2009

TRIAL

HIGHER SCHOOL CERTIFICATE

GIRRAWEEN HIGH SCHOOL

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

Attempt Questions 1 – 10

All questions are of equal value

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Marks

Question 1 (12 marks).

(a) Evaluate $3\sin\frac{\pi}{3}$ correct to 3 significant figures. **2**

(b) Factorise $3x^2 - 8x - 3$. **2**

(c) Simplify $\frac{m+3}{2} - \frac{m+2}{3}$. **2**

(d) Solve $|3x - 4| = 2$. **2**

(e) Expand and simplify $(\sqrt{5} - 1)(2\sqrt{5} - 5)$. **2**

(f) Find the sum of the first 30 terms of the arithmetic series **2**

$6 + 9 + 12 + \ldots$

Marks

Question 2 (12 marks). Start on a SEPARATE page.

(a) Differentiate with respect to x:

(i) $(x^3+3)^3$. **2**

(ii) $e^x(x+1)$. **2**

(iii) $\frac{\ln x}{x}$. **2**

(b) Let M be the midpoint of (1, 6) and (3, 8). **2**

Find the equation of the line through M with gradient $-\frac{1}{3}$.

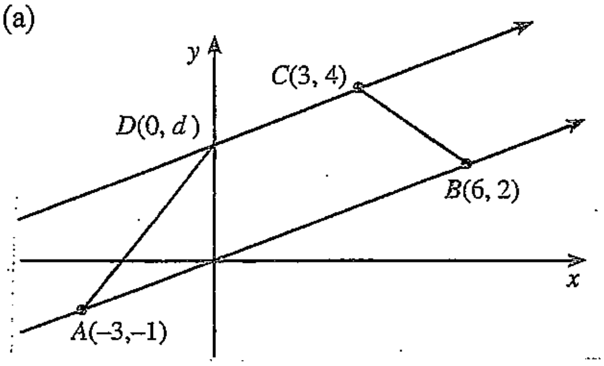
(c) (i) Find $\int \frac{dx}{x-3}$. **1**

(ii) Evaluate $\int_0^{\frac{2\pi}{3}} \sin\frac{x}{2}\,dx$. **3**

Marks

Question 3 (12 marks). Start on a SEPARATE page.

(a)



NOT
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In the diagram, ABCD is a quadrilateral.

(i) Find the gradient of AB. **1**

(ii) Find the value of d. **1**

(iii) Show that the equation of AB is $x-3y=0$. **2**

(iv) Find the perpendicular distance between C and the line AB. **2**

(v) Prove that AB = 3 CD. **2**

(vi) Find the area of quadrilateral ABCD.. **2**

(b) Solve $27^x=\sqrt{3}$. **2**

Question 4 (12 marks). Start on a SEPARATE page. **Marks**

(a) **2**

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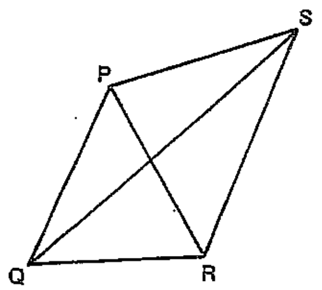


In the quadrilateral PQRS, $\angle PQR = \angle PRQ$ and $\angle PRS = \angle PSR$.

Copy or trace the diagram onto your answer page.

Prove that $\angle PQS = \angle PSQ$.

(b) An expedition sets out to travel from Mawson Base to the South Pole, a distance of 2420 km. On the first day, they travel 312 km. Subsequently, the distance travelled each day is 85% of that travelled the previous day.

(i) How far (to the nearest km) will they travel in the seventh day? **2**

(ii) How far (to the nearest km) will they be from Mawson Base after a week's travelling? **2**

(iii) Show that the expedition cannot reach the South Pole. **2**

(c) A, B are the points $(-1, 0)$ and $(5, 2)$ respectively. **4**
Show that the equation of the locus of the point $P(x, y)$ which moves so that PA is perpendicular to PB is

$$x^2 + y^2 - 4x - 2y - 5 = 0.$$

Marks

Question 5 (12 marks). Start on a SEPARATE page.

(a) The gradient of a curve is given by $\frac{dy}{dx} = \frac{2x}{x^2+1}$. **2**

The curve passes through the point (0, 2).

What is the equation of the curve?

(b) Consider the geometric series $1 + 3x + 9x^2 + 27x^3 + \ldots$

(i) For what values of x does this series have a limiting sum? **2**

(ii) The limiting sum of the series is 20. **2**
Find the value of x.

(c) A condenser discharges at a rate proportional to the charge present, that is, $\frac{dC}{dt} = -kC$, where k is a constant and C is the charge at time t seconds.

The charge reduces from 90 to 20 in 10 seconds.

(i) Show that $C = Ae^{-kt}$ satisfies the equation $\frac{dC}{dt} = -kC$. **1**

(ii) Find the value of k. **2**

(iii) What is the charge after 5 seconds? **1**

(iv) At what time does the charge reach 60? **2**

Question 6 (12 marks). Start on a SEPARATE page. **Marks**

(a)

(i) Show that $(\operatorname{cosec}^2 x - 1)\sin^2 x = \cos^2 x$. **2**

(ii) Hence, or otherwise, solve **3**

$$(\operatorname{cosec}^2 x - 1)\sin^2 x = \frac{3}{4} \text{ for } -\pi \le x \le \pi.$$

(b) A long trench is being dug by a party of soldiers who remove x cubic metres of soil in t minutes where **2**

$$x = 4t - \frac{t^2}{40}.$$

At what rate is the soil being removed at the end of half an hour?

(c) The position of a particle moving along the x – axis is given by

$$x = 1 - \cos(2t - 1).$$

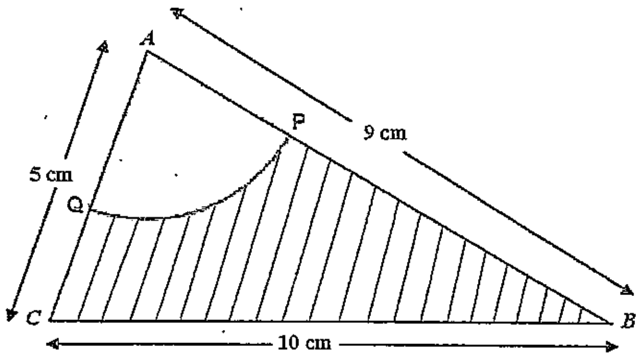
(i) Show that the particle is at rest when $t = 0.5$ **2**

(ii) When and where is the particle next at rest? **3**

Marks

Question 7 (12 marks). Start on a SEPARATE page.

(a)



Triangle ABC has AB = 9 cm, BC = 10 cm and CA = 5 cm.
A circle, centre A and radius 3 cm, intersects AB and AC at P and Q respectively, as shown in the diagram.

(i) Show that, to 3 decimal places, $\angle BAC = 1.504$ radians. **2**

(ii) Find the area of the sector APQ. **1**

(iii) Find the area of the shaded region BPQC. **2**

(iv) Find the perimeter of the shaded region BPQC. **2**

(b) Solve for x: $2\log x = \log(2x+8)$. **2**

(c) In a bag there are 20 marbles. The bag consists of 7 red marbles, 9 gold marbles and 4 blue marbles. One marble is drawn from the bag and not replaced, and then a second marble is drawn.
Using a tree diagram, or otherwise, find the probability of choosing :

(i) two gold marbles. **1**

(ii) marbles of different colour. **2**

Marks

Question 8 (12 marks). Start on a SEPARATE page.

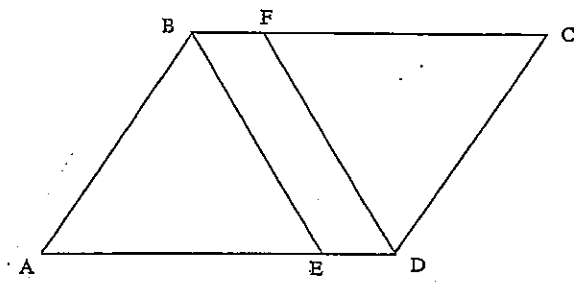
(a) Let $f(x) = x^3 - 6x^2 + 9x + 1$.

(i) Find the stationary points and determine their nature. **4**

(ii) Find any points of inflection. **2**

(iii) Sketch the graph of $f(x)$. **1**

(b)



In the diagram, ABCD is a parallelogram.

BE bisects $\angle ABC$ and FD bisects $\angle CDE$.

Copy or trace the diagram onto your answer page.

(i) Prove that Δ ABE is congruent to Δ CDF. **4**

(ii) Prove that BE = DF **1**

Marks

Question 9 (12 marks). Start on a SEPARATE page.

(a) (i) Sketch the curves $y = 4 - x^2$ and $y = 3x^2$ on the same diagram. **1**

(ii) If their point of intersection in the first quadrant is P, show that P has coordinates $(1, 3)$. **1**

(iii) If O is the origin and Q the point where $y = 4 - x^2$ meets the positive x – axis, find :

(α) the area bounded by OQ and the arcs OP, PQ of the two curves. **2**

(β) the exact volume of the solid formed when this area is rotated about the y – axis. **2**

(b) Ivana deposited \$ 20 000 at the beginning of January into an account which paid interest at the rate of $\frac{1}{2}\%$ per month compounded monthly. She withdrew \$50 each month from the account immediately after the interest was paid.

(i) How much money did she have in the account immediately after making the first instalment? **1**

(ii) Show that after making the n th withdrawal, her balance in the account is given by the expression **4**

$$\$\left(10000 \times 1.005^n + 10000\right).$$

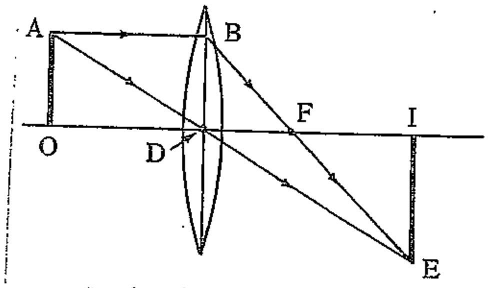
(iii) Find the maximum number of withdrawals needed for her account balance to show at least \$50 000. **1**

Marks

Question 10 (12 marks). Start on a SEPARATE page.

(a) Use the trapezoidal rule with 3 sub – intervals to find an approximation for $\int_1^4 \frac{2x}{(x^2+1)^2}\,dx$. **2**

(b)



The lens diagram shows two rays of light $A-B-E$ and $A-D-E$ which meet at E. OA is the object and IE is the image and they are both vertical.

$DF = f$ is the focal length of the lens. If $OD = u$ and $ID = v$:

(i) Use triangles AOD and EID, to prove that **2**

$$\frac{AO}{EI} = \frac{u}{v}.$$

(ii) Use triangles BDF and EIF, to prove that **2**

$$\frac{BD}{EI} = \frac{f}{v-f}.$$

(iii) Using the results of (i) and (ii) above, show that **2**

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

(iv) Show that the minimum distance between the object OA and the image IE is $4f$. **4**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } \; n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x < 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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