Gosford High School

# 2014

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

# Mathematics

- **General Instructions**
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

**Total Marks – 100**

**Section I** Pages 2 – 5

**10 marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

**Section II** Pages 6 – 15

**90 marks**

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

ection I

**10 marks**

**Attempt Questions 1 – 10.**

**Allow about 15 minutes for this section.**

Use the multiple-choice answer sheet for Questions 1 – 10.

1. An engine under development contains three components that are prone to failure. The probilities that each of the components will fail are $\frac{1}{10}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively. What is the probability that at least one part will fail?

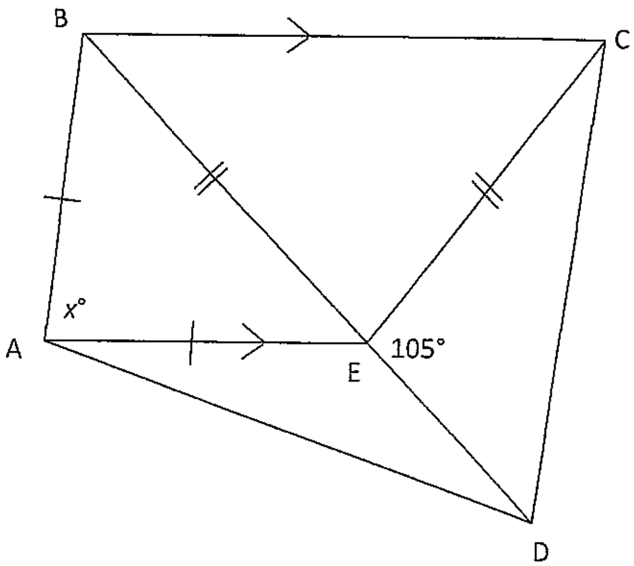
(A) $\frac{23}{50}$

(B) $\frac{1}{200}$

(C) $\frac{3}{100}$

(D) $\frac{11}{20}$

2. The vertices of quadrilateral ABCD meet at E such that $BC||AE$, BE is produced to D. $\angle CED = 105°$, $BE = CE$ and $AB = AE$. Determine the size of $x$.




(A) 105°

(B) 85°

(C) 75°

(D) 52.5°

3. Fully simplify the algebraic fraction: $\frac{x^3 - 8}{x^2 - 4}$.

(A) $\frac{x^2 - 2x + 4}{x - 2}$

(B) $x + 2$

(C) $\frac{x^2 + 4x + 4}{x + 2}$

(D) $\frac{x^2 + 2x + 4}{x + 2}$

4. What is the value of $\frac{dy}{dx}$ if $y = 4\sqrt{x}$ ?

(A) $\frac{dy}{dx} = 4$

(B) $\frac{dy}{dx} = 2\sqrt{x}$

(C) $\frac{dy}{dx} = \frac{2}{\sqrt{x}}$

(D) $\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$

5. During a two year study, the number of people using public transport increased by 6% in the first year and in the following year the number increased by a further 14% to 680 000. The number of people using public transport at the beginning of the study was approximately:

(A) 8 095

(B) 549 712

(C) 562 728

(D) 544 000

6. The quadratic function $3x^2 - 6x - 7$ has roots $\propto$ and $\beta$.

What is the value of $\propto^2 + \beta^2$?

(A) $\frac{26}{3}$

(B) $-\frac{7}{3}$

(C) 4

(D) 45

7. The primitive function of $\sec^2 5x + 1$ is:

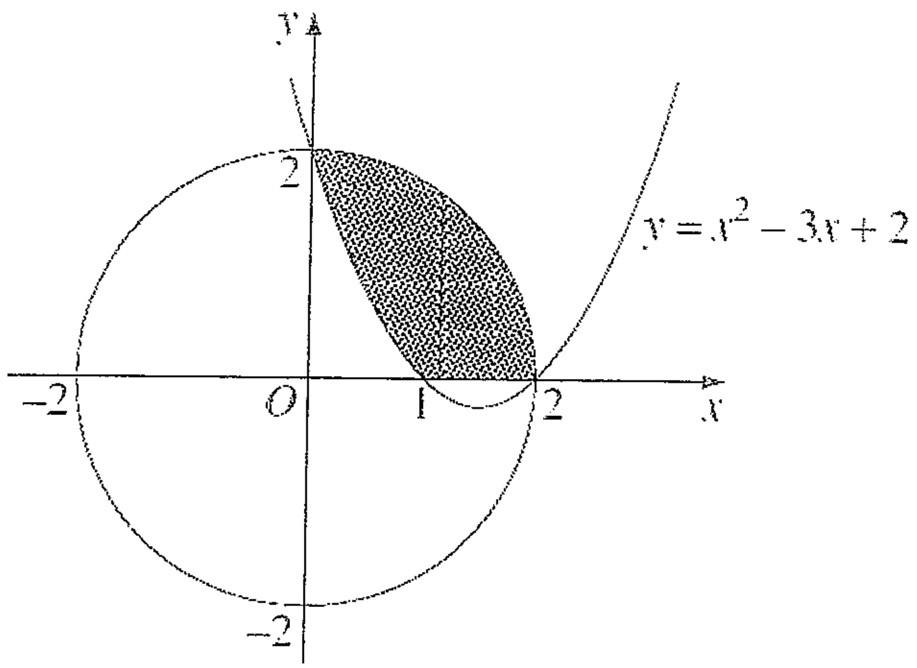
(A) $\frac{1}{5}\tan(5x) + x + C$

(B) $\frac{1}{5}\tan(x) + 5x + C$

(C) $\tan(5x) + x + C$

(D) $\tan(5x) + C$

8. Which set of inequalities best describes the shaded region.



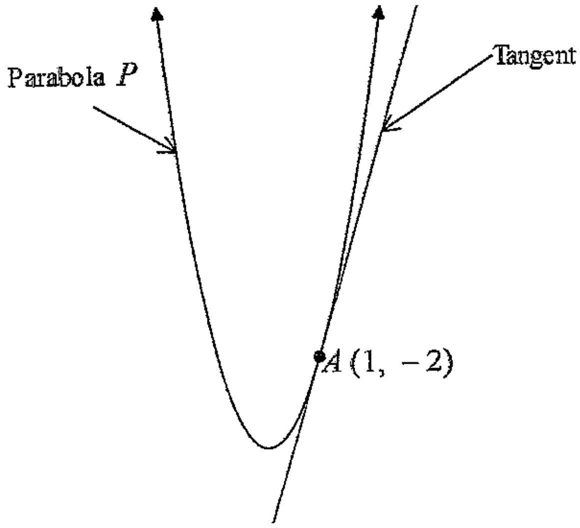

(A) $x^2 + y^2 \leq 2$ and $y \geq x^2 - 3x + 2$ and $y \geq 0$

(B) $x^2 + y^2 \leq 4$ and $y \leq x^2 - 3x + 2$ and $x \geq 0$

(C) $x^2 + y^2 \leq 2$ and $y \leq x^2 - 3x + 2$ and $y \geq 0$

(D) $x^2 + y^2 \leq 4$ and $y \geq x^2 - 3x + 2$ and $x \geq 0$

9. The diagram shows the parabola $P$ and its tangent at the point $A(1, -2)$.




Which of the following equations might represent the normal to the parabola at the point $A$?

(A) $x - 3y + 5 = 0$

(B) $2x - 3y + 1 = 0$

(C) $x + 3y + 5 = 0$

(D) $x + 3y - 5 = 0$

10. Differentiate $\log_e\left(\frac{x-5}{x+5}\right)$

(A) $\frac{25}{x^2-25}$

(B) $\frac{10}{x^2-25}$

(C) $\frac{1}{x-5} + \frac{1}{x+5}$

(D) $\frac{1}{x+5} - \frac{1}{x-5}$

## Section II

**90 marks**

**Attempt Questions 11 – 16.**

**Allow about 2hours and 45 minutes for this section.**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

**Question 11** (15 marks) Start on a new page.

(a) Evaluate $\log_{10} 12$ correct to 3 significant figures. **1**

(b) Solve $|2x - 3| = 4 - 3x$ **2**

(c) Differentiate the following functions with respect to $x$:

(i) $3xe^x$ **1**

(ii) $\frac{1}{(x^2+5)^3}$ **1**

(iii) $\frac{x^2+4}{3x-1}$ **1**

(d) If $f'(x) = 6x^2 + 5x - 1$ and $f(-1) = 5$ , find an expression for $f(x)$ . **2**

(e) A particle moves so that its displacement from the origin is given by:

$x = -t^2 + 7t + 8$ (where $x$ is displacement in metres and $t$ is time in seconds)

(i) What is the initial displacement of the particle? **1**

(ii) At what time will the particle be at the origin? **2**

(f) Consider the arithmetic sequence beginning with $-7, -3, 1 \ldots$:

(i) Find the $25^{th}$ term in the sequence. **1**

(ii) Show that the sum of the first 20 terms is 620 **1**

(iii) How many terms must be taken to give a sum of 221? **2**

**End of Question 11**

**Question 12** (15 marks) Start on a new page.

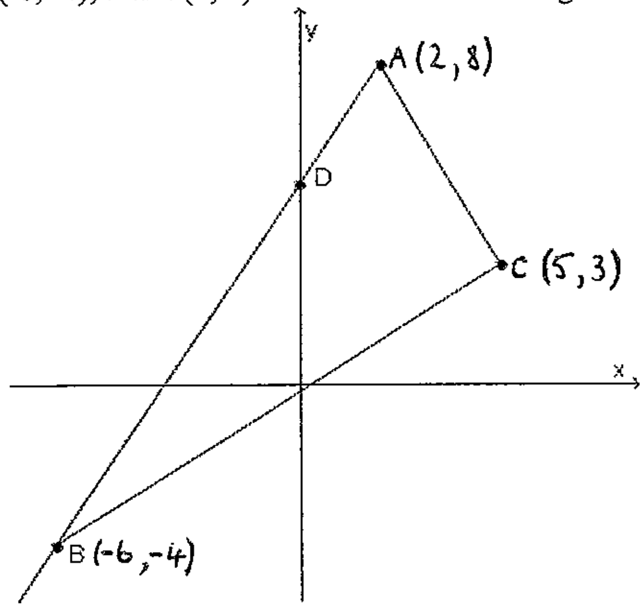
(a) Sammy sailed 15km south from point A to point B. He then sailed due west to C where he was on a bearing of $210°$ from A.

(i) Draw a diagram showing this information. **1**

(ii) Determine the exact distance AC **2**

(iii) Sammy then sailed NW for 8km to D. What is the distance from A to D correct to the nearest 100m. **2**

(b) The points $A(2, 8)$, $B(-6, -4)$, and $C(5, 3)$ are the vertices of triangle $ABC$.




(i) Find the equation of the line passing through $A$ and $B$. **2**

(ii) Find the coordinates of the point D. **1**

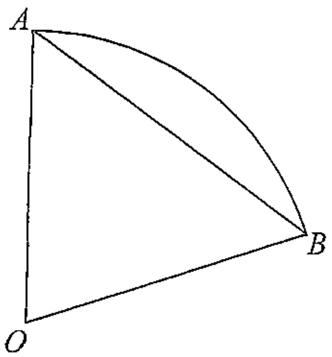
(iii) Find the perpendicular distance from point $C$ to the line $AB$. **2**

(iv) Find the exact distance between A and B. **2**

(v) Calculate the area of $\Delta ABC$. **1**

**Question 12 continues on page 9**

(c) Sector AOB has arc length $AB = 2\pi$ units and radii $OA = OB = 6$ units.




(i) Determine the size, in radians, of angle $A\hat{O}B$. 1

(ii) Show that the area of $\Delta AOB$ is $9\sqrt{3}$ square units. 1

**End of Question 12**

**Question 13** (15 marks) Start on a new page.

(a) A function is given by $f(x) = -x^3 + \frac{3x^2}{2} + 6x$

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion of $f(x)$. **1**

(iii) Hence, sketch the graph $y = f(x)$ showing the stationary points, point of inflexion and $y$ intercept. **3**

(iv) For what values of $x$ is the function decreasing? **1**

(b) Consider the function $f(x) = 4\sin 2\pi x$

(i) Determine the period and amplitude of $f(x)$ **2**

(ii) Determine the values of $x$ for which $f(x)$ is at its maximum in the domain $0 \le x \le 2$ **2**

(iii) Sketch $f(x)$ for $0 \le x \le 2$. **1**

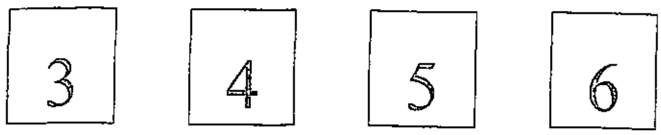
(c) Evaluate: $\lim\limits_{x \to 2} \frac{3x^2-12}{x-2}$ **2**

**End of Question 13**

**Question 14** (15 marks) Start on a new page.

(a) Use Simpson's rule to approximate $\int_1^5 \frac{dx}{x^2+1}$, using 4 sub-intervals to 4 decimal places. 3

(b) Four cards, numbered 3, 4, 5 and 6 are used in a game.




The four cards are placed face down and each player pays \$1 to take a turn to draw two cards, one at a time without replacement. The two cards make a 2-digit number, with the first card drawn being the first digit of their number.

If the cards form a number over 60, the player receives \$3 back (i.e. they win \$2), otherwise they receive nothing back.

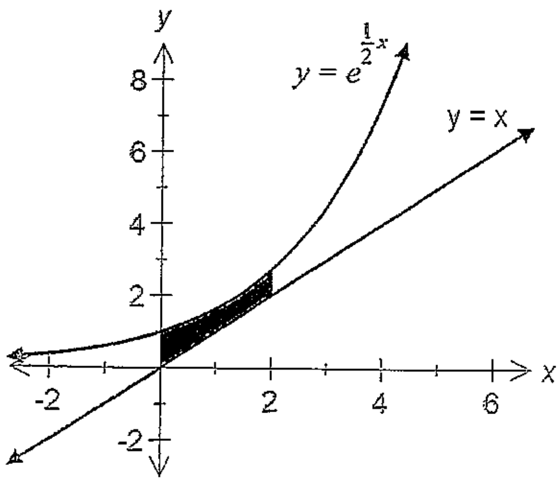
The four cards are then shuffled and replaced for the next turn.

(i) Use a diagram to show all of the possible 2-digit numbers that could be drawn. 2

(ii) What is the probability that a player will win on their first turn? 1

(iii) Calculate the probability that a player who brings \$5 to play will have \$3 left after 5 turns? 2

(c) The diagram shows the graphs of the functions $y = e^{\frac{1}{2}x}$ and $y = x$. The region between these 2 functions and the bounds $x = 0$ and $x = 2$ has been shaded. 3




Calculate the exact area of the shaded region.

**Question 14 continues on page 12**

**Question 14 (continued)**

(d) For the parabola with equation $16y = x^2 - 4x - 12$ :

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

**End of Question 14**

**Question 15** (15 marks) Start on a new page.

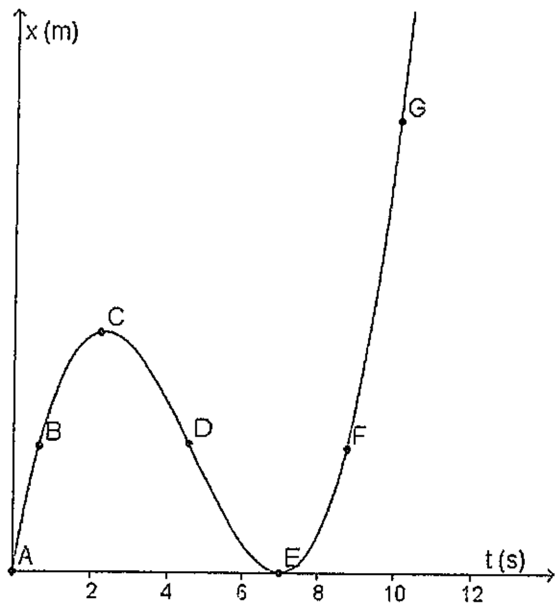
(a) The population of New South Wales in 2009 was 7.13 million. In 2012 the population had grown to approximately 7.3 million people.

(i) Assuming that the growth rate is proportional to the population, show that the annual growth rate is approximately 0.79%. **2**

(ii) Calculate the expected population of New South Wales in 2019 using this model. Give your answer rounded to the nearest hundred thousand. **1**

(iii) In what year will the population exceed 10 million? **2**

(b) The graph shows the displacement ($x$ metres) of a particle at time ($t$ seconds). The particle is moving horizontally.




(i) At which point, during the first 8 seconds is the particle at its maximum distance from the origin? **1**

(ii) Describe the motion of the particle at 4 seconds in terms of its displacement and velocity. **2**

(iii) By referring to the points marked on the graph, between what times is the acceleration of the particle negative? What feature of the graph tells us this? **2**

**Question 15 continues on page 14**

**Question 15 (continued)**

(c) Find the solutions to the equation: **3**

$$4\cos^2\theta = 6\sin\theta + 6 \text{ in the domain } 0 \leq \theta \leq 2\pi$$

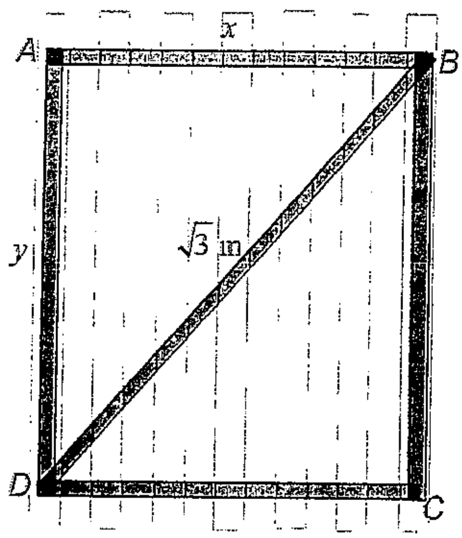
(d) Show that $\dfrac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ for all integers $n \geq 1$. **2**

**End of Question 15**

Question 16 (15 marks) Start on a new page.

(a) A swinging gate is to be constructed from timber palings. It will require a support frame using 5 pieces of timber: $AB$, $AD$, $BD$, $BC$ and $CD$.

$AB \parallel CD$ and $AD \parallel BC$. $AB = CD = x$ metres. $AD = BC = y$ metres. $BD$ is $\sqrt{3}$ metres long.




(i) Find an expression for $y$ in terms of $x$. **1**

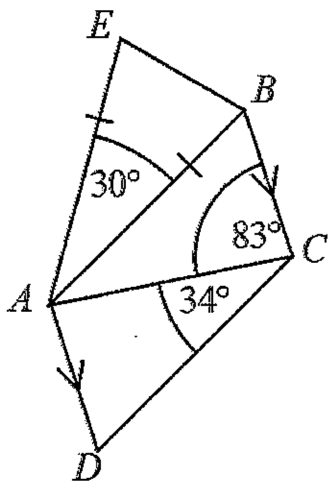
(ii) Show that the *total length* ($L$) of the timber pieces in the support frame is represented by $L = 2\left(x + \sqrt{3 - x^2} + \frac{\sqrt{3}}{2}\right)$. **1**

(iii) The gate will have its maximum strength when the *total length* ($L$) of its support frame is maximised. For what value of $x$ will the gate have maximum strength? Full working must be shown. **4**

**Question 16 continues on page 16**

**Question 16 (continued)**

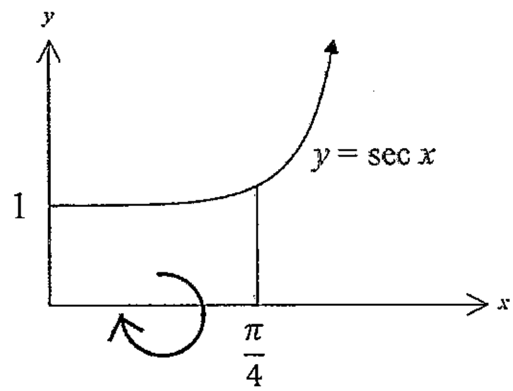
(b) In the diagram below: AD||BC , AE = AB, $\angle BAE = 30^\circ$, $\angle BCA = 83^\circ$, $\angle ACD = 34^\circ$, $\angle EBC = 138^\circ$.




(i) Prove that $AB \parallel DC$. **3**

(ii) Prove that $\Delta ABC \equiv \Delta ACD$. **3**

(c) The area bounded by the function $y = \sec x$, the $y$-axis and the line $x = \frac{\pi}{4}$ is rotated about the $x$-axis.




Find the volume of the solid formed. **3**

**End of Paper**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

## Trial HSC Examination 2014

## Mathematics Course

Name ______________________ Teacher ________________

## Section I – Multiple Choice Answer Sheet

**Allow about 15 minutes for this section**

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

# 2014 Trial HSC – Mathematics — Solutions

1. $P(\geq 1 \text{ fail}) = 1 - P(0 \text{ Fail})$

$= 1 - \left(\frac{9}{10} \times \frac{4}{5} \times \frac{3}{4}\right)$

$= 1 - \frac{27}{50}$

$= \frac{23}{50}$ (A)

2. $\angle EBC = \angle ECB$ (Equal base angles of isosceles $\triangle EBC$ opposite equal sides)

$\therefore 2\angle EBC = 105$ (Exterior angle of $\triangle EBC$)

$\angle EBC = 52 \cdot 5^\circ$

$\angle AEB = \angle CBE$ (Alternate angles on $BC \parallel AE$ are equal)

$= 52 \cdot 5^\circ$

$\angle ABE = \angle AEB$ (Equal base angles of isosceles $\triangle ABE$ opposite equal sides).

$= 52 \cdot 5^\circ$

$x + 52 \cdot 5 + 52 \cdot 5 = 180$ (Angle sum of $\triangle$)

$x = 75^\circ$ (C)

3. $\frac{x^3 - 8}{x^2 - 4} = \frac{(x-2)(x^2 + 2x + 4)}{(x+2)(x-2)}$

$= \frac{x^2 + 2x + 4}{x+2}$ (D)

4. $y = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} \times 4x^{-\frac{1}{2}}$

$= \frac{2}{\sqrt{x}}$ (C)

5. Number $\times 1.06 \times 1.14 = 680\,000$

number $= \frac{680\,000}{1.06 \times 1.14}$

$= 562727.57\ldots$

$\doteqdot 562728$ (C)

6. $3x^2 - 6x - 7$

$\alpha + \beta = -\frac{b}{a}$ $\qquad \alpha\beta = \frac{c}{a}$

$= -\frac{-6}{3}$ $\qquad = \frac{-7}{3}$

$= 2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2\left(\frac{-7}{3}\right)$

$= 4 + \frac{14}{3}$

$= \frac{26}{3}$ (A)

7. $\int \sec^2 5x + 1 \, dx$

$= \frac{1}{5} \tan 5x + x + c$ (A)

8. (D)

9. (C)

10. $\frac{d}{dx} \ln\left(\frac{x-5}{x+5}\right) = \frac{d}{dx} \ln(x-5) - \ln(x+5)$

$= \frac{1}{x-5} - \frac{1}{x+5}$

$= \frac{x+5 - x+5}{(x-5)(x+5)} = \frac{10}{\ldots}$ (B)

11

a) $\log_{10} 12 = 1.07918\ldots$

$= 1.08$ (3 sig. figs).

b) $|2x-3| = 4-3x$

$2x-3 = 4-3x$ or $-(2x-3) = 4-3x$

$5x = 7$ $\quad$ $-2x+3 = 4-3x$

$x = \frac{7}{5}$ $\quad$ $x = 1$

Check $LHS = |2(\frac{7}{5})-3|$ $\quad$ $LHS = |2(1)-3|$

$= \frac{1}{5}$ $\quad$ $= 1$

$RHS = 4-3(\frac{7}{5})$ $\quad$ $RHS = 4-3(1)$

$= -\frac{1}{5}$ $\quad$ $= 1$

$LHS \neq RHS$ $\quad$ $LHS = RHS$.

$\therefore x = 1$ is the only solution.

c) (i) $\frac{d}{dx}(3xe^x) = 3xe^x + 3e^x$

$= 3e^x(x+1)$

(ii) $\frac{d}{dx}\left(\frac{1}{(x^2+5)^3}\right) = \frac{d}{dx}(x^2+5)^{-3}$

$= -3(x^2+5)^{-4} \times 2x$

$= \frac{-6x}{(x^2+5)^4}$

(iii) $\frac{d}{dx}\left(\frac{x^2+4}{3x-1}\right) = \frac{(3x-1)\times 2x - 3(x^2+4)}{(3x-1)^2}$

$= \frac{6x^2-2x-3x^2-12}{(3x-1)^2}$

$= \frac{3x^2-2x-12}{(3x-1)^2}$

d) $f'(x) = 6x^2+5x-1$

$f(x) = \frac{6x^3}{3} + \frac{5x^2}{2} - x + c$

$= 2x^3 + \frac{5}{2}x^2 - x + c$

$f(-1) = 5$

$5 = 2(-1)^3 + \frac{5}{2}(-1)^2 - (-1) + c$

$5 = -2 + \frac{5}{2} + 1 + c$

$5 = 1\frac{1}{2} + c$

$c = 3\frac{1}{2}$

$\therefore f(x) = 2x^3 + \frac{5}{2}x^2 - x + \frac{7}{2}$

e) (i) Initial displacement $t=0$

$x = -(0)^2 + 7(0) + 8$

$x = 8$

(ii) Particle at origin $\Rightarrow x = 0$

$0 = -t^2 + 7t + 8$

$0 = t^2 - 7t - 8$

$0 = (t-8)(t+1)$

$t = -1, 8$

But $t > 0$ $\therefore t = 8$ seconds.

f) $a = -7$

$d = 4$

(i) $T_n = a + (n-1)d$

$T_{25} = -7 + (25-1) \times 4$

$= 89$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(-7) + (20-1)(4)]$

$= 620$

(iii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$221 = \frac{n}{2}[2(-7) + (n-1)(4)]$

$442 = n(-14 + 4n - 4)$

$442 = 4n^2 - 18n$

$4n^2 - 18n - 442 = 0$

$2n^2 - 9n - 221 = 0$

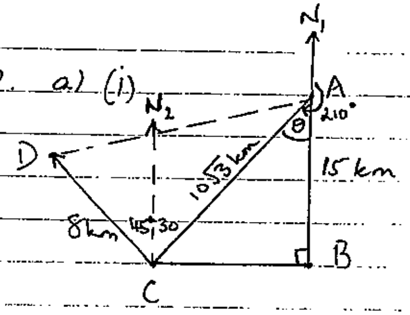
$(n-13)(2n+17) = 0$

$n = 13, -\frac{17}{2}$

but $n > 0$

$\therefore n = 13.$

12. a) (i)




(ii) Let $\theta = \angle CAB$

$\theta + 180 = 210°$

$\theta = 30°$

$\cos 30 = \frac{15}{AC}$

$\frac{\sqrt{3}}{2} = \frac{15}{AC}$

$AC = \frac{2 \times 15}{\sqrt{3}}$

$= \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{30\sqrt{3}}{3}$

$= 10\sqrt{3}$ km

(iii) $\angle N_2CA = \angle BAC = 30°$ (Alternate angles on parallel north lines)

$\angle DCA = 30° + 45°$ (D is NW of C)

$= 75°$

$DA^2 = 8^2 + (10\sqrt{3})^2 - 2(8)(10\sqrt{3})\cos 75$

$= 292.2739...$

$DA = 17.0960...$

$\doteqdot 17.1$ km (nearest 100 m)

b) (i) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$    $A(2,8)$ $B(-6,-4)$

$y - 8 = \frac{-4-8}{-6-2}(x-2)$

$y - 8 = \frac{3}{2}(x-2)$

$2y - 16 = 3x - 6$

$3x - 2y + 10 = 0$

(ii) D: $x = 0$

$3(0) - 2y + 10 = 0$

$2y = 10$

$y = 5$

$\therefore D(0,5)$

(iii) $C(5,3)$, $3x - 2y + 10 = 0$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|3\times5 + (-2)\times3 + 10|}{\sqrt{3^2+(-2)^2}}$

$= \frac{19}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$

$= \frac{19\sqrt{13}}{13}$ units

(iv) $d = \sqrt{(2--6)^2 + (8--4)^2}$

$= \sqrt{8^2 + 12^2}$

$= \sqrt{208}$

$= 4\sqrt{13}$ units.

(v) $A = \frac{1}{2} \times 4\sqrt{13} \times \frac{19\sqrt{13}}{13}$

$= \frac{2 \times 19 \times 13}{13}$

$= 38\ u^2$

c) (i) $l = r\theta$

$2\pi = 6\theta$

$\theta = \frac{\pi}{3}^c$

(ii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 6 \times 6 \times \sin\frac{\pi}{3}$

$= 18 \times \frac{\sqrt{3}}{2}$

$= 9\sqrt{3}\ u^2$

13. a (i) $f(x) = -x^3 + \frac{3x^2}{2} + 6x$

$f'(x) = -3x^2 + 3x + 6$

$f''(x) = -6x + 3$

Stationary points $f'(x) = 0$

$0 = -3x^2 + 3x + 6$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$\therefore x = -1, 2$

At $x = -1$, $y = -(-1)^3 + \frac{3(-1)^2}{2} + 6(-1)$

$= -3.5$

$f''(x) = -6(-1) + 3$

$> 0$

$\therefore (-1, -3.5)$ is a local minimum

At $x = 2$, $y = -(2)^3 + \frac{3(2)^2}{2} + 6(2)$

$= 10$

$f''(x) = -6(2) + 3$

$< 0$

$\therefore (2, 10)$ is a local maximum

(ii) Point of inflexion $f''(x) = 0$

$0 = -6(x) + 3$

$-3 = -6x$

$x = \frac{1}{2}$

$y = -(\frac{1}{2})^3 + \frac{3(\frac{1}{2})^2}{2} + 6(\frac{1}{2})$

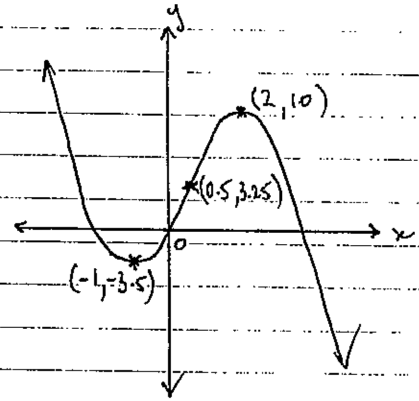
$= 3.25$

$(0.5, 3.25)$

Check:

| $x$ | $\frac{1}{2}^-$ | $\frac{1}{2}$ | $\frac{1}{2}^+$ |
|---|---|---|---|
| $f''(x)$ | $+$ | $0$ | $-$ |

(iii)




(iv) $x < -1$ and $x > 2$

b) $f(x) = 4 \sin 2\pi x$

(i) Amplitude $= 4$

Period $= 1$

(ii) $f(x) = 4 \sin 2\pi x$

$f'(x) = 8\pi \cos 2\pi x$ $\quad f''(x) = -16\pi^2 \sin 2\pi x$

$f'(x) = 0$

$0 = 8\pi \cos 2\pi x$

$0 = \cos 2\pi x$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

$2\pi x = \frac{\pi}{2}$

$x = \frac{1}{4}$ $\quad f''(x) = -16\pi^2 \sin(2\pi(\frac{1}{4}))$

$< 0$ $\quad \therefore$ maximum

$2\pi x = \frac{3\pi}{2}$

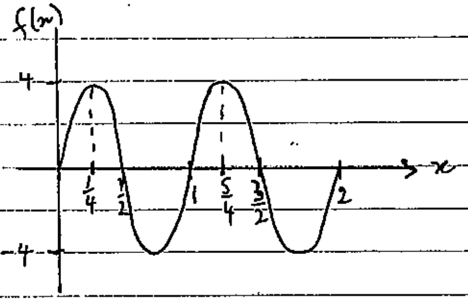
$x = \frac{3}{4}$ $\quad f''(x) = -16\pi^2 \sin(2\pi(\frac{3}{4}))$

Period = 1

$\therefore$ Local maximums at $x = \frac{1}{4},\ 1+\frac{1}{4}$

(iii)

$x = \frac{1}{4}, \frac{5}{4}$




c) $\lim_{x\to 2} \frac{3x^2-12}{x-2} = \lim_{x\to 2} \frac{3(x+2)(x-2)}{x-2}$

$= 3(2+2)$

$= 12$

14 a) $\int_1^5 \frac{1}{x^2+1}\,dx$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 0.5 | 0.2 | 0.1 | $\frac{1}{17}$ | $\frac{1}{26}$ |

$\int_1^5 \frac{1}{x^2+1}\,dx \doteqdot \frac{1}{3}\left[0.5 + 4\left(0.2+\frac{1}{17}\right) + 2(0.1) + \frac{1}{26}\right]$

$= 0.591251\ldots$

$= 0.5913$ (4 d.p.)

b) (i) 2nd card.

| | | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| | 3 | X | 34 | 35 | 36 |
| 1st card | 4 | 43 | X | 45 | 46 |
| | 5 | 53 | 54 | X | 56 |
| | 6 | 63 | 64 | 65 | X |

(ii) $P(W) = \frac{1}{4}$

(iii) Start with \$5.

5 turns cost $5 \times \$1$

\$0 remaining

1 win = +\$3 $\therefore$ 4 losses

$P(1\text{ win}, 4\text{ losses}) = \frac{1}{4} \times \left(\frac{3}{4}\right)^4 \times 5$

$= \frac{405}{1024}$

c) $A = \int_0^2 e^{\frac{1}{2}x}\,dx - \int_0^2 x\,dx$

$= \left[2e^{\frac{1}{2}x}\right]_0^2 - \left[\frac{x^2}{2}\right]_0^2$

$= \left(2e^{\frac{1}{2}(2)} - 2e^{\frac{1}{2}(0)}\right) - \left(\frac{(2)^2}{2} - \frac{(0)^2}{0}\right)$

$= 2e - 2 - 2 + 0$

$= 2e - 4$

$= 2(e-2)\ u^2$

d) $16y = x^2 - 4x - 12$

(i) $16y = (x-2)^2 - 4 - 12$

$16y + 16 = (x-2)^2$

$16(y+1) = (x-2)^2$

Vertex at $(2, -1)$

(ii) $4a = 16$

$a = 4$

$\therefore$ Focus $= (2, -1+4)$ (Concave up)

$= (2, 3)$

(iii) Directrix $y = -1 - 4$

$y = -5$

15. a)(i) $N = Ae^{kt}$

$t = 0 \quad N = 7.13 \times 10^6 = A.$

$t = 3 \quad N = 7.3 \times 10^6$

At $t = 3$

$7.3 \times 10^6 = 7.13 \times 10^6 e^{3k}$

$e^{3k} = \frac{7.3}{7.13}$

$3k = \ln\left(\frac{7.3}{7.13}\right)$

$k = \frac{\ln\left(\frac{7.3}{7.13}\right)}{3}$

$= 0.0078543\ldots$

$\doteqdot 0.79\%$ (2 d.p.)

(ii) $2019 \Rightarrow t = 10$

$N = 7.13 \times 10^6 e^{0.0078 \times 10}$

$= 7.7 \times 10^6$ (Nearest hundred thousand).

(iii) $10 \times 10^6 = 7.13 \times 10^6 e^{0.0079t}$

$e^{0.0079t} = \frac{10}{7.13}$

$t = \frac{\ln\left(\frac{10}{7.13}\right)}{0.0079}$

$= 43.068\ldots$

$\therefore$ During the ~~th~~ (2009 + 43) year

$\therefore$ During 2052.

b) (i) C

(ii) At 4s, it is in a positive direction from the origin (right) and has a negative velocity (moving towards the origin).

(iii) Between points A and D
The graph is concave down during this time.

c) $4\cos^2\theta = 6\sin\theta + 6$

$4(1-\sin^2\theta) = 6\sin\theta + 6$

$4 - 4\sin^2\theta - 6\sin\theta - 6 = 0$

$-4\sin^2\theta - 6\sin\theta - 2 = 0$

Let $x = \sin\theta$

$-4x^2 - 6x - 2 = 0$

$2x^2 + 3x + 1 = 0$

$(2x+1)(x+1) = 0$

$\therefore x = -\frac{1}{2}, -1$

$\sin\theta = -\frac{1}{2}$ $\quad \left(\sin\frac{\pi}{6} = \frac{1}{2}\right)$

$\theta = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 3\pi + \frac{\pi}{6}, 4\pi - \frac{\pi}{6}, \ldots$

$\sin\theta = -1$ $\quad \left(\sin\frac{\pi}{2} = 1\right.$

$\theta = \pi + \frac{\pi}{2}, 3\pi + \frac{\pi}{2}, \ldots$

But $0 \leq \theta \leq 2\pi$

$\therefore \theta = \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

d) $\frac{1}{\sqrt{n}+\sqrt{n+1}} \times \frac{\sqrt{n}-\sqrt{n+1}}{\sqrt{n}-\sqrt{n+1}} = \frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$

$= \frac{\sqrt{n}-\sqrt{n+1}}{-1}$

$= \sqrt{n+1} - \sqrt{n}$

16. a) (i) $(\sqrt{3})^2 = x^2 + y^2$

$y = \sqrt{(\sqrt{3})^2 - x^2}$

$= \sqrt{3 - x^2}$

(ii) $L = 2x + 2y + \sqrt{3}$

$= 2(x + y + \frac{\sqrt{3}}{2})$

Sub $y = \sqrt{3 - x^2}$

$L = 2(x + \sqrt{3 - x^2} + \frac{\sqrt{3}}{2})$

(iii)

$L = 2(x + (3 - x^2)^{\frac{1}{2}} + \frac{\sqrt{3}}{2})$

$\frac{dL}{dx} = 2\left(1 + \frac{1}{2}(3 - x^2)^{-\frac{1}{2}} \times -2x\right)$

$= 2 - \frac{2x}{\sqrt{3 - x^2}}$

Stationary point: $\frac{dL}{dx} = 0$

$0 = 2 + \frac{2x}{\sqrt{3 - x^2}}$

$\frac{2x}{\sqrt{3 - x^2}} = 2$

$2x = 2\sqrt{3 - x^2}$

$4x^2 = 4(3 - x^2)$

$4x^2 = 12 - 4x^2$

$8x^2 = 12$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$

But $x > 0$ (length

$\therefore x = \sqrt{\frac{3}{2}}$ m

Check if it's a maximum $\frac{d^2L}{dx^2} < 0$

$\frac{dL}{dx} = 2 - 2x(3 - x^2)^{-\frac{1}{2}}$

$\frac{d^2L}{dx^2} = -2x \times (-\frac{1}{2})(-2x)(3 - x^2)^{-\frac{3}{2}} + (-2)(3 - x^2)^{-\frac{1}{2}}$

$= -2x^2(3 - x^2)^{-\frac{3}{2}} - 2(3 - x^2)^{-\frac{1}{2}}$

$= -\frac{2x^2}{\sqrt{(3 - x^2)^3}} - \frac{2}{\sqrt{(3 - x^2)}}$

When $x = \sqrt{\frac{3}{2}}$

$\frac{d^2L}{dx^2} = -\frac{2(\frac{3}{2})}{\sqrt{(3 - \frac{3}{2})^3}} - \frac{2}{\sqrt{(3 - \frac{3}{2})}}$

$= -3.265$

$< 0$

$\therefore$ maximum

$\therefore$ Maximum strength when $x = \sqrt{\frac{3}{2}}$

b) (i) In $\triangle AEB$,

$\angle AEB = \angle ABE$ (Equal angles opposite equal sides $AE = AB$ (Given))

$\therefore 2\angle ABE + 30 = 180$ ($\angle$ sum of $\triangle$ is 180)

$\therefore \angle ABE = 75^\circ$

But

$\angle ABE + \angle ABC = 138^\circ$ ($\angle EBC = 138$ (Given))

$\therefore \angle ABC = 138 - 75$

$= 63^\circ$

$\angle ABC + \angle BCD = 63 + 83 + 34$

$= 180$

$\therefore AB \parallel DC$ (Cointerior angles add to $180^\circ$).

(ii) In $\triangle ABC$ and $\triangle CDA$,

A $\angle BAC = \angle DCA = 34^\circ$ (Alternate angles on $AB \parallel DC$)

A $\angle BCA = \angle DAC = 83^\circ$ (Alternate angles on $AD \parallel BC$)

S $AC$ is common

$\therefore \triangle ABC \equiv \triangle CDA$ (AAS).

c) $V = \pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx$

$= \pi \left[\tan x\right]_0^{\frac{\pi}{4}}$

$= \pi \left[\tan \frac{\pi}{4} - \tan 0\right]$

$= \pi \ u^3$