GIRRAWEEN HIGH SCHOOL

1999 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

MATHEMATICS

2 UNIT

Time Allowed – Three Hours
(3 minutes reading time)

DIRECTIONS TO CANDIDATES
- *All* questions may be attempted.
- *All* questions are of equal value.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Standard integrals are provided. Approved calculators may be used.
- Each question should be started on a new page.

Question 1 MAI

a) Solve: $3 - x < 4$

b) A car costing \$25 000 depreciates by 15% each year. Find its value after 2 years.

c) Find, correct to 3 decimal places, the value of:

$$\frac{\sqrt{2000}}{4.73+8.29}.$$

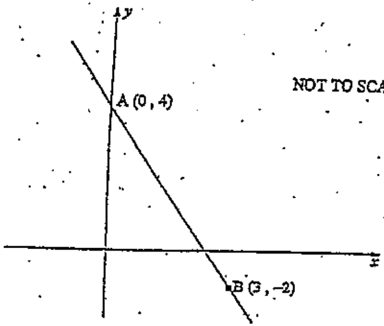
d) Solve: $3(x-2)=\frac{x+1}{2}$. 2

e) Factorise fully: $2x^3-16$. 2

f) If $\sqrt{24}+\sqrt{54}=a\sqrt{b}$, find a and b. 2

Question 2

Marks



A, B and C are the points (0, 4), (3, −2) and (−3, 0) respectively.

(a) Copy the diagram onto your paper and mark on the given information. 1

(b) Show that the equation of AB is $2x+y-4=0$. 2

(c) Find M, the midpoint of the interval joining B and C. 2

(d) Find the perpendicular distance of M to the line AB. 2

(e) Find the area of ΔAMB. 2

(f) Illustrate on your diagram the region satisfied simultaneously by 3

$y \le 4,\ x \ge 0,\ 2x+y-4 \le 0.$

Question 3

MARKS

a) Solve for x: $9^{x}-12(3^{x})+27=0$ using the substitution $u=3^{x}$. 2

b) Differentiate:

(i) $\ln(x^2+7)$ 1

(ii) x^2e^{3x} 2

(iii) $\dfrac{\sin x}{x}$ 2

c) (i) Find $\int (2+\sqrt{x})\,dx$ 2

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 3x\,dx$ 3

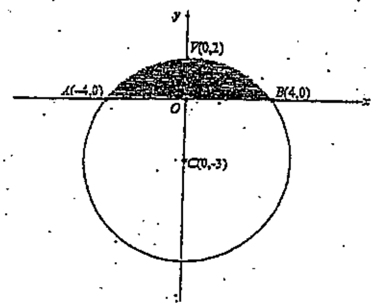
Question 4

a) For the Arithmetic Sequence 4, 7, 10, ..., find the fortieth (40^{th}) term and the sum of the first 40 terms. 3

b) Given that the fourth term of a Geometric Progression is 288 and the seventh term is $85\frac{1}{3}$, find the common ratio and the first term. 3

c) The diagram shows the graph of the circle $x^2+(y+3)^2=25$.

The shaded area shows the region whose area is represented by the integral $\int_{-4}^{4}(-3+\sqrt{25-x^2})dx$.



Copy the table of values of $y=-3+\sqrt{25-x^2}$ and use all five values of the

x	-4	-2	0	2	4
$f(x)$	0·000	1·583	2·000	1·583	0·000

function to find the approximate value of:

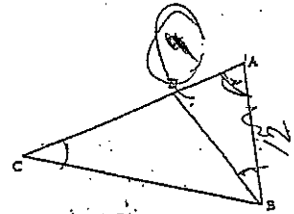
$$\int_{-4}^{4}(-3+\sqrt{25-x^2})dx$$

(i) by trapezoidal rule (to 3 decimal places); 3

(ii) by Simpson's rule (to 3 decimal places). 3

Question 5

a)

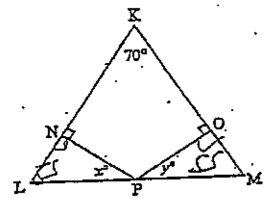


In the diagram above, $\angle ACB = \angle ABD$.

(i) Prove that triangle ABC is similar to triangle ADB. Give reasons for your answer.

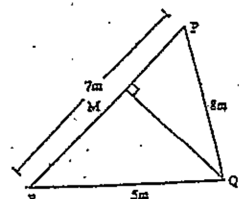
(ii) If $AD = 9$ and $AB = 12$, find AC.

b)



KLM is an isosceles triangle with KL = KM.
Perpendiculars NP and OP are drawn from point P, the midpoint of LM.
Given $\angle LKM = 70°$, prove that $x° = y°$.

c)



The triangle PQR has sides with lengths $PQ = 8m$, $PR = 7m$ and $QR = 5m$. M is the foot of the perpendicular drawn from Q to PR.

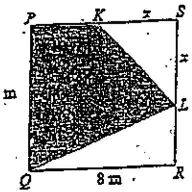
(i) Show that the angle PQR is equal to 60°.

(ii) Find the area of the triangle PQR as an exact value.

(iii) Hence, or otherwise, find the length of QM correct to 2 decimal places.

Question 7 MARKS

a) *PQRS* represents a square flower bed with sides 8 m long. The quadrilateral *PKLQ* represents the area covered with grass. Points *K* and *L* are on *PS* and *SR* respectively so that $KS = SL = x$.



NOT TO SCALE

(i) Find in terms of x the areas of triangles *KSL* and *QRL*. 2

(ii) Show that the area of the quadrilateral *PKLQ* is $A = 32 + 4x - \frac{x^2}{2}$ square metres. 2

(iii) What is the maximum possible area of the quadrilateral *PKLQ*? Justify your answer. 3

b) If $\log_a 5 = 1.03$ and $\log_a 2 = 0.64$, find the value of :

(i) $\log_a 10$ 1

(ii) $\log_a 2\sqrt{2}$ 1

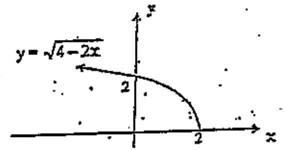
c)

(i) Given that $f(x) = \sin^2 x$, find $f'(x)$ and $f''(x)$ 2

(ii) Show that $\frac{f''(x) + 2f(x)}{f'(x)} = \cot x$. 1

Question 6 MARKS

a)

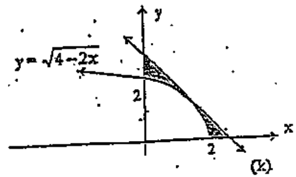


The diagram shows the graph of the curve $y = \sqrt{4-2x}$.
Copy this diagram.

(i) Find the equation of the tangent (k) at the point on the curve where $x = 1\frac{1}{2}$. 3

(ii) Show that the area bounded by the curve, x axis and y axis is $2\frac{2}{3}$ square units. 2

(iii) Using the result of part (ii), or otherwise, find the area of the shaded region between $y = \sqrt{4-2x}$ the tangent (k) and the x and y axes as shown in the diagram below. 3



b)

(i) A gambler has two custom made dice.

One die has only odd numbers (1, 1, 3, 3, 5 and 5) on its six faces while the other die has only even numbers (2, 2, 4, 4, 6 and 6) on its six faces. By drawing a tree diagram, or otherwise, determine the probability of getting a total of seven when the two dice are rolled together. 2

(ii) In a certain town, the probability of an adult catching a cold during 1994 is 0.4. Two adults in the town are chosen at random. 2

What is the probability that at least one of them will catch a cold in 1994?

Question 8

MARKS

a) A parabola P has equation $x^2 = 16(y-1)$.

Draw a neat sketch of P and clearly indicate on it:

(i) The coordinates of the vertex. 1

(ii) The coordinates of the focus. 1

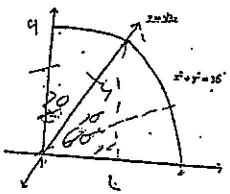
(iii) The equation of the directrix. 1

b) The acceleration of a particle at time t seconds is given by $a = 6 + e^{-t}$. Initially the particle is at the origin and is moving with a velocity of -1 m/s.

(i) Show that its displacement x from the origin is given by $x = 3t^2 + e^{-t} - 1$. 3

(ii) Find the velocity and displacement when $t = 3$. Give answers correct to two decimal places. 2

c)



(i) Show that the curve $x^2 + y^2 = 16$ and the line $y = \sqrt{3}x$ intersect at $x = 2$. 2

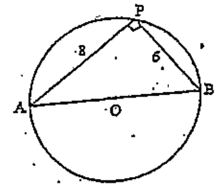
(ii) The region bounded by the line $y = \sqrt{3}x$, the curve $x^2 + y^2 = 16$ and above the x axis is revolved about the x axis. 2

Find the volume of the solid so formed. Leave the answer in terms of π.

Question 9

MARKS

a)



AB is the diameter of a circle whose centre is O. P is a point on the circumference such that the chord AP = 8 cm and the chord BP = 6 cm, $\angle APB = 90°$.
Calculate (correct to 3 significant figures): 3

(i) the values, in radians, of $\angle PAB$ and $\angle POB$.

(ii) the area of the sector bounded by OP, OB and the minor arc PB. 2

b) A function is defined by $y = 2x^3 - 6x - 1$.

(i) Find any turning points and determine their nature. 4

(ii) Find any point(s) of inflexion. 1

(iii) Draw a neat sketch of the curve showing all important features. 2

Question 10

a) According to a fictional story, scary dinosaurs may be created due to nuclear waste after a nuclear test. The number of these dinosaurs can increase according to the following formula.

$N = Ae^{kt}$ where A and k are constant and t is the time in months.

If after a nuclear test on the 1st of January 2000, 4 scary dinosaurs were created in a desert, and they reproduced so that after 9 months their total number became 2048.

(i) Find the constants A and k. 2

(ii) How many months will be needed in order that total number of dinosaurs reach 131 072. 3

b) Mr. Jones on his 25th birthday started making regular annual investments of $500 into an investment fund which paid interest at the rate of 9 % p.a. After retiring on his 60th birthday, he collected his investment payout with which he immediately repaid the balance owing on his housing loan. After that he was still left with a surplus of $795, which he decided to give as a present to his six grandchildren.

(i) Show that the amount of Mr. Jones owed on his housing loan just prior to his payout was $116 767.36. 4

(ii) The surplus of $795 was then divided amongst his six grandchildren. Each grandchild received a different amount, forming an arithmetic series with 6 terms. The oldest, Clark (13 yrs), received $186, then Susan (11 yrs), Jason (9 yrs), Greg (7 yrs), Beatrice (5 yrs) received progressively less, with the youngest, Debbie (3 yrs), the smallest amount.

(α) How much did Debbie receive? 1

(β) How much did other grandchildren receive? 2

1999 2 unit solutions

Question 1

a) $3 - x < 4$

$-x < 1$

$\therefore x > -1$ (2)

b) $A = P(1-R)^n$

$A = 25000(1-0.15)^2$

$= \$18062.50$

After 2 years the car is worth $18062.50 (2)

c) $\dfrac{\sqrt{2060}}{4.73+9.29} = 3.235$ corr to 3 decimal places (3)

d) $3(x-2) = \dfrac{x+1}{2}$

$6x - 12 = x + 1$

$5x = 13$

$x = \dfrac{13}{5}$ (2)

e) $2x^3 - 16 = 2(x^3 - 8)$

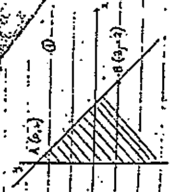
$= 2(x-2)(x^2 + 2x + 4)$ (2)

f) $\sqrt{24} + \sqrt{54} = 2\sqrt{6} + 3\sqrt{6}$

$= 5\sqrt{6}$ (2)

Question 2

a) (1)



b) $m_{AB} = \dfrac{4 - -2}{0 - 3} = -2$

equation AB: $y - y_1 = m(x - x_1)$

$y - 4 = -2(x - 0)$

$y - 4 = -2x$

$2x + y - 4 = 0$ proven (2)

c) midpoint$_{BC} = \left(\dfrac{-3+3}{2}, \dfrac{0+-2}{2}\right)$

$= (0, -1)$ (2)

d) M.p. $(0,-1)$ $2x + y - 4 = 0$

$d = \left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$= \dfrac{2(0) + 1(-1) - 4}{\sqrt{2^2+1^2}}$

$= \left|\dfrac{-5}{\sqrt{5}}\right| = \dfrac{5}{\sqrt{5}}$

$= \sqrt{5}$ (2)

e) $d_{AB} = \sqrt{(0-3)^2 + (4+2)^2}$

$= 3\sqrt{5}$

$A = \frac{1}{2} \times 3\sqrt{5} \times \sqrt{5}$

$= 7\frac{1}{2}$ units^2 (2)

f) Region indicated on diagram.

Substitute $(0,0)$

$2(0) + 0 - 4 \le 0$ true (3)