STANDARD INTEGRALS

$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$

$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE : $\ln x = \log_e x,\quad x > 0$

The Hills Grammar School Limited

TRIAL HIGHER SCHOOL CERTIFICATE

1989

MATHEMATICS

2/3 UNIT

Time allowed – Three hours
(Including reading time)

TEACHER RESPONSIBLE: MRS B SPENCER

Directions to candidates

- All questions may be attempted.
- All questions are of equal value.
- All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.
- Standard integrals are listed on the last page.
- Each question attempted is to be returned in a separate writing booklet clearly marked Question 1, Question 2 etc on the cover. Each booklet must show your Candidate's Number and the Centre Number.
- Additional writing booklets may be obtained from the Examination Supervisor.
- Approved silent calculators may be used.

QUESTION 1 (Start a new Booklet)

(a) Solve for x

$$\frac{x+6}{2} \leq x$$

(b) Simplify the expression

$$3a - 2(5 + a)$$

(c) Find, correct to two decimal places, the value of

$$\frac{\sqrt{18.7}}{2.65 + 3.61}$$

(d) Factorise $2x^2 - 8$

(e) The capacity V, of a cylindrical bucket, is given by

$$V = 100r - \frac{1}{2}\pi r^3$$

Find V, correct to two decimal places, if $r = 3$

QUESTION 2 (Start a new Booklet)

(a) Differentiate:

i) $x^2 + 2x - \frac{1}{x} + 3$

ii) $2x\,e^x$

iii) $\log_e (x+1)^2$

(b) Find, correct to one decimal place, the value of:

i) $\int_0^1 \frac{1}{2x+3}\,dx$ ii) $\int_0^{1/4} e^{4x}\,dx$

QUESTION 3 (Start a new booklet)

A triangle ABC has sides whose equations are

AC: $x - y + 4 = 0$

BC: $2x + y - 10 = 0$

AB: $x - 2y + 7 = 0$

C has co-ordinates (2,6) and A (−1,3)

i) Show that the triangle is right-angled.

ii) Find the co-ordinates of B

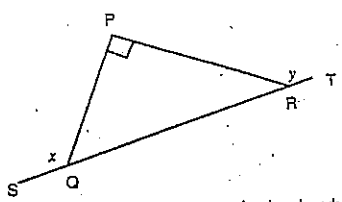
iii) Sketch the triangle on a set of co-ordinate axes (use about half a page) showing all the data.

iv) Obtain the length of the hypotenuse in *simplest* surd form.

v) Hence or otherwise find area of Δ ABC.

QUESTION 4 (Start a new booklet)

(a)



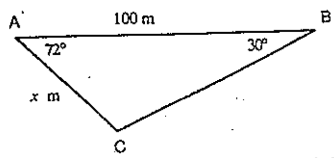
i) In your writing booklet draw a neat sketch and mark on it all given information.

ii) Prove $x + y = 3$ right angles.

(b) Simplify $\sqrt{2} + \frac{1}{\sqrt{50} - 7}$ expressing your final answer in the form $a + b\sqrt{2}$,

(c) Find the equation of the normal to the curve $y = \sqrt{x+1}$ at the point P (3,2).

(d)



ABC is a triangle in which AB = 100m A = 72°, B = 30° as in the above figure.
Use the sine rule to calculate x correct to three significant figures.

Question 3 *Start a new page* **Marks**

(a) Find primitives of:

(i) $\sec^2 7x$ — 1

(ii) $\dfrac{x^2}{x^3+3}$ — 2

(b) (i) On the same diagram draw graphs of the functions $y = x^2$ and $y = 5-4x$ showing all intercepts with the x and y- axes. — 2

(ii) Show that the graphs intersect at $x = 1$ and $x = -5$. — 1

(iii) Hence find the exact area bounded by the two functions. — 2

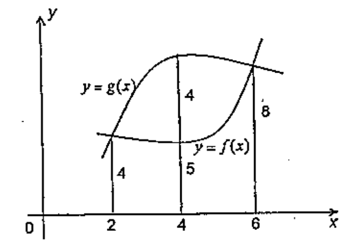
(c). Evaluate :

(i) $\displaystyle\int_{-2}^{-1}\left(\frac{1}{x^2}\right)dx$ — 2

(ii) $\displaystyle\int_0^{\frac{\pi}{3}}\cos(2x+\pi)\,dx$ — 2

Question 5 *Start a new page* **Marks**

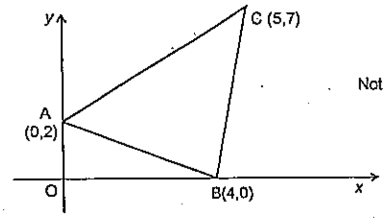
(a) — 3



Not to Scale

In the diagram above, the graphs of the functions $y = f(x)$ and $y = g(x)$ are shown. Using Simpson's Rule, find an approximate value for the area enclosed by the two curves.

(b)



Not to Scale

The diagram shows the points A (0,2), B (4,0) and C (5,7). Copy the diagram onto your worksheet.

(i) Find the coordinates of M, the midpoint of AB. — 1

(ii) Show that the gradient of AB is $-\dfrac{1}{2}$ — 1

(iii) Find the equation of the perpendicular bisector of AB. — 2

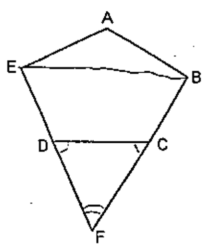
(iv) Show that the perpendicular bisector of AB passes through C. — 1

(v) What type of triangle is ABC? (Give a reason for your answer) — 1

(c) Solve : $2^{2x} - 15(2^x) - 16 = 0$ — 3

Question 7 *Start a new page* **Marks**

(a)



Not to Scale

ABCDE is a regular pentagon. BC and ED are produced to meet at F.

Copy or trace the diagram onto your working paper.

(i) Show that the size of each internal angle in the pentagon is 108° — 1

(ii) Show that triangle FCD is isosceles. — 1

(iii) Prove that triangle FCD is similar to triangle FBE. — 2

(iv) If the sides of the pentagon are each 5 centimetres and BE = 8 centimetres, determine the length of CF.. — 2

(b) For the curve represented by the equation $y = x^3 + 3x^2 - 1$

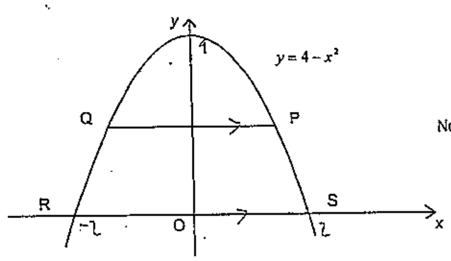
(i) Find $\dfrac{dy}{dx}$. — 1

(ii) Find all stationary points and determine their nature. — 3

(iii) Sketch the curve in the domain $-3 \le x \le 2$, showing the above information. — 2

Question 9 *Start a new page* **Marks**

(a)



Not to Scale

The parabola $y = 4 - x^2$ cuts the x-axis at R and S. The point P (x,y) lies on the parabola in the first quadrant. Q also lies on the parabola such that PQ is parallel to the x-axis.

(i) Write down the coordinates of R and S. — 1

(ii) Show that the area of trapezium PQRS is given by : — 2

$$A = (2+x)(4-x^2)$$

(iii) Hence find the value of x which gives a maximum value of A, justifying your answer — 3

(b) The size of the population, P, of a colony of whiteants after t days is given by the equation $P = 3000e^{kt}$

(i) What was the initial size of the colony? — 1

(ii) If there are 4000 whiteants in the colony after 1 day, find the value of k correct to 2 decimal places. — 2

(iii) What is the size of the colony after 2 days? — 2

(iv) When will the colony quadruple in size? (Answer to the nearest day) — 1