Student Name: ....

2005

YEAR 12 TRIAL HSC EXAMINATION

MATHEMATICS

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value.

Question 1. (Start this question on a new page) **Marks**

(a) Express 0.031997 correct to three significant figures. **1**

(b) Find a primitive of $\frac{2}{x}$ **1**

(c) Solve $(v-2)^2=16$ **2**

(d) Simplify $\frac{3x-2}{3}-\frac{3x-5}{4}$ **2**

(e) If $\sqrt{27}-\frac{1}{\sqrt{3}}=a\sqrt{3}$, find the value of a **2**

(f) Find the exact value of $\cos\frac{\pi}{6}+\sin\frac{3\pi}{4}$ **2**

(g) Find the values of x for which $x+1=|4-2x|$ **2**

Question 2. (Start this question on a new page) **Marks**

(a) On the number plane mark the origin O and the points $A(5,4)$, $B(-1,2)$, $C(-3,-7)$ and $D(3,-5)$, and then:

(i) Show that AB is parallel to DC **1**

(ii) Show that the length of AB is the same as DC. **1**

(iii) Show that the midpoint M of AC is also the midpoint of BD. **1**

(iv) Show that $ABCD$ is a parallelogram. **2**

(v) Show that the equation of DC is $x-3y-18=0$ **2**

(vi) Find the perpendicular distance from B to $x-3y-18=0$ **2**

(vii) Find the area of the parallelogram $ABCD$ **1**

(b) Find the length of the longer diagonal of a parallelogram with sides 7 cm and 9 cm and an acute angle of 50°. **2**

Question 3. (Start this question on a new page) **Marks**

(a) Draw a neat sketch of $y = 1 - |x|$ **1**

(b) Find the domain of $y = \sqrt{3 - 2x}$ **1**

(c) Differentiate with respect to x:

(i) $\dfrac{e^{2x}}{x}$ **2**

(ii) $\sin^2 3x$ **2**

(iii) $\ln(x^3 - 5)^7$ **2**

(d) Find $\int \dfrac{4}{1+3x} dx$ **2**

(e) Draw a neat sketch of the parabola $y^2 = 8x$ and write down the coordinates of the focus. **2**

Question 4. (Start this question on a new page) **Marks**

(a) Evaluate $\sum_{2}^{4}(2r - 3)$ **1**

(b) Differentiate $\dfrac{1}{x\sqrt{x}}$ **1**

(c) Given that $f(x) = \begin{cases} -1 & \text{if } x \le -1 \\ 3x+2 & \text{if } |x| < 1 \\ 7-2x & \text{if } x \ge 1 \end{cases}$, **2**

find the value of $f(-3) + f\left(-\frac{1}{3}\right) + f\left(3\frac{1}{2}\right)$

(d) Prove that $\dfrac{1}{1-\sin A} + \dfrac{1}{1+\sin A} = 2\sec^2 A$ **2**

(e) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$ **3**

(f) Find the geometric series whose second term is 6 and the sum to infinity is 49. **3**

Question 5. (Start this question on a new page) **Marks**

(a) A bag contains five red and five black balls. A ball is chosen at random from the bag. If it is red it is put to one side, and if it is black it is returned to the bag. A second drawing is then made from the bag.

(i) What is the probability that both balls are red? **1**

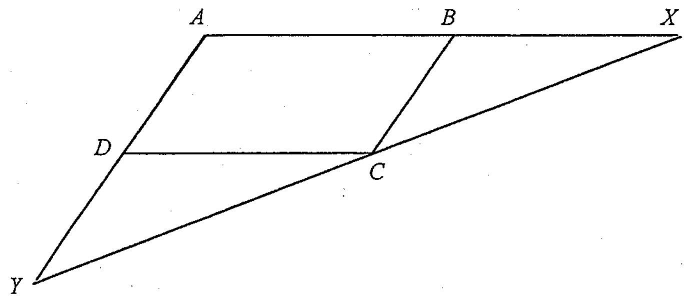
(ii) What is the probability of one ball of each colour? **2**

(b) Find the value of a if $\int_2^a (2x+1)\,dx = 14$ **2**

(c) ABCD is a parallelogram. Through C a straight line is drawn cutting AB, AD (both produced) at X, Y respectively.

(i) Show that $\angle CBX = \angle YDC$ **1**

(ii) Prove that ΔDCY is similar to ΔBXC and hence show that $\frac{XB}{AB} = \frac{AD}{DY}$. **3**



(d) Sketch the curve $y = 1 - \sin 2x$ for $0 \le x \le \pi$ **3**

Question 6. (Start this question on a new page) **Marks**

(a) The line $y = 2x + 9$ meets the parabola $y = x^2 + 2x$ at two points A and B. Find :

(i) The coordinates of A and B. **1**

(ii) the area between the curves $y = 2x + 9$ and $y = x^2 + 2x$ **3**

(b) A, B, C and D are respectively the points $(0,2)$, $(0,8)$, $(4,0)$ and $(6,0)$. Find the locus of the point $P(x,y)$ which moves so that the areas of the triangles PAB and PCD are equal in magnitude. **3**

(c) A closed tin rectangular box is to have a square base and a volume of 8 cubic metres. The length of the edge of the base is x metres.

(i) Express the height h m, of the box in terms of x. **1**

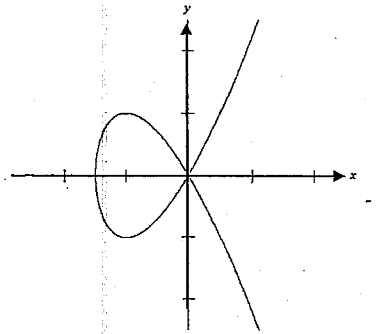
(ii) Show that the total surface area A square metres, is given by $A = \frac{32}{x} + 2x^2$ **1**

(iii) Find the value of x for which A is a minimum. Hence find the smallest area of tin sheet necessary to fulfil these specifications. **3**

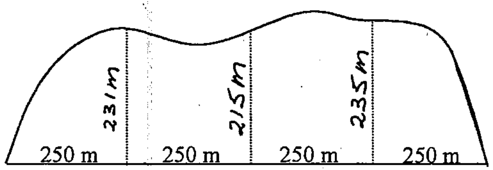
Question 7. (Start this question on a new page) **Marks**

(a) The curve with equation $y=\pm x\sqrt{(x+3)}$ is called **Tschirnhausen's cubic.** **4**

Find the volume of the solid generated when the area enclosed by the loop is rotated about the x-axis



(b) The diagram below represents an area of land bounded by a river and a straight fence which is 1 kilometre in length. Four subdivisions are made at equal distances along the straight fence as shown in the diagram. The distance from the fence to the river is indicated. Use the Trapezoidal rule with 5 function values to find the approximate area of the land. **3**



(c) Find the coordinates of the point on the curve $y=\frac{1}{2}x^2-3x+2$ at which the tangent is parallel to the line $4x-2y-7=0$. **2**

Question 7 part (d) is on the next page

(d) The curve $y=f(x)$ has a second derivative given by $\frac{d^2y}{dx^2}=(x-2)^2(x-3)$, find the x coordinate of any *possible* points of inflection and show that there is only one inflection. **3**

Question 8. (Start this question on a new page) **Marks**

(a) If water drains from a cylindrical tank according to the formula

$V=5000\left(1-\frac{t}{40}\right)^2$, where V is the volume of water in the tank at any time t. V is in litres and t in minutes.

(i) How much water is initially in the tank? **1**

(ii) How long will it take to empty the tank. **1**

(iii) Find the rate at which the water is flowing out of the tank after 10 minutes **2**

(b) The position of a particle moving along the x-axis is given by $x=8e^{-2t}-8+16t$, where t is the time in seconds and x is measured in cm.

(i) Show that the particle is at rest when $t=0$ **2**

(ii) What is the limiting velocity which the particle approaches as t increases? **1**

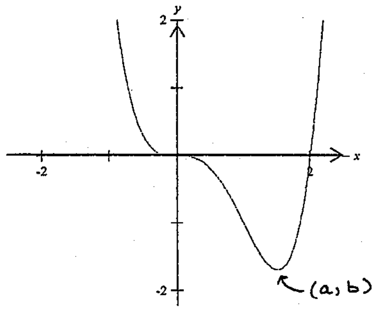
(iii) Show that the acceleration is $32-2v$ **2**

(c) A disease is spreading through the community. Let N be the number of people with the disease after t days. Let D be the rate at which the number of people who have the disease is increasing. It is known that $D=5+\left(\frac{40}{4+t}\right)^2$. **3**

Initially 20 people had the disease. How many would you expect to have the disease after 10 days?

Question 9. (Start this question on a new page) **Marks**

(a) The graph below is $y = f(x)$ **2**



On your answer sheet draw a neat sketch of the derivative $y = f'(x)$

Show clearly what happens at $x = 0$ and at $x = a$.

(b) Find the equation of the straight line k, such that the x axis is the bisector of the angle between the line with equation $5x + 4y = 1$ and the line, k. **2**

(c) The sum of the three middle terms of a nineteen term arithmetic series is 57 and the sum of the last three is 105, find the second term. **3**

(d) Xing Borrows \$240 000 in order to buy a house. Interest of 6% per annum on the loan is calculated monthly on the balance owing.

The equal repayments of $\$M$, are made monthly and the loan is to be repaid over 20 years.

(i) Show that A_2 the amount owing at the end of 2 months is given by $A_2 = 240000 \times 1.005^2 - M(1 + 1.005)$. **2**

(ii) Show that M is given by $M = \dfrac{1200 \times 1.005^{240}}{1.005^{240} - 1}$ **2**

(iii) Find the value of M correct to the nearest \$. **1**

Question 10. (Start this question on a new page) **Marks**

(a) The equation $x^2 + 3x - 2 = 0$ has roots α, β.

(i) Find $\alpha + \beta$ and $\alpha\beta$. **2**

(ii) Hence or otherwise find the equation with roots α^2, β^2. **2**

(b) Find expressions for the perpendicular distances from (x_1, y_1) to $7x - y + 9 = 0$ and to $x + y - 1 = 0$ and hence find the locus of the two lines bisecting the angles between the lines $7x - y + 9 = 0$ and $x + y - 1 = 0$. **4**

(c) Two circles have radii 4 cm. and 7 cm. respectively. Their centres are 8 cm. apart. **4**

Find the length of the arc of the smaller circle cut off by the larger circle.

End of Examination

MATHEMATICS

GOSFORD HIGH SCHOOL TRIAL HSC

Question 1 a) $\underline{0 \cdot 0320}$

b) $\int \frac{2}{x}\, dx$

$= 2\int \frac{1}{x}\, dx$

$= \underline{2 \ln x + c.}$

c) $(v-2)^2 = 16$

$v - 2 = \pm 4$

$v = 2 \pm 4$

$\underline{v = 6 \text{ OR } -2}.$

d) $\frac{3x-2}{3} - \frac{3x-5}{4}$

$= \frac{4(3x-2) - 3(3x-5)}{12}$

$= \frac{12x - 8 \quad -9x+15}{12}$

$= \frac{3x+7}{12}$

e) $\sqrt{27} - \frac{1}{\sqrt{3}} = \sqrt{9 \times 3} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 3\sqrt{3} - \frac{\sqrt{3}}{3}$

$= \frac{9\sqrt{3} - \sqrt{3}}{3}$

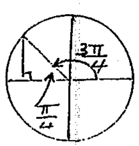
$= \frac{8\sqrt{3}}{3}$

$\therefore \underline{a = \frac{8}{3}}$

f) $\cos\frac{\pi}{6} + \sin\frac{3\pi}{4}$

$= \frac{\sqrt{3}}{2} + \sin\frac{\pi}{4}$

$= \underline{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}$



g) $x + 1 = |4 - 2x|$

$x+1 = 4 - 2x$ OR $x+1 = -(4-2x)$

$3x = 3$ OR $x+1 = -4+2x$

$x = 1$ $\quad 5 = x$

Checking

if $x = 1$ $\quad$ if $x = 5$

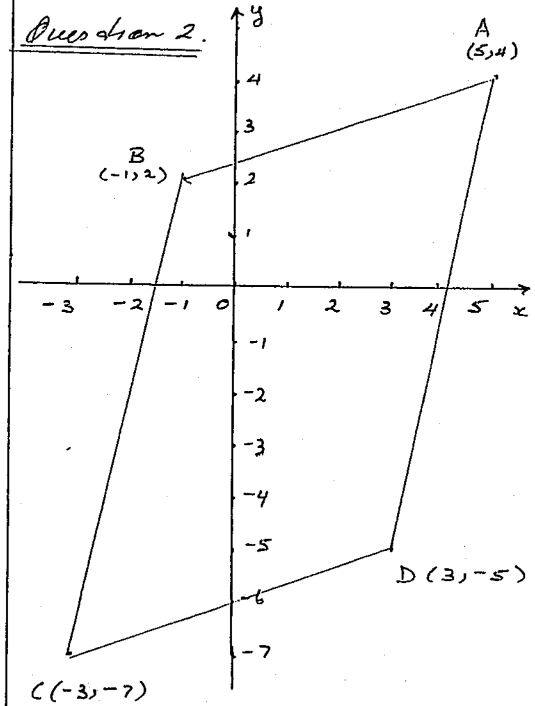
$1+1 = |4-2\times1|$ $\quad 5+1 = |4-2\times5|$

$2 = |4-2|$ $\quad 6 = |-6|$

True $\quad$ True

$\therefore \underline{x = 1 \text{ OR } x = 5}$

Question 2.



(i) $m_{AB} = \frac{4-2}{5-(-1)}$ $\quad m_{DC} = \frac{-5-(-7)}{3-(-3)}$

$= \frac{2}{6}$ $\quad = \frac{2}{6}$

$= \frac{1}{3}$ $\quad = \frac{1}{3}$

$\therefore m_{AB} = m_{DC}$

$\therefore \underline{AB \parallel DC}$

(ii) $d_{AB} = \sqrt{(5-(-1))^2 + (4-2)^2}$

$= \sqrt{6^2+2^2}$

$= \sqrt{40}$

$= \underline{2\sqrt{10}}$

$d_{DC} = \sqrt{(3-(-3))^2 + (-5-(-7))^2}$

$= \sqrt{6^2+2^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

$\therefore d_{AB} = d_{DC}$

$\therefore AB = DC$

(iii) Mid pt of AC

$= \frac{5+(-3)}{2}, \frac{4+(-7)}{2}$

$= \underline{(1, -\frac{3}{2})}$

Mid pt of BD

$= (\frac{-1+3}{2}, \frac{2+-5}{2})$

$= (1, -\frac{3}{2})$

$\therefore$ AC and BD have the same mid-pt.

(iv) ABCD is a //ogram

(α) because from (iii) the diagonals bisect each other

OR

(β) from (i) & (ii) $AB \parallel DC$ and $AB = DC$

i.e. one pair of opp. sides is both equal and parallel.

(v) Eqn of DC is

$y - y_1 = m(x - x_1)$

$y - (-5) = \frac{1}{3}(x - 3)$

$3y + 15 = x - 3$

$\underline{x - 3y - 18 = 0}$

(vi) $d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}} \right|$

$= \left| \frac{(-1) - 3(2) - 18}{\sqrt{1^2 + (-3)^2}} \right|$

$= \left| \frac{-1-6-18}{\sqrt{10}} \right|$

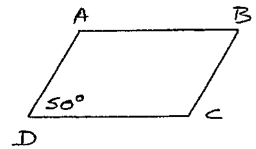
$= \underline{\frac{25}{\sqrt{10}}}$

(vii) Area = base × ⊥ height

$= 2\sqrt{10} \times \frac{25}{\sqrt{10}}$

$= \underline{50 \text{ unit}^2}.$

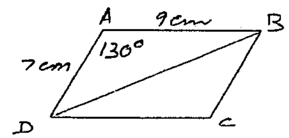
b)



$\angle D = 50°$

$\therefore \angle A = 130°$ since $\angle A + \angle D = 18$[0°]

co-int ∠'s $AB \parallel DC$

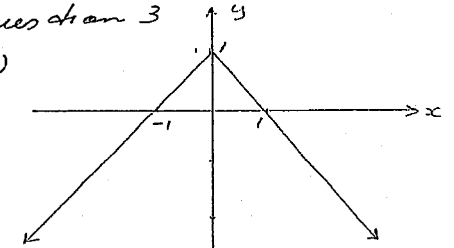


$DB^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \cos 130°$

$\underline{DB = (14.5 \text{ to 1 dec pl.})}$

Question 3

a)



b) Domain $3-2x \geq 0$

$3 \geq 2x$

$\frac{3}{2} \geq x$

Domain $x \leq \frac{3}{2}$

c) (i) $\frac{d}{dx}\left(\frac{e^{2x}}{x}\right) = \frac{x\,2e^{2x} - e^{2x}.1}{x^2}$

$= \frac{e^{2x}(2x-1)}{x^2}$

(ii) $\frac{d}{dx}(\sin 3x)^2$

$= 2(\sin 3x)^1 .\ 3\cos 3x$

$= 6\sin 3x \cos 3x$.

(iii) $\frac{d}{dx}\left(\ln (x^3-5)^7\right)$

$= \frac{d}{dx}\left(7 \ln(x^3-5)\right)$

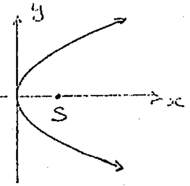
$= 7 . \frac{3x^2}{x^3-5}$

$= \frac{21x^2}{x^3-5}$

d) $\int \frac{4}{1+3x}\,dx = \frac{4}{3}\int \frac{3\,dx}{1+3x}$

$= \frac{4}{3}\ln(1+3x) + C$

e)



$y^2 = 8x$

$y^2 = 4ay$

$4a = 8$

$a = 2$

Focus $(2,0)$

Question 4

a) $\sum_{2}^{4} (2r-3)$

$= (2\times 2-3) + (2\times 3-3) + (2\times 4-3)$

$= 1 + 3 + 5$

$= 9.$

b) $\frac{d}{dx}\left(\frac{1}{x\sqrt{x}}\right) = \frac{d}{dx}\left(x^{-\frac{3}{2}}\right)$

$= -\frac{3}{2}x^{-\frac{5}{2}}$

$= \frac{-3}{2x^2\sqrt{x}}$

c) $f(-3) + f(-\frac{1}{3}) + f(3\frac{1}{2})$

$= -1 + 3(-\frac{1}{3}) + 2. + 7 - 2\times 3\frac{1}{2}$

$= -1 + 1 + 0$

$= 0$

d) $\frac{1}{1-\sin A} + \frac{1}{1+\sin A}$

$= \frac{1+\sin A + 1 - \sin A}{(1-\sin A)(1+\sin A)}$

$= \frac{2}{1-\sin^2 A}$

$= \frac{2}{\cos^2 A}$

$= 2\sec^2 A.$

e) $\int_0^{\pi/2} \sin 2x\,dx$

$= -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/2}$

$= -\frac{1}{2}\{\cos \pi - \cos 0\}$

$= -\frac{1}{2}\{-1-1\}$

$= -\frac{1}{2}\times(-2)$

$= 1.$

f) $ar = 6$ $\qquad \frac{a}{1-r} = 49$

$a = \frac{6}{r}$

$a = 49(1-r)$

Substitute $\frac{6}{r}$ for a in $a = 49(1-r)$

$\frac{6}{r} = 49(1-r)$

$6 = 49r - 49r^2$

$49r^2 - 49r + 6 = 0$

$(7r-1)(7r-6) = 0$

$\therefore r = \frac{1}{7}$ OR $r = \frac{6}{7}$

If $r = \frac{1}{7}$, $a\times\frac{1}{7} = 6$

$a = 42$

$\therefore$ Series is

$42, 6, \frac{6}{7}, \dots$

If $r = \frac{6}{7}$, $a\times\frac{6}{7} = 6$

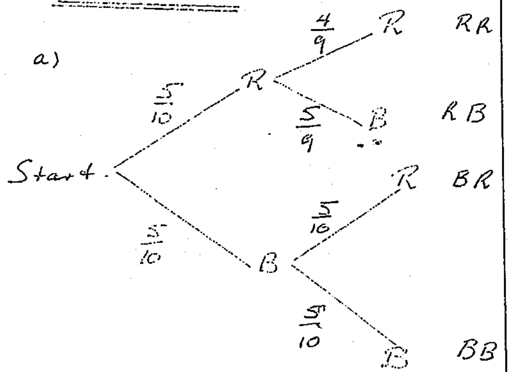
$a = 7.$

$\therefore$ Series is

$7, 6, \frac{36}{7}, \dots$

Question 5

a)



(i) $P(RR) = \frac{5}{10}\times\frac{4}{9}$

$= \frac{2}{9}$

(ii) P(one of each colour)

$= P(RB) + P(BR)$

$= \frac{5}{10}\times\frac{\ }{9} + \frac{5}{10}\times\frac{5}{10}$

$= \frac{25}{90} + \frac{25}{100}$

$= \frac{5}{18} + \frac{1}{4}$

$= \frac{10+9}{36}$

$= \frac{19}{36}$

b) $\int_2^a (2x+1)\,dx = 14$

$\left[x^2+x\right]_2^a = 14$

$a^2 + a - (2^2+2) = 14$

$a^2 + a - 6 = 14$

$a^2 + a - 20 = 0$

$(a+5)(a-4) = 0$

$a = -5$ OR $a = 4$.

c) (i) $\angle A = \angle CBX$ (corr. ∠'s DA//CB)

$\angle A = \angle YDC$ (corr. ∠'s DC//AB)

$\therefore \angle CBX = \angle YDC$

(ii)

In $\triangle DCY$ and $\triangle BXC$

(i) $\angle YDC = \angle CBX$ (proven above)

(ii) $\angle DCY = \angle BXC$ (corr. ∠'s DC//AX)

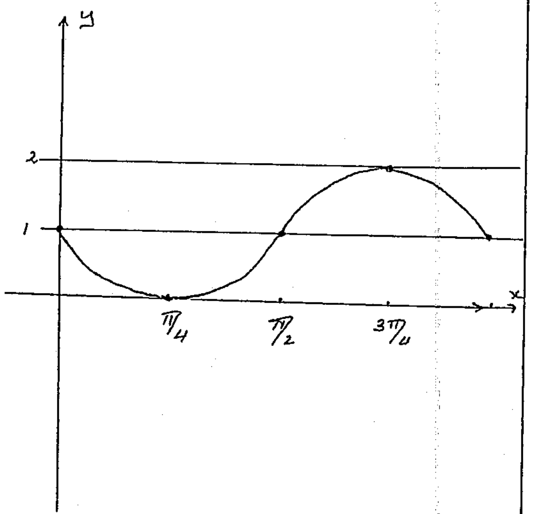
$\therefore \triangle DCY\ |||\ \triangle BXC$ (equiangular)

$\therefore \frac{XB}{DC} = \frac{BC}{DY}$ (corr sides of ||| △'s)

But AB = DC and BC = AD

opp. side of //ogram

$\therefore \frac{XB}{AB} = \frac{AD}{DY}$



x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sin 2x$	0	1	0	−1	0
$-\sin 2x$	0	−1	0	1	0
$-\sin 2x+1$	1	0	1	2	1

Question 6.

a) $y = 2x+9$

$y = x^2+2x$

$\therefore\ x^2+2x = 2x+9$

$x^2 = 9$

$x = \pm 3$

$\therefore$ A is $(3, 15)$

B is $(-3, 3)$

b) $A = \left| \int_{-3}^{3} \{x^2+2x-(2x+9)\}\,dx \right|$

$= \left| \int_{-3}^{3} (x^2-9)\,dx \right|$

$= \left[\frac{x^3}{3} - 9x \right]_{-3}^{3}$

$= \left| \left(\frac{27}{3} - 27\right) - \left(\frac{-27}{3}+27\right) \right|$

$= \left| 9-27-(-9+27) \right|$

$= |-18-18|$

$= |-36|$

$= 36$ unit^2.

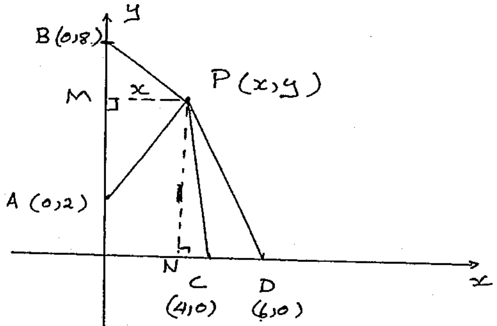
OR since x^2-9 is an even function

$\left| \int_{-3}^{3} (x^2-9)\,dx \right|$

$= 2 \left| \int_{0}^{3} (x^2-9)\,dx \right|$

$= 2 \times 18$

$= 36$ square units.



Area $\triangle PAB$ = Area $\triangle PCD$

$\frac{1}{2} AB \times MP = \frac{1}{2} CD \times PN$

$\frac{1}{2} \times 6 \times |x| = \frac{1}{2} \times 2 \times |y|$

$3|x| = |y|$

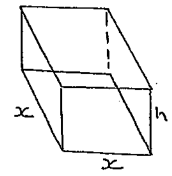
$\therefore\ y = \pm 3x$

Note P could be in any of the 4 quadrants

c)

(i) $x \cdot x \cdot h = 8$

$h = \frac{8}{x^2}$



(ii)

$A = 2x^2 + 4xh$

$= 2x^2 + 4x \times \frac{8}{x^2}$

$= 2x^2 + \frac{32}{x}$

$= 2x^2 + 32x^{-1}$

$\frac{dA}{dx} = 4x - 32x^{-2}$

$= 4x - \frac{32}{x^2}$

Stationary points occur when $\frac{dA}{dx} = 0$

i.e. $4x - \frac{32}{x^2} = 0$

$4x = \frac{32}{x^2}$

$4x^3 = 32$

$x^3 = 8$

$x = 2$

$\frac{d^2A}{dx^2} = 4 + 64x^{-3}$

$= 4 + \frac{64}{x^3}$

at $x = 2$, $\frac{d^2A}{dx^2} = 4 + \frac{64}{8}$

> 0

$\therefore$ A is a minimum when $x = 2$

If $x = 2$, $A = 2\times 2^2 + \frac{32}{2}$

$= 8+16$

$= 24$

Minimum Area $= 24\ m^2$

Question 7.

a) $V = \pi \int_{-3}^{0} y^2\,dx$

$= \pi \int_{-3}^{0} x^2(x+3)\,dx$

$= \pi \int_{-3}^{0} (x^3+3x^2)\,dx$

$= \pi \left[\frac{x^4}{4} + x^3 \right]_{-3}^{0}$

$= \pi \left\{ (0+0) - \left(\frac{(-3)^4}{4} + (-3)^3 \right) \right\}$

$= \pi \left\{ -\left(\frac{81}{4} - 27 \right) \right\}$

$= \pi \left\{ -\frac{81}{4} + 27 \right\}$

$= \pi \left(\frac{-81+108}{4} \right)$

$= \frac{27\pi}{4}$ cubic units

b) Area $\doteq \frac{h}{2} \{ y_0 + y_n + 2y_{res}$

Area $\doteq \frac{h}{2} \{ y_0 + y_4 + 2(y_1+y_2+y_3$

$= \frac{250}{2} \{ 0+0+2(231+215+235$

$= 170250\ m^2$

$= 17.025$ ha.

c) $4x - 2y - 7 = 0$

$4x - 7 = 2y$

$y = 2x - \frac{7}{2}$

gradient of line $= 2$

$y = \frac{1}{2}x^2 - 3x + 2$

$\frac{dy}{dx} = x - 3$

We want $x - 3 = 2$

$x = 5$

If $x = 5$, $y = -\frac{1}{2}$

$\therefore$ Point is $(5, -\frac{1}{2})$

d) Possible inflexion when $\frac{d^2y}{dx^2} = 0$

$\therefore (x-2)^2(x-3) = 0$

$\therefore x-2=0$ OR $x-3=0$

$x = 2$ OR $x = 3$

at $x = 1\cdot 9$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

at $x = 2\cdot 1$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

No change in concavity

$\therefore$ No inflexion

at $x = 2\cdot 9$, $\frac{d^2y}{dx^2} = (+)(-) < 0$

at $x = 3\cdot 1$, $\frac{d^2y}{dx^2} = (+)(+) > 0$

Change in concavity

$\therefore$ inflexion at $x = 3$

$\therefore$ Only one inflexion at $x = 3$.

Question 8.

a)

(i)

at $t=0$

$V = 5000\left(1-\frac{0}{40}\right)^2$

$= 5000$ litres

(ii) If the tank is empty $V=0$

$\therefore 0 = 5000\left(1-\frac{t}{40}\right)^2$

$1-\frac{t}{40} = 0$

$40 = t$

$\therefore$ the tank will be empty when $t = 40$

(iii) $V = 5000\left(1-\frac{t}{40}\right)^2$

$\frac{dV}{dt} = 5000 \times 2\left(1-\frac{t}{40}\right)\times\left(\frac{-1}{40}\right)$

$= -250\left(1-\frac{t}{40}\right)$

at $t = 10$

$\frac{dV}{dt} = -250\left(1-\frac{10}{40}\right)$

$= -250 \times \frac{3}{4}$

$= 187\cdot 5$ litres/minute

b) $x = 8e^{-2t} - 8 + 16t$

(i)

$v = \frac{dx}{dt}$

$= 8(-2e^{-2t}) + 16$

$= -16e^{-2t} + 16$

at $t=0$

$v = -16e^0 + 16$

$= -16+16$

$= 0$

$\therefore$ particle is at rest when $t=0$

(ii) $\lim_{t\to\infty}(-16e^{-2t} + 16)$

$= -16\times 0 + 16$

$= 16$ cm/sec

(iii) $a = \frac{dv}{dt}$

$= -16(-2e^{-2t})$

$= 32e^{-2t}$

But $v = -16e^{-2t} + 16$

$e^{-2t} = \frac{16-v}{16}$

$\therefore a = 32\left(\frac{16-v}{16}\right) \Rightarrow a = 32-2v$

c) $D = 5 + \left(\frac{40}{4+t}\right)^2$

$\frac{dN}{dt} = 5 + 1600(4+t)^{-2}$

$N = \int\{5 + 1600(4+t)^{-2}\}dt$

$N = 5t + 1600\frac{(4+t)^{-1}}{-1} + c$

$N = 5t - \frac{1600}{(4+t)} + c$

at $t=0$, $N = 20$

$20 = 0 - \frac{1600}{4} + c$

$20 + 400 = c$

$c = 420$

$N = 5t - \frac{1600}{(4+t)} + 420$

If $t = 10$

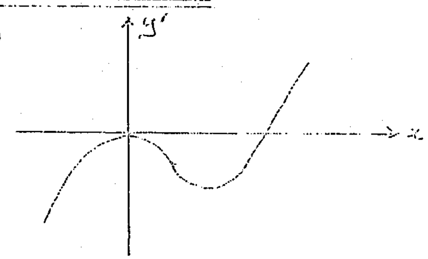
$N = 5\times 10 - \frac{1600}{14} + 420$

$\doteqdot 356$

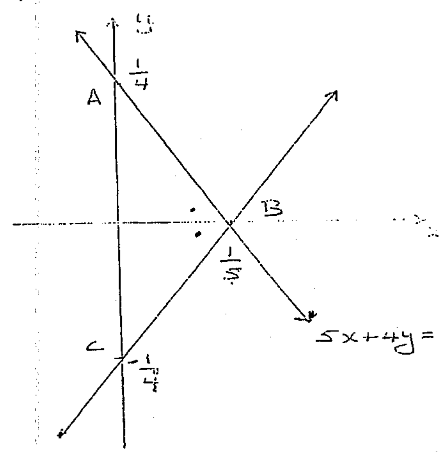
Approximately 356 people will have the disease after 10 days.

Question 9.

a)



b)



Equ of BC is $\frac{x}{a} + \frac{y}{b} = 1$

i.e. $\frac{x}{\left(\frac{1}{5}\right)} + \frac{y}{\left(-\frac{1}{4}\right)} = 1$

$5x - 4y = 1.$

c) $T_9 + T_{10} + T_{11} = 57$

$a+8d + a+9d + a+10d = 57$

$3a + 27d = 57$

$a + 9d = 19$

$T_{17} + T_{18} + T_{19} = 105$

$a+16d + a+17d + a+18d = 105$

$3a + 51d = 105$

$a + 17d = 35$

Solve $a+17d = 35$ and $a+9d = 19$

$8d = 16$

$d = 2$

$\therefore a + 9\times 2 = 19$

$a = 1$

$T_2 = a + d$

$= 1+2$

$= 3.$

d) $r = \frac{6}{12}\%$

$= 0.5\%$

$A_1 = 240\,000 + 0.5\% \text{ of } 240000 - M$

$= 240000(1+0.005) - M$

$= 240000 \times 1.005 - M$

$A_2 = A_1 + 0.5\% \text{ of } A_1 - M$

$= A_1(1+0.005) - M$

$= A_1 \times 1.005 - M$

$= (240000 \times 1.005 - M) \times 1.005 - M$

$= 240000 \times 1.005^2 - 1.005M - M$

$= 240\,000 \times 1.005^2 - M(1+1.005)$

$A_3 = A_2 \times 1.005 - M$

$= 240\,000 \times 1.005^3 - M(1+1.005+1.005^2)$

$A_n = 240000 \times 1.005^n - M(1+1.005+1.005^2+\ldots \ldots 1.005^{n-1})$

$= 240000 \times 1.005^n - M \times 1\frac{(1.005^{240}-1)}{(1.005-1)}$

$= 240000 \times 1.005^n - M \times \frac{(1.005^{240}-1)}{0.005}$

$= 240000 \times 1.005^n - M \times 200(1.005^{240}-1)$

Note $0.005 = \frac{5}{1000} = \frac{1}{200}$

If $A_n = 0$

then

$200M(1.005^{240}-1) = 240000 \times 1.005^{240}$

$M = \frac{240\,000 \times 1.005^{240}}{200 \times (1.005^{240}-1)}$

$= \frac{1200 \times 1.005^{240}}{(1.005^{240}-1)}$

(iii) $M = \$1719.43$

Question 10

a) $\alpha + \beta = -\frac{b}{a} = -3$ $\qquad \alpha\beta = \frac{c}{a} = -2$

b) Equation with roots α^2 & β^2 is

$x^2 - (\alpha^2+\beta^2)x + \alpha^2\beta^2 = 0$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-3)^2 - 2(-2)$

$= 9+4$

$= 13$

$\alpha^2\beta^2 = (\alpha\beta)^2$

$= (-2)^2$

$= 4$

$\therefore$ Required equation is

$x^2 - 13x + 4 = 0$

b) from (x_1, y_1) to $7x - y + 9 = 0$

$D_1 = \left|\frac{7x_1 - y_1 + 9}{\sqrt{7^2+(-1)^2}}\right|$

$D_1 = \left|\frac{7x_1 - y_1 + 9}{\sqrt{50}}\right|$

from (x_1, y_1) to

$x + y - 1 = 0$

$D_2 = \left|\frac{x_1 + y_1 - 1}{\sqrt{1^2+1^2}}\right|$

$= \left|\frac{x_1 + y_1 - 1}{\sqrt{2}}\right|$

Required locus is such that the perpendicular distance from (x, y) to both lines is equal

$\therefore \left|\frac{7x-y+9}{5\sqrt{2}}\right| = \left|\frac{x+y-1}{\sqrt{2}}\right|$

$\therefore \frac{7x-y+9}{5\sqrt{2}} = \frac{x+y-1}{\sqrt{2}}$

$7x - y + 9 = 5x + 5y - 5$

$2x - 6y + 14 = 0$

$x - 3y + 7 = 0$

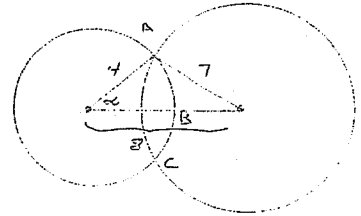
OR $\frac{7x-y+9}{5\sqrt{2}} = -\frac{(x+y-1)}{\sqrt{2}}$

$7x - y + 9 = -5x - 5y + 5$

$12x + 4y + 4 = 0$

$3x + y + 1 = 0$

c)



$\cos\alpha = \frac{4^2+8^2-7^2}{2 \times 4 \times 8}$

$\alpha \doteqdot 1.065^c$ $\quad (\doteqdot 61°2')$

$\therefore$ Arc $ABC = r\theta$

$= 4 \times 2\alpha$

$= 4 \times (2 \times 1.065^c)$

$= 8.52$ cm