GIRRAWEEN HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2007

MATHEMATICS

Time allowed - Three hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are on sheet provided.
- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question 1 , Question 2 , etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

Question 1 (12 marks) **Marks**

(a) Evaluate $\sqrt{\dfrac{762.8}{2.7 \times 3.5}}$ correct to 3 significant figures. **2**

(b) Factorise $5x^2 - 16x - 3$ **2**

(c) Find the primitive for e^{2x} . **2**

(d) Find the values of x for which $|2x-3| < 7$ **2**

(e) Express $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ in the form $a + b\sqrt{6}$ where a and b are integers. **2**

(f) Karan pays $153.00 for a DVD player which has been discounted by 15%. What was the original price of the DVD player? **2**

Question 2 (12 marks)

(a) Differentiate with respect to x:

(i) $x^2 e^x$ **2**

(ii) $\dfrac{3x}{\cos x}$ **2**

(b) Find:

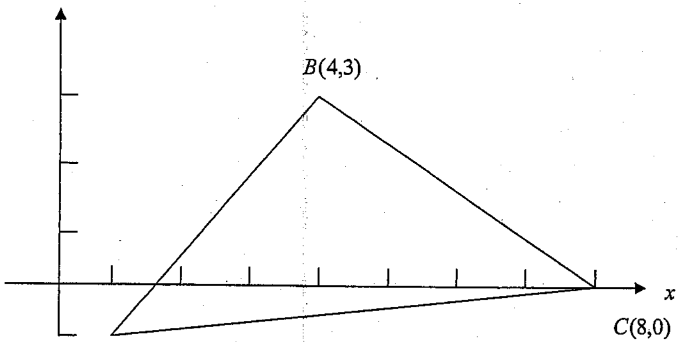
(i) $\displaystyle\int \frac{10x}{x^2+5}\,dx$ **2**

(ii) $\displaystyle\int_0^{\frac{\pi}{8}} 5\sec^2 2x\,dx$ **3**

(c) Find the equation of the tangent to $f(x) = e^{2x-4}$ at the point where $x = 2$. **3**

Question 3 (12 marks)

(a)



In the diagram above, $A, B,$ and C are the points $(1,-1), (4,3)$ and $(8,0)$ respectively.

Copy the diagram on to your own paper and answer the following questions:

(i) Find the gradient of the line AC. **2**

(ii) Find D, the midpoint of AC. **1**

(iii) Show that the equation of the line through B which is perpendicular to AC is $7x + y - 31 = 0$. **3**

(iv) Show that D lies on the line in part (iii). **1**

(v) Show that ΔABC is isosceles. **3**

Question 3 (continued)

(b) Find the angle θ in the diagram below: **2**

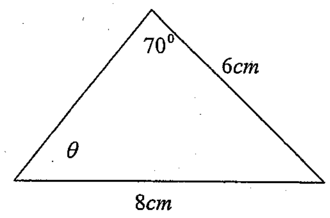


Diagram not to scale

Question 4 (12 marks)

(a) Evaluate $\sum_{n=4}^{6} \frac{1}{n-2}$ **1**

(b) Use the change of base rule to find $\log_3 7$. **1**

(c) A sector of a circle has an area of $8\pi\ cm^2$. The arc at the circumference of this sector is 2π cm long. Find

(i) The radius of the circle. **2**

(ii) The angle subtended by the arc at the centre of the circle. **1**

(d) (i) Find the focus of the parabola $x^2 = 12(y-1)$ **2**

(ii) Find the volume of the solid of revolution formed when the area between $x^2 = 12(y-1)$ and the y axis is rotated about the y axis between $y = 1$ and $y = 3$. **3**

(e) For what values of k does the equation $4x^2 - 4x + k = 0$ have real roots? **2**

Question 5 (12 marks)

(a) For the function $f(x) = 4x^2(2x+3)$

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **2**

(iii) Sketch the graph of f(x) showing all stationary points, points of inflexion and intercepts with the co-ordinate axes. **2**

(b) The probability that Rusty will beat Danielle in a set of tennis is $\frac{3}{5}$. On a particular day they play 3 sets of tennis.

(i) What is the probability that Rusty will win all 3 sets? **1**

(ii) Draw a probability tree to illustrate the possible results of the 3 sets. **2**

(iii) What is the probability that Danielle will win exactly 2 sets? **1**

(iv) What is the probability that Danielle will win at least 1 set? **1**

Question 6 (12 marks)

(a) A farmer is delivering loads of cement from a pile at the end of an irrigation ditch 3 kilometres long to points 120 metres apart along the ditch. After delivering each load, the farmer must return to the pile at the end of the ditch to collect the next load. He starts at the pile and delivers his first load to the first point (120 metres away) then after returning to the pile delivers his second load to the second point (240 metres away) and so on.

(i) How far along the ditch is the 12^{th} load delivered? **2**

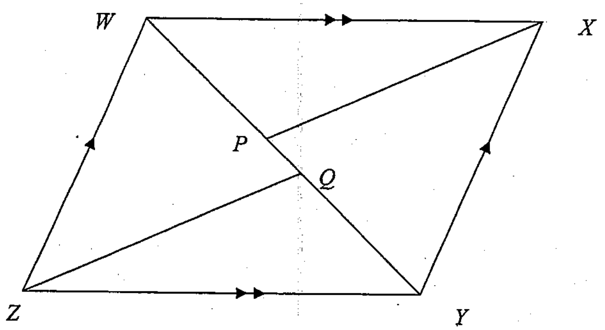
(ii) How many loads are delivered along the entire length of the 3km ditch? (The last load is delivered to the very end of the ditch.) **2**

(iii) How many km has the farmer travelled in order to deliver all of the loads, then return to the end of the ditch where the pile was? **2**

Question 6 (continued)

(b) $WXYZ$ is a parallelogram. XP bisects $\angle WXY$ and ZQ bisects $\angle WZY$.

Copy the diagram on to your answer sheet and answer the following questions:



(i) Explain why $\angle WXY = \angle WZY$. **2**

(ii) Prove $\Delta WXP \equiv \Delta YZQ$ **3**

(iii) Hence find the length of PQ given $WY = 20cm$ and $QY = 8cm$. **1**

Question 7 (12 marks)

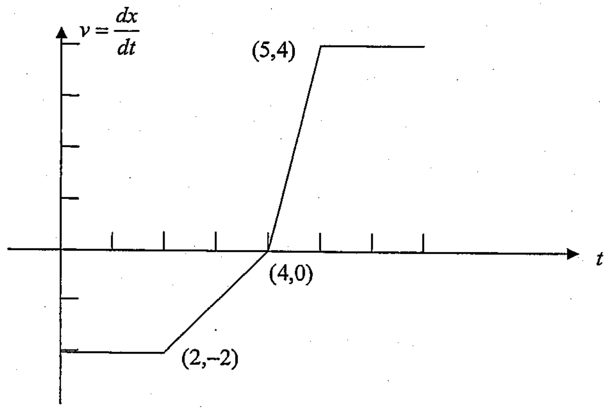
(a) If α and β are the roots of the quadratic equation $2x^2 - 3x + 7 = 0$ find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(b) Below is the graph of the velocity of a particle in metres per second. Initially the particle is at the origin.



(i) When is the particle furthest from the origin? **1**

(ii) How far, and in what direction, is the particle from the origin after 7 seconds? **2**

(iii) Sketch the acceleration of the particle from time $t = 0$ to time $t = 7$. **2**

Question 7 (continued)

(c) (i) Differentiate $f(x) = \cos^3 x$. **2**

(ii) Hence find $\int_0^{\frac{\pi}{3}} \cos^2 x \sin x.dx$ **2**

Question 8 (12 marks)

(a) Use Simpson's Rule with 5 function values to find an approximation **3**

for $\int_0^1 \ln(x+1).dx$

(b) The population of a town is growing according to the formula

$\frac{dP}{dt} = kP$.

(i) Show that $P = Ae^{kt}$ is a solution to this differential equation. **1**

(ii) If the town's population was 3000 in 1980 and 5000 in 1990 **2**

find values for A and k given 1980 is when $t = 0$.

(iii) Find the town's population in 2007. **1**

(c) An arithmetic series has $T_3 = 60$ and $T_7 = 95$. Find the sum of the **3**

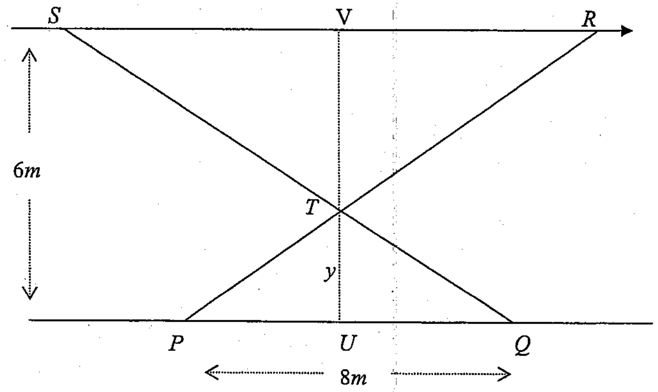
first 10 terms.

(d) The limiting sum of the series $1 + 3^x + 3^{2x} + 3^{3x} + \ldots$ is equal to $\frac{9}{8}$. **2**

Find the value of x.

Question 9 (12 marks)

(a) In the diagram below PQ and SR are parallel railings which are $6m$ apart. The points P and Q are fixed $8m$ apart on the lower railing. Two crossbars PR and QS intersect at T as shown in the diagram. The line through T perpendicular to PQ intersects PQ at U and SR at V. The length of UT is y metres.



(i) By using similar triangles or otherwise show that $\frac{SR}{PQ} = \frac{VT}{UT}$. **3**

(ii) Show that $SR = \frac{48}{y} - 8$. **1**

(iii) Hence show that the total area of ΔPTQ and ΔRTS is **2**

given by $\frac{144}{y} + 8y - 48$.

(iv) Find the value of y that minimises A. Justify your answer. **3**

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Trial Exam

Mathematics — 2007

Solutions:

Q.(1) (a) $\sqrt{\dfrac{762\cdot 8}{2\cdot 7\times 3\cdot 5}} = 8\cdot 9844\ldots$

$\doteqdot 8\cdot 98$ (to 3SF). 2

(b) $5x^2-16x-3 = 5\left(x-\dfrac{8-\sqrt{79}}{5}\right)\left(x-\dfrac{8+\sqrt{79}}{5}\right)$ 2

using quadratic formula $x = \dfrac{16\pm\sqrt{16^2-4\times 5\times -3}}{2\times 5}$.

(c) $\int e^{2x}\,dx = \frac{1}{2}e^{2x}+C$. 2

(d) $|2x-3| < 7$

$-7 < 2x-3 < 7$

$-4 < 2x < 10$

$-2 < x < 5$. 2

(e) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$ 2

$= \dfrac{3+\sqrt{6}}{1}$

$= 3+\sqrt{6}$.

(f) \$153·00 = 85 % of original price. 2

\$1·80 = 1 %

\$180·00 = Original price.

GHS 2007 Mathematics Trial

Solutions p.2

Q.(2)(a)(i) $\dfrac{d}{dx}\left(x^2e^x\right)$

$= 2xe^x + x^2e^x$

$= e^x(2x+x^2)$ 2

or $= xe^x(2+x)$.

(ii) $\dfrac{d}{dx}\left(\dfrac{3x}{\cos x}\right)$ 2

$= \dfrac{\cos x \times 3 - 3x\times -\sin x}{\cos^2 x}$

$= \dfrac{3\cos x + 3x\sin x}{\cos^2 x}$.

or $= \dfrac{3(\cos x + x\sin x)}{\cos^2 x}$.

(b)(i) $\int \dfrac{10x}{x^2+5}\,dx$

$= 5\int \dfrac{2x}{x^2+5}\,dx$ 2

$= 5\ln(x^2+5)+C$.

(ii) $\int_0^{\frac{\pi}{8}} 5\sec^2 2x\,dx$ 3

$= \left[\frac{5}{2}\tan 2x\right]_0^{\frac{\pi}{8}}$

$= \frac{5}{2}\tan\frac{\pi}{4} - \frac{5}{2}\tan 0$

$= \frac{5}{2}$.

(c) $f(x) = e^{2x-4}$

$f'(x) = 2e^{2x-4}$

Where $x = 2$,

$f(x) = e^{2\times 2-4}$

$= e^0$

$= 1$.

$f'(x) = 2e^{2x-4}$ 3

$= 2e^0$

$= 2$.

So tangent is line passing through $(2,1)$ with $m=2$.

By $y-y_1 = m(x-x_1)$

$y-1 = 2(x-2)$

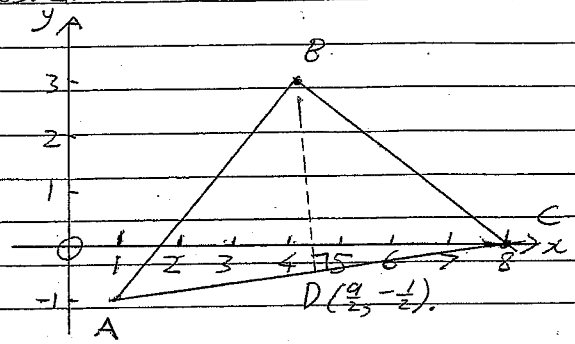
$y-1 = 2x-4$

$y = 2x-3$

Or in general form:

$2x-y-3 = 0$.

Q.(3)(a)



(i) $m\,AC = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0+1}{8-1}$

$= \frac{1}{7}$. (2)

(ii) D, midpoint of AC

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{1+8}{2}, \frac{-1+0}{2}\right)$ (1)

$= \left(\frac{9}{2}, -\frac{1}{2}\right)$

(iii) Line through B perpendicular to AC:

$m = -7$.

By $y - y_1 = m(x - x_1)$ (3)

$y - 3 = -7(x - 4)$

$y - 3 = -7x + 28$

$7x + y - 31 = 0$.

(iv) Substituting co-ordinates of D into $7x+y-31=0$

$7\left(\frac{9}{2}\right) - \frac{1}{2} - 31 = 0$. (1)

$\therefore$ D is on line $7x+y-31=0$

(v) Showing $\triangle ABC$ is isosceles:

BD common

AD = DC [as D is midpoint of AC]

$\angle ADB = \angle BDC$ [proven in (iii) and (iv)].

$\therefore \triangle ADB \equiv \triangle CDB$ [SAS].

AB = BC [matching sides in $\equiv \triangle$]

Hence $\triangle ABC$ is isosceles. (3)

Alternatively,

distance $AB = \sqrt{25} = 5$.

distance $BC = \sqrt{25} = 5$

AB = BC

$\triangle ABC$ is isosceles.

(b) By $\frac{\sin A^\circ}{a} = \frac{\sin B^\circ}{b}$ (2)

$\frac{\sin\theta}{6} = \frac{\sin 70^\circ}{8}$

$\sin\theta = \left(\frac{\sin 70^\circ \times 6}{8}\right)$

$\theta = 44^\circ 49'$ [to nearest minute].

Q.(4)(a) $\sum_{n=4}^{6} \frac{1}{n-2}$

$= \frac{1}{4-2} + \frac{1}{5-2} + \frac{1}{6-2}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$= 1\frac{1}{12}$ or $\frac{13}{12}$ (1)

(b) $\log_3 7 = \frac{\ln 7}{\ln 3}$ (1)

$\doteqdot 1.77$ (2DP).

(c)(i) Sector area:

$\frac{1}{2}r^2\theta = 8\pi$.

Radius:

$r\theta = 2\pi$.

$\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{8\pi}{2\pi}$

$\frac{1}{2}r = 4$

$r = 8$cm. (2)

(ii) Angle θ:

$r = 8$, $r\theta = 2\pi$

$\theta = \frac{2\pi}{8}$

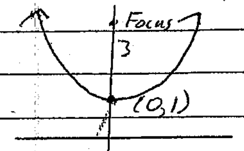
Angle subtended $= \frac{\pi}{4}$. (1)

(d)(i) $x^2 = 12(y-1)$

Vertex $= (0,1)$

Focal length: $4a = 12$

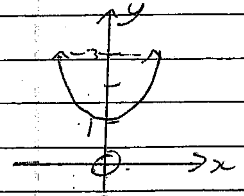
$a = 3$.



(2)

Focus $= (0,4)$

(ii)



$V = \pi\int x^2\,dy$

$= \pi\int_{y=1}^{y=3} 12(y-1)\,dy$

$= \pi\int_1^3 12y - 12\,dy$ (3)

$= \pi\left[6y^2 - 12y\right]_1^3$

$= \pi\left[(6\times3^2 - 12\times3) - (6\times1^2 - 12\times1)\right]$

$= 24\pi$ cubic units.

$4x^2 - 4x + k = 0$ has real roots:

$\therefore \Delta = b^2 - 4ac \geq 0$

$(-4)^2 - 4\times4\times k \geq 0$ (2)

$16 - 16k \geq 0$

$16 \geq 16k$

$1 \geq k$

$\therefore 4x^2 - 4x + k = 0$ will have real roots where $k \leq 1$.

Q.(5)(a)(i) $f(x)=4x^2(2x+3)$
$=8x^3+12x^2$

$f'(x)=24x^2+24x$.

Stationary points are where $f'(x)=0$

$24x^2+24x=0$

$24x(x+1)=0$

$x=0$ or $x=-1$. (3)

If $x=0$, $y=4(0)^2[2(0)+3]=0$.

If $x=-1$, $y=4(-1)^2[2(-1)+3]=4$

Stationary points at $(0,0)$ & $(-1,4)$

Nature of stationary points: $f''(x)=48x+24$.

At $x=-1$, $f''(x)=48(-1)+24=-24$.

$(-1,4)$ is a LOCAL MAXIMUM.

At $x=0$, $f''(x)=48(0)+24=24$.

$(0,0)$ is a LOCAL MINIMUM.

(ii) Point of inflexion: $f''(x)=0$.

$48x+24=0$

$48x=-24$

$x=-\frac{1}{2}$ (2)

$y=4(-\frac{1}{2})^2[2\times-\frac{1}{2}+3]$

$=(-\frac{1}{2},2)$.

Testing point of inflexion:

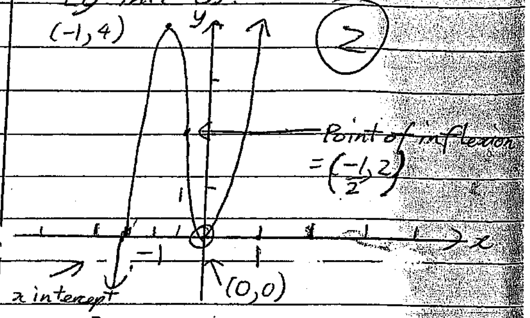
At $x=-1$, $f''(x)=48(-1)+24$
$=-24$.

~~At $x=0$, $f''(x)=48(0)+24$~~
$=24$.

$\therefore f''(x)$ changes sign $\Rightarrow$
$(-\frac{1}{2},2)$ is a point of inflexion.

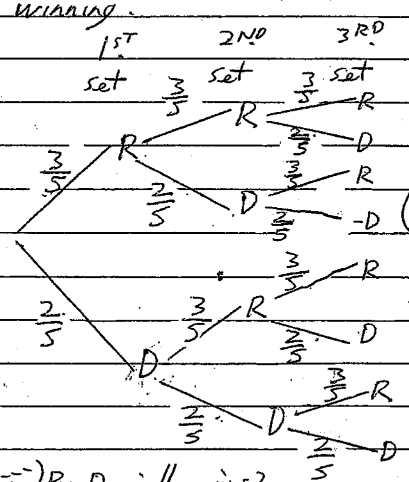
(iii) Note: x intercepts $=0$ & where $2x+3=0$
i.e. $x=-\frac{3}{2}$.

[y int. $=0$]. (2)



(b)(i) Pr[Rusty winning]
$=0\cdot6\times0\cdot6\times0\cdot6$
$=0\cdot216$. (1)

(ii) Probability Tree:
R = Rusty winning, D = Danielle winning.

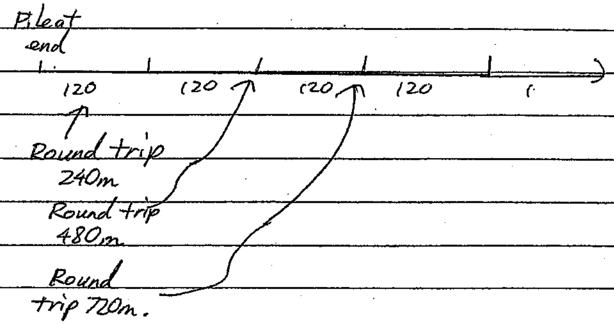


(2)

(iii) Pr Danielle wins 2
$=0\cdot4\times0\cdot4\times0\cdot6\ \times3$
$=0\cdot288$. (1)

Pr Danielle wins at least 1 set
$=1-\text{Pr}[\text{Rusty wins all 3}]$
$=1-0\cdot216\ =\underline{0\cdot784}$ (1)

Q.(6)(a)



(2)

(i) 12TH load delivered $12\times120\text{m}=1440\text{m}$ along ditch.

(ii) Total no. of loads $=3000\div120$
$=25$ loads. (2)

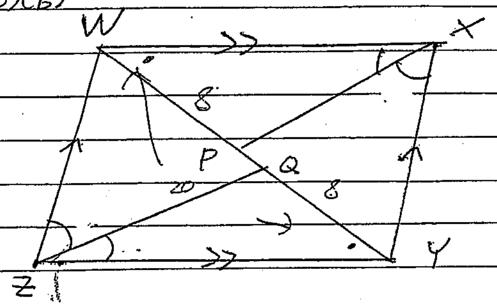
(iii) Farmer travels $240+480+720+\ldots+6000\text{m}$.

By $S_n=\frac{n}{2}(a+L)$ (2)

Total distance $=\frac{25}{2}(240+6000)$
$=78000\text{m}$ or 78km.

Note: Could also do $S_n=\frac{n}{2}(2a+(n-1)d)$
with $n=25$, $a=240$, $d=240$

Q.(6)(b)



(i) $\angle WXY = \angle WZY$ (opposite angles of parallelogram =). (2)

(ii) Hence $\angle WXP = \angle PXY = \angle YZQ = \angle PZQ$
[as XP bisects $\angle WXY$ & ZQ bisects $\angle WZY$].
$\angle XWY = \angle ZYW$ [alternate $\angle$'s in $\parallel$ lines =].
$WX = ZY$ (opposite sides of parallelogram =)
$\therefore \triangle WXP \equiv \triangle YZQ$ (AAS). (3)

(iii) $WP = 8$cm [matching sides in $\equiv \triangle WXP$ and $\triangle YZQ$ =].
Hence $PQ = \underline{4\text{cm}}$. (1)

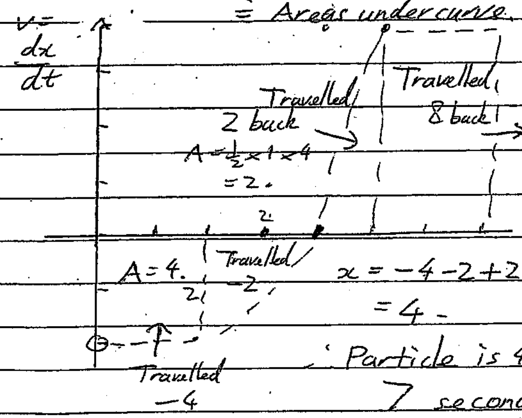
Q.(7)(a)(i) $\alpha+\beta = \frac{-b}{a}$ (1)
$= \frac{3}{2}$

(ii) $\alpha\beta = \frac{c}{a}$ (1)
$= \frac{7}{2}$.

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= \left(\frac{3}{2}\right)^2 - 2\times\frac{7}{2}$ (1)
$= -4\frac{3}{4}$.

(b)(i) Particle furthest from origin when $v=0$
($t = 4$ seconds). (1)

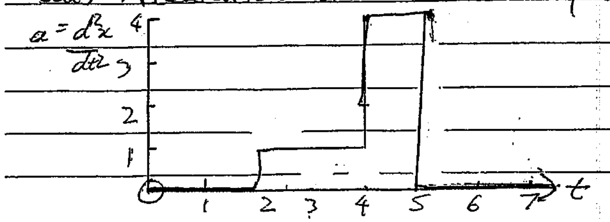
(ii) x will be $\int v.dt$
$\doteqdot$ Areas under curve.



(2)

$x = -4-2+2+8$
$= 4$.
$\therefore$ Particle is 4m to RIGHT of origin after 7 seconds.

(iii) Acceleration $= \frac{dv}{dt}$ = GRADIENT of velocity: (2)



Q.(7)(c) (i) $f(x) = \cos^3 x$.

$f'(x) = 3\cos^2 x \times -\sin x$

$= -3\cos^2 x \sin x$. (2)

(ii) Hence $\int_0^{\frac{\pi}{3}} \cos^2 x \sin x \, dx$

$= -\frac{1}{3}\int_0^{\frac{\pi}{3}} -3\cos^2 x \sin x \, dx$

$= -\frac{1}{3}\left[\cos^3 x\right]_0^{\frac{\pi}{3}}$

$= -\frac{1}{3}\left[\cos^3\left(\frac{\pi}{3}\right) - \cos^3(0)\right]$ (2)

$= -\frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - 1^3\right]$

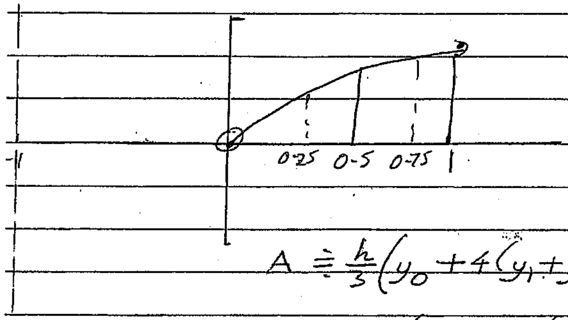
$= \frac{7}{24}$.

Solutions

Q.(8)(a) $\int_0^1 \ln(x+1) \, dx$

$h = \frac{1}{4} = 0.25$.

$y_0 = \ln 1 = 0$.



$A \doteqdot \frac{h}{3}\left(y_0 + 4(y_1 + y_3) + 2y_2 + y_5\right)$

$= \frac{0.25}{3}\left(\ln 1 + 4\left(\ln 1.25 + \ln 1.75\right) + 2\ln 1.5 + \ln 2\right)$

$= 0.386\ldots$

$\therefore \int_0^1 \ln(x+1) \, dx \doteqdot 0.386$ [3SF]. (3)

(b)

(i) If $P = Ae^{kt}$

$\frac{dP}{dt} = kAe^{kt}$ & $kP = kAe^{kt}$ (1)

Hence $\frac{dP}{dt} = kP$

(ii) $P = 3000$ when $t = 0$ (2)

$3000 = Ae^0$

$\therefore 3000 = A$.

$P = 3000e^{kt}$

$P = 5000$ when $t = 10$

$5000 = 3000e^{10k}$

$\frac{5}{3} = e^{10k}$

$\ln\left(\frac{5}{3}\right) = 10k$.

$\frac{1}{10}\ln\left(\frac{5}{3}\right) = k$.

$k \doteqdot 0.05108\ldots$

(iii) Population in '07 ($t = 27$) (1)

$= 3000e^{0.05108\ldots \times 27}$

$= 11\,915.51\ldots$

Population $\doteqdot 11\,900$

(to nearest 100 people).

Q.(8)(c) $T_7 = a+6d = 95$ (1)

$T_3 = a+2d = 60$ (2) (1)−(2)

$4d = 35.$

$d = 8{\cdot}75.$

From $T_3 = a+2d = 60$

$a + 2\times 8{\cdot}75 = 60$

$a = 42{\cdot}5.$ 3

Sum of first 10 terms:

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{10}{2}[2\times 42{\cdot}5 + 9\times 8{\cdot}75]$

$= 818{\cdot}75.$

(d) Limiting sum $= \frac{a}{1-r} = \frac{9}{8}$

$\frac{1}{1-3^x} = \frac{9}{8}$

$\times 8(1-3^x)$

$8 = 9(1-3^x)$

$8 = 9 - 9\times 3^x$ $+9\times 3^x - 8$

$9\times 3^x = 1.$

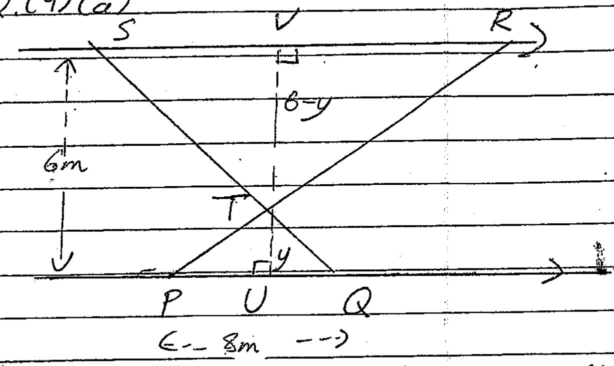
As $9 = 3^2$, & $1 = 3^0$

$3^{x+2} = 3^0$

$x+2 = 0$ 2

$x = -2.$

Q.(9)(a)



3

(i) $\angle PTQ = \angle STR$ [Vertically opposite $\angle$'s =].

$\angle TSR = \angle TQP$ [alternate $\angle$'s in // lines =].

$\angle TRS = \angle TPQ$ [" " "]. Any 2 of

$\therefore \triangle TSR \,|||\, \triangle TQP$ [2 pairs of matching $\angle$'s =].

$\therefore \frac{SR}{PQ} = \frac{ST}{TQ}$ [ratio of matching sides in ||| $\triangle$].

$SV \perp VT$ [data] This is 1

$QU \perp UT$ ["]. Any 2 of

$\angle STV = \angle QTU$ [vertically opposite $\angle$'s =].

$\angle TSR = \angle TQ.U$ [proven earlier]

$\therefore \triangle STV \equiv \triangle QTU$ [2 pairs of matching $\angle$'s =].

$\therefore \frac{ST}{TQ} = \frac{VT}{UT}$ [ratio of matching sides in ||| $\triangle$'s].

As $\frac{ST}{TQ} = \frac{SR}{PQ}$ [proven earlier].

$\therefore \frac{SR}{PQ} = \frac{VT}{UT}.$

(ii) As $\frac{SR}{PQ} = \frac{VT}{UT}$ 1

$\frac{SR}{8} = \frac{6-y}{y}$

$SR = \frac{48-8y}{y}$

$= \frac{48}{y} - 8$

Q.(9)(a)(iii) Total area of $\triangle PTQ$ & $\triangle RTS$

$$= \frac{1}{2} \times 8xy + \frac{1}{2} \times \left(\frac{48}{y} - 8\right) \times (6-y)$$

$$= 4y + \frac{144}{y} + 4y - 48$$ (2)

$$= \frac{144}{y} + 8y - 48$$

(iv) Value of y that minimises A:

Find where $\frac{dA}{dy} = 0$.

$$-\frac{144}{y^2} + 8 = 0.$$

$$8y^2 - 144 = 0$$ (3)

$$y^2 - 18 = 0$$

$$y = \pm\sqrt{18}$$

$$= \pm 3\sqrt{2}$$

→ As y is a measurement, $y = 3\sqrt{2}$.

Justifying: This is a minimum if $\frac{d^2A}{dy^2} > 0$.

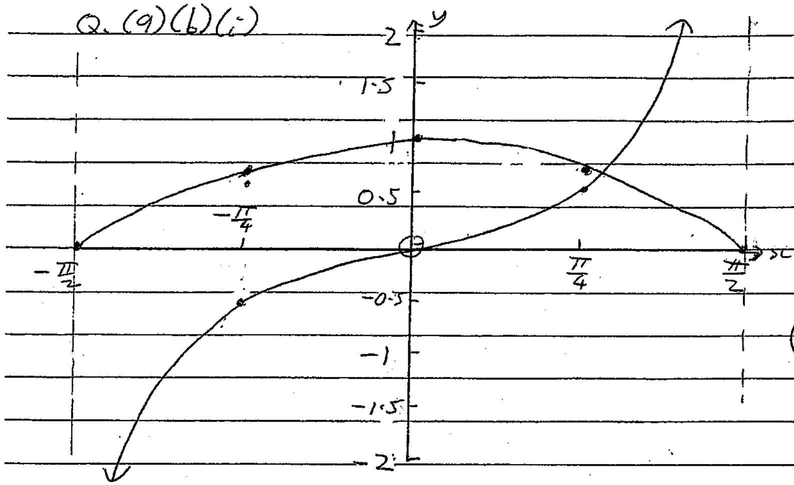
$$\frac{d^2A}{dy^2} = \frac{288}{y^3}$$

Where $y = 3\sqrt{2}$, $\frac{d^2A}{dy^2} = \frac{288}{(3\sqrt{2})^3}$

$$\doteqdot 33.9.$$

→ As $\frac{d^2A}{dy^2} > 0$ where $y = 3\sqrt{2}$, area is a MINIMUM.

Q.(9)(b)(i)



(1)

(ii) $\cos x = \frac{1}{2}\tan x$

$$\cos x = \frac{\sin x}{2\cos x} \quad \left[\text{as } \tan x = \frac{\sin x}{\cos x}\right].$$

$$2\cos^2 x = \sin x$$ (2)

$$2(1-\sin^2 x) = \sin x \quad [\text{as } \cos^2 x + \sin^2 x = 1].$$

$$2 - 2\sin^2 x - \sin x = 0$$

$$2\sin^2 x + \sin x - 2 = 0.$$

This is a quadratic equation in $\sin x$:

$$\sin x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 2 \times -2}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{17}}{4}.$$

→ Only possible solution (from graph) is in 1st quadrant.

So $\sin x = \frac{-1+\sqrt{17}}{4}$ $\doteqdot 0.78..$ $x \doteqdot 0.895.$	$\cos x \doteqdot 0.624..$ $\frac{1}{2}\tan x \doteqdot 0.624..$	Co-ordinates $= (0.895, 0.624)$

Q. (10)(a) Limiting sum $= \dfrac{a}{1-r}$

$= \dfrac{1}{1-\sin^2 x}$

$= \dfrac{1}{\cos^2 x}$ [as $\cos^2 x + \sin^2 x = 1$].

$= \sec^2 x$ (3)

$= 1 + \tan^2 x$. [as $1 + \tan^2 x = \sec^2 x$].

(b)(i) Amount left to be repaid: 6 % P.A. = 0.5 % per month

Month:	Start of month:	End:
1	\$400 000	$\$400\,000 \times 1.005 - P$
2		$(\$400\,000 \times 1.005 - P) \times 1.005 - P$
		$= \$400\,000 \times 1.005^2 - P \times 1.005 - P$
3		$(\$400\,000 \times 1.005^2 - P \times 1.005 - P) \times 1.005 - P$
		$= \$400\,000 \times 1.005^3 - P \times 1.005^2 - P \times 1.005 - P$
		$= \$400\,000 \times 1.005^3 - P(1 + 1.005 + 1.005^2)$

(2)

(ii) After 20 years [240 months]

Amount left to be repaid $= 0$

$400\,000 \times 1.005^{240} - P(1 + 1.005 + 1.005^2 + \dots + 1.005^{239}) = 0$ (1) (3)

$1 + 1.005 + 1.005^2 + \dots$ [By $S_n = \dfrac{a(r^n - 1)}{r-1}$]

$= \dfrac{1(1.005^{240} - 1)}{1.005 - 1}$

$\doteqdot 462.04\ldots$ [Keep in calculator].

Sub. in (1): $400\,000 \times 1.005^{240} - 462.04\ldots P = 0$

$\$400\,000 \times 1.005^{240} = 462.04\ldots P$

$\$2\,865.72 = P$

They repay \$2 865.72 [pay \$2 865.70] per month.

Q. (10)(b)(iii) Repaying loan at \$4000 per month:

→ Time = n months. $P = \$4000$

Amount left to be repaid:

$\$400\,000 \times 1.005^n - \$4000(1 + 1.005 + \dots + 1.005^{n-1}) = 0$ (1)

By $S_n = \dfrac{a(r^n - 1)}{r-1}$

$1 + 1.005 + \dots + 1.005^{n-1} = \dfrac{1(1.005^n - 1)}{0.005}$

$= 200(1.005^n - 1)$ (4)

$= 200 \times 1.005^n - 200$.

Sub. in (1):

$\$400\,000 \times 1.005^n - 4000 \times (200 \times 1.005^n - 200) = 0$

$400\,000 \times 1.005^n - 800\,000 \times 1.005^n + 800\,000 = 0$

$400\,000 \times 1.005^n = 800\,000$.

$1.005^n = 2$

$\ln(1.005^n) = \ln 2$

$n \ln(1.005) = \ln 2$

$n = \dfrac{\ln 2}{\ln(1.005)}$

$n = 138.97\ldots$

→ The loan will be paid off in 139 months

[11 years 7 months].