Student Number:

Section 1 – Multiple Choice Questions

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1. The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

A. -5
B. 5
C. 13
D. 42

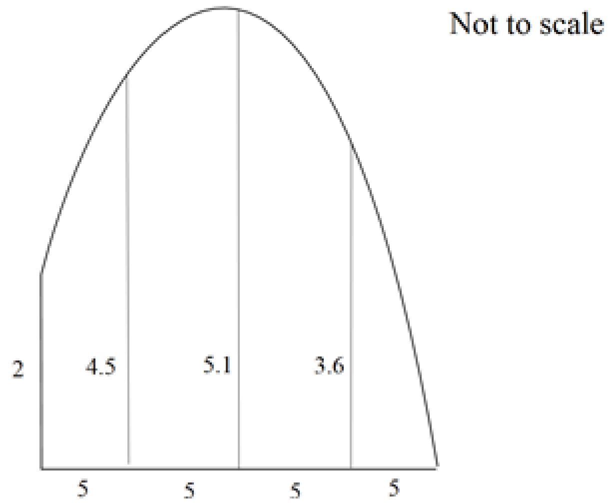
2. The graph of the function $f(x) = 3x^{\frac{5}{2}}$ is reflected in the x axis and then translated 3 units to the right and 4 units down. The equation of the transformed function is

A. $y = 3(-x-3)^{\frac{5}{2}} - 4$
B. $y = -3(x-3)^{\frac{5}{2}} - 4$
C. $y = -3(x+3)^{\frac{5}{2}} - 4$
D. $y = 3(-x+3)^{\frac{5}{2}} - 4$

3. What is the solution to the equation $\cos 2x = \dfrac{1}{2}$ in the domain $[-\pi, \pi]$?

A. $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{-5\pi}{6}, \dfrac{-\pi}{6}$
B. $x = \dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{-11\pi}{12}, \dfrac{-\pi}{12}$
C. $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$
D. $x = \dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}$

4. The diagram below shows a native garden. All measurements are in metres.



What is an approximate value for the area of the native garden using the trapezoidal rule with 4 intervals?

A. 31 m^2
B. 62 m^2
C. 71 m^2
D. 74 m^2

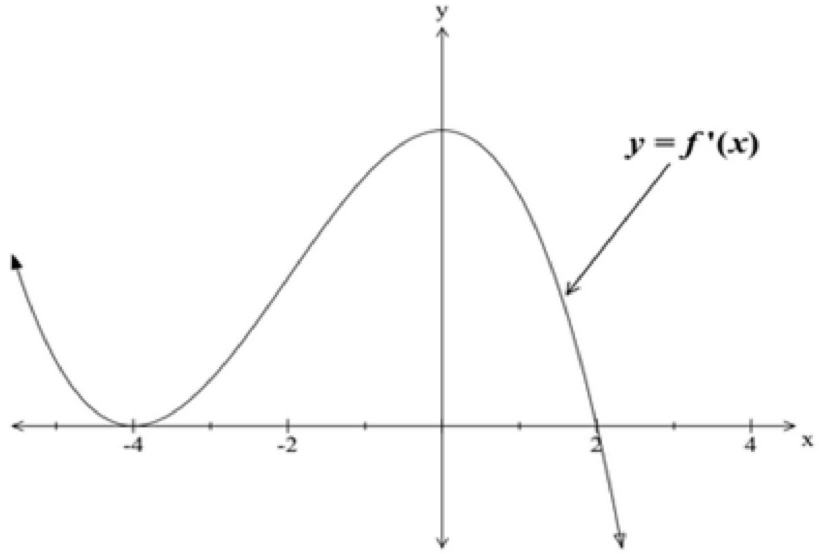
5. What is the domain and range of the function $y = \dfrac{1}{\sqrt{x-9}}$?

A. Domain: $[9,\infty)$, Range: $(0,\infty)$
B. Domain: $(9,\infty)$, Range: $(0,\infty)$
C. Domain: $[-\infty,\infty]$, Range: $[-\infty,\infty]$
D. Domain: $[-3,3]$, Range: $(-\infty,0)$

6. At what angle is the line $y = -\sqrt{3}x$ inclined to the positive side of the x axis?

A. 30°
B. 60°
C. 120°
D. 150°

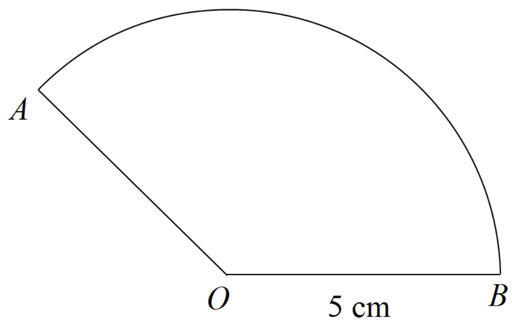
7.



The diagram above represents a sketch of the gradient function of the curve $y = f(x)$.
Which of the following is a true statement? The curve $y = f(x)$ has

A. a minimum turning point at $x = -4$
B. a horizontal point of inflexion at $x = 2$
C. a horizontal point of inflexion at $x = -4$
D. a minimum turning point at $x = 2$.

8. AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of $10\pi\ cm^2$.



Not to scale

What is the arc length of the sector, in centimetres?

A. 2π
B. 4π
C. 6π
D. 10π

9. The discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X = x)$	a	3a	5a	7a

The expected value of X is

A. $\frac{1}{16}$

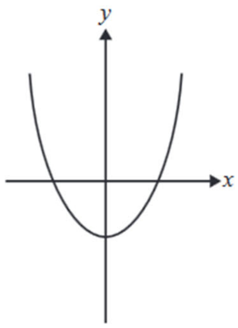
B. 2

C. $\frac{35}{16}$

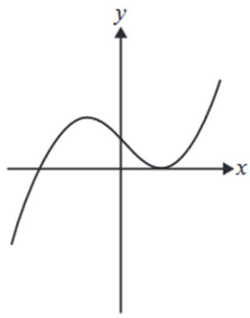
D. $\frac{17}{8}$

10. If $f'(x) = 3x^2 - 4$, which of the following graphs could represent the graph of $y = f(x)$?

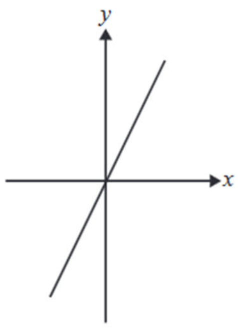
A.



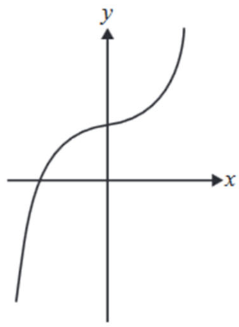
B.



C.



D.

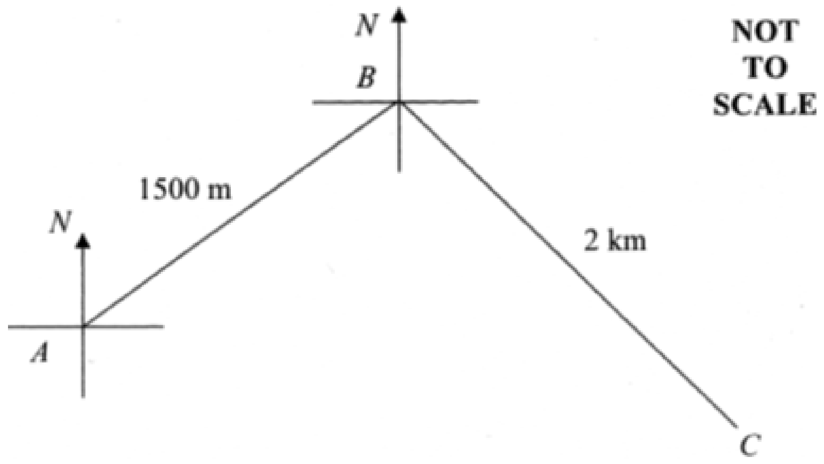


Student Number:

Section 2 – Part A

Question 1 (3 marks)

From a starting point A, Roy rows 1500 m on a bearing of $68°30'$ to point B. He then turns and continues on a bearing of $145°30'$ for 2 km to a point C. He then returns to his starting point A.



(a) What is the size of $\angle ABC$? **1**

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(b) Calculate AC, the distance Roy rows back to his starting point. Answer to the nearest metre. **2**

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Question 2 (6 marks)

Differentiate the following expressions. Simplify your answers, if possible. **2**

(a) $\dfrac{e^{3x}}{x-1}$

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(b) $\tan x \cos x$ **3**

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Question 2 continues on page 7

(c) $\log_4\left(x^2+3\right)$ **1**

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Question 3 (2 marks)

Find the following

(a) $\int \sin\left(\frac{x}{3}\right) dx$ **1**

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(b) $\int x^4\left(x^5-2\right)^3 \, dx$ **1**

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Question 4 (2 marks)

Evaluate exactly $\int_0^1 \frac{x-1}{x^2-2x+4} \, dx$, simplify your answer fully. **2**

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Student Number:

Section 2 – Part B

Question 5 (2 marks)

For what values of k does the quadratic equation $5x^2 - 2x + (8k - 15) = 0$ have real roots? **2**

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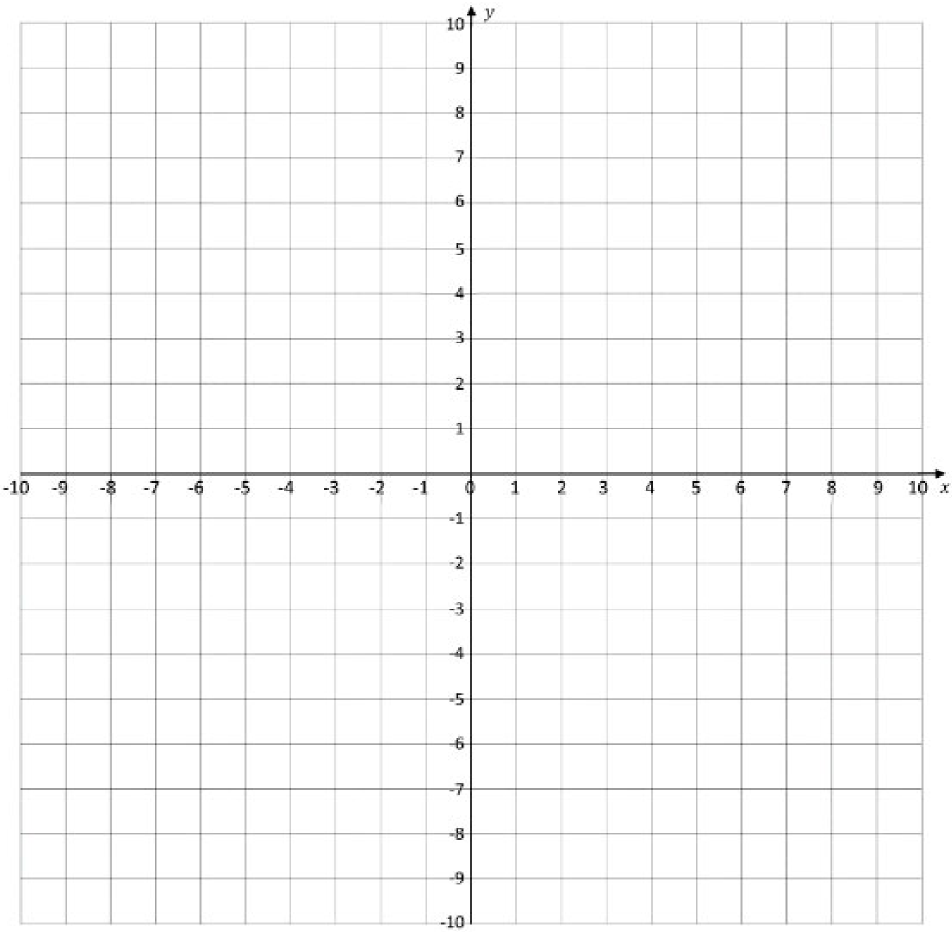
Question 6 (3 marks)

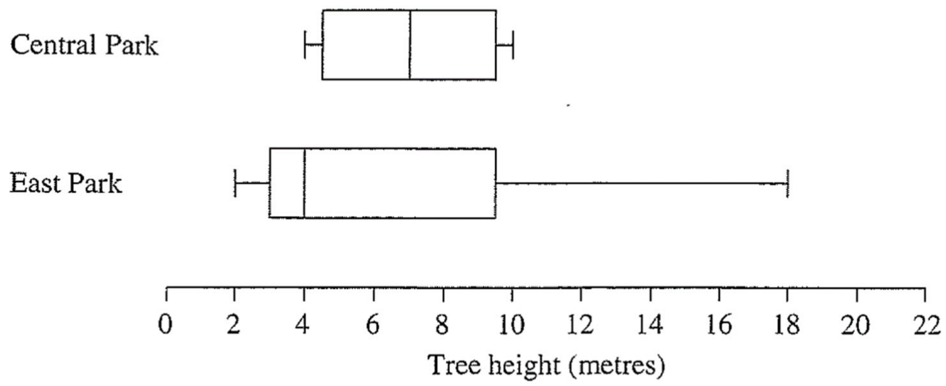
Sketch the graph of $y = 2 - \dfrac{4}{x+1}$ on the axes below, showing all intercepts with the coordinate axes and all asymptotes. **3**



Question 7 (3 marks)

Annie and her biology class went to two large city parks and measured the heights of the trees in metres.

In Central Park there were 25 trees. In East Park there were 27 trees. The data sets were displayed in two box-and-whisker plots.



Compare and contrast the two data sets by examining the shape and skewness of the distributions, and the measures of location and spread. **3**

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Question 8 (3 marks)

Find the equation of the normal to the curve $y = \ln\left(\dfrac{2x-1}{x+1}\right)$, at the point where $x = 2$. **3**

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Question 9 (2 marks)

A circle is given by the equation $x^2 + y^2 + 4x - 10y = -16$. Find the centre and radius of this circle. **2**

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Student Number:

Section 2 – Part C

Question 10 (15 marks)

Let $f(x)=(x+2)(x-2)^3$.

(a) Find the x and y - intercepts of $y=f(x)$. **2**

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(b) Show that $f'(x)=4(x-2)^2(x+1)$ and $f''(x)=12x(x-2)$. **2**

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Question 10 continues on page 14

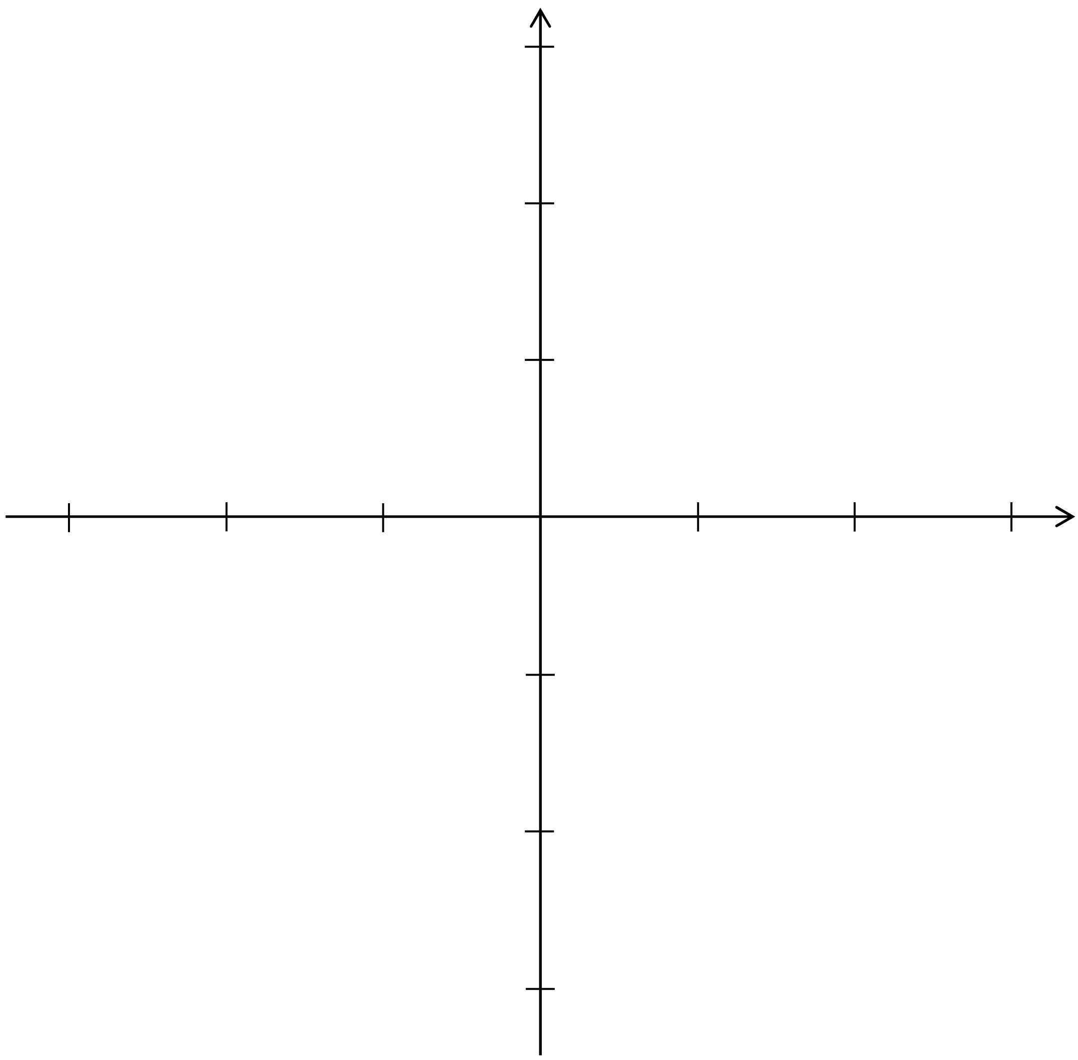
(c) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **3**

Justify your answers fully.

(d) Find the coordinates of all points of inflection of $y = f(x)$. **2**

Question 10 continues on page 15

(e) Sketch the graph of $y = f(x)$ on the axes below, showing all the features found above. **2**
(*Hint: use different scales on the axis)*



(f) State in the correct order, the transformations required to obtain the graph of **2**

$$y = f\left(2\left(x - \frac{1}{4}\right)\right).$$

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(g) On the set of axes provided in part (e), sketch the graph of $y = f\left(2\left(x - \frac{1}{4}\right)\right)$, **2**

showing coordinates of x - intercepts, stationary points and inflection points.

Student Number:

Section 2 – Part D

Question 11 (5 marks)

Tom planted a silky oak tree three years ago when it was 80 cm tall. At the end of the first year after planting, it was 130 cm tall, that is it grew 50 cm. Each year's growth was 90% of the previous year's.

(a) What was the growth of the silky oak after 3 years? **1**

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(b) Assuming that it maintains the present growth pattern, explain why the tree will never reach a height of 6 metres. **2**

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(c) In which year will the silky oak reach a height of 5 metres? **2**

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Question 12 (4 marks)

A leak from a tanker has accidentally contaminated a farmer's paddock with a toxic chemical. The chemical concentration in the soil was 6 kL/ha immediately after the accident. One year later the concentration in the soil was measured to be 2.4 kL/ha.
It is known that the concentration, C , is given by:

$$C = C_0 e^{-kt} \ ,$$

where C_0 and k are constants and t is measured in years.

(a) Show that $C_0 = 6$ and $k = -\ln\left(\frac{2}{5}\right)$. **2**

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(b) It will not be safe for the farmer to plant a new crop until the concentration falls below 0.2 kL/ha. How long, to the nearest month, after the spill does the farmer need to wait for the paddock to be safe to use. **2**

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Question 13 (5 marks)

Alice and Raoul take turns throwing darts at a dartboard. The winner is whoever hits the bullseye first. Alice has a $\frac{1}{30}$ chance of hitting the bullseye, while Raoul has a $\frac{1}{40}$ chance of hitting the bullseye. Alice throws the first dart.

(a) Draw a tree diagram for the first four throws of the games (two throws forAlice and two for Raoul). **1**

(b) What is the probability that Alice wins on her first or second throw? **1**

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(c) What is the probability that Alice will eventually win the game. **3**

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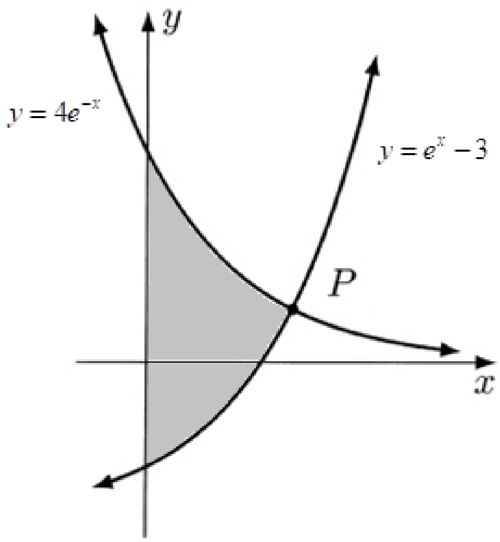
Student Number:

Section 2 – Part E

Question 14 (5 marks)

The diagram shows the graphs of $y = 4e^{-x}$ and $y = e^{x} - 3$.



(a) Show that the curves intersect when $e^{2x} - 3e^{x} - 4 = 0$. **1**

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(b) Hence, show the x coordinate of the point P is $x = \ln 4$. **2**

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Question 14 continues on page 21

(c) Find the exact shaded area between the two curves . **2**

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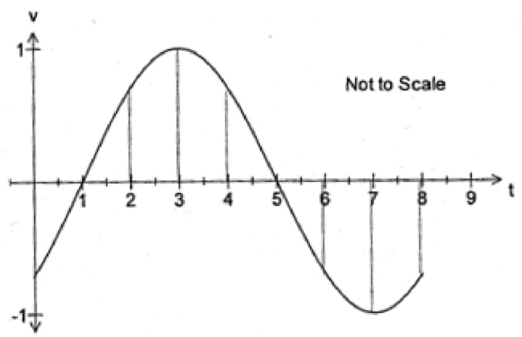
Question 15 (3 marks)

The diagram shows the velocity-time graph for a particle moving in a straight line.



State the times between t=0 and t=8 at which:

(a) The acceleration is zero **1**

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(b) The speed is increasing **2**

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Question 16 (7 marks)

A particle moves in a straight line so that after t seconds ($(t \geq 0)$ its velocity v is given by $v = \left(\frac{2}{1+t} - t\right) m/s$. The displacement of the particle from the origin is given by x metres.

(a) Find the acceleration of the particle when $t = 0$. **1**

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(b) If the particle is initially at the origin, find the displacement as a function of t . **2**

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(c) When is the particle stationary? **2**

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(d) How far does the particle travel in the first 2 seconds? Give your answer correct to three significant figures. **2**

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Student Number:

Section 2 – Part F

Question 17 (7 marks)

A tank initially holds 3600 litres of water. The water drains from the bottom of the tank. The tank takes 60 minutes to empty. A mathematical model predicts that the volume V litres of water that will remain in the tank after t minutes is given by

$V = 3600\left(1 - \frac{t}{60}\right)^2$, where $0 \le t \le 60$.

(a) What volume does the model predict will remain after 10 minutes? **1**

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(b) How long will it take for the tank to become half full (to nearest minute)? **2**

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(c) At what rate does the model predict that the water will drain from the tank after twenty minutes? **2**

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(d) At what time does the model predict that the water will drain from the tank at its fastest rate? **2**

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Question 18 (7 marks)

The average monthly temperature, $T°$, for a town in NSW can be modeled on the formula

$T = 7\sin\left(nx + 1.5\right) + 13$,

where $n =$ a constant value and

$x =$ the number of the month of the year (i.e. January=1, February=2, …..).

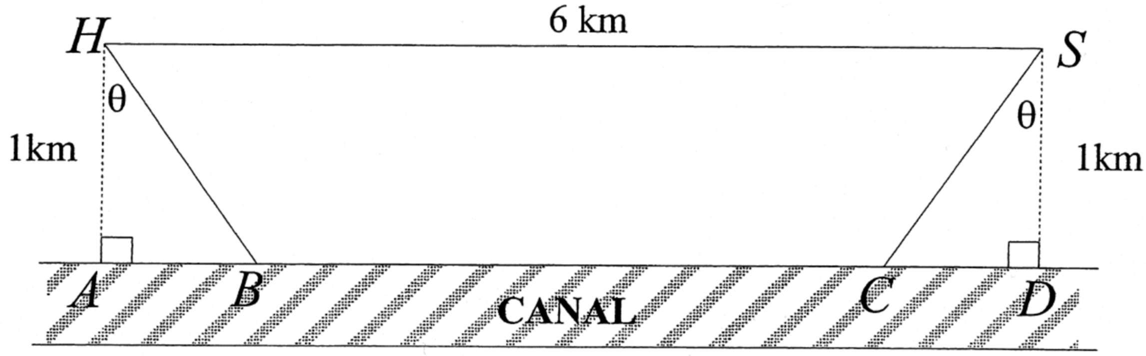
(a) According to the model, what are the maximum and minimum average monthly temperatures in this town? **2**

(b) The period of the function is 12. Determine the value of n correct to 2 decimal places. **1**

(c) Which month has the lowest average monthly temperature? **2**

(d) Graph the function $T = 7\sin\left(nx + 1.5\right) + 13$ for $1 \le x \le 12$. **2**

Question 19 (6 marks)



The diagram above shows that the distance between a boy's home H and his school S is 6 km. A canal $ABCD$ is 1 km from both his home and school. In winter the canal is frozen, he takes an alternate route $HBCS$, walking HB, skating BC and walking CS. His walking speed is 4 km/h and his skating speed is 12 km/h. Let $\angle AHB = \angle DSC = \theta$.

(a) Show that the time taken for this alternate route is $T = \dfrac{1}{2\cos\theta} + \dfrac{1}{2} - \dfrac{\tan\theta}{6}$. **2**

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Question 19 continues on page 28

(b) Find, to the nearest minute, the value of θ which minimizes the time taken for the journey to school. **4**

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END OF EXAMINATION

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Section 1 – Multiple Choice Questions

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1. The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

A. -5
B. 5
C. 13
D. 42

$T_4: a + 3d = 27$ ①
$T_7: a + 6d = 12$ ②
$3d = -15$ ② − ①
$d = -5$

2. The graph of the function $f(x) = 3x^{\frac{5}{2}}$ is reflected in the x axis and then translated 3 units to the right and 4 units down. The equation of the transformed function is

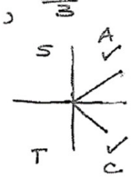
A. $y = 3(-x-3)^{\frac{5}{2}} - 4$
B. $y = -3(x-3)^{\frac{5}{2}} - 4$
C. $y = -3(x+3)^{\frac{5}{2}} - 4$
D. $y = 3(-x+3)^{\frac{5}{2}} - 4$

$f(x) = 3x^{5/2}$
after reflection : $y = -3x^{5/2}$
after translation 3 units right : $y = -3(x-3)^{5/2}$
after translation 4 units down : $y = -3(x-3)^{5/2} - 4$

3. What is the solution to the equation $\cos 2x = \dfrac{1}{2}$ in the domain $[-\pi, \pi]$?

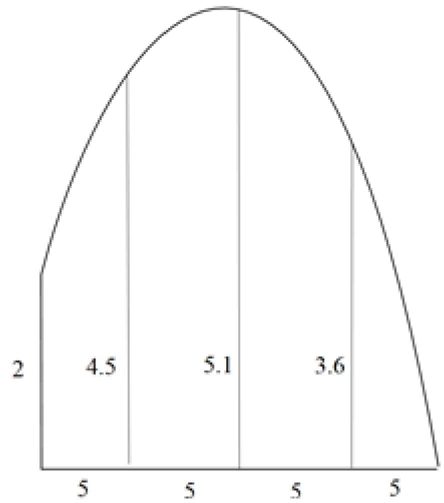
A. $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{-5\pi}{6}, \dfrac{-\pi}{6}$
B. $x = \dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{-11\pi}{12}, \dfrac{-\pi}{12}$
C. $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$
D. $x = \dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}$

$\cos 2x = \frac{1}{2}$ $\quad -2\pi \le 2x \le 2\pi$
related $\angle = \frac{\pi}{3}$
$\therefore\ 2x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, -\frac{\pi}{3}, -2\pi + \frac{\pi}{3}$
$\therefore\ 2x = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{3}$
$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-\pi}{6}, -\frac{5\pi}{6}$



4. The diagram below shows a native garden. All measurements are in metres.

Not to scale



What is an approximate value for the area of the native garden using the trapezoidal rule with 4 intervals?

A. 31 m^2
B. 62 m^2
C. 71 m^2
D. 74 m^2

$h = 5$

Area $\doteqdot \frac{h}{2}\left[y_1 + y_5 + 2(y_2 + y_3 + y_4)\right]$

$= \frac{5}{2}\left[2 + 0 + 2(4.5 + 5.1 + 3.6)\right]$

$= 71 \ m^2$

5. What is the domain and range of the function $y = \dfrac{1}{\sqrt{x-9}}$?

A. Domain: $[9,\infty)$, Range: $(0, \infty)$
B. Domain: $(9,\infty)$, Range: $(0, \infty)$
C. Domain: $[-\infty,\infty]$, Range: $[-\infty,\infty]$
D. Domain: $[-3,3]$, Range: $(-\infty,0)$

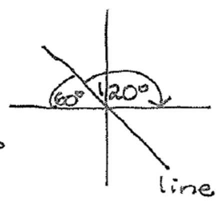
$x - 9 > 0$ & $y > 0$
$x > 9$

$\therefore$ Domain: $(9, \infty)$
Range: $(0, \infty)$

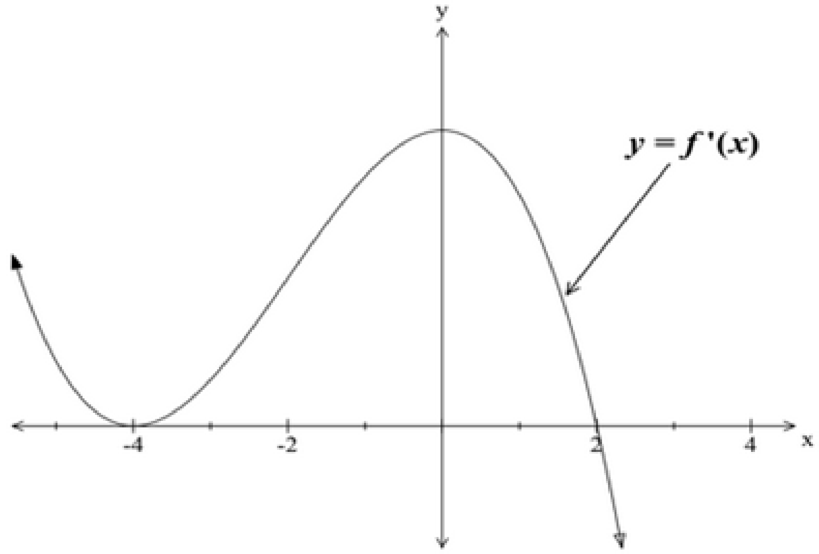
6. At what angle is the line $y = -\sqrt{3}x$ inclined to the positive side of the x axis?

A. 30°
B. 60°
C. 120°
D. 150°

$m = -\sqrt{3}$
$\tan\theta = m$
$\tan\theta = -\sqrt{3}$
related $\angle = \frac{\pi}{3} = 60°$



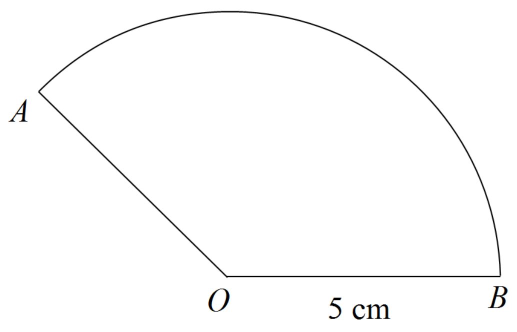
7.



The diagram above represents a sketch of the gradient function of the curve $y = f(x)$. Which of the following is a true statement? The curve $y = f(x)$ has

A. a minimum turning point at $x = -4$
B. a horizontal point of inflexion at $x = 2$
C. a horizontal point of inflexion at $x = -4$
D. a minimum turning point at $x = 2$.

8. AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of $10\pi\ cm^2$.



Not to scale

What is the arc length of the sector, in centimetres?

A. 2π
B. 4π
C. 6π
D. 10π

$A = \frac{1}{2}r^2\theta$

$10\pi = \frac{1}{2} \times 5^2 \times \theta$

$\theta = \frac{10\pi}{12.5} = \frac{4\pi}{5}$

$l = r\theta$

$= \cancel{5} \times \frac{4\pi}{\cancel{5}}$

$l = 4\pi$

9. The discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X=x)$	a	3a	5a	7a

The expected value of X is

A. $\frac{1}{16}$

B. 2

C. $\frac{35}{16}$

D. $\frac{17}{8}$

$P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$

i.e. $a+3a+5a+7a=1$

$16a=1$

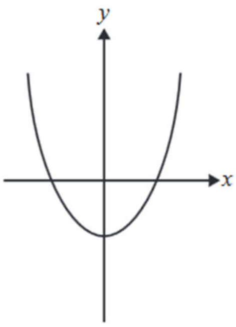
$a=\frac{1}{16}$

$\therefore E(X)=\left(0\times\frac{1}{16}\right)+\left(1\times\frac{3}{16}\right)+\left(2\times\frac{5}{16}\right)+\left(3\times\frac{7}{16}\right)$

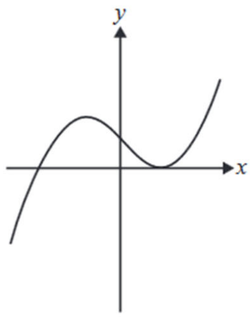
$=\frac{17}{8}$

10. If $f'(x)=3x^2-4$, which of the following graphs could represent the graph of $y=f(x)$?

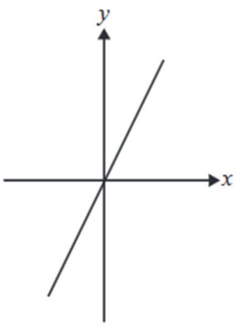
A.



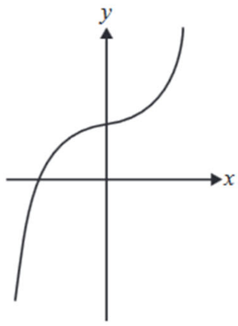
B.

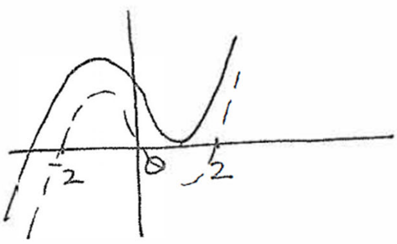


C.



D.





$f'(x)=3x^2-4$

$f(x)=x^3-4x+c$

$=x(x^2-4)+c$

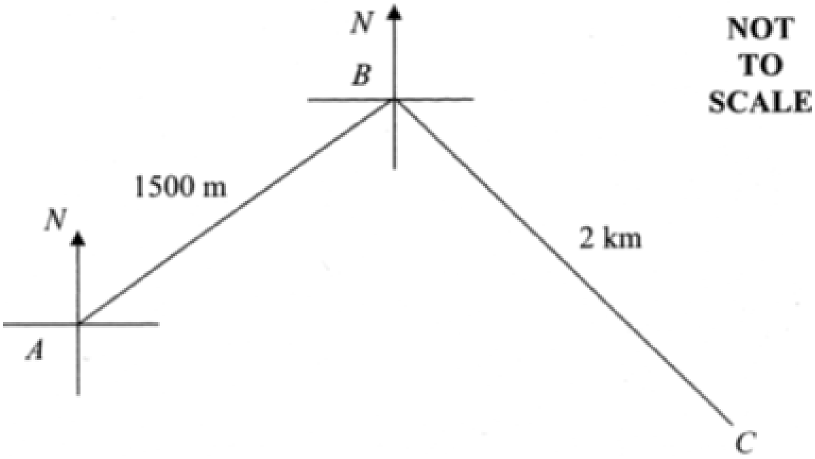
$=x(x+2)(x-2)+c$

$\therefore$ This is function $y=x(x+2)(x-2)$ translated 'c' units vertically

Section 2 – Part A

Question 1 (3 marks)

From a starting point A, Roy rows 1500 m on a bearing of $68°30'$ to point B. He then turns and continues on a bearing of $145°30'$ for 2 km to a point C. He then returns to his starting point A.



(a) What is the size of $\angle ABC$?

Criteria	Marks
Provides correct solution	1

$\angle ABC = 103°$

Marker's Comments:
Overall, this was well done.

(b) Calculate AC, the distance Roy rows back to his starting point. Answer to the nearest metre.

Criteria	Marks
Provides correct solution	2
Correctly substitutes into the cosine rule	1

$AC^2 = 1500^2 + 2000^2 - 2(1500)(2000)\cos 103°$

$AC^2 \doteqdot 7599706 \cdot 326$

$AC \doteqdot 2756 \cdot 7565$

$\therefore$ Roy rows back a distance of 2757 m (nearest m)

Marker's Comments:

Carried error from (a) was considered.

Some could not remember the cosine rule - it is on the Reference Sheet!

A common error was forgetting to find the $\sqrt{}$.

Question 2 (6 marks)

Differentiate the following expressions. Simplify your answers, if possible.

(a) $\dfrac{e^{3x}}{x-1}$

Criteria	Marks
Provides correct solution in simplified form (Factorisation of numerator is not required)	2
Attempts to use the quotient rule	1

let $y = \dfrac{e^{3x}}{x-1}$

$\dfrac{dy}{dx} = \dfrac{(x-1)3e^{3x} - e^{3x}(1)}{(x-1)^2}$

$= \dfrac{3xe^{3x} - 3e^{3x} - e^{3x}}{(x-1)^2}$

$= \dfrac{3xe^{3x} - 4e^{3x}}{(x-1)^2} = \dfrac{e^{3x}(3x-4)}{(x-1)^2}$

Marker's Comments:

Well done.

Some students needed to take care in simplifying the numerator.

(b) $\tan x \cos x$

Method 2

let $y = \tan x \cos x$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$y = \sin x$

$\therefore \dfrac{dy}{dx} = \cos x$

Criteria	Marks
Provides correct solution in simplified form	3
Attempts to substitute correct trigonometric identities	2
Attempts to use the product rule	1

let $y = \tan x \cos x$

$\dfrac{dy}{dx} = (\cos x)\sec^2 x + \tan x(-\sin x)$

$= \dfrac{\cos x}{\cos^2 x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x}$

$= \cos x$

Marker's Comments:

Some students did not simplify the derivative after applying the product rule. Only 1 mark was awarded for the first step opposite.

Please take note of the easier method.

(c) $\log_4\left(x^2+3\right)$

Criteria	Marks
Provides correct solution	1

let $y = \log_4(x^2+3)$ $\therefore \frac{dy}{dx} = \frac{1}{\ln 4} \cdot \frac{2x}{x^2+3}$

$= \frac{\ln(x^2+3)}{\ln 4}$ $= \frac{2x}{(\ln 4)(x^2+3)}$

$= \frac{x}{(\ln 2)(x^2+3)}$

Marker's Comments:

Overall, well done.

Question 3 (2 marks)

Find the following

(a) $\int \sin\left(\frac{x}{3}\right) dx$

Criteria	Marks
Provides correct solution	1

$= -3\cos\left(\frac{x}{3}\right) + C$

Marker's Comments: Many students did not include the constant of integration C. This time, marks were not deducted for omitting see, however, You may lose marks in the HSC if you forget to write C.

(b) $\int x^4\left(x^5-2\right)^3 dx$

Criteria	Marks
Provides correct solution	1

$= \frac{1}{20}\int 4 \times 5x^4 (x^5-2)^3 dx$

$= \frac{1}{20}(x^5-2)^4 + C$

Marker's Comments:

Many students could not do this integration. Please practise the 'reverse chain rule'.

Question 4 (2 marks)

Evaluate exactly $\int_0^1 \frac{x-1}{x^2-2x+4}\,dx$, simplify your answer fully.

Criteria	Marks
Provides correct solution(answer simplified)	2
Obtains correct primitive	1

$= \int_0^1 \frac{1}{2}\left(\frac{2x-2}{x^2-2x+4}\right)dx$

$= \left[\frac{1}{2}\ln\left|x^2-2x+4\right|\right]_0^1$

$= \frac{1}{2}\ln 3 - \frac{1}{2}\ln 4$

$= \frac{1}{2}(\ln 3 - \ln 4) = \frac{1}{2}\ln\left(\frac{3}{4}\right) = \ln\left(\frac{\sqrt{3}}{2}\right)$

Marker's Comments:

Overall, well done.

Some students lost 1 mark for not attempting to simplify the answer.

Section 2 – Part B

Question 5 (2 marks)

For what values of k does the quadratic equation $5x^2 - 2x + (8k - 15) = 0$ have real roots?

Criteria	Marks
Provides correct solution	2
Finds the correct discriminant	1

$\Delta \geq 0$

$(-2)^2 - 4(5)(8k-15) \geq 0$

$4 - 160k + 300 \geq 0$

$-160k \geq -304$

$k \leq \frac{-304}{-160}$

$k \leq \frac{19}{10}$

Marker's Comments:

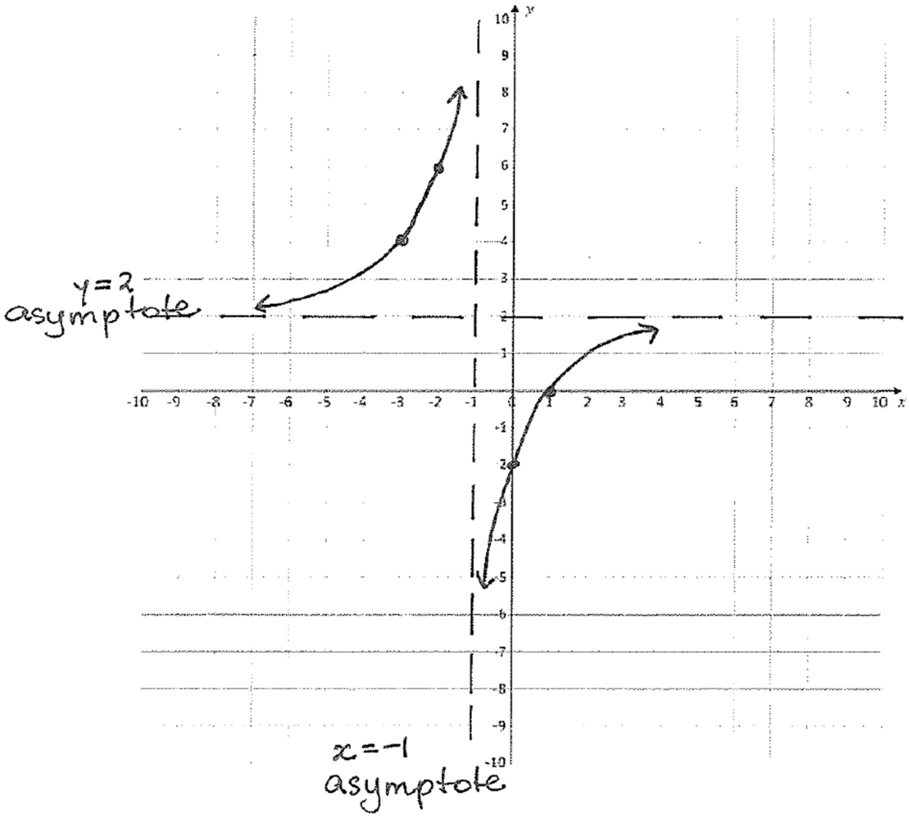
Many students failed to realise that when $\Delta = 0$, there are two real and equal roots.

One mark was taken off if just $\Delta > 0$ instead of $\Delta \geq 0$.

Question 6 (3 marks)

Sketch the graph of $y = 2 - \frac{4}{x+1}$ on the axes below, showing all intercepts with the coordinate axes and all asymptotes.

Criteria	Marks
Provides correct graph (Each branch of graph should be shown to pass through at least two correct points)	3
Gives only two correct features out of shape, asymptotes, intercepts	2
Gives only one correct feature out of shape, asymptotes, intercepts	1



Marker's Comments:

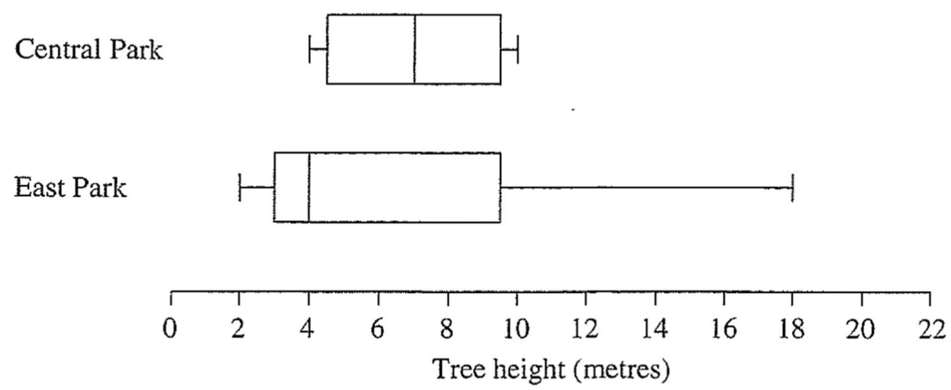
Mostly well done.

Make sure to check the shape of the LHS branch by substituting values in.

Question 7 (3 marks)

Annie and her biology class went to two large city parks and measured the heights of the trees in metres.

In Central Park there were 25 trees. In East Park there were 27 trees. The data sets were displayed in two box-and-whisker plots.



Compare and contrast the two data sets by examining the shape and skewness of the distributions, and the measures of location and spread.

Criteria	Marks
Compares and contrasts correctly all features asked for (shape and skewness, measures of location and spread) of the data sets	3
Compares and contrasts correctly only two features of the data sets	2
Compares and contrasts correctly only one feature of the data sets	1

1. The Central Park tree distribution is symmetrical whereas the East Park distribution is positively skewed.
2. The median height of the trees in Central Park is greater than that of East Park (7 compared to 4). In fact, we can see that the median height of the trees in East Park is the same as the shortest tree in Central Park meaning 50% of the trees in East Park are shorter than the shortest tree in Central Park.
3. East Park has a bigger range (16 compared to 6). However, their interquartile ranges are more similar (≑ 6.5 compared to 5). East Park's tree heights are slightly less consistent.

Marker's Comments:

Not answered very well. Many students just listed characteristics of each data set without making any inferences.

Students who received 3 marks were able to correctly identify the features (skew, location and spread) and also interpret what these meant in terms of the tree heights at the two parks.

Question 8 (3 marks)

Find the equation of the normal to the curve $y = \ln\left(\dfrac{2x-1}{x+1}\right)$, at the point where $x = 2$.

Criteria	Marks
Provides correct equation of the normal	3
Obtains the correct gradient of the normal	2
Obtains the correct derivative of the given function	1

$y = \ln(2x-1) - \ln(x+1)$

$\dfrac{dy}{dx} = \dfrac{2}{2x-1} - \dfrac{1}{x+1}$

at $x=2$, $\dfrac{dy}{dx} = \dfrac{2}{3} - \dfrac{1}{3}$

$= \frac{1}{3}$ (m of tangent)

$\therefore$ m of normal $= -3$ (perp. lines $m_1 m_2 = -1$)

If $x = 2$, $y = \ln\left(\frac{3}{3}\right) = \ln 1 = 0$

Using $y - y_1 = m(x - x_1)$

$y - 0 = -3(x-2)$

$y = -3x + 6$ is the equation of the required normal.

Marker's Comments:

Many students forgot that they could use their logarithm rules to make their differentiating a lot simpler instead of using quotient rule and then dividing by $\frac{2x-1}{x+1}$.

Some students also did not read the question properly and gave the equation of the **tangent** instead.

Question 9 (2 marks)

A circle is given by the equation $x^2 + y^2 + 4x - 10y = -16$. Find the centre and radius of this circle.

Criteria	Marks
Provides the correct equation, centre and radius	2
Provides one correct feature (centre or radius)	1

$x^2 + 4x + 4 + y^2 - 10y + 25 = -16 + 4 + 25$

$(x+2)^2 + (y-5)^2 = 13$

$\therefore$ centre $= (-2, 5)$ radius $= \sqrt{13}$

Marker's Comments:

Completing the square not well done.

Section 2 – Part C

Question 10 (15 marks)

Let $f(x)=(x+2)(x-2)^3$.

(a) Find the x and y - intercepts of $y=f(x)$.

Criteria	Marks
Provides the correct x and y intercepts	2
Obtains either the correct x or y intercepts	1

x-intercepts $\therefore$ sub $y=0$

$(x+2)(x-2)^3=0$

when $x=-2, 2$

y-intercepts $\therefore$ sub $x=0$

$y=(2)(-2)^3=-16$

Marker's Comments:

Generally answered well.

(b) Show that $f'(x)=4(x-2)^2(x+1)$ and $f''(x)=12x(x-2)$.

Criteria	Marks
Correctly proves both formulae	2
Correctly proves one formula	1

$f'(x) = (x-2)^3(1) + (x+2).3(x-2)^2.1$

$= (x-2)^3 + 3(x+2)(x-2)^2$

$= (x-2)^2[(x-2) + 3(x+2)]$

$= (x-2)^2[x-2+3x+6]$

$= (x-2)^2(4x+4)$

$= 4(x-2)^2(x+1)$

$f''(x) = (x+1).4.2(x-2)^1.1 + 4(x-2)^2.1$

$= 8(x+1)(x-2) + 4(x-2)^2$

$= 4(x-2)[2(x+1) + (x-2)]$

$= 4(x-2)[2x+2+x-2]$

$= 4(x-2)(3x)$

$= 12x(x-2)$

Marker's Comments:

Generally answered well but some students expanded instead of taking out a common factor. This complicated the process and often led to careless errors.

(c) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature.

Justify your answers fully.

Criteria	Marks
Obtains the correct stationary points and justifies the nature of both stationary points correctly	3
Obtains the correct stationary points and justifies the nature of one stationary point correctly	2
Obtains the correct stationary points	1

Stationary points occur when $f'(x)=0$

$4(x-2)^2(x+1)=0$

$\therefore$ at $x=-1, 2$

If $x=-1$, $f(-1)=-27$

If $x=2$, $f(2)=0$

$\therefore$ stationary points are $(-1,-27)$ and $(2,0)$

Nature:

$f''(-1)=12(-1)(-3)=36>0$ concave up $\therefore$ $(-1,-27)$ is a minimum turning point

$f''(2)=12(2)(0)=0$ $\therefore$ at $x=2$ possible horizontal inflexion.

x	1.5	2	2.5
$f''(x)$	-3	0	15
	$\cap$	$\cdot$	$\cup$

concavity changes $\therefore$ $(2,0)$ is a horizontal inflexion.

Marker's Comments:
Many students did not qualify that there was a possible POI at $\frac{dy}{dx}=0$ and simply concluded that it was. Tests were not carried out to determine if stationary points were max/ min turning points or a horizontal POI.
Mark was awarded if testing was conducted in part (c).

(d) Find the coordinates of all points of inflection of $y=f(x)$.

Criteria	Marks
Justifies a second point of inflexion exists by testing for change in concavity	2
Obtains second possible point of inflection	1

$f''(x)=12x(x-2)=0$

when $x=0$, $x=2$

We have already established $(2,0)$ is a horizontal inflexion.

x	-0.5	0	0.5
$f''(x)$	15	0	-9
	$\cup$	$\cdot$	$\cap$

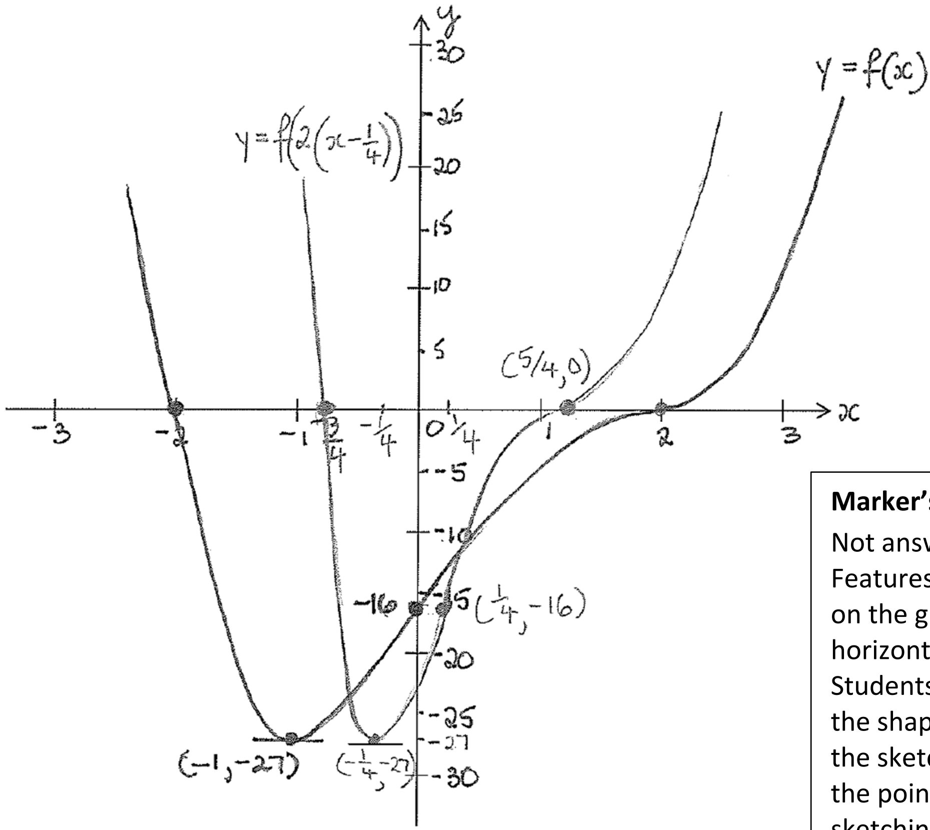
$\therefore$ $(0,-16)$ is a point of inflexion.

concavity changes

Marker's Comments:
Not answered very well. Many students did not test f'' values to determine if the around the point was a POI

(e) Sketch the graph of $y=f(x)$ on the axes below, showing all the features found above.
(*Hint: use different scales on the axis)*

Criteria	Marks
Correct shape of the curve and correctly shows all 4 important points found in parts (c) and (d)	2
Correct shape of the curve and correctly shows at least 2 of the 4 important points found in parts (c) and (d)	1



Marker's Comments:
Not answered very well.
Features were not correctly shown on the graph, particularly the horizontal point of inflexion.
Students did not correctly recall the shape, were very careless with the sketches, and some did not use the points from part (d) when sketching

(f) State in the correct order, the transformations required to obtain the graph of $y=f\left(2\left(x-\frac{1}{4}\right)\right)$.

Criteria	Marks
Correctly identifies both transformations AND gives correct order of them	2
States at least one transformation correctly	1

Marker's Comments:
There was a lot of confusion in the order of the transformation. Many misunderstood the horizontal translation as a vertical one

Horizontal dilation by factor of $\frac{1}{2}$.
THEN : Horizontal translation to the right $1/4$ of a unit.

(g) On the set of axes provided in part (e), sketch the graph of $y=f\left(2\left(x-\frac{1}{4}\right)\right)$, showing coordinates of x- intercepts, stationary points and inflection points.

Criteria	Marks
Sketches correct graph and correctly shows all of the 4 important points found after applying transformations	2
Sketches correct shape and shows at least 2 of the 4 important points found after applying transformations	1

Marker's Comments:
Not answered very well. Many students were unable to make the connection between their answer in part (f) when sketching.

Section 2 – Part D

Question 11 (5 marks)

Tom planted a silky oak tree three years ago when it was 80 cm tall. At the end of the first year after planting, it was 130 cm tall, that is it grew 50 cm. Each year's growth was 90% of the previous year's.

(a) What was the growth of the silky oak after 3 years?

Criteria	Marks
Provides correct solution	1

$80 \quad 50 \quad 50(0.9) \quad 50(0.9)^2 \quad 50(0.9)^3 \ldots$

year 1 year 2 year 3

Growth after 3 years $= 50 + 50(0.9) + 50(0.9)^2 = 135.5$ cm

Marker's Comments: Many students did not read the question carefully. It asked for 'growth' and not height.

(b) Assuming that it maintains the present growth pattern, explain why the tree will never reach a height of 6 metres.

Criteria	Marks
Provides correct solution (i.e. calculates S_∞ correctly, realises the limiting height i.e. adds 80 cm and concludes correctly)	2
Identifies S_∞ needs to be calculated and calculates it correctly	1

$50, 50(0.9), 50(0.9)^2, \ldots \Rightarrow$ G.P. $r = 0.9$, infinite series

S_∞ exists as $|r| < 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{50}{1-0.9}$

$= 500$ cm

$\therefore$ limiting growth is 500 cm & limiting height is 580 cm < 600 cm

$\therefore$ Tree will never reach a height of 6 m

Marker's Comments:

Generally, this question was not answered well.

1 mark was deducted if the limiting height was not considered.

(c) In which year will the silky oak reach a height of 5 metres?

Criteria	Marks
Provides correct solution	2
Substitutes correctly into S_n for a GP	1

To reach a height of 5 m, tree needs to grow 420 cm.

$S_n = \frac{a(1-r^n)}{1-r}$

$420 = \frac{50(1-0.9^n)}{1-0.9}$

$42 = 50(1-0.9^n)$

$\frac{42}{50} = 1-0.9^n$

$0.9^n = 1-\frac{42}{50}$

$0.9^n = \frac{8}{50}$

$\log_{0.9}\left(\frac{4}{25}\right) = n$

$n = \frac{\ln(0.16)}{\ln(0.9)}$

$n \doteqdot 17.39344$

$\therefore$ Tree will reach 5 metres during 18^{th} year

Marker's Comments:

The main error was not realising the tree needed to grow 420 cm and not 500 cm.

Question 12 (4 marks)

A leak from a tanker has accidentally contaminated a farmer's paddock with a toxic chemical. The chemical concentration in the soil was 6 kL/ha immediately after the accident. One year later the concentration in the soil was measured to be 2.4 kL/ha.
It is known that the concentration, C , is given by:

$$C = C_0\, e^{-kt} \ ,$$

where C_0 and k are constants and t is measured in years.

(a) Show that $C_0 = 6$ and $k = -\ln\left(\frac{2}{5}\right)$.

Criteria	Marks
Shows the value for both C_0 and k correctly	2
Shows the value for C_0 or k correctly	1

At $t=0$, $C = 6$ kL

$C = C_0 e^{-kt}$

$6 = C_0 e^0$

$6 = C_0(1)$

$\therefore C_0 = 6$

At $t=1$, $C = 2.4$ kL

$2.4 = 6e^{-k}$

$0.4 = e^{-k}$

$\ln(0.4) = -k$

$k = -\ln(0.4)$

$k = -\ln\left(\frac{2}{5}\right)$

Marker's Comments:

This is a 'show' question!

All necessary steps needed to be written.

(b) It will not be safe for the farmer to plant a new crop until the concentration falls below 0.2 kL/ha. How long, to the nearest month, after the spill does the farmer need to wait for the paddock to be safe to use.

Criteria	Marks
Finds t and states how long the farmer should leave the field	2
Showing some progress to be able to finding t	1

$C = 6e^{-(-\ln(\frac{2}{5}))t}$

$\therefore C = 6e^{\ln(\frac{2}{5})t}$

$0.2 = 6e^{\ln(\frac{2}{5})t}$

$\frac{1}{30} = e^{\ln(2/5)t}$

$\ln\left(\frac{1}{30}\right) = \ln\left(\frac{2}{5}\right)t$

$\therefore t = \dfrac{\ln(\frac{1}{30})}{\ln(2/5)}$

$t \doteqdot 3.711919441$

$\therefore$ Paddock will be safe after 3 years and 9 months

Marker's Comments:

Overall, well done.

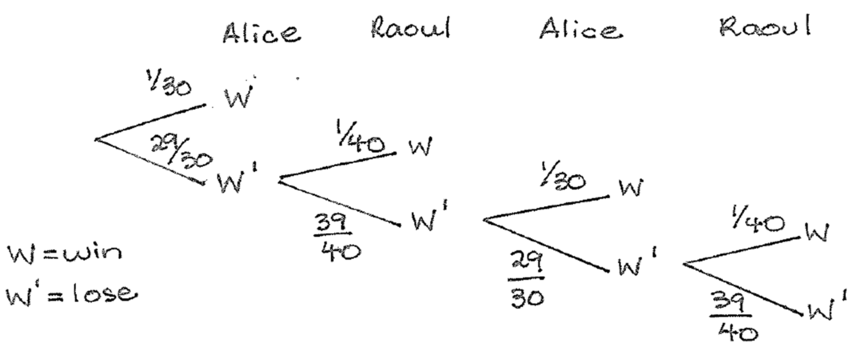
Some students attempted to solve an inequality and made errors with the inequality symbol. Hence, marks were deducted.

Question 13 (5 marks)

Alice and Raoul take turns throwing darts at a dartboard. The winner is whoever hits the bullseye first. Alice has a $\frac{1}{30}$ chance of hitting the bullseye, while Raoul has a $\frac{1}{40}$ chance of hitting the bullseye. Alice throws the first dart.

(a) Draw a tree diagram for the first four throws of the games (two throws forAlice and two for Raoul).

Criteria	Marks
Provides correct tree diagram	1



Marker's Comments:

Poorly done.

Many students failed to realise that the game stopped after hitting the bullseye.

(b) What is the probability that Alice wins on her first or second throw?

Criteria	Marks
Provides correct solution	1

$$P\begin{pmatrix}\text{Alice wins on} \\ \text{1st or 2nd} \\ \text{throw}\end{pmatrix} = \frac{1}{30} + \left(\frac{29}{30}\right)\left(\frac{39}{40}\right)\left(\frac{1}{30}\right)$$

$$= \frac{259}{4000}$$

Marker's Comments:

Not well done. Many students forgot to consider BOTH Alice and Raoul.

(c) What is the probability that Alice will eventually win the game.

Criteria	Marks
Provides correct solution	3
Identifies S_∞ is required and finds correct ratio	2
Obtains the correct geometric series	1

P(Alice wins game)

$$= \frac{1}{30} + \left(\frac{29}{30}\right)\left(\frac{39}{40}\right)\left(\frac{1}{30}\right) + \left(\frac{29}{30}\right)\left(\frac{39}{40}\right)\left(\frac{29}{30}\right)\left(\frac{39}{40}\right)\left(\frac{1}{30}\right) + \cdots$$

geometric series $a = \frac{1}{30}$, $r = \left(\frac{29}{30}\right)\left(\frac{39}{40}\right)$

S_∞ exists as $|r|<1$

$$= \frac{a}{1-r}$$

$$= \frac{\frac{1}{30}}{1-\left(\frac{29}{30}\right)\left(\frac{39}{40}\right)}$$

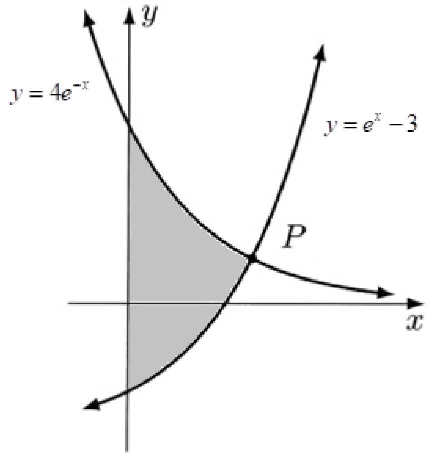
$$= \frac{40}{69}$$

Marker's Comments:

Not well done. Many students could not answer this question.

Section 2 – Part E

Question 14 (5 marks)



The diagram shows the graphs of $y = 4e^{-x}$ and $y = e^x - 3$.

(a) Show that the curves intersect when $e^{2x} - 3e^x - 4 = 0$.

Criteria	Marks
Provides correct solution	1

$4e^{-x} = e^x - 3$

$\frac{4}{e^x} = e^x - 3$

$4 = e^x(e^x - 3)$

$4 = e^{2x} - 3e^x \quad \therefore \quad e^{2x} - 3e^x - 4 = 0$

Marker's Comments:

Well done

(b) Hence, show the x coordinate of the point P is $x = \ln 4$.

Criteria	Marks
Provides correct solution	2
Reduces equation from(a) into correct quadratic form and solves for u	1

let $u = e^x$, $u^2 = e^{2x}$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4$, $u = -1$

$e^x = 4$, $e^x = -1$

$x = \ln 4$, $e^x \neq -1$ as $e^x > 0$

$\therefore$ x coordinate of P is $x = \ln 4$.

Marker's Comments:

Mostly well done.

Some students made errors when factorising the quadratic in u. Others failed to explain why $e^x \neq -1$

(c) Find the exact shaded area between the two curves .

Criteria	Marks
Provides correct solution	2
Obtains correct primitive	1

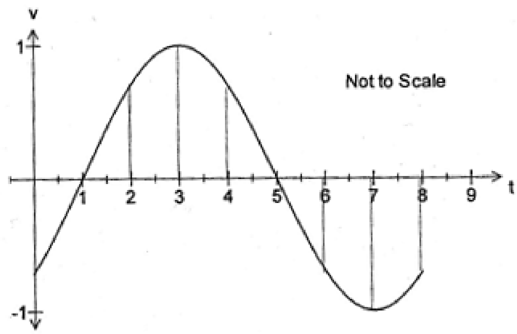
$$A = \int_0^{\ln 4} (4e^{-x} - e^{x} + 3)\,dx$$
$$= \left[-4e^{-x} - e^{x} + 3x\right]_0^{\ln 4}$$
$$= -4e^{-\ln 4} - e^{\ln 4} + 3\ln 4 - (-4e^0 - e^0 + 3(0))$$
$$= -4e^{\ln(1/4)} - e^{\ln 4} + 3\ln 4 + 4e^0 + e^0$$
$$= -4\left(\tfrac{1}{4}\right) - 4 + 3\ln 4 + 4 + 1$$
$$= 3\ln 4 \text{ units}^2$$

Marker's Comments:

Well done. Some students persist in writing decimal approximations to definite integrals.

Question 15 (3 marks)

The diagram shows the velocity-time graph for a particle moving in a straight line.



State the times between t=0 and t=8 at which:

(a) The acceleration is zero

Criteria	Marks
Provides both correct answers	1

$t=3,\ t=7$

Marker's Comments:

Well done

(b) The speed is increasing

Criteria	Marks
Obtains both correct answers	2
Obtains one correct answer	1

$\left.\begin{array}{l} a>0 \\ \text{and} \\ v>0 \end{array}\right\} \therefore 1<t<3 \qquad \left.\begin{array}{l} a<0 \\ \text{and} \\ v<0 \end{array}\right\} \therefore 5<t<7$

Marker's Comments:

Poorly done. Not many students recognised the relationships between a and v that determine the answer

Question 16 (7 marks)

A particle moves in a straight line so that after t seconds ($(t \geq 0)$ its velocity v is given by $v = \left(\frac{2}{1+t} - t\right) m/s$. The displacement of the particle from the origin is given by x metres.

(a) Find the acceleration of the particle when $t = 0$.

Criteria	Marks
Provides correct solution	1

$v = 2(1+t)^{-1} - t$

$a = -2(1+t)^{-2} - 1$

$a = \frac{-2}{(1+t)^2} - 1$

at $t=0$, $a = -2-1$

$= -3 \text{ ms}^{-2}$

Marker's Comments:

Well done

(b) If the particle is initially at the origin, find the displacement as a function of t .

Criteria	Marks
Finds constant of integration and provides correct function for displacement	2
Obtains the correct primitive function with constant of integration	1

$x = \int v \, dt$

$x = 2\ln|1+t| - \frac{t^2}{2} + c$

at $t=0$, $x=0$

$\therefore 0 = 2\ln 1 - 0 + c$

$0 = c$

$\therefore x = 2\ln|1+t| - \frac{t^2}{2}$

Marker's Comments:

Many students failed to show c=0

(c) When is the particle stationary?

Criteria	Marks
Provides correct solution	2
Obtains correct equation to solve in terms of t	1

stationary when $v = 0$

i.e. $\frac{2}{1+t} - t = 0$

$\frac{2 - t(1+t)}{1+t} = 0$

$2 - t - t^2 = 0$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$\therefore t = 1 \quad (t \geq 0)$

Marker's Comments:

Mostly well done. Some students factorised incorrectly. Others forgot to exclude the negative value of time.

(d) How far does the particle travel in the first 2 seconds? Give your answer correct to three significant figures.

Criteria	**Marks**
Provides correct solution	2
Considers and obtains correct displacements of particle at $t = 0,\ t = 1,\ t = 2s$	1

At $t = 0$, $x = 0$

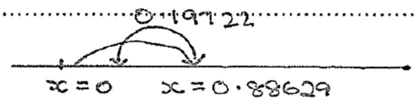
At $t = 1$, $x = 2\ln 2 - \frac{1}{2} \doteqdot 0.88629$

At $t = 2$, $x = 2\ln 3 - 2 \doteqdot 0.19722$

$\therefore$ distance travelled in first 2 seconds

$\doteqdot 0.88629 + (0.88629 - 0.19722) \doteqdot 1.57536$

$\doteqdot 1.58$ metres



Marker's Comments:

Poorly done. Only a few students obtained the correct solution.

Many students found the displacement at t=0 , t=1 or t=2 but did not know how to combine them to get correct answer.

Section 2 – Part F

Question 17 (7 marks)
A tank initially holds 3600 litres of water. The water drains from the bottom of the tank. The tank takes 60 minutes to empty. A mathematical model predicts that the volume V litres of water that will remain in the tank after t minutes is given by

$$V = 3600\left(1-\frac{t}{60}\right)^2, \text{ where } 0 \le t \le 60 .$$

(a) What volume does the model predict will remain after 10 minutes?

Criteria	Marks
Provides correct solution	1

When $t=10$,

$V = 3600\left(1-\frac{10}{60}\right)^2$

$V = 2500$ litres

Marker's Comments:

Generally answered well

(b) How long will it take for the tank to become half full (to nearest minute)?

Criteria	Marks
Provides correct solution, explaining why only one solution for t	2
Showing some progress to be able to finding t	1

$3600\left(1-\frac{t}{60}\right)^2 = 1800$

$\left(1-\frac{t}{60}\right)^2 = \frac{1}{2}$

$1-\frac{t}{60} = \pm\frac{1}{\sqrt{2}}$

$1 \mp \frac{1}{\sqrt{2}} = \frac{t}{60}$

$t = 60\left(1 \mp \frac{1}{\sqrt{2}}\right)$

$t = 60\left(1-\frac{1}{\sqrt{2}}\right)$ or $t = 60\left(1+\frac{1}{\sqrt{2}}\right)$

$t \doteqdot 17.5736$ $\quad$ $t \doteqdot 102.4264$

$\therefore t \doteqdot 17$ minutes as $0 \le t \le 60$

Marker's Comments:

A significant number of answers neglected the fact that there are 2 cases. A mark was deducted in these instances.

To get 2 marks you need to show that $t \neq 102$ since $0 \le t \le 60$.

(c) At what rate does the model predict that the water will drain from the tank after twenty minutes?

Criteria	Marks
Provides correct solution	2
Provides correct differentiation	1

$\frac{dV}{dt} = 7200\left(1-\frac{t}{60}\right)\left(-\frac{1}{60}\right)$

$= -120\left(1-\frac{t}{60}\right)$

$= -120 + 2t$. At $t=20$, $\frac{dV}{dt} = -80$

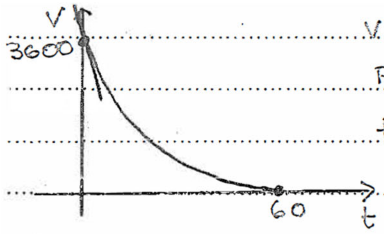
$\therefore$ After 20 minutes, water is draining at 80 L/min

Marker's Comments:

Generally answered well.

(d) At what time does the model predict that the water will drain from the tank at its fastest rate?

Criteria	Marks
Correct answer for t with correct reasoning	2
Correct answer for t	1



$V = 3600\left(1-\frac{t}{60}\right)^2$

From the graph, slope of the tangent to the curve $\left(\left|\frac{dV}{dt}\right|\right)$ is greatest when $t=0$

$\therefore$ maximum draining rate when $t=0$ (i.e. 120 L/min)

Marker's Comments:

A significant number of responses incorrectly let $\frac{dv}{dt} = 0$, however this will tell you when the flow rate has stopped/is a minimum.

Question 18 (7 marks)

The average monthly temperature, $T°$, for a town in NSW can be modeled on the formula

$T = 7\sin(nx + 1.5) + 13$,

where $n =$ a constant value and

$x =$ the number of the month of the year (i.e. January=1, February=2, …..).

(a) According to the model, what are the maximum and minimum average monthly temperatures in this town?

Criteria	Marks
Gives the correct maximum and correct minimum	2
Gives either the correct maximum or correct minimum	1

maximum T : $T = 7(1) + 13 = 20°$

minimum T : $T = 7(-1) + 13 = 6°$

Marker's Comments:

Generally answered well.

(b) The period of the function is 12. Determine the value of n correct to 2 decimal places.

Criteria	Marks
Gives the correct answer. If given, accept the exact answer	1

$12 = \frac{2\pi}{n}$

$12n = 2\pi$

$n = \frac{2\pi}{12} = \frac{\pi}{6} = 0.52$ (2 dec pls)

Marker's Comments:

Generally answered well

(c) Which month has the lowest average monthly temperature?

Criteria	Marks
Finds correct x and states the correct month	2
Showing some progress to be able to finding x	1

$nx + 1.5 = \frac{3\pi}{2}$, $n = \frac{\pi}{6}$

$x = \boxed{\dfrac{\frac{3\pi}{2} - 1.5}{n}}$ $\therefore$ June is the 6th month.

$x \doteqdot 6.1$ So it is the month with the coldest average temperature

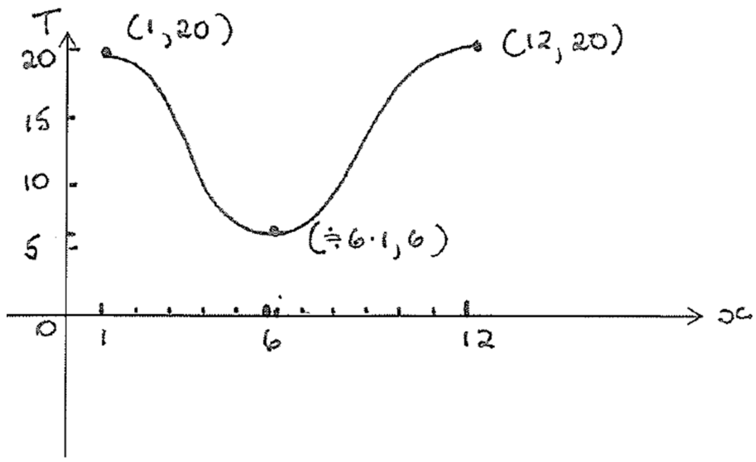
Marker's Comments:

To get 1 mark, the response must recognise that $nx + 1.5 = \frac{3\pi}{2}$, since, for a sine curve this will give the value of x that results in a minimum.

To get 2 marks the correct answer must be obtained.

(d) Graph the function $T = 7\sin\left(nx + 1.5\right) + 13$ for $1 \le x \le 12$.

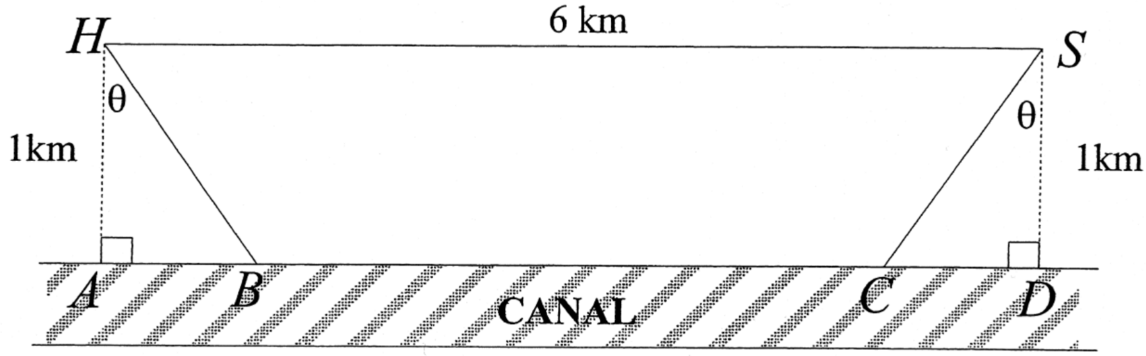
Criteria	Marks
Provides correct graph	2
Gives the correct range or domain	1



Marker's Comments:

1 Mark was awarded if the student recognised the correct range, domain or period/shape.

Question 19 (6 marks)



The diagram above shows that the distance between a boy's home H and his school S is 6 km. A canal $ABCD$ is 1 km from both his home and school. In winter the canal is frozen, he takes an alternate route $HBCS$, walking HB, skating BC and walking CS. His walking speed is 4 km/h and his skating speed is 12 km/h. Let $\angle AHB = \angle DSC = \theta$.

(a) Show that the time taken for this alternate route is $T = \dfrac{1}{2\cos\theta} + \dfrac{1}{2} - \dfrac{\tan\theta}{6}$.

Criteria	Marks
Obtains, with working, correct distances travelled and times taken	2
Obtains, with working, the correct distances travelled	1

In ΔHAB : $\cos\theta = \frac{1}{HB}$ and $\tan\theta = \frac{AB}{1}$

$HB = \frac{1}{\cos\theta}$ $AB = \tan\theta$

Similarly, in ΔSDC : $SD = \frac{1}{\cos\theta}$ and $DC = \tan\theta$

$\therefore$ Distance walked $= 2HB$

$= \frac{2}{\cos\theta}$

$\therefore$ Time walked $= \left(\frac{2}{\cos\theta}\right) \div 4 = \frac{1}{2\cos\theta}$ hours

Distance skated $= 6 - 2\tan\theta$

$\therefore$ Time skated $= \frac{6 - 2\tan\theta}{12}$ hours

So, $T = \frac{1}{2\cos\theta} + \frac{6 - 2\tan\theta}{12}$

$T = \frac{1}{2\cos\theta} + \frac{1}{2} - \frac{\tan\theta}{6}$ as required.

Marker's Comments:

1 Mark was awarded if the student obtained correct expressions for the distances walked and skated.

1 Marks were awarded if they correctly converted the expressions to account for time, rather than distance.

(b) Find, to the nearest minute, the value of θ which minimizes the time taken for the journey to school.

Criteria	Marks
Provides correct solution (must include testing θ gives minimum T	4
Solves correctly for θ	3
Gives correct equation to solve for θ	2
Shows correct use of chain rule and correct derivative of $\tan\theta$	1

$$T = \frac{(\cos\theta)^{-1}}{2} + \frac{1}{2} - \frac{\tan\theta}{6}$$

$$\frac{dT}{d\theta} = -\frac{(-\sin\theta)(\cos\theta)^{-2}}{2} - \frac{\sec^2\theta}{6}$$

$$= \frac{\sin\theta}{2\cos^2\theta} - \frac{1}{6\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta}\left(3\sin\theta - 1\right)$$

For minimum T, $\frac{dT}{d\theta} = 0$

when $3\sin\theta - 1 = 0$, $\frac{1}{\cos^2\theta} = \sec^2\theta \neq 0$

$\sin\theta = \frac{1}{3}$

$\therefore \theta \doteqdot 19°28'$

AND

θ	$10°$	$19°28'$	$30°$
$\frac{dT}{d\theta}$	-0.94	0	$1/9$
slope	\	—	/

$\therefore$ minimum turning point at $\theta = 19°28'$

Marker's Comments:

Most common errors were incorrect differentiation or neglecting to conduct a final check for the minimum using an appropriate method (second derivative or table of slopes).