefore PB = 6$$
$$OB = 3\sqrt{5} \quad \text{given}$$
$$OP^2 = 6^2 + \left(3\sqrt{5}\right)^2$$
$$\therefore OP = 9$$



$$3a = 9$$
$$\therefore a = 3 \quad ❶$$

$$\cos\theta = \frac{3\sqrt{5}}{9}$$
$$\therefore \cos\theta = \frac{\sqrt{5}}{3} \quad ❶$$

$$V = \frac{3^3\pi}{3}\left(27 \times \left(\frac{\sqrt{5}}{3} \right)^3 - 9 \times \frac{\sqrt{5}}{3} + 2 \right)$$
$$= 9\pi\left(5\sqrt{5} - 3\sqrt{5} + 2\right)$$
$$= 18\pi\left(\sqrt{5} + 1\right) \text{ units}^3 \quad ❶$$

Comments

Do not know how to find $\cos\theta$

Using decimal numbers instead of exact value