2018

Higher School Certificate

Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Board approved calculators may be used.
- Write using black or blue pen
- A reference sheet is provided
- All necessary working should be shown in Question 11 – 16
- Write your student number and/or name at the top of every page

Total marks – 100

Section I - Pages 2 – 6

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 7 – 14

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

Section I

10 marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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1 What is 0.00367254 written in scientific notation, correct to 4 significant figures?

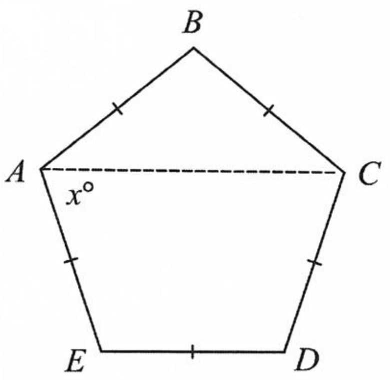
(A) 3.672×10^{-2}

(B) 3.673×10^{-2}

(C) 3.673×10^{-3}

(D) 3.7×10^{-3}

2 $ABCDE$ is a regular pentagon and $\angle CAE = x°$.



What is the value of x?

(A) 36°

(B) 72°

(C) 84°

(D) 108°

3 Which of the following is equal to $\sec^2 x - \tan^2 x - \cos^2 x$?

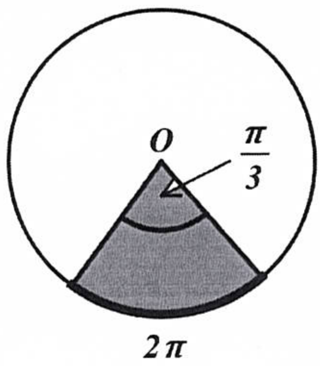
(A) $1 - \tan^2 x$

(B) $-\tan^2 x$

(C) $\cot^2 x$

(D) $\sin^2 x$

4 A circle with centre O has a sector with arc length of 2π units and a sector angle of $\frac{\pi}{3}$.



What is the area of the shaded sector?

(A) 6π

(B) 12π

(C) $\frac{\pi^2}{5400}$

(D) $\frac{\pi^3}{5400}$

5 Which of the following is the solution to $3^x = 50$?

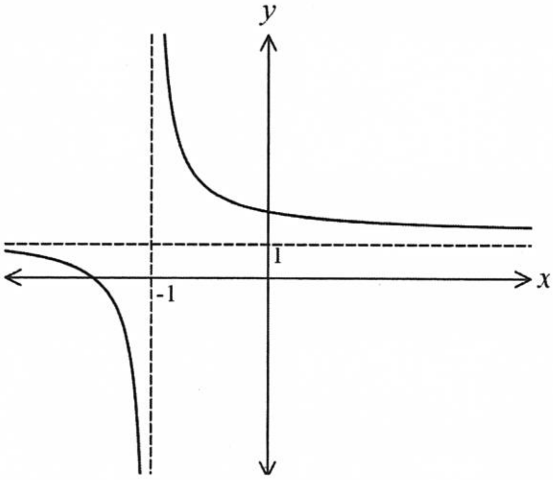
(A) $x = 1.70$

(B) $x = 3.56$

(C) $x = 3.60$

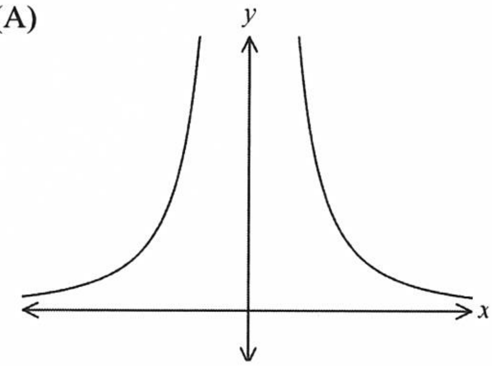
(D) $x = 3.91$

6 The diagram shows the graph of $y = f(x)$.

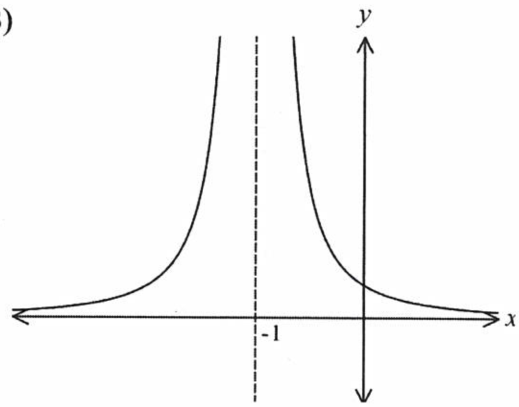


Which graph represents $f'(x)$?

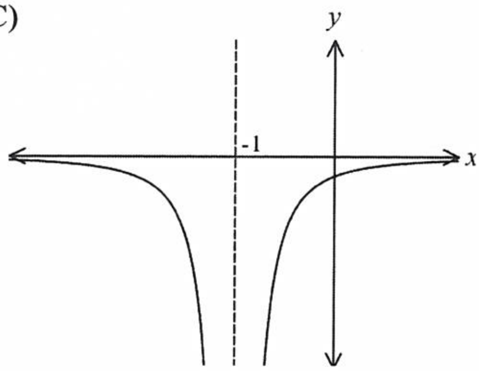
(A)



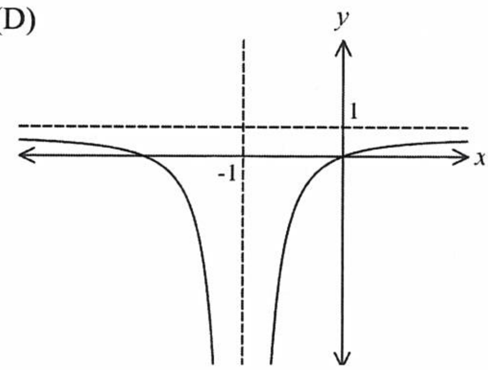
(B)



(C)



(D)



7 If $a > 0$ and the function $f(x) = ax^3 + bx^2 + cx + d$ is always increasing, which of the following conditions must apply to a, b and c?

(A) $b^2 - ac < 0$

(B) $b^2 - 2ac < 0$

(C) $b^2 - 3ac < 0$

(D) $b^2 - 4ac < 0$

8 The geometric series $a + ar + ar^2 + ar^3 + \cdots$ has limiting sum S.

What is the limiting sum of the geometric series $a^2 + a^2r^2 + a^2r^4 + a^2r^6 + \cdots$?

(A) S^2

(B) $\dfrac{aS}{1+r}$

(C) $\dfrac{S^2}{1+r}$

(D) $\dfrac{aS(1-r)}{1+r}$

9 Which of the following is a correct expression for $f'(x)$, given $f(x) = \left[\ln\left(x^2+1\right)\right]^3$?

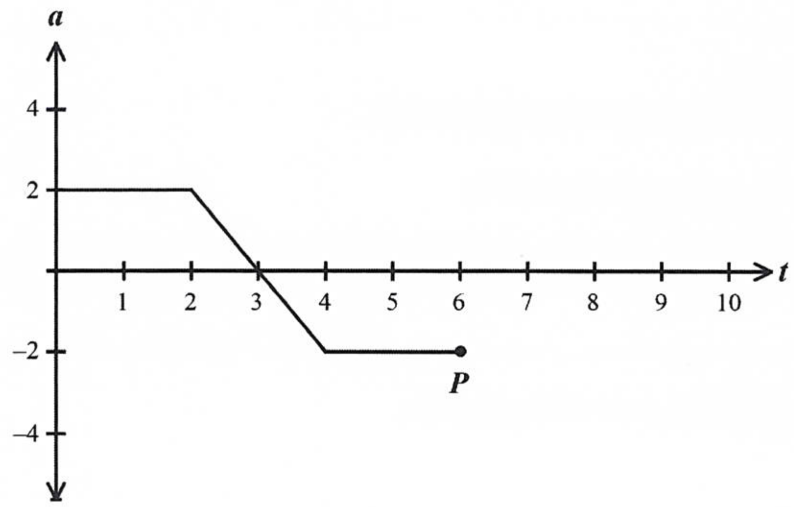
(A) $\dfrac{6x}{x^2+1}$

(B) $\dfrac{8x^3}{\left(x^2+1\right)^3}$

(C) $3\left[\ln\left(x^2+1\right)\right]^2$

(D) $\dfrac{6x}{x^2+1}\left[\ln\left(x^2+1\right)\right]^2$

10 A particle is moving in a straight line. The acceleration/time graph is shown below.



If the particle was originally stationary at the origin, which of the following statements best describes the particle at point *P*?

(A) *P* is stationary at a point on the right of the origin.

(B) *P* has positive velocity and negative acceleration.

(C) *P* has negative velocity and negative acceleration.

(D) *P* has negative velocity and zero acceleration.

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a new writing booklet.

In Questions 11–16, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Solve $\frac{2x}{3}-1=\frac{x}{2}$. **2**

(b) Evaluate $\lim_{m\to 4}\left(\frac{2m^2-9m+4}{m^2-16}\right)$. **2**

(c) If $\left(2\sqrt{2}-\sqrt{6}\right)^2=a-b\sqrt{3}$, find the values of a and b. **2**

(d) Differentiate $\frac{3x+2}{2x^3-5x^2}$. **2**

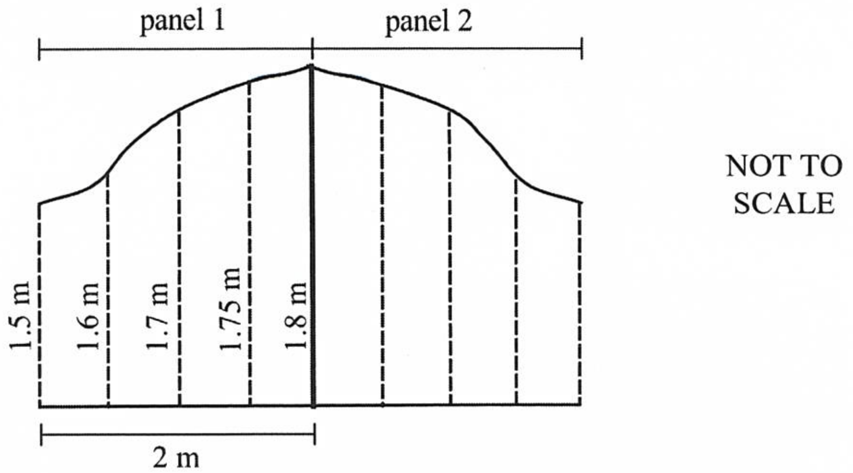
(e) Solve the inequality $|2x-3|\le 3$. **2**

(f) Evaluate $\int_0^1 \frac{1}{2x+1}\,dx$ **3**

(g) Find the equation of the tangent to the curve $y=2(\tan x-1)$ at the point $\left(\frac{\pi}{4},0\right)$. **2**

Question 12 (15 marks) Start a new writing booklet.

(a) A gate is being installed for the drive-way of a home. **3**
The gate consists of two steel panels that are exactly the same.
Each panel is 2 m in length and is divided into 4 equal intervals.



Using Simpson's rule with 5 vertical lengths, calculate the total area of the gate.

(b) Find the domain and range of $y=\sqrt{x^2-1}$ **3**

(c) Solve $8\sin^2 x=\operatorname{cosec} x$ for $0\le x\le 2\pi$ **2**

(d) The roots of the equation $2x^2-3x+8=0$ are α and β.

(i) Find the value of $\alpha+\beta$. **1**

(ii) Find the value of $\alpha\beta$. **1**

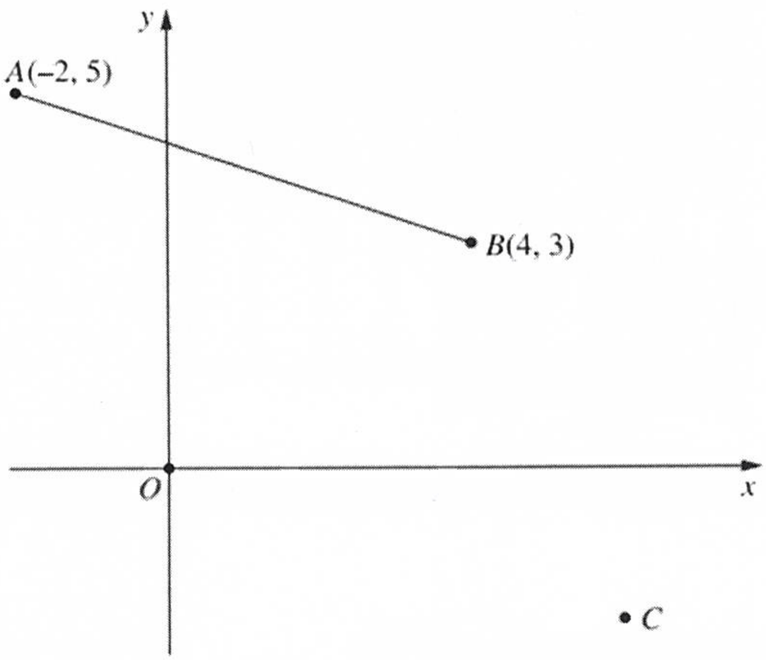
(iii) Find the value of $\alpha^2+\beta^2$. **2**

(e) The region bounded by the curve $y=e^x$ and the x axis between $x=0$ and $x=\log_e 3$ is rotated through one complete revolution about the x axis. **3**

Find, in simplest exact form, the volume of the solid formed.

Question 13 (15 marks) Start a new writing booklet.

(a) The diagram shows the points $A(-2,5)$, $B(4,3)$ and $O(0,0)$. The point C is the fourth vertex of the parallelogram $OABC$.



(i) Show that the equation of AB is $x+3y-13=0$. **2**

(ii) Show that the length of AB is $2\sqrt{10}$. **1**

(iii) Calculate the perpendicular distance from O to AB. **2**

(iv) Calculate the area of parallelogram $OABC$. **1**

(v) Find the coordinates of C. **1**

(b) Consider the curve $y=-x^3+3x+2$.

(i) Find the stationary points and determine their nature. **3**

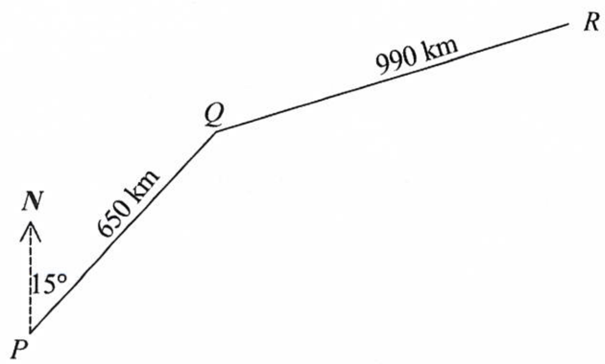
(ii) Find any point(s) of inflexion. **2**

(c) Shade the region in the Cartesian plane which simultaneously satisfies the inequalities $y \geq 3^x$ and $y > 4-x^2$. **3**

Question 14 (15 marks) Start a new writing booklet.

(a) An aeroplane flies directly from town P to town Q. The distance from P to Q is 650 kilometres. The bearing of Q from P is 015°.

At town Q, the aeroplane turns onto a bearing of 040° and heads to town R which is 990 kilometres from town Q.

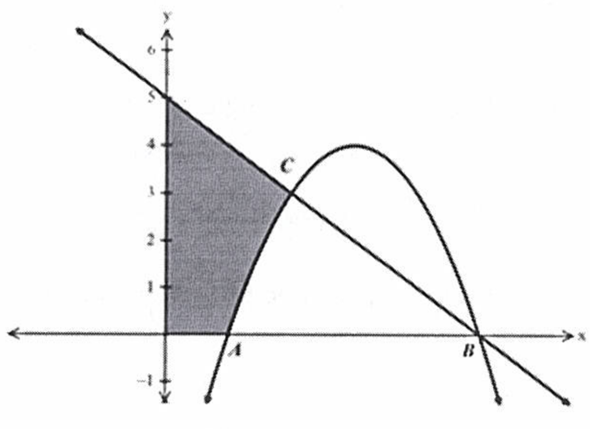


NOT TO SCALE

(i) Show that $\angle PQR = 155^\circ$ **2**

(ii) Calculate the distance from town P to town R, correct to the nearest kilometre. **2**

(b) The parabola $y = -x^2 + 6x - 5$ meets the x axis at points $A(1,0)$ and $B(5,0)$. **3**
The line $y = -x + 5$ meets the parabola at points B and $C(2,3)$.

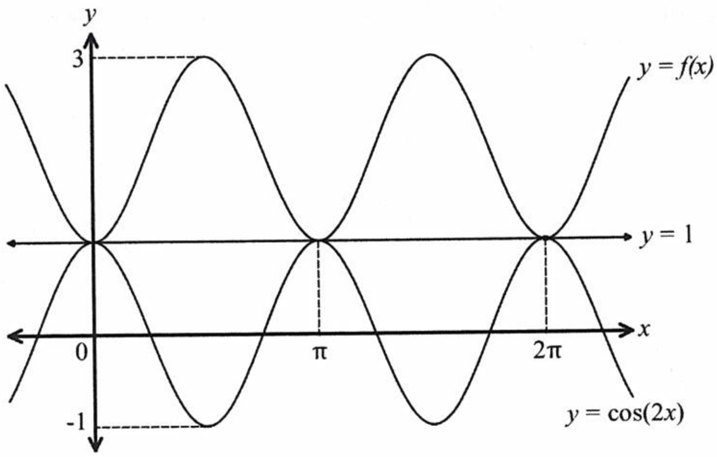


Find the area of the shaded region.

Question 14 continues on page 11

(c) The diagram below shows the graph of $y = \cos(2x)$ and $y = f(x)$ from $x = 0$ to $x = 2\pi$. **2**

The graph of $y = f(x)$ is a reflection of $y = \cos(2x)$ across the line $y = 1$.



Find the equation for the graph of $y = f(x)$.

(d) Consider the parabola $y = x^2 - 8x + 4$.

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(e) In March 1958, 24 koalas were introduced on Kangaroo Island. By March 2018, the number of koalas had risen to 5000.

Assume that the number N of koalas is increasing exponentially and satisfies the equation $N = 24e^{kt}$, where k is a constant and t is measured in years from March 1958.

(i) Show that $k = 0.0890$, correct to 4 significant figures. **2**

(ii) Predict the number of koalas that will be present on Kangaroo Island in March 2024. **1**

End of Question 14

Question 15 (15 marks) Start a new writing booklet.

(a) The point $P(x, y)$ moves such that the gradient of AP is twice the gradient of BP, where the points A and B are $(0, -5)$ and $(-2, 3)$ respectively. **3**

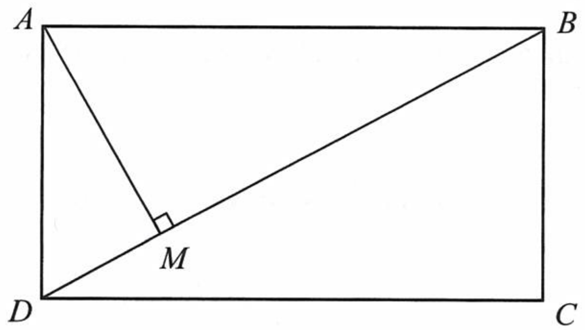
Find the equation of the locus of P.

(b) A series is given by **2**

$$\log_{10} e + \log_{10} e^2 + \log_{10} e^3 + \cdots + \log_{10} e^n$$

Show that this is an arithmetic series.

(c) $ABCD$ is a rectangle with $AB = 12$ cm and $AD = 9$ cm. AM is perpendicular to BD.



(i) Find the length of BD. **1**

(ii) Prove that ΔABM is similar to ΔDBA. **2**

(iii) Hence find the length of BM. **2**

Question 15 continues on page 13

(d) At the start of the month, Katrina opens a bank account and deposits \$300 into the account.

At the start of each subsequent month, Katrina makes a deposit which is 1.5% more than the previous deposit.

At the end of every month, the bank pays Katrina interest at a rate of 3% per annum on the balance of the account.

(i) Show that the balance of the account at the end of the second month is **2**

$$\$300(1.0025)^2 + \$300(1.015)(1.0025)$$

(ii) Show that the balance of the account at the end of the n^{th} month is given by **2**

$$\$300(1.0025)^n \left(\frac{\left(\frac{1.015}{1.0025}\right)^n - 1}{\left(\frac{1.015}{1.0025}\right) - 1} \right)$$

(iii) Calculate the balance of the account at the end of the 60^{th} month, correct to the nearest dollar. **1**

End of Question 15

Question 16 (15 marks) Start a new writing booklet.

(a) A particle travelling in a straight line is initially at rest.
Its acceleration as a function of time t (seconds), is given by $a = 4\sin 2t$.

(i) Show that the velocity of the particle is given by $v = 2 - 2\cos(2t)$ **2**

(ii) Sketch the graph of the velocity as a function of time, for $0 \le t \le \pi$ **2**

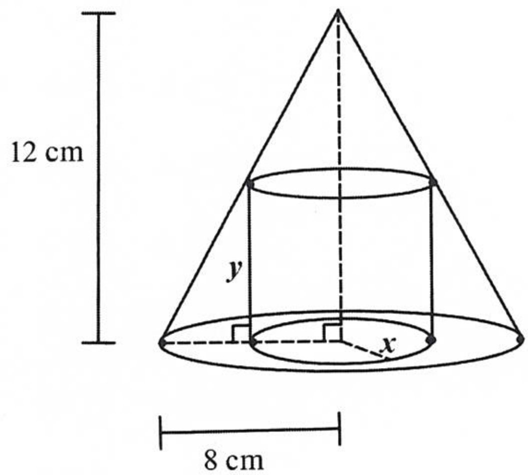
(iii) Explain why the particle never changes direction. **1**

(iv) Find the total distance travelled in the first π seconds. **1**

(b) (i) Find $\dfrac{d}{dx}\left[x\tan x + \log(\cos x)\right]$. **2**

(ii) Hence find $\displaystyle\int_0^{\frac{\pi}{4}} \left(x\sec^2 x - \frac{\sin x}{\cos x} \right) dx$. **3**

(c) The diagram below shows a cylinder of radius x cm and height y cm, inscribed in a cone. The cone has a radius of 8 cm and its height is 12 cm.



(i) Show that $y = \dfrac{3}{2}(8 - x)$. **1**

(ii) Show that the volume of the cylinder is given by $V = \dfrac{3}{2}\pi x^2(8 - x)$ cm^3. **1**

(iii) Find the value of x for which the volume is a maximum. **2**

End of paper

GHS Mathematics Trial HSC 2018 - Solutions

1. C	5. B	9. D
2. B	6. C	10. A
3. D	7. C	
4. A	8. B	

Question 11

a) $\frac{2x}{3} - 1 = \frac{x}{2}$

$4x - 6 = 3x$

$x = 6$

b) $\lim_{m \to 4} \left(\frac{2m^2 - 9m + 4}{m^2 - 16} \right) = \lim_{m \to 4} \frac{(2m-1)(m-4)}{(m+4)(m-4)}$

$= \frac{2(4) - 1}{4 + 4}$

$= \frac{7}{8}$

c) $(2\sqrt{2} - \sqrt{6})^2 = 8 - 4\sqrt{12} + 6$

$= 14 - 8\sqrt{3}$

$\therefore a = 14, \ b = 8$

d) $\frac{d}{dx}\left(\frac{3x + 2}{2x^3 - 5x^2} \right) = \frac{(2x^3 - 5x^2) \times 3 - (3x+2)(6x^2 - 10x)}{(2x^3 - 5x^2)^2}$

$= \frac{6x^3 - 15x^2 - 18x^3 + 30x^2 - 12x^2 + 20x}{(2x^3 - 5x^2)^2}$

$= \frac{20x + 3x^2 - 12x^3}{(2x^3 - 5x^2)^2}$

e) $|2x - 3| \le 3$

$2x - 3 \le 3$ or $2x - 3 \ge -3$

$2x \le 6$ $\quad$ $2x \ge 0$

$x \le 3$ $\quad$ $x \ge 0$

$\therefore 0 \le x \le 3$

f) $\int_0^1 \frac{1}{2x+1} \, dx = \frac{1}{2} \int_0^1 \frac{2}{2x+1} \, dx$

$= \frac{1}{2} \left[\ln(2x+1) \right]_0^1$

$= \frac{1}{2} \left[\ln 3 - \ln 1 \right]$

$= \frac{1}{2} \ln 3$

g) $y = 2\tan x - 2$

$y' = 2\sec^2 x$

$m_T = 2\sec^2 \frac{\pi}{4}$

$= 4$

$y - y_1 = m(x - x_1)$

$y - 0 = 4\left(x - \frac{\pi}{4}\right)$

$y = 4x - \pi$

CBDA | BCCB | DA

Question 12

a) $A \approx \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2)\right] \times 2$

$A \approx \frac{0.5}{3}\left[1.5 + 1.8 + 4(1.6 + 1.75) + 2(1.7)\right] \times 2$

$= 6.7\ m^2$

b) $y = \sqrt{x^2 - 1}$

Domain: $x^2 - 1 \geq 0$

$x \leq -1$ or $x \geq 1$

Range: $y \geq 0$

c) $8\sin^2 x = \operatorname{cosec} x \quad , \quad 0 \leq x \leq 2\pi$

$8\sin^3 x = 1$

$\sin^3 x = \frac{1}{8}$

$\sin x = \frac{1}{2}$

* S	A *
T	C

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

d) $2x^2 - 3x + 8 = 0$

(i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{3}{2}$

(ii) $\alpha\beta = \frac{c}{a}$

$= 4$

(iii) $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2 \times 4$

$= -\frac{23}{4}$

e) $V = \pi \int_0^{\log_e 3} (e^x)^2 \, dx$

$= \pi \int_0^{\ln 3} e^{2x} \, dx$

$= \frac{\pi}{2}\left[e^{2x}\right]_0^{\ln 3}$

$= \frac{\pi}{2}\left[e^{2\ln 3} - e^0\right]$

$= \frac{\pi}{2}\left[e^{\ln 9} - e^0\right]$

$= \frac{\pi}{2}\left[9 - 1\right]$

$= 4\pi$ units^3

Question 13

a) (i) $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

$\dfrac{y-5}{x+2} = \dfrac{3-5}{4+2}$

$6(y-5) = -2(x+2)$

$6y - 30 = -2x - 4$

$2x + 6y - 26 = 0$

$x + 3y - 13 = 0$

(ii) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(4+2)^2 + (3-5)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$

(iii) $d_\perp = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$d_\perp = \dfrac{|1\times 0 + 3\times 0 - 13|}{\sqrt{1^2+3^2}}$

$= \dfrac{13}{\sqrt{10}}$

(iv) Area $= b \times h$

$= 2\sqrt{10} \times \dfrac{13}{\sqrt{10}}$

$= 26$ units^2

(v) $C\,(6, -2)$

b) $y = -x^3 + 3x + 2$

(i) $y' = -3x^2 + 3$

Stat. pts when $y' = 0$

$-3x^2 + 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

When $x = 1$, $y = -1 + 3 + 2 = 4$

When $x = -1$, $y = 1 - 3 + 2 = 0$

$y'' = -6x$

When $x = 1$, $y'' = -6$

When $x = -1$, $y'' = 6$

$\therefore$ Max turning point at $(1, 4)$, min turning point at $(-1, 0)$

(ii) Possible points of inflexion when $y'' = 0$

$-6x = 0$

$x = 0$

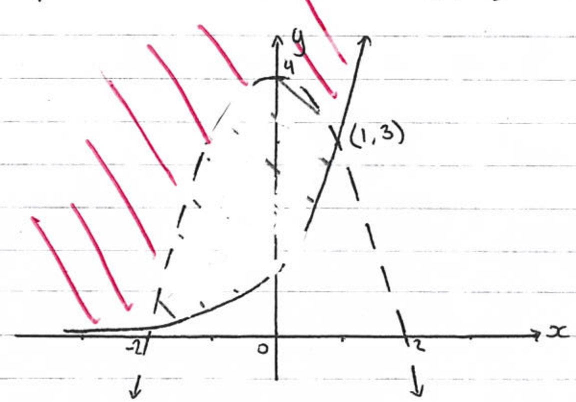
Check concavity

x	-0.5	0	0.5
y''	>0	0	<0

When $x = 0$, $y = 0 + 0 + 2 = 2$

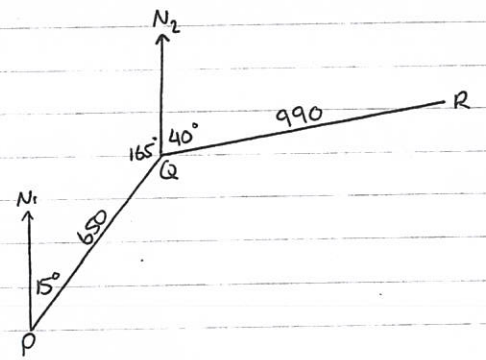
$\therefore$ point of inflexion at $(0, 2)$

c)



Question 14

a)



(i) $\angle PQN_2 = 165°$ (co-interior ∠'s in ∥ lines)

$\angle RQN_2 = 40°$ (bearing of R from Q)

$\angle PQR = 360 - 165 - 40$ (∠'s in a revolution)

$= 155°$

(ii) $PR^2 = 650^2 + 990^2 - 2 \times 650 \times 990 \times \cos 155°$

$= 2\,569\,018 \cdot 122 \ldots$

$PR = 1602 \cdot 8156 \ldots$

≈ 1603 km

b) $\text{Area} = \frac{2}{2}(3+5) - \int_1^2 -x^2 + 6x - 5 \, dx$

$= 8 - \left[\frac{-x^3}{3} + 3x^2 - 5x\right]_1^2$

$= 8 - \left[\frac{-8}{3} + 12 - 10 - \left(\frac{-1}{3} + 3 - 5\right)\right]$

$= \frac{19}{3}$ units^2

c) $y = 2 - \cos 2x$

d) $y = x^2 - 8x + 4$

(i) $y = x^2 - 8x + 16 - 12$

$y + 12 = (x-4)^2$

Vertex $(4, -12)$

(ii) $4a = 1$

$a = \frac{1}{4}$

Focus $(4, -11\frac{3}{4})$

e) (i) $N = 24e^{kt}$

When $t = 60$, $N = 5000$

$5000 = 24e^{60k}$

$e^{60k} = \frac{5000}{24}$

$60k = \ln\left(\frac{5000}{24}\right)$

$k = \frac{\ln\left(\frac{5000}{24}\right)}{60}$

$k = 0.0889856\ldots$ 0.08899 *

$= 0.0890$ to 4 sig. figs.

(ii) $N = 24e^{0.0890 \times 66}$

$= 8536$

or if using complete k value,

$N = 24e^{66k}$

$= 8528$

* Various answers according to rounded response part e) i)

Question 15

a) $A(0,-5)$, $B(-2,3)$, $P(x,y)$

$m_{AP} = 2 \times m_{BP}$

$\frac{y+5}{x-0} = 2 \times \frac{y-3}{x+2}$

$(y+5)(x+2) = 2(y-3)(x)$

$xy + 2y + 5x + 10 = 2xy - 6x$

$11x + 2y - xy + 10 = 0$

b) $\log_{10} e^2 - \log_{10} e = 2\log_{10} e - \log_{10} e$

$= \log_{10} e$

$\log_{10} e^3 - \log_{10} e^2 = 3\log_{10} e - 2\log_{10} e$

$= \log_{10} e$

$\therefore T_3 - T_2 = T_2 - T_1$

$\therefore$ it is an arithmetic series with $d = \log_{10} e$

c) (i) $BD^2 = 12^2 + 9^2$

$= 225$

$BD = 15$ cm

(ii) Let $\angle ABM = x$

$\angle BAM = 90 - x$ ($\angle$ sum of $\triangle BAM$)

$\angle DAM = x$ (adjacent complementary $\angle$'s)

$\angle AMB = \angle DAB = 90°$

$\angle ABM = \angle DBA = x$ (as above)

$\therefore \triangle ABM \ ||| \ \triangle DBA$ (equiangular)

(iii) $\frac{BM}{AB} = \frac{AB}{BD}$

$\frac{BM}{12} = \frac{12}{15}$

$BM = \frac{12^2}{15}$

$= 9.6$ cm

d) (i) $r = 3\%$ p.a.

$= 0.0025$ per month

$A_1 = 300 \times 1.0025$

$A_2 = (A_1 + 300 \times 1.015) \times 1.0025$

$= 300 \times 1.0025^2 + 300 \times 1.015 \times 1.0025$

(ii) $A_3 = (A_2 + 300 \times 1.015^2) \times 1.0025$

$= (300 \times 1.0025^2 + 300 \times 1.015 \times 1.0025 + 300 \times 1.015^2) \times 1.0025$

$= 300 \times 1.0025^3 + 300 \times 1.015 \times 1.0025^2 + 300 \times 1.015^2 \times 1.002$

$= 300(1.0025^3 + 1.015 \times 1.0025^2 + 1.015^2 \times 1.0025)$

$A_n = 300(1.0025^n + 1.015 \times 1.0025^{n-1} + 1.015^2 \times 1.0025^{n-2} + \ldots + 1.015^{n-1} \times 1.0025)$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1.0025^n\left[\left(\frac{1.015}{1.0025}\right)^n - 1\right]}{\left(\frac{1.015}{1.0025} - 1\right)}$

$\therefore A_n = 300(1.0025)^n \left[\frac{\left(\frac{1.015}{1.0025}\right)^n - 1}{\left(\frac{1.015}{1.0025}\right) - 1}\right]$

(iii) $A_{60} = 300 \times 1.0025^{60} \times \left[\dfrac{\left(\frac{1.015}{1.0025}\right)^{60} - 1}{\left(\frac{1.015}{1.0025}\right) - 1} \right]$

$= \$30\ 835.37$

$\approx \$30\ 835$

Question 16

a) (i) $\frac{dv}{dt} = 4 \sin 2t$

$v = \int 4 \sin 2t \, dt$

$v = -2 \cos 2t + c$

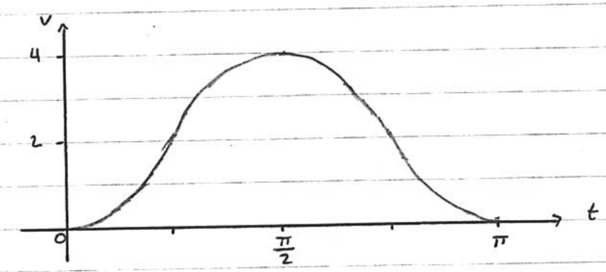
When $t = 0$, $v = 0$

$0 = -2 \cos 0 + c$

$c = 2$

$\therefore v = 2 - 2\cos 2t$

(ii)



(iii) Velocity is never negative

(iv) Distance = area under curve

$= (4 \times \pi) \div 2$

$= 2\pi$ units

b) (i) $\frac{d}{dx}\left[x \tan x + \log(\cos x)\right]$

$= \tan x \times 1 + x \times \sec^2 x - \frac{\sin x}{\cos x}$

$= \tan x + x \sec^2 x - \tan x$

$= x \sec^2 x$

(ii) $\int_0^{\frac{\pi}{4}} x\sec^2 x - \frac{\sin x}{\cos x}\, dx$

$= \left[x\tan x + \log(\cos x) + \log(\cos x)\right]_0^{\frac{\pi}{4}}$

$= \left[x\tan x + 2\log(\cos x)\right]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4} \times \tan\frac{\pi}{4} + 2\log(\cos\frac{\pi}{4}) - \left[0 + 2\log(\cos 0)\right]$

$= \frac{\pi}{4} + 2\log\left(\frac{1}{\sqrt{2}}\right) - 2\log 1$

$= \frac{\pi}{4} + 2\log\left(2^{-\frac{1}{2}}\right)$

$= \frac{\pi}{4} - \log 2$

c) (i) Using similar triangles,

$\frac{y}{12} = \frac{8-x}{8}$

$y = \frac{12(8-x)}{8}$

$y = \frac{3}{2}(8-x)$

(ii) $V = \pi r^2 h$

$V = \pi x^2 y$

$= \pi x^2 \times \frac{3}{2}(8-x)$

$= \frac{3}{2}\pi x^2 (8-x)$

(iii) $V(x) = 12\pi x^2 - \frac{3}{2}\pi x^3$

$V'(x) = 24\pi x - \frac{9}{2}\pi x^2$

Max or min when $V'(x) = 0$

$0 = 24\pi x - \frac{9}{2}\pi x^2$

$0 = \frac{3}{2}\pi x(16 - 3x)$

$x = 0, \frac{16}{3}$

But $x > 0$

Check concavity

$V''(x) = 24\pi - 9\pi x$

$V''\left(\frac{16}{3}\right) = 24\pi - 9\pi\left(\frac{16}{3}\right)$

$= -24\pi$

< 0

$\therefore$ max value when $x = \frac{16}{3}$