# Gosford High School

**2019** TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided.
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

**Total Marks – 100**

**Section I - 10 marks** (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

**Section 2 – 90 marks** (pages 8 – 18)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

**Section I**

**10 marks**

**Attempt Questions 1 – 10.**

**Allow about 15 minutes for this section.**

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. What is the value of $3ln7$ correct to two significant figures?

   (A) 5.84

   (B) 2.54

   (C) 5.8

   (D) 2.5

2. Find the gradient of the normal to the function $y = e^{-x}$ at the point $x = ln2$.

   (A) $-2$

   (B) $-\frac{1}{2}$

   (C) $2$

   (D) $\frac{1}{2}$

3. What is the $\int \tan^2 x\, dx$

   (A) $\sec^4 x + c$

   (B) $tanx - x + c$

   (C) $\cot x + c$

   (D) $\ln(\cos^2 x) + c$

4. For the angle $\theta$, $tan\theta = \frac{1}{2}$ and $cos\theta < 0$. What is the value of $sin\theta$ ?

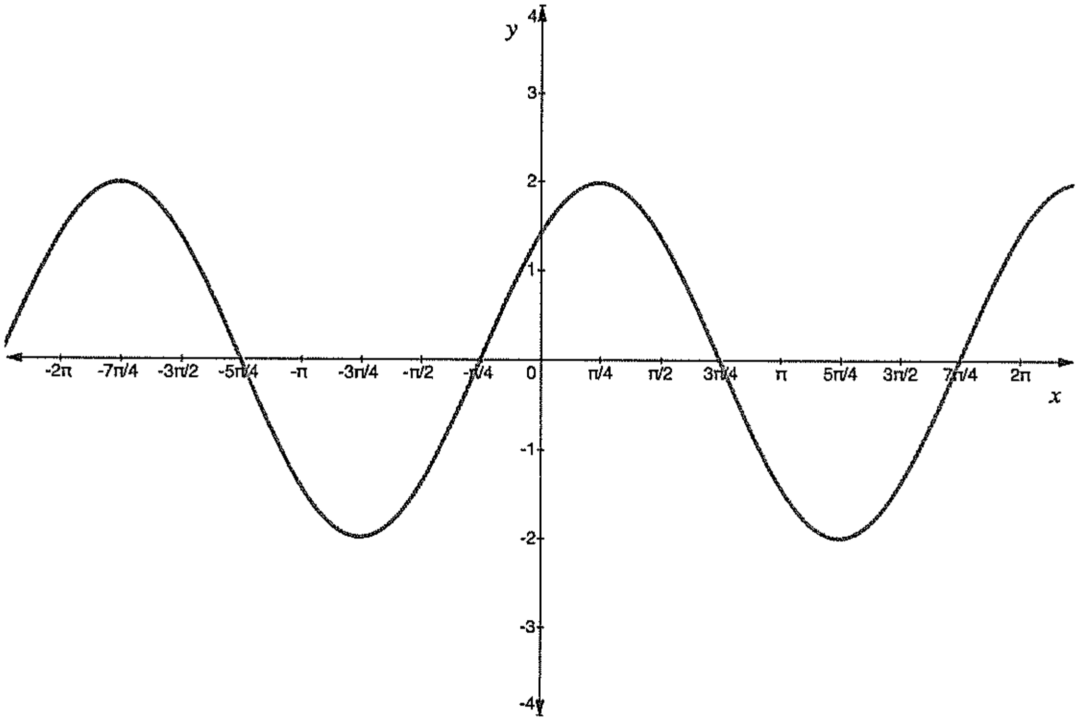
(A) $\frac{\pi}{3}$

(B) $\frac{-1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{-1}{\sqrt{5}}$

5.




The graph of $y = a\cos(x - k)$, *where a and k are constants*, is drawn above. The values of $a$ and $k$ are

(A) $a = -2$ and $k = \frac{\pi}{4}$

(B) $a = 2$ and $k = \frac{-\pi}{4}$

(C) $a = 2$ and $k = \frac{\pi}{4}$

(D) $a = -2$ and $k = \frac{-\pi}{4}$

6. The point $P$ moves such that it is always 3 units from a fixed point.
Which of the following equations might describe the locus of $P$?

(A) $y = \pm 3$

(B) $(x-1)^2 + (y+3)^2 = 9$

(C) $y = \dfrac{3}{x}$

(D) $x^2 = 12y$

7. Evaluate $\lim_{x \to 0} \left( x + \frac{sin2x}{x} \right)$

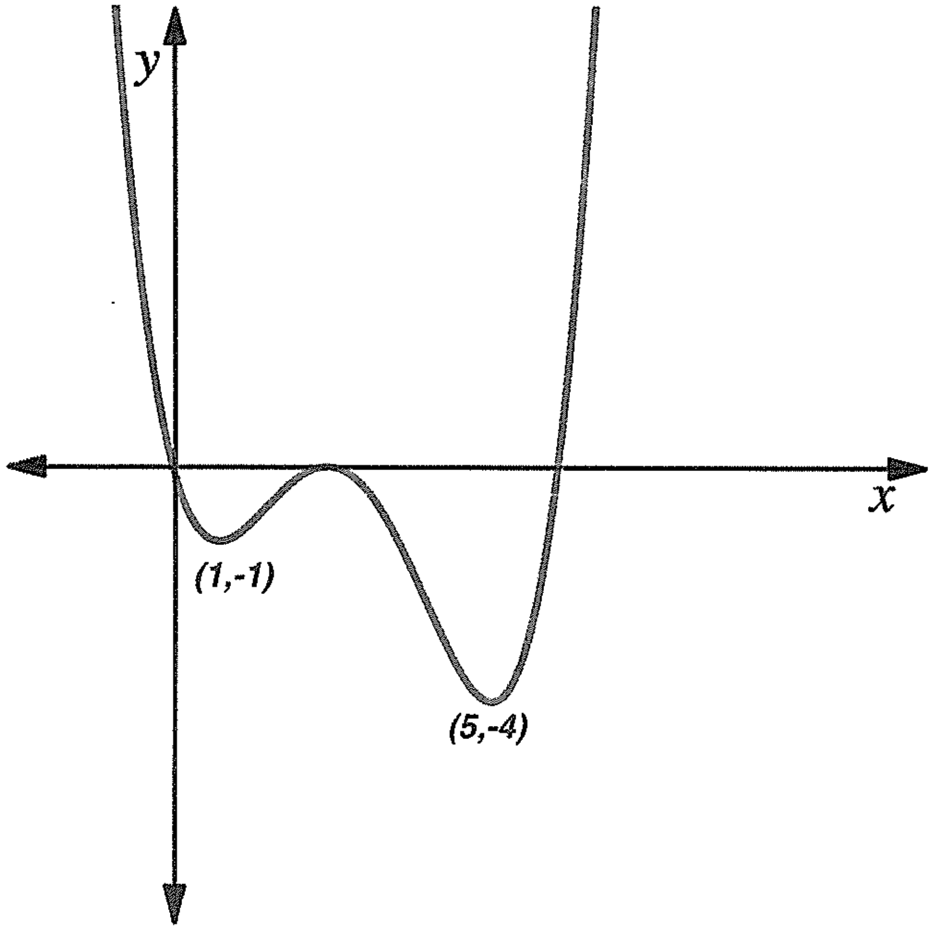
(A) 0

(B) $\infty$

(C) 2

(D) $\frac{1}{2}$

8. The graph below shows the function $y = f(x)$.




$f(x) + k > 0$ *for all* $x$. If $k$ is a constant, then $k$ is given by

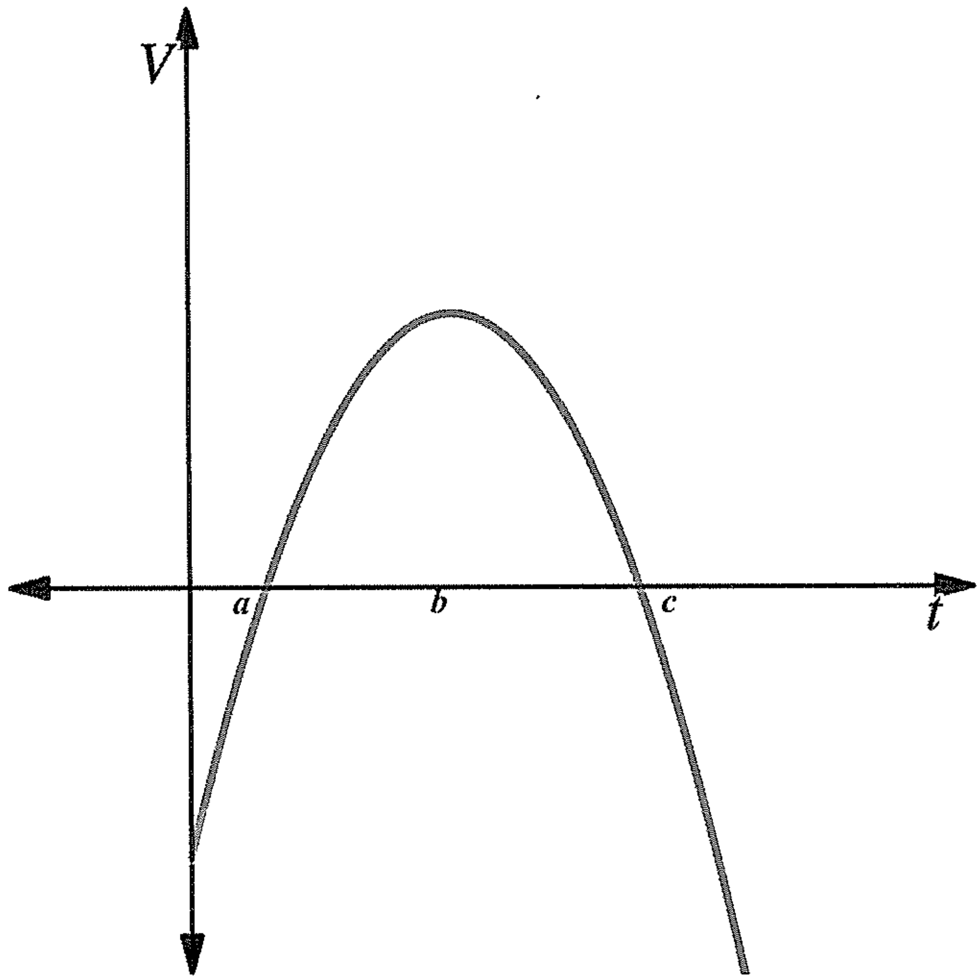
(A) $1 < k < 5$

(B) $-1 < k < -4$

(C) $k > 4$

(D) $k < 5$

9. The graph below shows the velocity of a particle as it moves along the $x$-axis. The velocity is measured in metres per second and time is measured in seconds.




When $t = 0$, the particle's displacement, $x$, is $-2\,m$ and its velocity, V, is $10\ ms^{-1}$.

At what times is the **acceleration** zero?

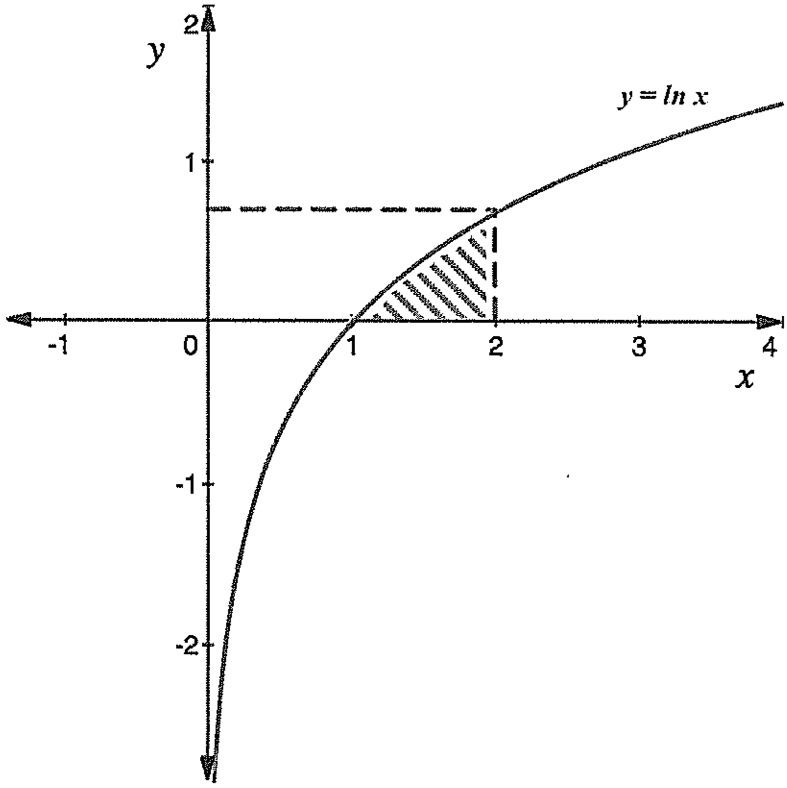
(A) $t = a$ and $c$

(B) $t = b$

(C) $0 \le t \le b$

(D) cannot be determined

10. The graph of $y = lnx$ is shown below




The shaded area is given by

(A) $\int_{1}^{2} e^{x} dx$

(B) $2\ln 2 - \int_{0}^{ln2} e^{x} dx$

(C) $\ln 4 - \int_{0}^{ln2} e^{y} dy$

(D) $\int_{0}^{ln2} e^{y} dy$

**Section II**

**90 marks**

**Attempt Questions 11 – 16.**

**Allow about 2 hours and 45 minutes for this section.**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use the Question 11 writing booklet.

(a) $\frac{1}{\sqrt{3}-\sqrt{2}} = \sqrt{a}+\sqrt{b}$ Find $a$ and $b$ given both are real integers. . **2**

(b) Differentiate $\frac{1+x+x^2}{x}$ with respect to $x$. **2**

(c) Find $\int \frac{x^2}{3x^3-1}\,dx$ . **2**

(d) Solve $-2-5x<13$ . **1**

(e) Find the points of intersection of $y=4-x^2+2x$ and $x+y=0$ . **3**

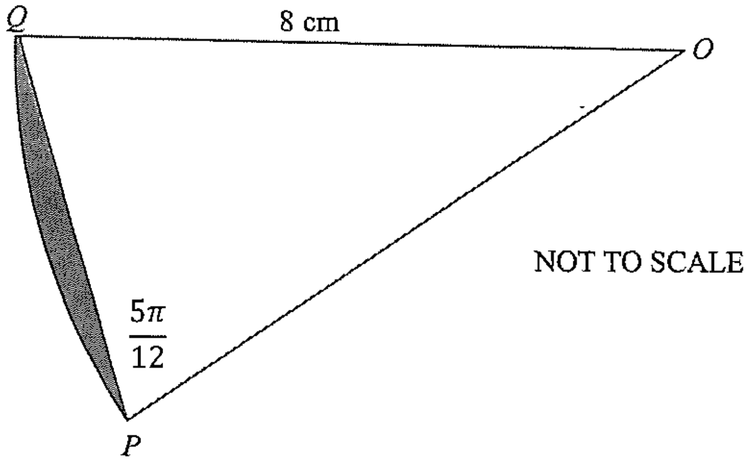
(f) Differentiate $e^{3x}\tan x$ (expressing your answer in simplest factored form) **2**

**Question 11 continues on page 9**

(g) A sector $OPQ$ of a circle, is shown in the diagram.

The circle has its centre at $O$ and a radius of 8 cm.

$\angle OPQ = \frac{5\pi}{12}$




(i) By using the Sine Rule, show that the length of the interval $PQ$ is given by $\dfrac{4}{\sin\left(\frac{5\pi}{12}\right)}$ **2**

(ii) Calculate the perimeter of the shaded segment, correct to 1 decimal place. **1**

**End of Question 11**

**Question 12** (15 marks) Use the Question 12 writing booklet.

(a) Solve $x = \sqrt{2 + x}$ **2**

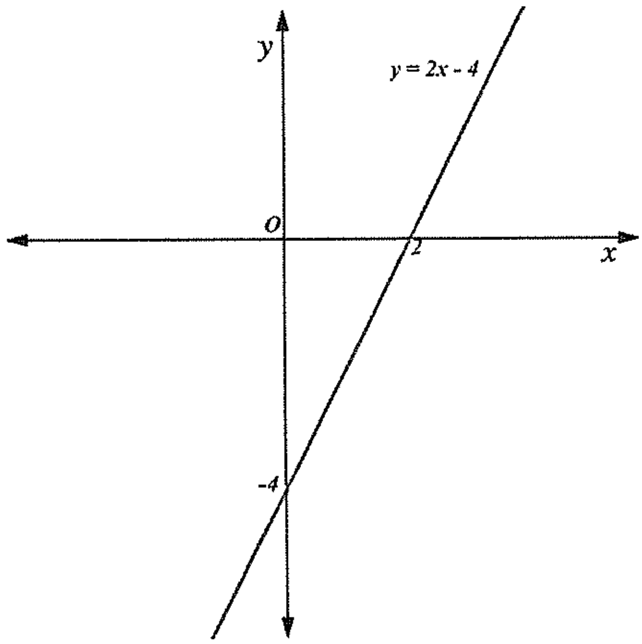
(b) If $\alpha$ and $\beta$ are the roots of $2x^2 + 3x - 6 = 0$, evaluate

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(c)




Consider the graph drawn above of the line $y = 2x - 4$ and the origin marked as $O$.

Let $P$ be a point on the line $y = 2x - 4$ such that the length of $OP$, the line segment from the origin $O$ to $P$, is a minimum.

(i) Show that the coordinates of P are $\left(\frac{8}{5}, -\frac{4}{5}\right)$ **2**

(ii) Find the **exact** distance OP, as a simplified surd. **2**

**Question 12 continues on page 11**

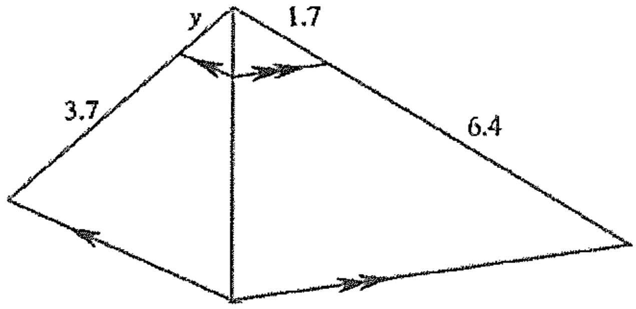
**Question 12 continued**

(d) The fifth term of an arithmetic series is 22. The sum of the first five terms is 50.

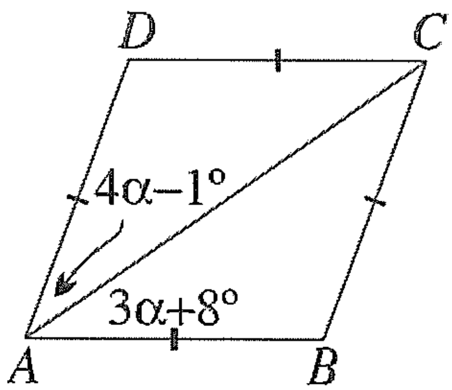
(i) Find the values of $a$, the first term and $d$, the common difference for this series. **2**

(ii) How many terms of this series are required to reach a sum greater than 1000? **2**

(e) From the diagram below, find the value of $y$ to 2 decimal places. (Reasons **not** required) **1**




(f) Find the value of $\alpha$ **1**




**End of Question 12**

**Question 13** (15 marks) Use the Question 13 writing booklet.

(a) Consider the curve $y = 3x^2 - x^3$

(i) Find the stationary points of the curve and determine their nature. **3**

(ii) Show that there is a point of inflexion and determine its coordinates. **2**

(iii) Sketch the curve, carefully labelling $x$ and $y$ intercepts, stationary points and the point of inflexion. **2**

(iv) Hence, or otherwise, find the values of $x$ for which $\frac{dy}{dx} < 0$. **1**

(b) Australia's population was 19 million at the start of the year 2000. By the beginning of 2019, it had grown to 25 million. Assuming that the population of Australia is increasing exponentially, it can be represented by an equation in the form $P = Ae^{kt}$, where A and $k$ are constants, and $t$ is measured in years from the beginning of 2000

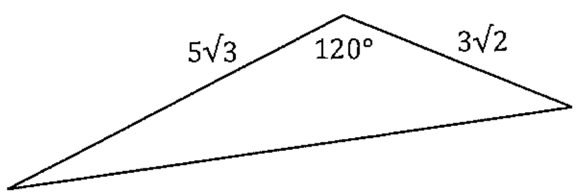
(i) Show that $P = Ae^{kt}$ satisfies the equation $\frac{dP}{dt} = kP$. **1**

(ii) What is the value of $A$? **1**

(iii) Show that $k = 0.0144$ (correct to 4 decimal places) **2**

(iv) A newspaper article, published in 1980, forecast that the Australian population would reach 50 million by the year 2050. Using calculations to justify your response, state whether you agree or disagree with this article. **2**

(c) Find the **exact area** of the triangle drawn below. **1**




**End of Question 13**

**Question 14** (15 marks) Use the Question 14 writing booklet.

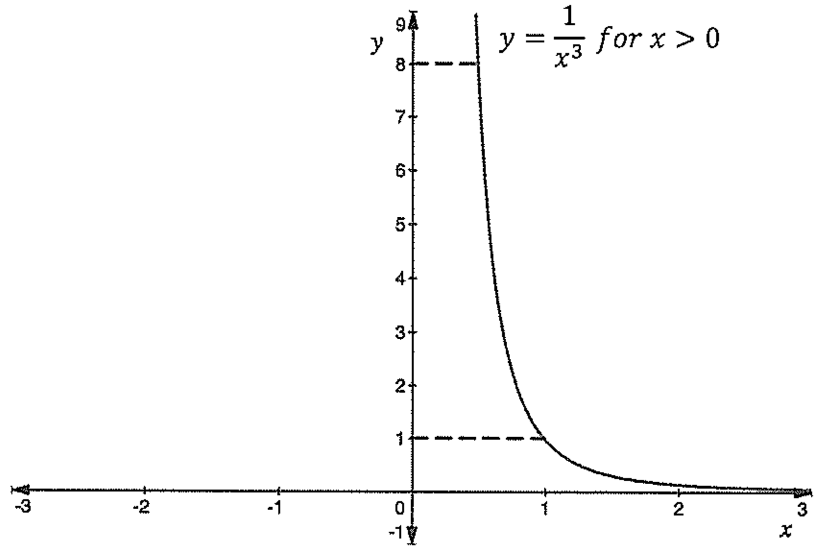
(a) Solve the equation $2\sin^2 x + cosx = 2$ where $0 \leq x \leq 2\pi$. **3**

(b) Kelsey borrowed \$600 000 for the purchase of a home. The interest rate on the loan is 3.6% per annum and the loan term is 30 years. Let $\$A_n$ be the amount owing at the end of n months and $\$M$ be the monthly repayment amount.

(i) Show that $A_2 = 600\,000(1 \cdot 003)^2 - M(1 \cdot 003 + 1)$ **1**

(ii) Show that $A_n = 600\,000(1 \cdot 003)^n - M\left[\frac{1 \cdot 003^n - 1}{0.003}\right]$ **2**

(c)

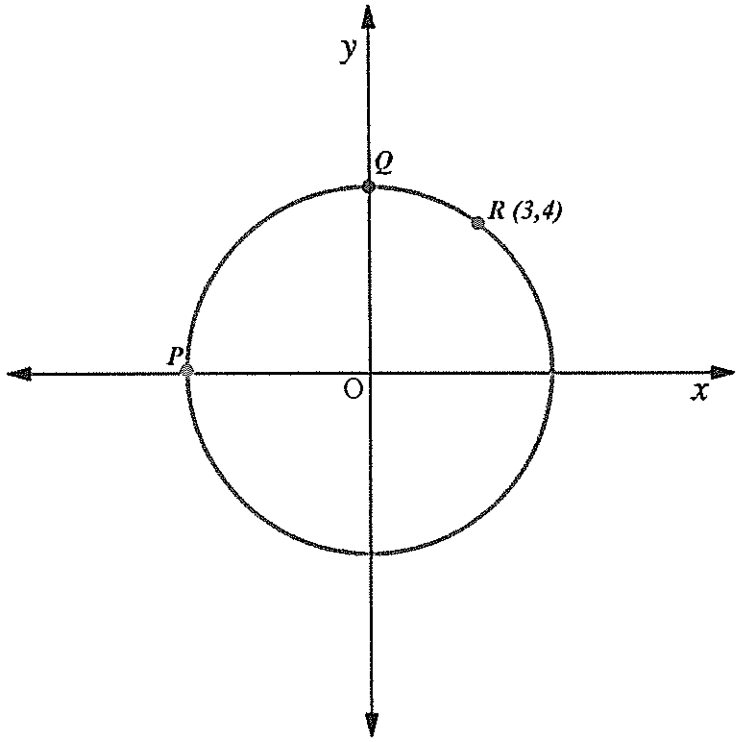



In the diagram above, the region bounded by the curve $y = \frac{1}{x^3}$ $for\ x > 0$, the y-axis and the lines $y = 1$ and $y = 8$, is revolved about the y-axis to form a solid. **3**

Find the **exact volume** of the solid.

**Question 14 continues on page 14**

(d) The diagram below shows a circle with equation $x^2 + y^2 = 25$




(i) State the coordinates of P and Q. 2

(ii) $R(3,4)$ lies on the circle as shown. Find the gradient PR 1

(iii) M is the midpoint of PR and O is the origin. Prove that OM and PR are perpendicular. 2

(iv) Show that the equation of PR is $x - 2y + 5 = 0$ 1

**End of Question 14**

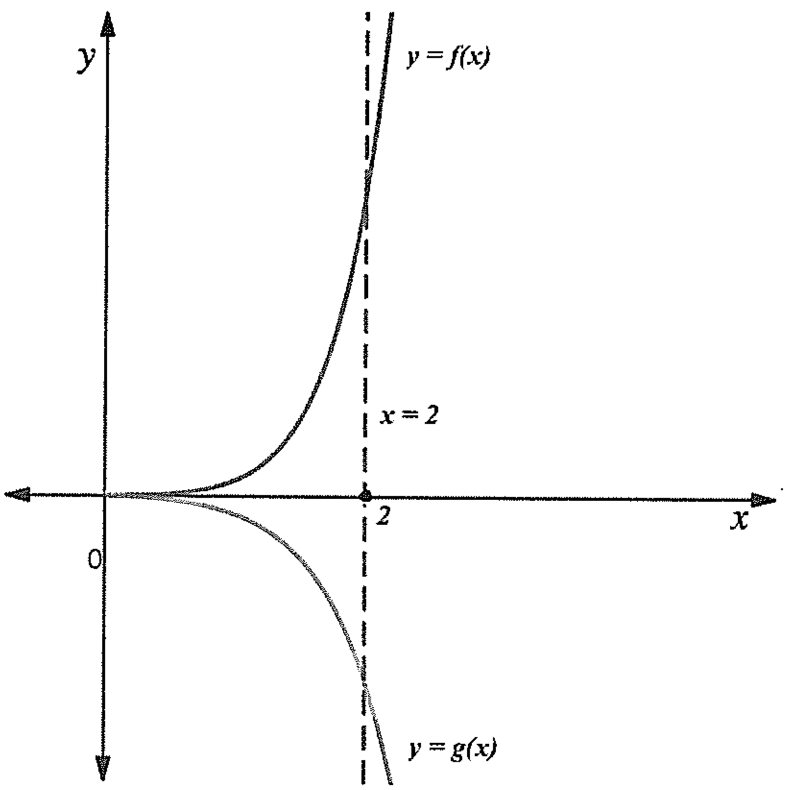
**Question 15** (15 marks) Use the Question 15 writing booklet.

(a) $f(x) = x^2 e^{kx}$ where $k$ is a positive constant.

(i) Show that $f'(x) = xe^{kx}(kx + 2)$ **1**

The graph of $f(x) = x^2 e^{kx}$ and $g(x) = \frac{-2xe^{kx}}{k}$ and the line $x = 2$ is drawn below.

$f(x) = g(x)$ at only one point, that is at $(0,0)$.




Let $A$ be the area of the region bounded by the curves $y = f(x)$, $y = g(x)$ and the line $x = 2$.

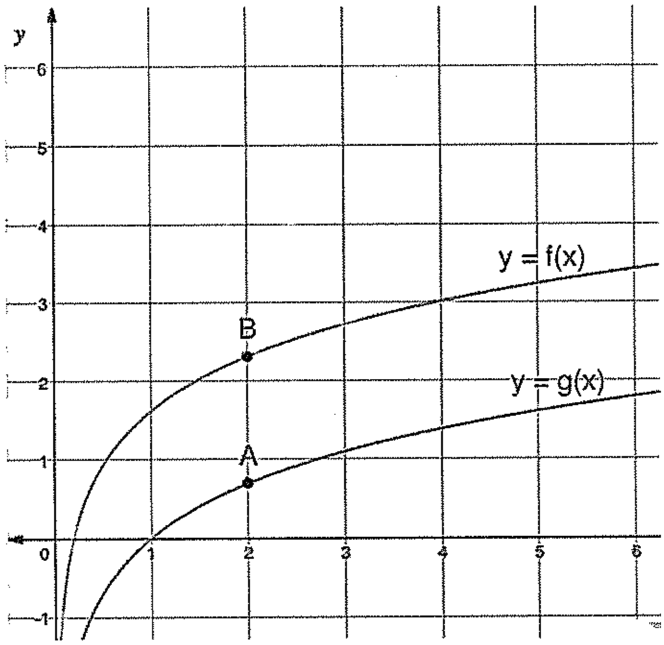
(ii) Write down a definite integral that gives the value of $A$. **1**

(iii) Using the result from **part (i)**, or otherwise, find the value of $k$ such that $A = \frac{16}{k}$ **3**

**Q15 continues on page 16**

**Question 15 continued**

(b) Consider the graphs of $f(x) = \ln(5x)$ and $g(x) = \ln(x)$ as shown below.




There is a point labelled **A** at $(2, f(2))$ and a point labelled **B** at $(2, g(2))$

(i) What is the exact distance between points **A** and **B**? **1**

(ii) Find the value of $x$ such that $g'(x) = 2$ **1**

(iii) Solve $\ln(5x) + \ln(x) = \ln(125)$ **2**

(c) Using the Trapezoidal Rule, with 5 function values,

Evaluate $\displaystyle\int_0^4 \sqrt{4-x}\,dx$ to 3 decimal places. **2**

(d) A particle is initially at the origin and its velocity function is given by

$v = \dfrac{t(t-3)(t-6)}{6}$, (where $t$ is in seconds and $v$ is in metres per second.)

(i) Explain why the particle starts at rest. **1**

(ii) Find the first time, after $t = 0$, that the particle is at rest and its position at that time. **3**

**End of Question 15**

**Question 16** (15 marks) Use the Question 16 writing booklet.

(a) Consider the parabola $20y = x^2 - 4x + 24$.

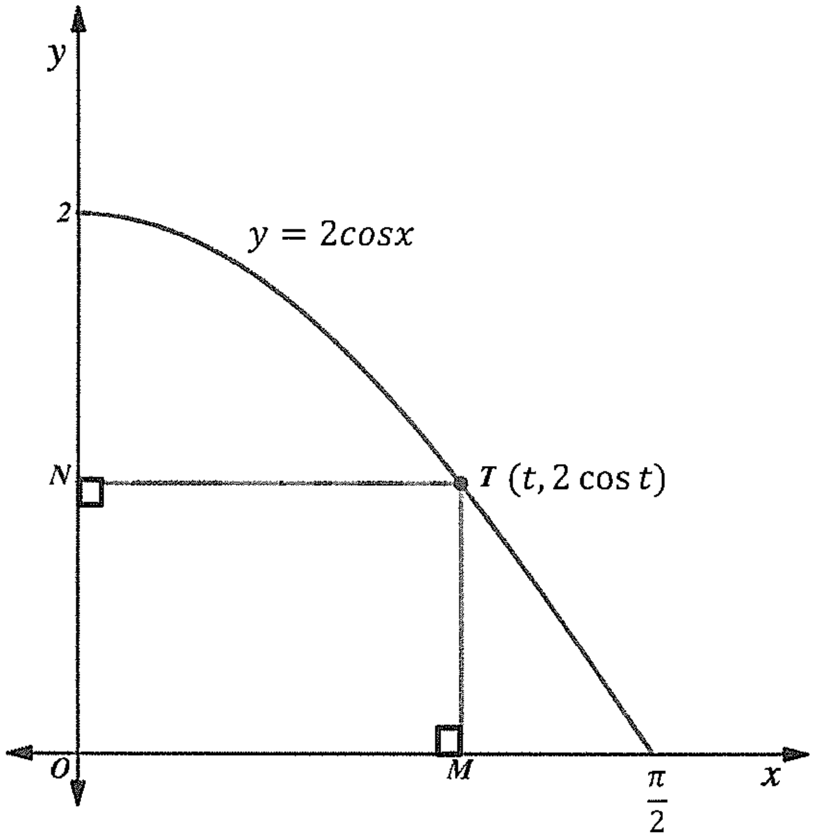
(i) Find the coordinates of the focus **2**

(ii) Find the slope of the tangent to the parabola at the point $x = 12$ **1**

(iii) Sketch, on the number plane, the region that satisfies

$20y \leq x^2 - 4x + 24$ **1**

(b)




In this diagram, T $(t, 2\cos t)$ is a variable point on $y = 2cosx$, where $0 \leq t \leq \frac{\pi}{2}$

TM and TN are perpendicular to the coordinate axes

(i) Show that the sum of the lengths of ON, NT, TM and OM is given by $f(t) = 2t + 4\cos t$. **1**

(ii) Show that $f(t)$ has a maximum value and find this value in exact form. **3**

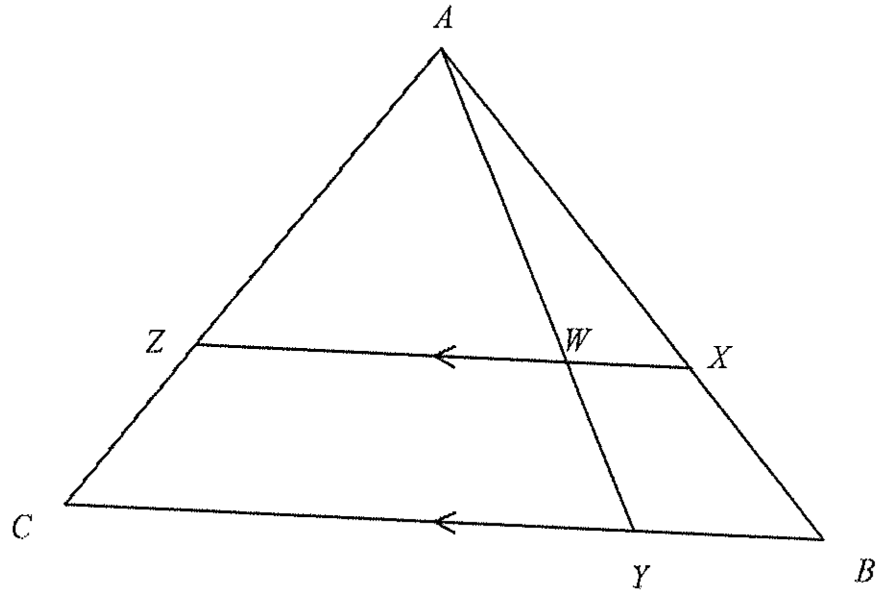
(iii) What is the least value of $f(t)$ giving reasoning for your answer. **2**

**Question 16 continues on page 18**

**Question 16 continued**

(c) In $\Delta ABC$, $AB$ has been divided by point $X$ and $AC$ has been divided by point $Z$ such that $ZX \parallel CB$. $CB$ has been divided by point $Y$. $AY$ meets $ZX$ at $W$.

$AX = p$ and $XB = q$




(i) Show that $\Delta ACY \parallel\!\mid \Delta AZW$. **2**

(ii) Show that $\dfrac{\text{AZ}}{\text{AC}} = \dfrac{p}{p+q}$. **2**

(iii) If $A_1$ is the area of $\Delta AZW$ and $A_2$ is the area of $\Delta ACY$, then it is known that

$$\frac{A_2}{A_1} = \left(\frac{p+q}{p}\right)^2 \quad \textbf{(YOU DON'T NEED TO PROVE THIS)}$$

- By using the above information, show that the area of the trapezium $ZWYC$ is given by $A_1\left(\left(\dfrac{p+q}{p}\right)^2 - 1\right)$. **1**

**End of Paper**

2019 Gosford trial HSC Mathematics

Multiple choice

1 (C)

2 $\frac{dy}{dx} = -e^{-x}$

at $x = \ln 2$

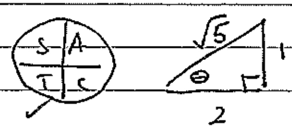
$= \frac{-1}{e^x}$ $\therefore m_T = \frac{-1}{2}$ $\therefore m_n = 2$

(C)

3 $1 + \tan^2 x = \sec^2 x$ $\therefore \tan^2 x = \sec^2 x - 1$

$\int \sec^2 x - 1 \, dx = \tan x - x + c$

(B)

4 


$\sin\theta = -\frac{1}{\sqrt{5}}$

(D)

5 (C)

6 (B)

7 $\lim_{x \to 0} \left( x + \frac{2 \sin 2x}{2x} \right)$

$= 0 + 2$

$= 2$ (C)

8 (C)

9 (B)

10 $(2 \times \ln 2) - \int_0^{\ln 2} x \, dy$

$y = \ln x$

$x = e^y$

$\therefore \ln 4 - \int_0^{\ln 2} e^y \, dy$ (C)

Q11

(a) $\frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{1}$ $a = 3$, $b = 2$

(b) $\frac{d}{dx}\, x^{-1} + 1 + x = -x^{-2} + 1$.

$= \frac{-1}{x^2} + 1$

(c) $\frac{1}{9}\int \frac{9x^2}{3x^3-1}\, dx = \frac{1}{9} \ln|3x^3-1| + C$

(d) $-5x < 15$

$x > -3$

(e) $y = -x$ $\therefore -x = 4 - x^2 + 2x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ OR $x = -1$

$y = -4$ $y = 1$

$\therefore$ intersect at $(4, -4)$ and $(-1, 1)$

(f) $\frac{d}{dx} = e^{3x} . \sec^2 x + \tan x . 3e^{3x}$

$= e^{3x}(\sec^2 x + 3\tan x)$

(g) $OP = PO = 8$ (radii) $\angle PQO = \frac{5\pi}{12}$ (base $\angle$'s isosceles $\triangle OQP$)

$\therefore \angle QOP = \frac{\pi}{6}$

Q11 (continued)

$\frac{8}{\sin\left(\frac{5\pi}{12}\right)} = \frac{QP}{\sin\left(\frac{\pi}{6}\right)}$

$QP = \frac{8\sin\left(\frac{\pi}{6}\right)}{\sin\left(\frac{5\pi}{12}\right)}$

$= \frac{8 \times \frac{1}{2}}{\sin\left(\frac{5\pi}{12}\right)}$

$= \frac{4}{\sin\left(\frac{5\pi}{12}\right)}$

(ii) $P = \left(\frac{4}{\sin\left(\frac{5\pi}{12}\right)}\right) + \left(8 \times \frac{\pi}{6}\right)$

$= 8.3$

Q12 (a) $x^2 = 2 + x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

check $x = 2$ $\quad 2 = \sqrt{2+2}$ TRUE

check $x = -1$ $\quad -1 = \sqrt{2-1}$ FALSE

$\therefore$ $x = 2$ only solution

(b) $2x^2 + 3x - 6 = 0$

(i) $\alpha + \beta = -\frac{3}{2}$

(ii) $\alpha\beta = -\frac{6}{2}$

$= -3$

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{-3}{2}\right)^2 - 2(-3)$

$= \frac{9}{4} + 6$

$= \frac{33}{4}$

(c) Gradient $y = 2x - 4$ is 2.

Shortest distance is $\perp$ distance

$\therefore m_{OP} = -\frac{1}{2}$

$\therefore$ equation OP is $y = -\frac{1}{2}x$

Solve $2x - 4 = -\frac{1}{2}x$

$4x - 8 = -x$

$5x = 8$

$x = \frac{8}{5}$ $\quad y = -\frac{1}{2}\left(\frac{8}{5}\right)$

$= -\frac{4}{5}$

$\therefore$ P is $\left(\frac{8}{5}, -\frac{4}{5}\right)$

(ii) OP is the $\perp$ distance from $(0, 0)$ and $2x - y - 4 = 0$

$d = \frac{|(2)(0) + (-1)(0) - 4|}{\sqrt{4+1}}$

$= \frac{4}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{4\sqrt{5}}{5}$

(d) $\boxed{a + 4d = 22}$ (1) $\quad \frac{5}{2}(2a + 4d) = 50$

$10a + 20d = 100$

$\boxed{a + 2d = 10}$ (2)

Solving (1) + (2) simultaneously

$a + 4d = 22$

$a + 2d = 10$

$2d = 12$

$d = 6$ $\quad a + 12 = 10$

$a = -2$

Q12 (d) continued

(ii) $\frac{n}{2}\left(2(-2)+(n-1)6\right) > 1000$

$n(-4+6n-6) > 2000$

$n(6n-10) > 2000$

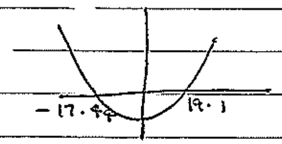
$6n^2 - 10n - 2000 > 0$

$3n^2 - 5n - 1000 > 0$

$n = \frac{5 \pm \sqrt{25 - 4 \times 3 \times (-1000)}}{6}$

$n = \frac{5 \pm \sqrt{12025}}{6}$

$n \doteqdot 19.1$ OR $-17.44$




$\therefore$ Will need 20 terms to exceed 1000

(e) $\frac{y}{3.7} = \frac{1.7}{6.4}$

$6.4y = 3.7 \times 1.7$

$y = \frac{3.7 \times 1.7}{6.4}$

$y = 0.98$

(f) $4\alpha - 1 = 3\alpha + 8$

$\alpha = 9$

Q13/ $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

(i) st pts occur when $\frac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0$ OR $x = 2$

$y = 0$ $y = 4$

$\frac{d^2y}{dx^2} = 6 - 6x$

at $x = 0$ $\frac{d^2y}{dx^2} > 0 \therefore$ Min st pt at $(0,0)$

at $x = 2$ $\frac{d^2y}{dx^2} < 0 \therefore$ Max st pt at $(2,4)$

(ii) Possible Point of inflexion when $\frac{d^2y}{dx^2} = 0$

$6 - 6x = 0$

$6 = 6x$

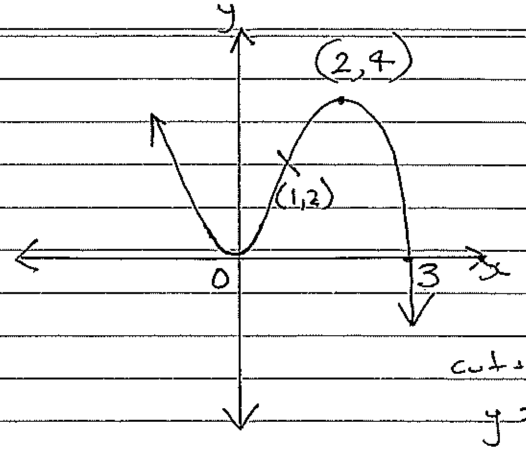
$x = 1$

| $x$ | $1^-$ | $1$ | $1^+$ |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | + | 0 | − |

$\therefore$ concavity changes either side of $x = 1$

$\therefore (1,2)$ is a possible point of inflexion

(iii)




cuts $x$ axis when

$y = 0$

$0 = 3x^2 - x^3$

$= x^2(3-x)$

$x = 0$ OR $x = 3$

(iv) $x < 0$ OR $x > 2$

(b) (i) $P = Ae^{kt}$

$\frac{dP}{dt} = k.Ae^{kt}$

$= k.P$

(ii) $A = 19$ million

(iii) $25 = 19e^{19k}$

$\frac{25}{19} = e^{19k}$

$19k = \ln\left(\frac{25}{19}\right)$

$k = \frac{\ln\left(\frac{25}{19}\right)}{19}$

$k = 0.0144$ (to 4 dec places)

(iv) $50 = 19e^{0.0144t}$

$\frac{50}{19} = e^{0.0144t}$

$\left(\ln \frac{50}{19}\right) \div 0.0144 = t$

$t \doteqdot 67.2$ years

Statement is false as won't reach 50 million until around 2068

(c) $A = \frac{1}{2} \times 5\sqrt{3} \times 3\sqrt{2} \sin 120°$

$= \frac{15\sqrt{6}}{2} \times \sin 60°$

$= \frac{15\sqrt{6}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{15\sqrt{18}}{4}$

$= \frac{15 \times 3\sqrt{2}}{4}$

$= \frac{45\sqrt{2}}{4}$

Q14/(a) $2(1-\cos^2 x) + \cos x = 2$

$2 - 2\cos^2 x + \cos x - 2 = 0$

$0 = 2\cos^2 x - \cos x$

$0 = \cos x(2\cos x - 1)$

$\therefore \cos x = 0 \qquad \cos x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

(b)(i) Let $A_1$ = amount owing after 1 month

$A_1 = 600{,}000(1.003) - M$

$\therefore A_2 = A_1(1.003) - M$

$= [600{,}000(1.003) - M]1.003 - M$

$= 600{,}000(1.003)^2 - 1.003M - M$

$= 600{,}000(1.003)^2 - M(1.003 + 1)$

(ii) $A_n = 600{,}000(1.003)^n - M(1.003^{n-1} + 1.003^{n-2} + \dots + 1.003 + 1)$

$A_n = 600{,}000(1.003)^n - M(S_n)$

where $S_n = \frac{a(r^n - 1)}{r-1}$

where $a = 1 \quad r = 1.003$

Q14 (continued) (c)

$S_n = \frac{1(1.003^n - 1)}{1.003 - 1}$

$= \boxed{\frac{1.003^n - 1}{0.003}}$

$\therefore A_n = 600{,}000(1.003)^n - M\boxed{\frac{1.003^n - 1}{0.003}}$

(c) $V = \pi\int_1^8 x^2\,dy$

$y = \frac{1}{x^3}$

$x^3 = \frac{1}{y}$

$(x^3)^{\frac{2}{3}} = \left(\frac{1}{y}\right)^{\frac{2}{3}}$

$\therefore V = \pi\int_1^8 y^{-\frac{2}{3}}\,dy \longleftarrow x^2 = y^{-\frac{2}{3}}$

$= \pi\left[3y^{\frac{1}{3}}\right]_1^8$

$= 3\pi\left[8^{\frac{1}{3}} - 1^{\frac{1}{3}}\right]$

$= 3\pi[2 - 1]$

$= 3\pi \text{ units}^3$

Q14/ (d)

(i) P is $(-5,0)$

Q is $(0,5)$

(ii) $m_{PR} = \frac{0-4}{-5-3}$

$= \frac{-4}{-8}$

$= \frac{1}{2}$

(iii) M is $\left(\frac{-5+3}{2}, \frac{0+4}{2}\right)$

M is $(-1,2)$

$m_{OM} = \frac{0-2}{0+1}$ $\quad m_{PR} = \frac{1}{2}$

$= -2$

$m_{OM} \times m_{PR} = -2 \times \frac{1}{2}$

$= -1$

$\therefore$ lines are $\perp$

(iv) $y - 0 = \frac{1}{2}(x+5)$

$2y = x+5$

$0 = x - 2y + 5$

Q15/ (a) $f(x) = x^2 e^{kx}$

(i) $f'(x) = x^2 . ke^{kx} + e^{kx} . 2x$

$= xe^{kx}(kx+2)$ as required

(ii) $A = \int_0^2 f(x) - g(x) \, dx$

$= \int_0^2 x^2 e^{kx} + \frac{2}{k} x e^{kx} \, dx$

(iii) $x^2 e^{kx} + \frac{2}{k} x e^{kx} = \frac{1}{k} x e^{kx}(kx+2)$

$\therefore A = \frac{1}{k} \int_0^2 x e^{kx}(kx+2) \, dx$

From part (i) $\int x e^{kx}(kx+2) \, dx$

$= x^2 e^{kx} + c$

$\therefore A = \frac{1}{k}\left[x^2 e^{kx}\right]_0^2$

$A = \frac{1}{k}\left[4e^{2k} - 0\right]$

$\boxed{A = \frac{4e^{2k}}{k}}$

Q15 (a) (continued)

If $A = \frac{16}{K}$

$$\frac{16}{K} = \frac{4e^{2k}}{K}$$

$$\frac{16}{4} = e^{2k}$$

$$e^{2k} = 4$$

$$2k = \ln 4$$

$$k = \frac{\ln 4}{2}$$

$$k = \tfrac{1}{2}\ln 4$$

$$k = \ln 4^{\frac{1}{2}}$$

$$k = \ln 2$$

(b) (i) $\ln 5$

(ii) $g'(x) = \frac{1}{x}$

If $g'(x) = 2$

$$2 = \frac{1}{x}$$

$$\therefore x = \frac{1}{2}$$

(iii) $\ln 5x^2 = \ln 125$

$$5x^2 = 125$$

$$x^2 = 25$$

$$x = \pm 5 \quad \text{but } x > 0$$

$$\therefore x = 5 \text{ only}$$

Q15 (continued)

(c)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | $\sqrt{3}$ | $\sqrt{2}$ | 1 | 0 |

$$\therefore \int_0^4 \sqrt{4-x}\,dx \doteq \frac{1}{2}\left[2+0+2\left(\sqrt{3}+\sqrt{2}+1\right)\right]$$

By Trapezoidal Rule

$$\doteq 5.146$$

(d) $v = \frac{t(t-3)(t-6)}{6}$

(i) When $t=0$ $\quad v = \frac{0(-3)(-6)}{6}$

$$= 0$$

$\therefore$ as $v=0$ at $t=0$, particle starts at rest.

(ii) Next at rest when $t=3$

$$v = \frac{(t^2-3t)(t-6)}{6}$$

$$v = \frac{1}{6}\left(t^3-6t^2-3t^2+18t\right)$$

$$= \frac{1}{6}\left(t^3-9t^2+18t\right)$$

$$x = \frac{1}{6}\int t^3-9t^2+18t\,dt$$

$$x = \frac{1}{6}\left[\frac{t^4}{4}-3t^3+9t^2\right]+C$$

when $t = 0 \quad x = 0$

$\therefore c = 0$

$\therefore x = \frac{1}{6}\left[\frac{t^4}{4} - 3t^3 + 9t^2\right]$

when $t = 3$

$x = \frac{1}{6}\left[\frac{81}{4} - 3(27) + 9(9)\right]$

$x = \frac{27}{8}$

$\therefore$ Next at rest when $t = 3$ and the particle is $\frac{27}{8}$ m to the right of the origin.

Q16

(i) $20y - 24 = x^2 - 4x$

$20y - 24 + 4 = x^2 - 4x + 4$

$20y - 20 = (x-2)^2$

$20(y-1) = (x-2)^2$

$\therefore (x-2)^2 = 4(5)(y-1)$

Vertex is $(2,1)$ focal length $= 5$

$\therefore$ Focus is $(2,6)$

(ii) $y = \frac{1}{20}\left[x^2 - 4x + 24\right]$

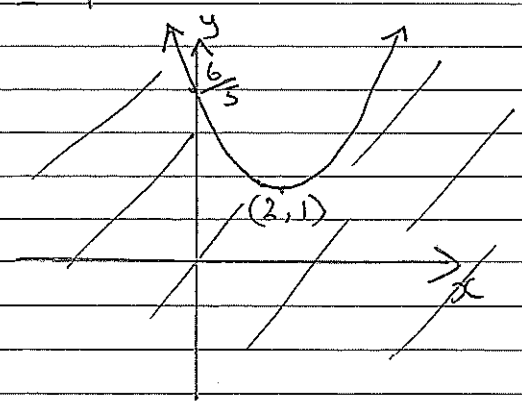
$\frac{dy}{dx} = \frac{1}{20}\left[2x - 4\right]$

when $x = 12$

$m_T = \frac{1}{20}\left[24 - 4\right]$

$= 1$

(iii)



Q16 (b)

(i) $ON = 2\cos t$

$NT = t$

$TM = 2\cos t$

$OM = t$

$\therefore ON + NT + TM + OM = 2t + 4\cos t$

(ii) $f(t) = 2t + 4\cos t$

$f'(t) = 2 - 4\sin t$

St pts occur when $f'(t) = 0$

$4\sin t = 2$

$\sin t = \frac{1}{2}$ but $0 \le t \le \frac{\pi}{2}$

$\therefore t = \frac{\pi}{6}$

$f''(t) = -4\cos t$ at $t = \frac{\pi}{6}$

$f''(t) = -4\cos\frac{\pi}{6}$

$= -2\sqrt{3}$

As $f''(t) < 0$ at $t = \frac{\pi}{6}$

there is a maximum at $t = \frac{\pi}{6}$

$f(t) = 2t + 4\cos t$ (when $t = \frac{\pi}{6}$)

$f(t) = \frac{\pi}{3} + 4\cos\frac{\pi}{6}$

$= \frac{\pi}{3} + \frac{4\sqrt{3}}{2}$

$= \frac{\pi}{3} + 2\sqrt{3}$

Q16 (continued)

(b)(iii) least values (from diagram) must occur at the end points

$P(0) = 2(0) + 4\cos 0$

$= 4$

$P\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right) + 4\cos\left(\frac{\pi}{2}\right)$

$= \pi$

But $\pi < 4$ hence least value occurs when $t = \frac{\pi}{2}$

and the least value of $f(t)$ is $\pi$.

(c)(i) $\angle AZW = \angle ACY$ (corresponding $\angle$'s $ZX \parallel CB$)

$\angle CAY$ is common

$\therefore \triangle ACY \text{ ||| } \triangle AZW$ (equiangular)

(ii) $\frac{AX}{XB} = \frac{AW}{WY} = \frac{AZ}{ZC}$ (ratio of intercepts theorem)

But $\frac{AX}{XB} = \frac{p}{q}$ and $\frac{AX}{AB} = \frac{AW}{AY} = \frac{AZ}{AC}$

(ratio of intercepts)
(or by similar $\Delta$'s)

But $\frac{AX}{AB} = \frac{p}{p+q} = \frac{AZ}{AC}$ (as required)

(iii) Area trapezium $= A_2 - A_1$

$$\frac{A_2}{A_1} = \left(\frac{p+q}{p}\right)^2 \quad \text{(given)}$$

$$\therefore \quad A_2 = A_1\left(\frac{p+q}{p}\right)^2$$

$$\therefore \quad \text{Area trapezium} = A_1\left(\frac{p+q}{p}\right)^2 - A_1$$

$$= A_1\left[\left(\frac{p+q}{p}\right)^2 - 1\right]$$