Student Number: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2019

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4

Mathematics

Time allowed: 3 hours

(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11, 12, 13
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	14, 15
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	16

Total Marks 100

Section I 10 marks

Multiple Choice, attempt all questions

Allow about 15 minutes for this section

Section II 90 Marks

Attempt Questions 11-16

Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

1. Find the value of e^8 correct to four significant figures.

   (A) 2980
   (B) 2980.9580
   (C) 2981
   (D) 21.75

2. Differentiate $(2x+1)^5$

   (A) $5(2x+1)^4$
   (B) $10(2x+1)^4$
   (C) $\frac{5}{2}(2x+1)^4$
   (D) $\frac{(2x+1)^6}{12}$

3. Find $\int (2x+1)^{\frac{1}{3}}\,dx$

   (A) $-\frac{2}{3}(2x+1)^{-\frac{1}{3}}+C$
   (B) $\frac{3}{4}(2x+1)^{\frac{4}{3}}+C$
   (C) $\frac{3}{8}(2x+1)^{\frac{4}{3}}+C$
   (D) $\frac{8}{3}(2x+1)^{\frac{4}{3}}+C$

4. Differentiate $y=\frac{\tan x}{x}$ with respect to x

   (A) $\frac{dy}{dx}=\frac{\sec x^2-\tan x}{x^2}$
   (B) $\frac{dy}{dx}=\frac{\tan x-\sec^2 x}{x^2}$
   (C) $\frac{dy}{dx}=\frac{\sec^2 x-\tan x}{x^2}$
   (D) $\frac{dy}{dx}=\frac{x\sec^2 x-\tan x}{x^2}$

5. Solve $\cos x = \frac{\sqrt{3}}{2}$ for $-\pi \le x \le \pi$

(A) $x = -\frac{\pi}{3}, \frac{\pi}{3}$

(B) $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$

(C) $x = -\frac{\pi}{6}, \frac{\pi}{6}$

(D) $x = -\frac{5\pi}{6}, \frac{5\pi}{6}$

6. Factorise $x^2y - xy^2 - x + y$

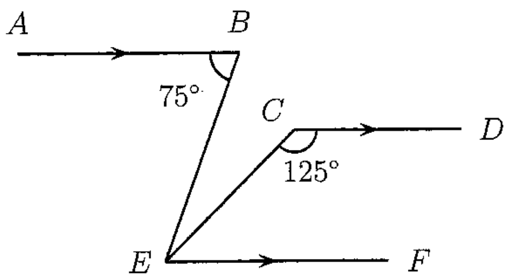
(A) $(xy-1)(x+y)$

(B) $(xy-1)(x-y)$

(C) $(xy+1)(x+y)$

(D) $(xy+1)(x-y)$

7. AB, CD and EF are parallel lines. $\angle ABE = 75°$ and $\angle DCE = 125°$



Calculate the size of $\angle BEC$

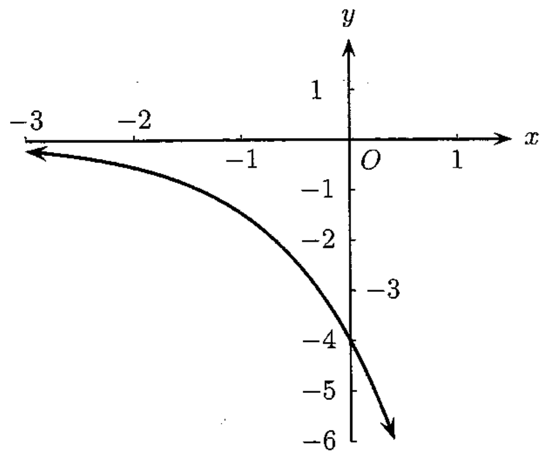
(A) 55°

(B) 50°

(C) 25°

(D) 20°

8. Which of the following could be the equation of the curve below?



(A) $y = -4e^{-x}$

(B) $y = 3 - e^{x}$

(C) $y = -4e^{x}$

(D) $y = -4 - e^{x}$

9. Two ordinary dice are rolled. What is the probability that the sum of the numbers on the top faces is at least 6?

(A) $\frac{5}{18}$

(B) $\frac{13}{18}$

(C) $\frac{27}{36}$

(D) $\frac{28}{36}$

10. What are the coordinates of the focus of the parabola $x^2 = -6(y+1)$?

(A) $\left(0, -2\frac{1}{2}\right)$

(B) $(0, -1)$

(C) $\left(0, \frac{1}{2}\right)$

(D) $(0, -7)$

Section II

90 marks
Attempt Questions 11 to 16
Allow about 2 hours 45 minutes for this section

Start each question in a new writing booklet. Extra writing booklets are available.
In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a new writing booklet.**

(a) Solve $|4x+1|=5$ **2**

(b) Evaluate $\sum_{n=2}^{4} n^2$ **1**

(c) The fourth term of a geometric series is 1 and the ninth term is 32. Find the common ratio **2**

(d) A market gardener plants cabbages in rows. The first row has 35 cabbages. The second row has 39 cabbages. Each succeeding row has 4 more cabbages than the previous row.

i) Calculate the number of cabbages in the 12^{th} row **1**

ii) Which row would be the first to contain more than 200 cabbages **2**

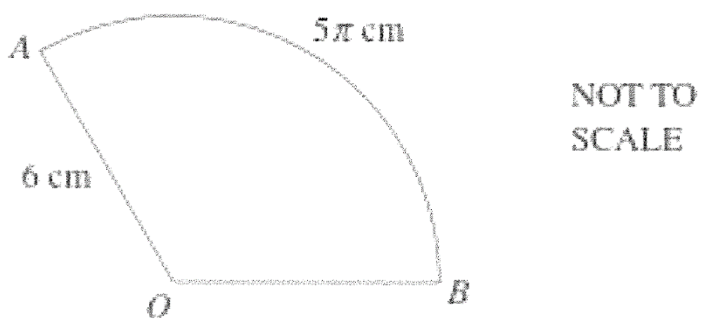
iii) The farmer plants only 945 cabbages. How many rows are needed? **2**

(e) In the series $y+y^2+y^3+...$

i) For what values of y does a limiting sum exist? **1**

ii) If $y=\dfrac{2}{3}$, find the limiting sum **1**

(f) AOB is a sector of a circle, centre O and radius $6\ cm$. The length of the arc AB is 5π cm.



Calculate the exact area of the sector AOB. **3**

Question 12 (15 marks) **Start a new writing booklet.**

(a)

i) State the period and amplitude of $y = 3\sin 2x$ **2**

ii) On the same diagram in the domain $0 \le x \le 2\pi$, draw $\frac{1}{3}$ page graphs of $y = 3\sin 2x$ and $y = 1 - \cos x$ **3**

iii) For the equation $3\sin 2x = 1 - \cos x$, how many solutions are there in the domain $0 \le x \le 2\pi$ **1**

(b) Find the exact gradient of the tangent to the curve $y = \sin x$ at the point where $x = \frac{\pi}{6}$. **2**

(c) Find the indefinite integral of $2\sin\left(\pi + 3x\right)$ **2**

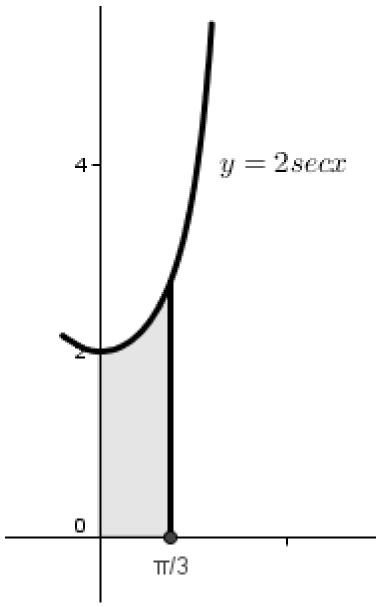
(d) Find the area bounded by the curve $y = \cos 2x$, the $x-$axis and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$.

Answer to 2 decimal places. **3**

(e) Solve $\sqrt{2}\sin\theta = 1$ for $0 \le \theta \le 2\pi$ **2**

Question 13 (15 marks) **Start a new writing booklet.**

(a) In the diagram, the shaded region is bounded by the curve $y = 2\sec x$, the coordinate axes and the line $x = \frac{\pi}{3}$. The shaded region is rotated about the $x-$axis. Calculate the exact volume of the solid of revolution formed. **3**

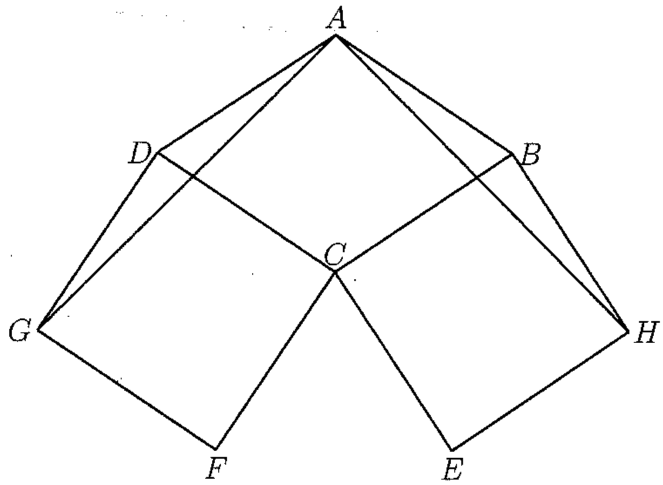


(b) Find the exact value of $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos\frac{x}{2}\, dx$ **2**

(c) What is the equation of the tangent to $y = x + e^{2x}$ at the point $x = 0$ **2**

(d) By first finding the points of intersection, find the area of the region enclosed by the line $y = 2x + 3$ and the parabola $y = x^2$ **3**

(e) In the diagram $ABCD$ is a rhombus, and $BHEC$ and $DCFG$ are squares.



i) Show that ΔAGD is congruent to ΔAHB **3**

ii) Hence, show that $\angle AGF = \angle AHE$ **2**

Question 14 (15 marks) **Start a new writing booklet.**

(a) The position x cm of an object moving in a straight line at time t seconds, is given by: $x = 6t^2 - t^3 + 4$

i) Find the times at which the object is at rest **2**

ii) Find the distance travelled between these stationary times **2**

iii) Find the total distance travelled in the first six seconds **2**

iv) Find the velocity when the acceleration is zero **2**

(b) A function $f(x)$ is defined by $f(x) = x^3 + 6x^2 + 15x$

i) Evaluate $f(-3)$ and $f(1)$ **1**

ii) Show that the curve of $y = f(x)$ is always increasing **2**

iii) Show that there is a point of inflexion at $(-2, -14)$ **2**

iv) Sketch the curve $y = f(x)$, clearly indicating the intercepts and the point of inflexion **2**

Question 15 (15 marks) **Start a new writing booklet.**

(a) Find $\int \frac{6}{3x+1}\,dx$ **2**

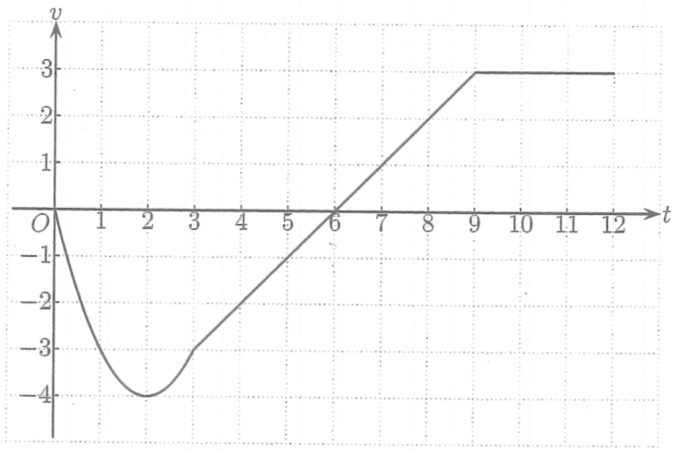
(b) Use Simpson's Rule, with 5 function values, to find an approximation to $\int_0^4 xe^x\,dx$ to 2 decimal places **3**

(c) Air pressure P, measured in kilopascals (kPa), at an altitude of h metres above sea level can be approximated using the formula $P = 101e^{-kh}$, where k is a constant.
The air pressure is $90\ kPa$ at an altitude of $1000\,m$.

i) Find the air pressure at an altitude of $5000\,m$ to 2 decimal places **2**

ii) Find the altitude above sea level, where the air pressure is $53\,kPa$, to the nearest m **2**

(d) A particle moves along the $x-$axis. Initially it is at rest at the origin. The graph shows its velocity v in ms^{-1} as a function of time t for $0 \le t \le 12$.



i) Given that the velocity $v = t^2 - 4t$ for $0 \le t \le 3$, find the displacement of the particle during this time. **2**

ii) Find the time at which the particle returns to the origin. **1**

iii) Sketch the graph of the displacement of this particle for $0 \le t \le 12$ **3**

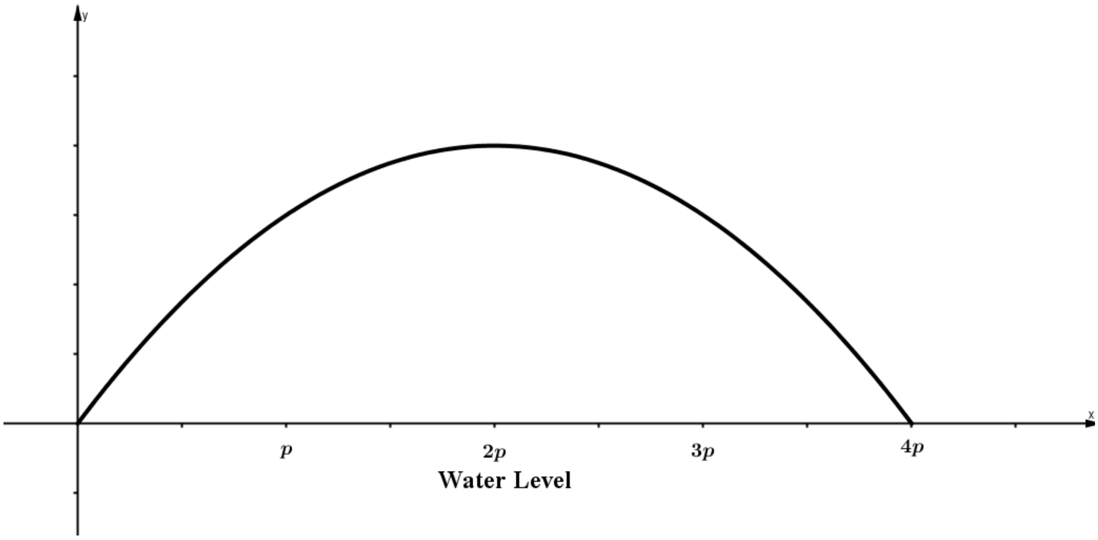
Question 16 (15 marks) **Start a new writing booklet.**

(a) The line $3x+4y+32=0$ is a tangent to a circle centre $(2,-3)$. Find the exact diameter of the circle. **2**

(b) A concrete arch is constructed over a river. The arch is symmetrical in shape with maximum height occurring at $x=2p$ as shown in the diagram.

The shape of the arch can be expressed as part of the curve $y=f(x)$, where

$f''(x)=-e^{10p}\left(e^{x-2p}+e^{2p-x}\right)$ $\quad f'(0)=-f'(4p)$ $\quad$ and $\quad f(0)=0$



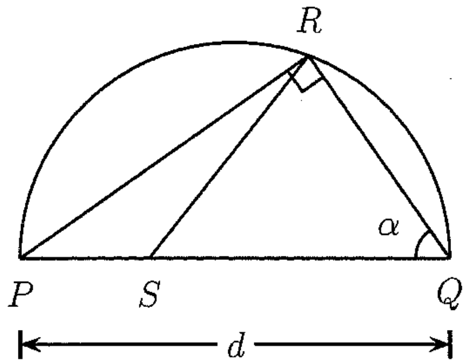
i) Find $f'(x)$ in terms of x and p **2**

ii) Find the height of the arch $y=f(x)$ in terms of x and p **2**

(c)

ΔPQR is a right angled triangle inscribed in a semi circle such that R is a variable point on the circumference.

The point S lies on PQ such that $SQ=kQR$, where k is a positive constant. Let $PQ=d\,cm$ and $\angle PQR=\alpha$ radians



i) Show that the area of ΔSQR is $A=\frac{1}{2}kd^2\cos^2\alpha\sin\alpha$ **3**

ii) Show that $\frac{dA}{d\alpha}=\frac{1}{2}kd^2\left(3\cos^3\alpha-2\cos\alpha\right)$ **3**

iii) Find the greatest possible area of ΔSQR in terms of k and d? **3**

~ End of Examination ~

Student Number: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2019

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 4

Mathematics

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11, 12, 13
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	14, 15
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	16

SOLUTIONS

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

---

1. Find the value of e^8 correct to four significant figures.

   (A) 2980
   (B) 2980.9580
   (C) 2981
   (D) 21.75

$$e^8 = 2980.957$$
$$\approx 2981$$
$\Rightarrow C$

2. Differentiate $(2x+1)^5$

   (A) $5(2x+1)^4$
   (B) $10(2x+1)^4$
   (C) $\frac{5}{2}(2x+1)^4$
   (D) $\frac{(2x+1)^6}{12}$

$$\frac{d(2x+1)^5}{dx}$$
$$= 5(2x+1)^4 \times 2$$
$$= 10(2x+1)^4$$
$\Rightarrow B$

3. Find $\int (2x+1)^{\frac{1}{3}}\,dx$

   (A) $-\frac{2}{3}(2x+1)^{-\frac{1}{3}} + C$
   (B) $\frac{3}{4}(2x+1)^{\frac{4}{3}} + C$
   (C) $\frac{3}{8}(2x+1)^{\frac{4}{3}} + C$
   (D) $\frac{8}{3}(2x+1)^{\frac{4}{3}} + C$

$$\int (2x+1)^{\frac{1}{3}}\,dx = \frac{(2x+1)^{\frac{4}{3}}}{\frac{4}{3}\times 2} + c$$
$$= \frac{3}{8}(2x+1)^{\frac{4}{3}} + c$$
$\Rightarrow C$

4. Differentiate $y=\frac{\tan x}{x}$ with respect to x

(A) $\frac{dy}{dx}=\frac{\sec x^2-\tan x}{x^2}$

(B) $\frac{dy}{dx}=\frac{\tan x-\sec^2 x}{x^2}$

(C) $\frac{dy}{dx}=\frac{\sec^2 x-\tan x}{x^2}$

(D) $\frac{dy}{dx}=\frac{x\sec^2 x-\tan x}{x^2}$

$$\frac{d\left(\frac{\tan x}{x}\right)}{dx}$$
$$=\frac{x.\frac{d(\tan x)}{dx}-\tan x.\frac{d(x)}{dx}}{(x)^2} \qquad \Rightarrow D$$
$$=\frac{x.\sec^2 x-\tan x}{x^2}$$

5. Solve $\cos x=\frac{\sqrt{3}}{2}$ for $-\pi\le x\le\pi$

(A) $x=-\frac{\pi}{3},\frac{\pi}{3}$

(B) $x=-\frac{2\pi}{3},\frac{2\pi}{3}$

(C) $x=-\frac{\pi}{6},\frac{\pi}{6}$

(D) $x=-\frac{5\pi}{6},\frac{5\pi}{6}$

$$\cos x=\frac{\sqrt{3}}{2}$$
$$x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \qquad \Rightarrow C$$
$$=\frac{\pi}{6}(\text{in } Q's\, 1\,\&\, 4)$$
$$=\frac{\pi}{6},\frac{-\pi}{6}$$

6. Factorise x^2y-xy^2-x+y

(A) $(xy-1)(x+y)$

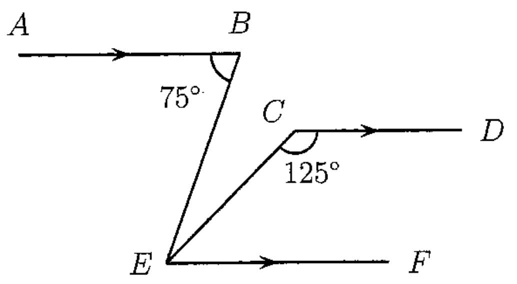
(B) $(xy-1)(x-y)$

(C) $(xy+1)(x+y)$

(D) $(xy+1)(x-y)$

$$x^2y-xy^2-x+y$$
$$=xy(x-y)-1(x-y) \qquad \Rightarrow B$$
$$=(x-y)(xy-1)$$

7. AB, CD and EF are parallel lines.
$\angle ABE = 75°$ and $\angle DCE = 125°$



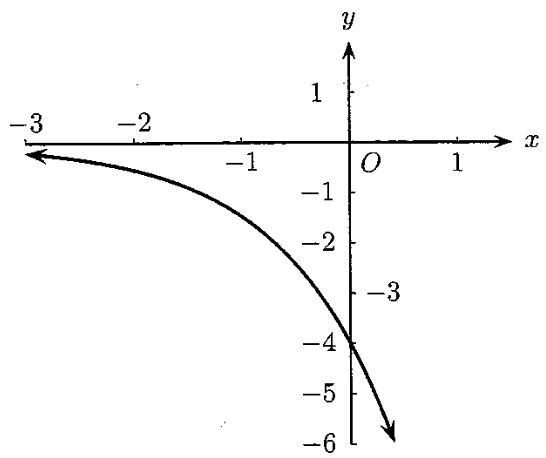
Calculate the size of $\angle BEC$

(A) 55°
(B) 50°
(C) 25°
(D) 20°

$$\angle BEC = 75 - (180 - 125)$$
$$= 20°$$

$\Rightarrow D$

8. Which of the following could be the equation of the curve below?



(A) $y = -4e^{-x}$

(B) $y = 3 - e^{x}$

(C) $y = -4e^{x}$

(D) $y = -4 - e^{x}$

$y = ke^{x}$

At $x = 0, y = -4,$ hence

$-4 = ke^{0}$

$k = -4$

$\therefore y = -4e^{x}$

$\Rightarrow C$

9. Two ordinary dice are rolled. What is the probability that the sum of the numbers on the top faces is at least 6?

(A) $\frac{5}{18}$

(B) $\frac{13}{18}$

(C) $\frac{27}{36}$

(D) $\frac{28}{36}$

$$P(\geq 6) = \frac{1+2+3+4+5+6+5}{36}$$
$$= \frac{26}{36} \qquad \Rightarrow B$$
$$= \frac{13}{18}$$

10. What are the coordinates of the focus of the parabola $x^2 = -6(y+1)$?

(A) $\left(0, -2\frac{1}{2}\right)$

(B) $(0, -1)$

(C) $\left(0, \frac{1}{2}\right)$

(D) $(0, -7)$

$$4a = -6$$
$$a = \frac{-3}{2}$$
$$V \text{ is} (0, -1) \qquad \Rightarrow A$$
$$\therefore S = (0, -1 + a)$$
$$= \left(0, \frac{-5}{2}\right)$$

Section II

90 marks
Attempt Questions 11 to 16
Allow about 2 hours 45 minutes for this section

Start each question in a new writing booklet. Extra writing booklets are available.
In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)	**Marking Criteria.**	**Markers Comments**

(a) Solve $|4x+1|=5$ **2**

Soln: $4x+1=5 \quad 4x+1=-5$ $4x=4 \quad 4x=-6$ $x=1 \quad x=\frac{-3}{2}$	**2** *Provides correct solution* **1** *Provides one correct x value*	Students should be aware there are two cases.

(b) Evaluate $\sum_{n=2}^{4} n^2$ **1**

Soln: $\sum_{n=2}^{4} n^2 = 2^2+3^2+4^2$ $=4+9+16$ $=29$	**1** *Provides correct solution*	Some students were unaware of how to evaluate sigma notation.

(c) The fourth term of a geometric series is 1 and the ninth term is 32. Find the common ratio **2**

Soln: $T_4=1 \quad T_9=32$ $ar^3=1$ ① $\quad ar^8=32$ ② ②÷①: $\quad r^5=32$ $r=2$	**2** *Provides correct solution* **1** *Correctly obtains geometric term equations*	Well answered.

(d) A market gardener plants cabbages in rows. The first row has 35 cabbages. The second row has 39 cabbages. Each succeeding row has 4 more cabbages than the previous row.

i) Calculate the number of cabbages in the 12[th] row **1**

Soln $d=4$ $T_{12}=a+(n-1)d$ $=35+4(12-1)$ $=79$	**1** *Provides correct solution*	

ii) Which row would be the first to contain more than 200 cabbages **2**

Soln $T_n \geq 200$ $35+4(n-1)\geq 200$ $35+4n-4\geq 200$ $4n\geq 169$ $n\geq 42.25$ $\therefore 43\text{rd row exceeds } 200$	**2** *Provides correct solution* **1** *Correctly obtains the inequality* $35+4(n-1)\geq 200$	Care needs to be exercised in stating what row would contain more than 200 cabbages.

iii) The farmer plants only 945 cabbages. How many rows are needed? **2**

Soln $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$ $945=\frac{n}{2}\left[2\times 35+4(n-1)\right]$ $1890=n(70+4n-4)$ $=n(66+4n)$ $0=4n^2+66n-1890$ $n=\frac{-66\pm\sqrt{66^2-4\times 35\times(-1890)}}{2\times 35}$ $=\frac{-66\pm\sqrt{34596}}{70}$ $=\frac{-66\pm 186}{70}$ $=15,\frac{-63}{2}$ But $n>0$, hence 15 rows needed.	**2** *Provides correct solution* **1** *Correctly obtains the quadratic* $0=4n^2+66n-1890$	Many students did not recognise the question was summating the cabbages. Students should be aware that T_n had already been asked in part (ii) and it would be unlikely the same concept would be repeated.

(e) In the series $y + y^2 + y^3 + ...$

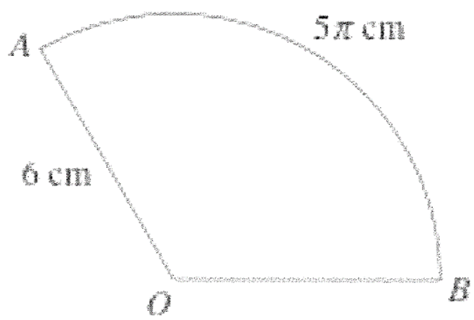
i) For what values of y does a limiting sum exist? **1**

Soln	
For convergence: $-1 < y < 1$	**1** *Provides correct solution*

ii) If $y = \frac{2}{3}$, find the limiting sum **1**

Soln	
$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{2/3}{1-2/3} \\ &= \frac{2/3}{1/3} \\ &= 2 \end{aligned}$$	**1** *Provides correct solution*

(f) AOB is a sector of a circle, centre O and radius $6\ cm$. The length of the arc AB is 5π cm.



NOT TO SCALE

Calculate the exact area of the sector AOB. **3**

Soln	
$$\begin{aligned} l &= r\theta \\ 5\pi &= 6\theta \\ \theta &= \frac{5\pi}{6} \\ A &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}.6^2.\frac{5\pi}{6} \\ &= 15\pi \text{ units}^2 \end{aligned}$$	**3** *Provides correct solution* **2** *Obtains correct θ value and substitutes all values into area formula correctly* **1** *Obtains correct θ value*

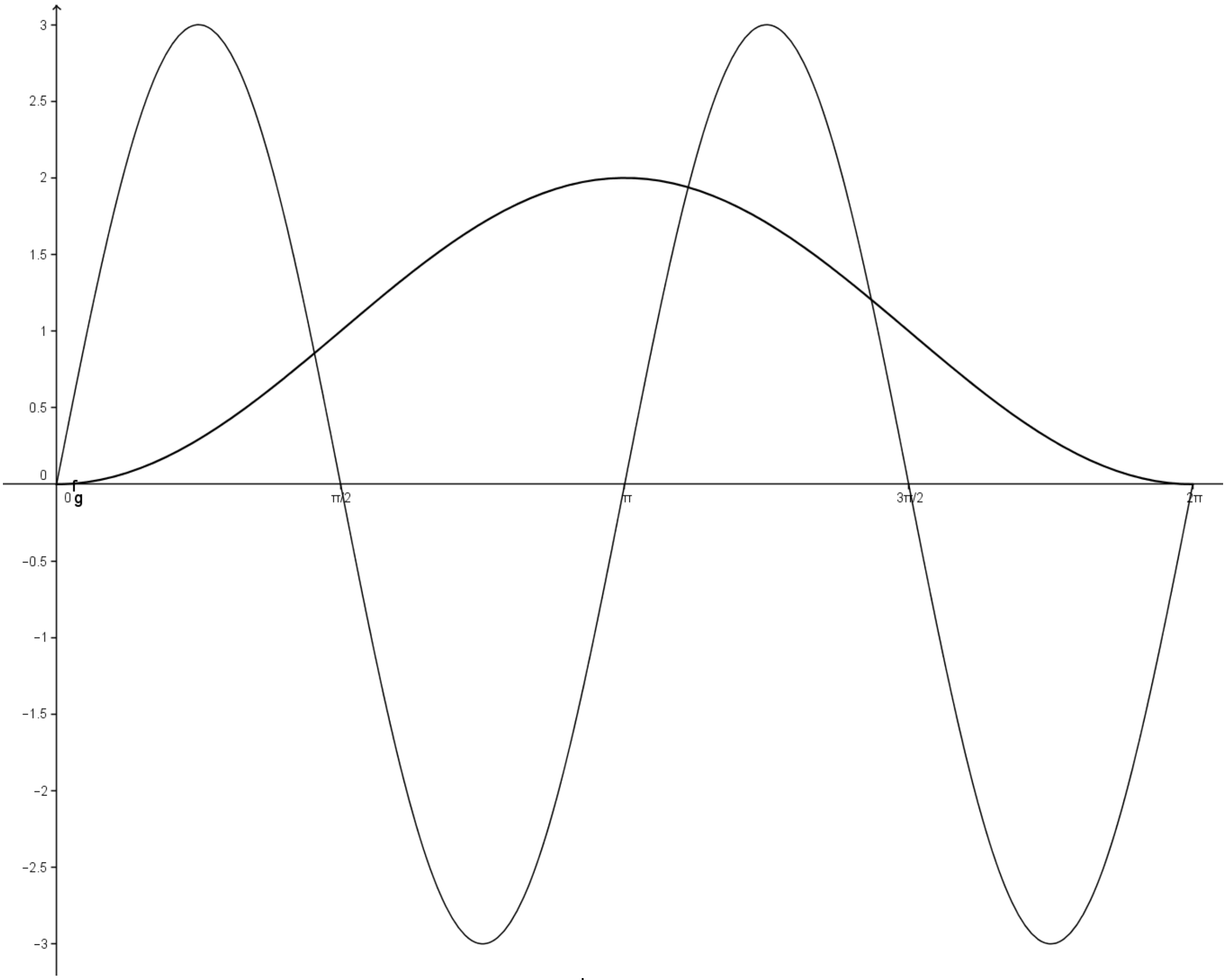
Question 12 (15 marks) **Marking Criteria.** **Markers Comments**

(a)

i) State the period and amplitude of $y = 3\sin 2x$ **2**

Soln		
Amplitude = 3 Period $= \dfrac{2\pi}{2}$ $= \pi$	**2** *Provides correct solution* **1** *One of amplitude or period correct*	Mostly done well.

ii) On the same diagram in the domain $0 \le x \le 2\pi$, draw $\frac{1}{3}$ page graphs of $y = 3\sin 2x$ and $y = 1 - \cos x$ **3**



3 *Graphs drawn correctly, showing endpoints and intercepts. Domain and range correct.* **2** *Correct domain and range, and one graph with correct shape, endpoints and intercepts.* **1** *Either correct domain and range, or one correct graph showing shape, endpoints and intercepts*	Mostly done well.

iii) For the equation $3\sin 2x = 1 - \cos x$, how many solutions are there in the domain $0 \le x \le 2\pi$ **1**

Soln From graphs, there are 5.	**1** *Provides correct solution*	Some students neglected the end points. Otherwise done well.

(b) Find the exact gradient of the tangent to the curve $y = \sin x$ at the point where $x = \frac{\pi}{6}$. **2**

Soln $y = \sin x$ $\frac{dy}{dx} = \cos x$ $At\, x = \frac{\pi}{6}$ $\frac{dy}{dx} = \cos\frac{\pi}{6}$ $= \frac{\sqrt{3}}{2}$	**2** *Provides correct solution* **1** *Correctly differentiates* $y = \sin x$	Some students didn't read the question properly. Make sure you don't waste time finding the equation of the line if it only wants the gradient.

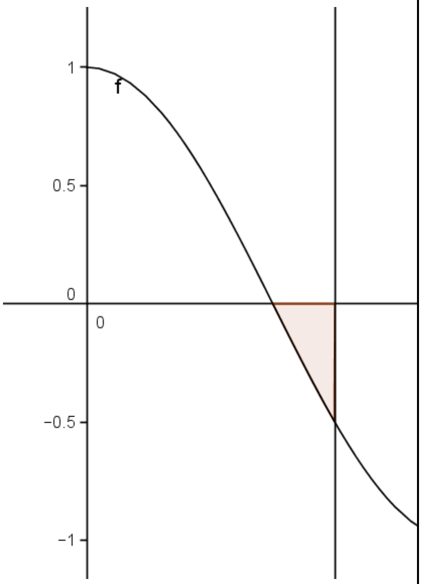
(c) Find the indefinite integral of $2\sin(\pi + 3x)$ **2**

Soln $\int 2\sin(\pi + 3x)\,dx$ $= \frac{-2}{3}\cos(\pi + 3x) + c$	**2** *Provides correct solution* **1** *Partially correct answer*	Mostly done well. Very few students forgot their constant of integration.

(d) Find the area bounded by the curve $y = \cos 2x$, the $x-$axis and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$.

Answer to 2 decimal places. **3**

Soln:



$$A = \left| \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 2x \, dx \right|$$

$$= \left| \left[\frac{1}{2} \sin(2x) \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \right|$$

$$= \left| \frac{1}{2} \left(\sin \frac{2\pi}{3} - \sin \frac{2\pi}{4} \right) \right|$$

$$\approx |-0.06698729811|$$

$$\approx 0.07 \ (2\text{dp})$$

3 *Provides correct solution*

2 *Expression in terms of π*

1 *correct integral and correct limits*

Students were still awarded the mark if they found the exact value. Recall that area cannot be negative.

(e) Solve $\sqrt{2} \sin \theta = 1$ for $0 \le \theta \le 2\pi$ **2**

Soln:

$$\sqrt{2} \sin \theta = 1$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$= \frac{\pi}{4}$$

Sin positive in Q1 &2, hence

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

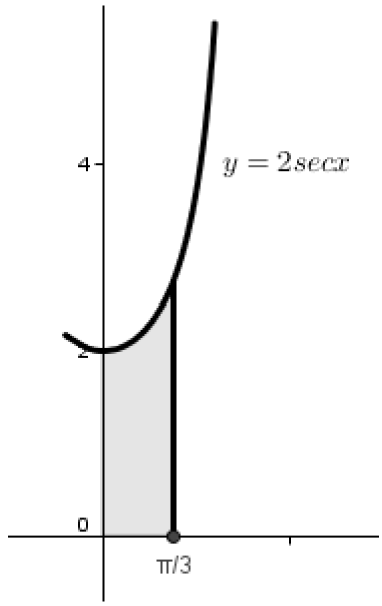
2 *Provides correct solution*

1 *Partial correct answer*

Mostly done well but some students provided 1 or 4 solutions instead of 2. Recall your ASTC rule. Sin is positive in the first and second quadrant.

Question 13 (15 marks) **Marking Criteria.** **Markers Comments**

(a) In the diagram, the shaded region is bounded by the curve $y = 2\sec x$, the coordinate axes and the line $x = \dfrac{\pi}{3}$. The shaded region is rotated about the $x-$axis . Calculate the exact volume of the solid of revolution formed. **3**



Solution	Marking Criteria	Markers Comments
Soln: $V = \pi\int_0^{\frac{\pi}{3}} (2\sec x)^2\, dx$ $= \pi\int_0^{\frac{\pi}{3}} 4\sec^2 x\, dx$ $= 4\pi\left[\tan x\right]_0^{\frac{\pi}{3}}$ $= 4\pi\left(\tan\frac{\pi}{3} - \tan 0\right)$ $= 4\pi\left(\sqrt{3} - 0\right)$ $= 4\sqrt{3}\pi$	**3** *Provides correct solution* **2** *Unsimplified expression in terms of π, such as* $4\pi\left(\tan\frac{\pi}{3} - \tan 0\right)$ **1** *integral in terms of x and correct limits*	Some students didn't square 2 and many forgot π

(b) Find the exact value of $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos\frac{x}{2}\, dx$ **2**

Soln:	
$I = \left[2\sin\frac{x}{2}\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$ $= 2\left(\sin\frac{2\pi}{6} - \sin\frac{\pi}{6}\right)$ $= 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ $= \sqrt{3} - 1$	**2** *Provides correct solution* **1** *integral in terms of x and correct limits*

(c) What is the equation of the tangent to $y = x + e^{2x}$ at the point $x = 0$ **2**

Soln:	
$y = x + e^{2x}$ $\frac{dy}{dx} = 1 + 2e^{2x}$ $At\ x = 0:$ $y = 0 + e^{0}$ $= 1, \text{and}$ $\frac{dy}{dx} = 1 + 2e^{0}$ $= 3$ $y - 1 = 3(x - 0)$ $y = 3x + 1$	**2** *Provides correct solution* **1** *Partial correct answer*

(d) By first finding the points of intersection, find the area of the region enclosed by the line $y = 2x + 3$ and the parabola $y = x^2$ **3**

Soln:

$x^2 = 2x + 3$

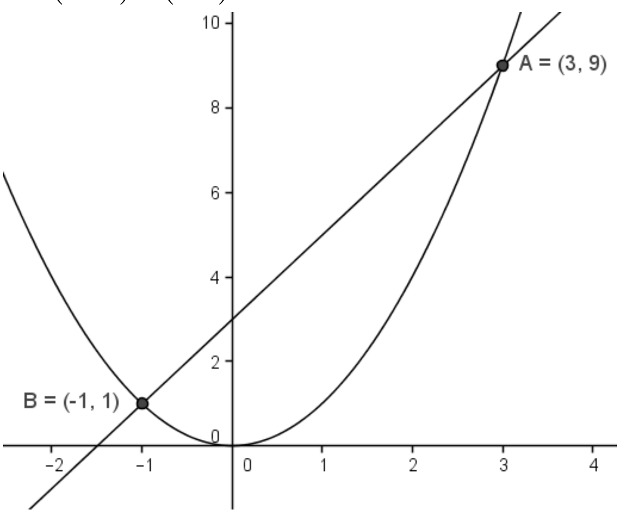
$0 = x^2 - 2x - 3$

$= (x-3)(x+1)$

$x = -1, 3$

$y = 1, 9$

$\therefore$ Intersection points are $(-1,1)$ & $(3,9)$



$$A = \int_{-1}^{3} 2x + 3 - x^2 \, dx$$

$$= \left[x^2 + 3x - \frac{1}{3}x^3 \right]_{-1}^{3}$$

$$= \left(3^2 + 3\times 3 - \frac{3^3}{3} \right) - \left((-1)^2 + 3\times(-1) - \frac{(-1)^3}{3} \right)$$

$$= 9 - \left(-1\frac{2}{3} \right)$$

$$= 10\frac{2}{3} \text{ units}^2$$

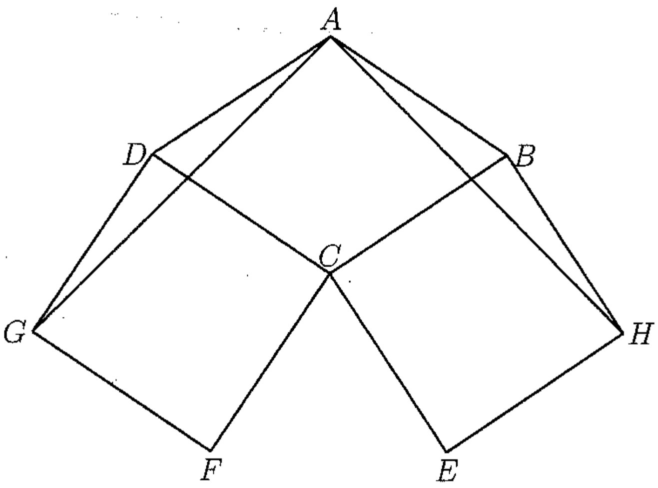
3 *Provides correct solution*

2 *Integral in terms of x and correct limits*

1 *Partial correct answer*

Many students split the integral into two parts unnecessarily

(e) In the diagram $ABCD$ is a rhombus, and $BHEC$ and $DCFG$ are squares.



i) Show that ΔAGD is congruent to ΔAHB **3**

Soln:

$DC = BC\left(\text{eq. sides of Rhombus}\right)$

Thus squares $DCFG$ & $BCEH$ are congruent.

Hence $\angle GDC = 90$ and $\angle CBH = 90$.

$\angle CDA = \angle ABC\left(\text{opp.} \angle\text{'s Rhomb. eq.}\right)$

$\angle ADG = \angle GDC + \angle CDA$

$\quad = \angle CBH + \angle ABC$

$\quad = \angle ABH$

In $\angle' s AGD, AHB$

$AD = AB\left(\text{eq. sides of Rhombus}\right)$

$DG = BH\left(\text{sides of cong. squares}\right)$

$\angle ADG = \angle ABC\left(\text{shown above}\right)$

$\therefore \Delta AGD \equiv \Delta AHB\left(\text{SAS}\right)$

3 *Provides correct solution*

2 *Solution contains one error*

1 *Partially correct solution*

Many students assumed that isosceles triangles with the same side lengths have equal base angles

ii) Hence, show that $\angle AGF = \angle AHE$ **2**

Soln:

$\angle AGD = \angle AHB\left(\text{corr } \angle\text{'s cong. } \Delta\text{'s}\right)$

$\angle DGF = \angle BHE\left(=90^\circ\right)$

$\angle AGF = \angle DGF - \angle AGD$

$\angle AHE = \angle BHE - \angle AHB$

$\therefore \angle AGF = \angle AHE$

2 *Provides correct solution*

1 *Partially correct solution*

Question 14 (15 marks)	**Marking Criteria.**	**Markers Comments**

(a) The position x cm of an object moving in a straight line at time t seconds, is given by: $x = 6t^2 - t^3 + 4$

i) Find the times at which the object is at rest **2**

Soln: $\frac{dy}{dx} = 12t - 3t^2$ At rest when $\frac{dy}{dx} = 0$ $0 = 12t - 3t^2$ $= 3t(4-t)$ $t = 0, 4$	**2** *Provides correct solution* **1** *Correct derivative*	Well answered.

ii) Find the distance travelled between these stationary times **2**

Soln: displacements at time $t = 0 \Rightarrow x = 4$ $t = 4 \Rightarrow$ $x = 6\times 4^2 - 4^3 + 4$ $= 36$ Hence distance travelled is $36 - 4$ $= 32cm$	**2** *Provides correct solution* **1** *Partially correct solution*	Well answered

iii) Find the total distance travelled in the first six seconds **2**

Soln: From (ii), distance after 4 sec is 32cm. Displacement when $t = 6$ sec: $x = 6\times 6^2 - 6^3 + 4$ $= 4$ Hence another $32cm$ travelled. Total distance is $64cm$.	**2** *Provides correct solution* **1** *Partially correct solution*	Some careless errors. Some students who used integration did not realise that two separate definite integrals needed to be added and evaluated. Students lost 1 mark for not including absolute value from the beginning of their working when they calculated the distance travelled from t=4 to t=6 s. Wrong setting out resulted in loss of marks.

iv) Find the velocity when the acceleration is zero **2**

Soln: $\frac{d^2x}{dt^2} = 12 - 6t$ $0 = 12 - 6t$ $t = 2$ $\frac{dy}{dx} = 12 \times 2 - 3 \times 2^2$ $= 12\, cm/s$	**2** *Provides correct solution* **1** *Calculates time when acceleration is zero*	Well answered. However, some students forgot to include units.

(b) A function $f(x)$ is defined by $f(x) = x^3 + 6x^2 + 15x$

i) Evaluate $f(-3)$ and $f(1)$ **1**

Soln: $f(-3) = (-3)^3 + 6(-3)^2 + 15(-3)$ $= -18$ $f(1) = (1)^3 + 6(1)^2 + 15(1)$ $= 22$	**1** *Provides correct solution*	Well answered

ii) Show that the curve of $y = f(x)$ is always increasing **2**

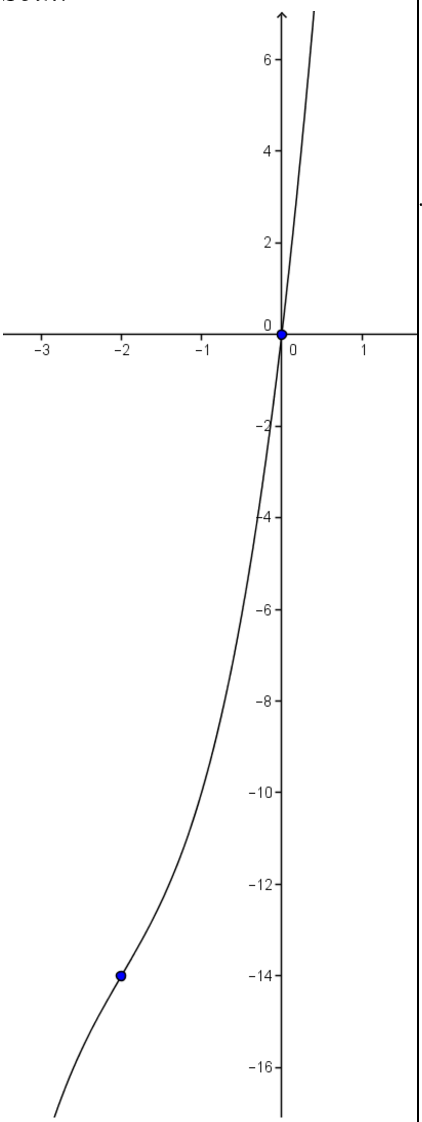
Soln: Show $f'(x) > 0$ for all x. $f'(x) = 3x^2 + 12x + 15$ $\Delta = 12^2 - 4 \times 3 \times 15$ $= -36$ < 0 Leading co-efficient >0, thus $f'(x) > 0$ for all x. Hence the curve is always increasing.	**2** *Provides correct solution* **1** *Calculates $f'(x)$ and discriminant of $f'(x)$*	Poorly answered. Students need to learn that there are 2 conditions that must be satisfied to prove f '(x) is a positive definite function: $\Delta < 0$ AND leading coefficient of quadratic expression 'a'> 0.

iii) Show that there is a point of inflexion at $(-2,-14)$ **2**

Soln: $f''(x)=6x+12$ Possible POI when $f''(x)=0$ $0=6x+12$ $x=-2$ Testing nature: x -2.5 -2 -1.5 $f''(x)$ -3 0 3 <0 >0 Hence change in concavity $f(-2)=(-2)^3+6(-2)^2+15(-2)$ $=-14$ Thus $(-2,14)$ is a POI.	**2** *Provides correct solution* **1** *Tests change in concavity*	One mark was deducted for failing to test for concavity changing about the point of inflexion. Another mark was deducted for not evaluating the y coordinate of the point of inflexion. This is a SHOW question!

iv) Sketch the curve $y = f\left(x\right)$, clearly indicating the intercepts and the point of inflexion **2**

Soln:



2 *Correct graph with intercept and POI marked*

1 *Correct shape, but some missing features, or infection point shown as a stationary point*

Poorly done! There was no scale shown on axes. The shape of the curve around the point of inflexion was incorrect. Axes were not labelled. Graphs were too small. The points on the function calculated in b)i) should have been shown on the sketch but marks were not lost if this was not done

Question 15 (15 marks)	**Marking Criteria.**	**Markers Comments**

(a) Find $\int \frac{6}{3x+1}\,dx$ **2**

Soln: $= 2\ln(3x+1)+c$	**2** *Provides correct solution including the constant* **1** *Partially correct solution*	Some students omitted the constant.

(b) Use Simpson's Rule, with 5 function values, to find an approximation to $\int_0^4 xe^x\,dx$ to 2 decimal places **3**

Soln:

$$A \approx \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+..)+2(y_2+y_4+...)\right]$$

$$h = \frac{4-0}{4}$$

$$=1$$

$$y_0 = 0, y_1 = e, y_2 = 2e^2, y_3 = 3e^3, y_4 = 4e^4$$

$$A = \frac{1}{3}\left[(0+4e^4)+4(e+3e^3)+2.2e^2\right]$$

$$\approx 166.62$$

3 *Provides correct solution*

2 *Calculates h and y values correctly*

1 *Substitution into correct Simpson's Rule*

Generally well answered.

(c) Air pressure P, measured in kilopascals (kPa), at an altitude of h metres above sea level can be approximated using the formula $P = 101e^{-kh}$, where k is a constant.
The air pressure is $90\ kPa$ at an altitude of $1000\,m$.

i) Find the air pressure at an altitude of $5000\,m$ to 2 decimal places **2**

Soln:

$$P = 90, h = 1000$$

$$90 = 101e^{-1000k}$$

$$\frac{90}{101} = e^{-1000k}$$

$$-1000k = \ln\left(\frac{90}{101}\right)$$

$$k = \frac{-1}{1000}\ln\left(\frac{90}{101}\right)$$

$$P = 101e^{-5000\times\frac{-1}{1000}\ln\left(\frac{90}{101}\right)}$$

$$= 101e^{5\ln\left(\frac{90}{101}\right)}$$

$$= 56.74$$

2 *Provides correct solution*

1 *Calculation of k correct*

Very well answered but students should remember to use the exact value of k and not an approximation when evaluating.

ii) Find the altitude above sea level, where the air pressure is $53\,kPa$, to the nearest m. **2**

Soln:

$$53 = 101e^{-\frac{-1}{1000}\ln\left(\frac{90}{101}\right)h}$$

$$\frac{53}{101} = e^{\frac{1}{1000}\ln\left(\frac{90}{101}\right)h}$$

$$\ln\left(\frac{53}{101}\right) = \frac{1}{1000}\ln\left(\frac{90}{101}\right)h$$

$$h = \frac{1000\ln\left(\frac{53}{101}\right)}{\ln\left(\frac{90}{101}\right)}$$

$$= 5592.09$$

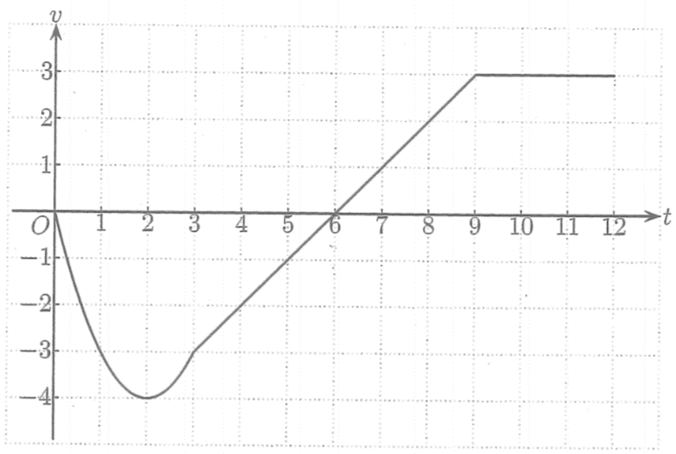
$$\approx 5592m$$

2 *Provides correct solution*

1 *Partially correct solution*

As above.

(d) A particle moves along the $x-$axis. Initially it is at rest at the origin. The graph shows its velocity v in ms^{-1} as a function of time t for $0 \le t \le 12$.



i) Given that the velocity $v = t^2 - 4t$ for $0 \le t \le 3$, find the displacement of the particle during this time. **2**

Soln:

Displacement is given by

$$x(t) = \int v\,dt$$

$$= \int t^2 - 4t\,dt$$

$$= \frac{1}{3}t^3 - 2t^2 + c$$

$$t = 0, x = 0$$

$$0 = c$$

$$x(t) = \frac{1}{3}t^3 - 2t^2$$

Alt: $x(3) = \frac{1}{3}\times 3^3 - 2\times 3^2$

$$= -9$$

2 *Provides correct solution*

1 *Correct integral and c value*

Technically displacement during this time is given as an equation while the displacement after 3 seconds would be -9 *m*. No penalty was applied for stating the displacement after 3 seconds.

Many students who provided a numerical value were confused with distance and incorrectly stated the displacement is +9 *m*.

Do not use absolute value for displacement.

ii) Find the time at which the particle returns to the origin. **1**

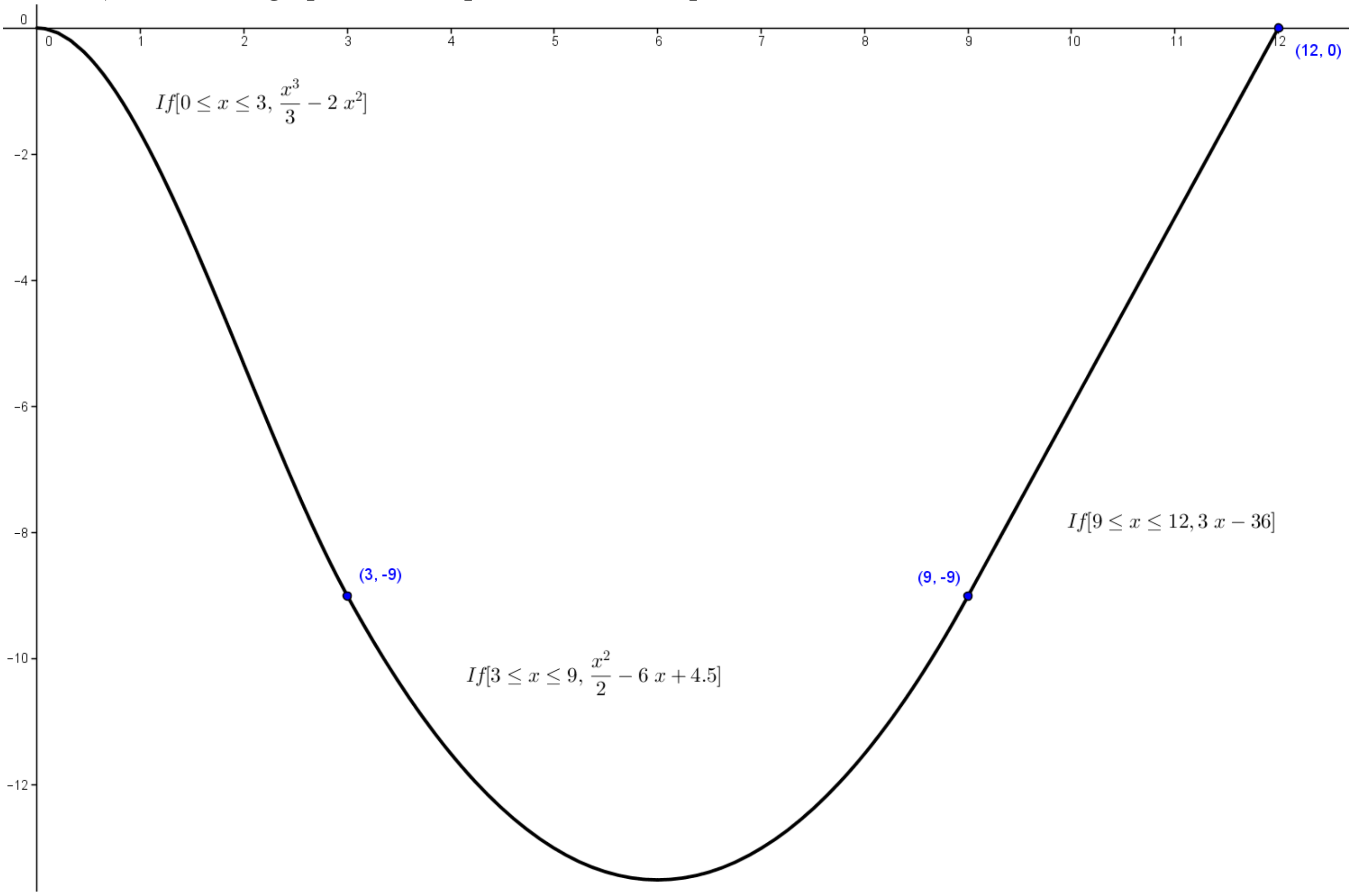
Soln:

When the area under the curve equal the area above the curve, the particle will return to the origin.
For $3 \le t \le 9$, the areas (above/below) cancel out.
For $0 \le t \le 3$, (i) Alt gives $A = -9$ and for $9 \le t \le 12$ there is a 3×3 square above the x-axis, so these cancel out.
Hence the particle returns to the origin at $t = 12$.

1 *Provides correct solution*

Many students tried to use the formula from part (i) to answer this question. However the formula only applies to the first 3 seconds and not all the graph.

iii) Sketch the graph of the displacement of this particle for $0 \le t \le 12$ **3**



3 *Graphs drawn with correct shape, labels, intercepts and the points where the equations change.*

2 *Graphs drawn with correct shape, some points shown.*

1 *Partial correct solution*

1 mark was awarded to students who could demonstrate 1 element of the solution. This may have been a correct shape of one of the sections or the correct points shown above.

Many students were unable to identify when the particle returned to the origin.

Question 16 (15 marks) **Marking Criteria.** **Markers Comments**

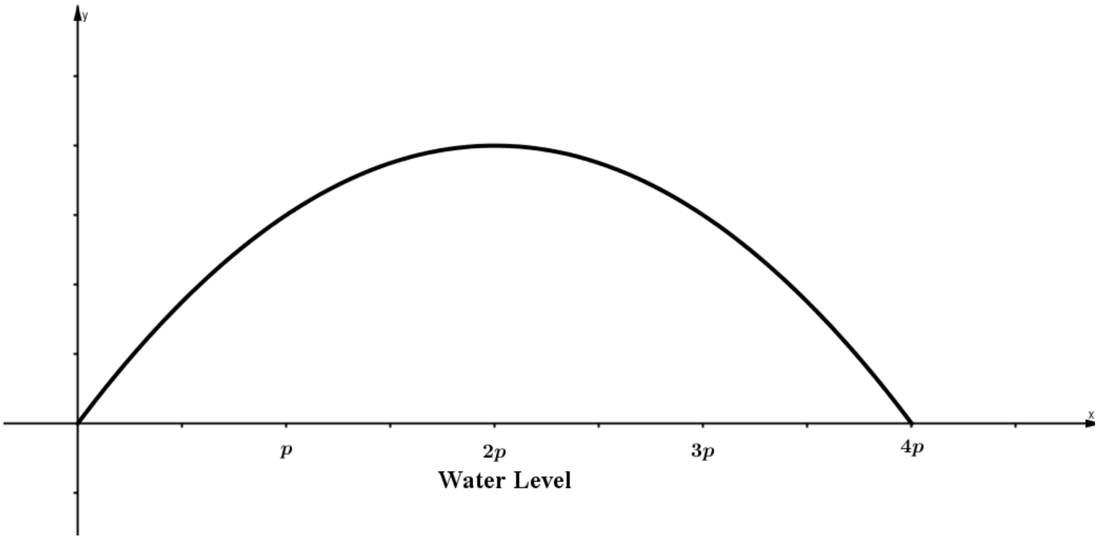
(a) The line $3x+4y+32=0$ is a tangent to a circle centre $(2,-3)$. Find the exact diameter of the circle. **2**

Solution	Marking Criteria	Markers Comments
Soln: Perpendicular distance is: $d=\frac{\lvert 3(2)+4(-3)+32\rvert}{\sqrt{3^2+4^2}}$ $=\frac{26}{5}$ Diameter: $=\frac{26}{5}\times 2$ $=\frac{52}{5}$ $=10\frac{2}{5}$	**2** *Provides correct solution* **1** *Correct substitution into perpendicular distance formula*	

(b) A concrete arch is constructed over a river. The arch is symmetrical in shape with maximum height occurring at $x=2p$ as shown in the diagram.

The shape of the arch can be expressed as part of the curve $y=f(x)$, where

$f''(x)=-e^{10p}\left(e^{x-2p}+e^{2p-x}\right)$ $\quad f'(0)=-f'(4p)$ $\quad$ and $\quad f(0)=0$



i) Find $f'(x)$ in terms of x and p **2**

Solution	Marking Criteria	Markers Comments
Soln: $f''(x)=-e^{10p}\left(e^{x-2p}+e^{2p-x}\right)$ $=-e^{10p+x-2p}-e^{10p+2p-x}$ $=-e^{8p+x}-e^{12p-x}$ $\therefore f'(x)=-e^{8p+x}+e^{12p-x}+c$ $f'(0)=-e^{8p}+e^{12p}+c$ $f'(4p)=-e^{12p}+e^{8p}+c$ *(Continued over)*	**2** *Provides correct solution* **1** *Partial correct solution*	

$$-e^{8p}+e^{12p}+c=-\left(-e^{12p}+e^{8p}+c\right)$$
$$2c=e^{12p}-e^{12p}-e^{8p}+e^{8p}$$
$$=0$$
$$c=0$$
$$\therefore f'(x)=-e^{8p+x}+e^{12p-x}$$

ii) Find the height of the arch $y=f(x)$ in terms of x and p **2**

Soln:

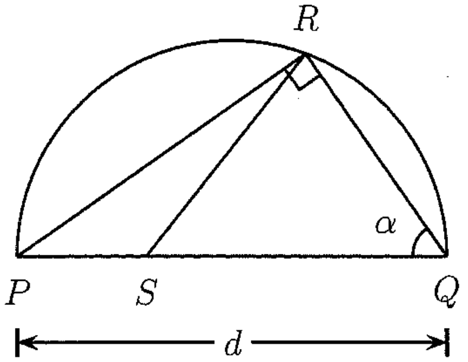
$$\int f'(x)\,dx$$
$$=\int -e^{8p+x}+e^{12p-x}\,dx$$
$$=-e^{8p+x}-e^{12p-x}+k$$
$$x=0, f(0)=0:$$
$$0=-e^{8p}-e^{12p}+k$$
$$k=e^{8p}+e^{12p}$$
$$f(x)=-e^{8p+x}-e^{12p-x}+e^{8p}+e^{12p}$$

2 *Provides correct solution*

1 *Partial correct solution*

(c)

ΔPQR is a right angled triangle inscribed in a semi circle such that R is a variable point on the circumference.
The point S lies on PQ such that $SQ=kQR$, where k is a positive constant. Let $PQ=d\ cm$ and $\angle PQR=\alpha$ radians



i) Show that the area of ΔSQR is $A=\frac{1}{2}kd^2\cos^2\alpha\sin\alpha$ **3**

Soln:

$$A_{\Delta SQR}=\frac{1}{2}.QR.SR.\sin\alpha$$
$$=\frac{1}{2}.QR.k.QR.\sin\alpha$$
$$=\frac{k}{2}.(QR)^2.\sin\alpha$$

(Continued over)

3 *Provides correct solution*

2 *Correct expression for* $\cos\alpha$ *in terms of QR and PQ (or d) amd correct area equation in terms of* $\sin\alpha$

1 *Correct area equation in terms of* $\sin\alpha$

From ΔPQR:

$$\cos\alpha = \frac{QR}{PQ}$$
$$= \frac{QR}{d}$$
$$QR = d\cos\alpha$$

Hence

$$A_{\Delta SQR} = \frac{k}{2}.\left(d\cos\alpha\right)^2.\sin\alpha$$
$$= \frac{kd^2}{2}.\cos^2\alpha.\sin\alpha$$

ii) Show that $\dfrac{dA}{d\alpha} = \dfrac{1}{2}kd^2\left(3\cos^3\alpha - 2\cos\alpha\right)$ **3**

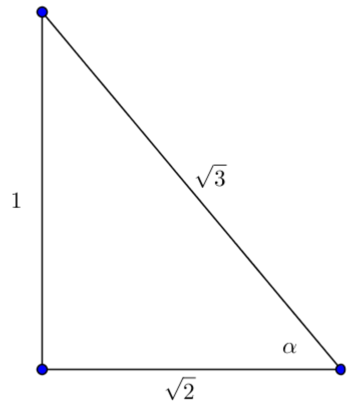
Soln:	
$A = \frac{kd^2}{2}.\cos^2\alpha.\sin\alpha$	**3** *Provides correct solution*
$\frac{dA}{d\alpha} = \frac{kd^2}{2}\left[\cos^3\alpha + \sin\alpha.2\cos\alpha.\left(-\sin\alpha\right)\right]$	**2** *Correct product rule expression for* $\frac{dA}{d\alpha}$
$= \frac{kd^2}{2}\left(\cos^3\alpha - 2\cos\alpha\sin^2\alpha\right)$	**1** *Substitutes* $1-\cos^2\alpha$ *for* $\sin^2\alpha$
$= \frac{kd^2}{2}\left(\cos^3\alpha - 2\cos\alpha\left(1-\cos^2\alpha\right)\right)$	
$= \frac{kd^2}{2}\left(\cos^3\alpha - 2\cos\alpha + 2\cos^3\alpha\right)$	
$= \frac{kd^2}{2}\left(3\cos^3\alpha - 2\cos\alpha\right)$	

iii) Find the greatest possible area of ΔSQR in terms of k and d ? **3**

Soln:	
$\frac{dA}{d\alpha} = 0$	**3** *Provides correct solution*
$0 = \frac{kd^2}{2}\left(3\cos^3\alpha - 2\cos\alpha\right)$	**2** *correct calculation of* $\cos\alpha$ *and* $\sin\alpha$.
$= \cos\alpha\left(3\cos^2\alpha - 2\right)$	**1** *Solving* $A' = 0$ *and stating* $\cos\alpha = 0$ *and* $\cos\alpha = \pm\sqrt{\frac{2}{3}}$
$\cos\alpha = 0$ $3\cos^2\alpha - 2 = 0$	
$\alpha = \cos^{-1}0$ $\cos^2\alpha = \frac{2}{3}$	
$= \frac{\pi}{2}$ $\cos\alpha = \pm\sqrt{\frac{2}{3}}$	

But $0 < \alpha < \frac{\pi}{2}$, so $\alpha = \frac{\pi}{2}$ and $\alpha = \cos^{-1}\left(-\sqrt{\frac{2}{3}}\right)$ fall outside this range.

Hence $\alpha = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ is the only solution.



So $\sin\alpha = \frac{1}{\sqrt{3}}$.

Hence:

$$A = \frac{kd^2}{2}.\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2.\frac{1}{\sqrt{3}}$$

$$= \frac{kd^2}{2}.\frac{2}{3}.\frac{1}{\sqrt{3}}$$

$$= \frac{kd^2}{3\sqrt{3}}$$