THE HILLS GRAMMAR SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2004

MATHEMATICS

Time Allowed: Three hours (plus 5 minutes reading time)

General Instructions:

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown for every question
- Begin each question in a new booklet

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value

	01	02	02	03	03	04	04	05	06	06	
Q	1	2	3	4	5	6	7	8	9	10	Total
M											

Students are advised that this is a Trial Examination only and cannot in any way guarantee the content or the format of the Higher School Certificate Examination.

QUESTION 1 (12 marks)

(a) Simplify $3x - 6 - (4 - 2x)$. **1**

(b) Fully simplify $\sqrt{27} - \sqrt{3} + \sqrt{18}$. **2**

(c) Solve $|x - 1| = 2x - 1$. **2**

(d) Express $0 \cdot \dot{6}\dot{9}$ as a fraction in simplest terms. **2**

(e) Solve simultaneously

$$3x - y = 5$$
$$5x + 3y = -8$$

3

(f) Find the integers a and b such that

$$\frac{\sqrt{5}}{2 - \sqrt{5}} = a + b\sqrt{5}.$$

2

QUESTION 2 (12 marks)

If A (5, 3), B (–2, 5) and C (4, –3) are points on a number plane find the:

(a) distance from A to C in surd form; **2**

(b) gradient of the line joining A and C; **2**

(c) equation of the line passing through A and C; **2**

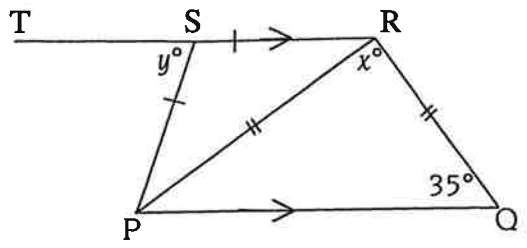
(d) perpendicular distance from B to the line passing through A and C; **2**

(e) area of the triangle ABC; **2**

(f) equation of the line perpendicular to the line AC passing through the midpoint of AC. **2**

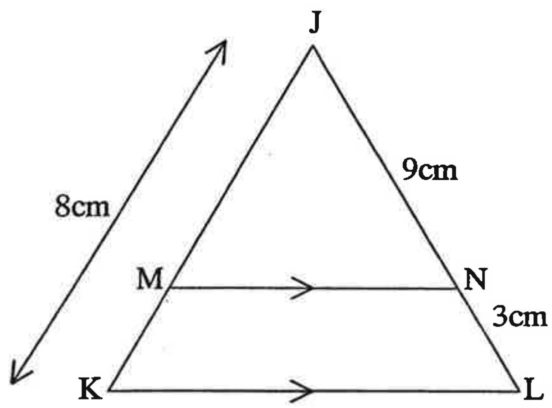
QUESTION 3 **(12 marks)**

(a)



Find $x°$ and $y°$ in the diagram above. **3**

(b)



In the triangle JKL, JK = 8cm, JN = 9cm and NL = 3cm.

(i) Prove that Δ JMN is similar to Δ JKL. **2**

(ii) Find the length of MK. **2**

Question 3 continued

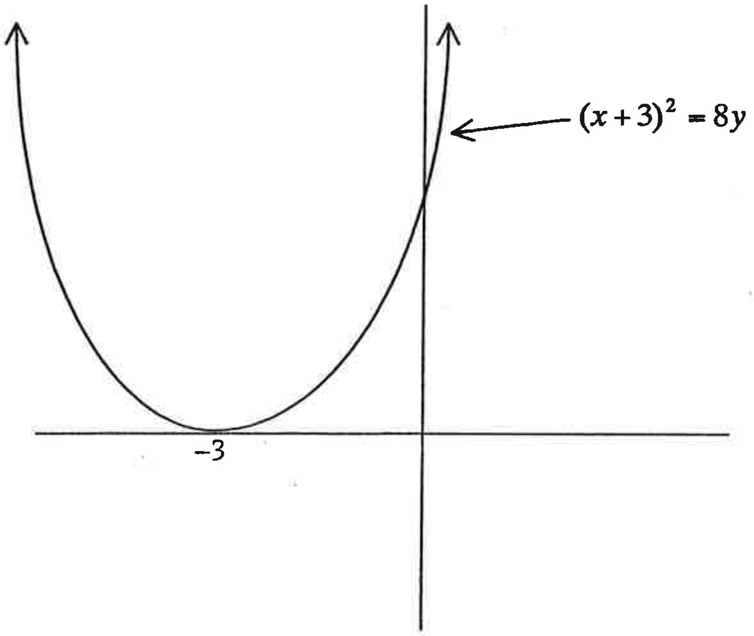
(c) For this parabola find the:

(i) co-ordinates of the vertex; **1**

(ii) focal length; **2**

(iii) co-ordinates of the focus; **1**

(iv) equation of the directrix. **1**



QUESTION 4 (12 marks)

(a) Find the exact arc length of a circle with radius 3cm and an angle subtended at the centre of 60°. **2**

(b) Solve $2\cos x = -\sqrt{3}$ for $0 \le x \le 2\pi$. **3**

(c) (i) Sketch $y = 3\cos 2x$ and $y = x$ on the same set of axes for $-\pi \le x \le \pi$. **3**

(ii) How many solutions does $3\cos 2x = x$ have? **1**
You are NOT required to find the solutions.

(d) Find the equation of the normal to the curve $y = x\sin x$ at the point $(\pi, 0)$. **3**

QUESTION 5 (12 marks)

(a) Solve $3^{3x-2} = 11$ correct to 2 decimal places. **2**

(b) If $y = \log_e\left(\dfrac{2x+1}{3x-1}\right)$ find $\dfrac{dy}{dx}$. **2**

Hint: use properties of logarithms before differentiating.

(d) Find (i) $\int \frac{1}{x} + \frac{1}{x^2}\,dx$ **1**

(ii) $\int \frac{4x}{x^2+1}\,dx$. **2**

(e) Differentiate with respect to x

$y = x^2 \ln x$. **2**

(f) Show that $\dfrac{2}{x+4} + \dfrac{3}{x-4} = \dfrac{5x+4}{x^2-16}$. **3**

Hence find $\int \frac{5x+4}{x^2-16}\,dx$.

QUESTION 6 **(12 marks)**

Evaluate each of the following integrals:

(a) $\int x^6 - 2x^4 \, dx$. **2**

(b) $\int_{-1}^{2} \frac{x^4 + 3x^3 + x^2}{x}$. **3**

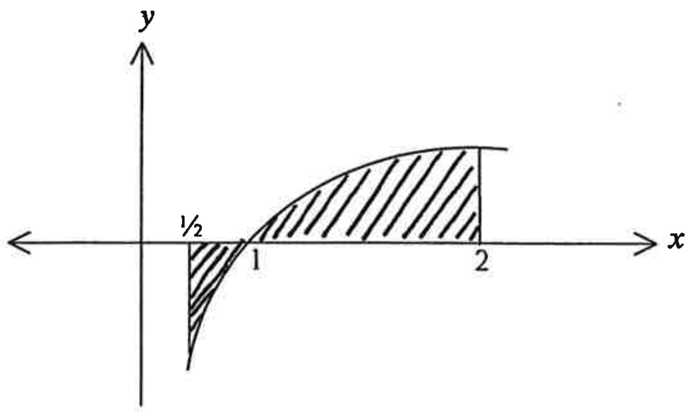
(c) $\int \sqrt{3-x} \, dx$. **2**

(d) The table shows the values of a function $f(x)$ for five x values. **3**

x	1	1·5	2	2·5	3
$f(x)$	1·011	1·179	1·322	1·447	1·559

Approximate the value of $\int_{1}^{3} f(x)\,dx$ using the five function values and the Trapezoidal rule.

(e) Bob used $A = \int_{\frac{1}{2}}^{2} f(x)\,dx$ to find the shaded area in the diagram below. **2**



Is this the correct method to use? If not, why not?

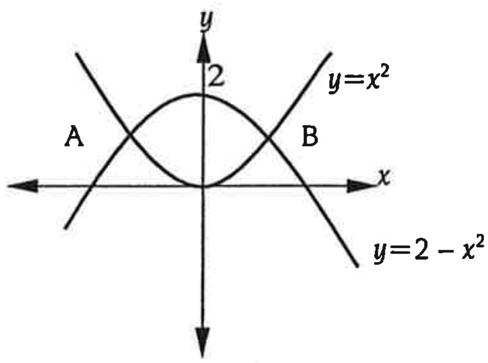
QUESTION 7 (12 marks)

(a) Consider the function $f(x) = 4\sin 2x$.

(i) Sketch $f(x)$ for $0 \le x \le 2\pi$. **2**

(ii) Using the result of part (i) and symmetry, or otherwise, find the area contained between the function $f(x)$ and the x-axis for $0 \le x \le 2\pi$. **2**

(b) The curves $y = x^2$ and $y = 2 - x^2$ intersect at two points, A and B.



(i) Find the coordinates of A and B. **2**

(ii) Find the area bounded by the curves $y = x^2$ and $y = 2 - x^2$. **3**

(c) Find the volume formed when the curve $y = \dfrac{\sqrt[3]{x}}{2}$ is rotated about the y-axis between $y = 0$ and $y = 2$. **3**

QUESTION 8 (12 marks)

(a) An arithmetic series has a third term of 14 and a seventh term of 30.
Find the first term and common difference of the series. **3**

(b) Consider the series

$$-2+4(\pi-3)-8(\pi-3)^2+16(\pi-3)^3-+\cdots.$$

(i) Explain why the geometric series has a limiting sum. **1**

(ii) Find the exact value of the limiting sum. **2**

(c) The velocity, in metres per second, of a particle moving in a straight line is given by $\dot{x}=t^3-7t-7$.

(i) Find an expression for the displacement if the particle was initially 6 m to the left of the origin. **2**

(ii) Calculate the acceleration of the particle after 3 seconds. **1**

(iii) Find the displacement and velocity of the particle after 3 seconds. **2**

(iv) Describe the motion of the particle when $t = 3$ seconds. **1**

QUESTION 9 (12 marks)

Observational evidence suggests that the level of "interest" students have in Mathematics lessons varies during a typical lesson according to $I(t) = 5te^{-2t}$, where I is the interest level, and t is time in hours.

(a) Show that the first derivative, $\dfrac{dI}{dt} = 5e^{-2t}(1-2t)$. **1**

(b) Show that the second derivative, $\dfrac{d^2I}{dt^2} = 20e^{-2t}(t-1)$. **2**

(c) At what time during the lesson is the student interest at a maximum? **3**

(d) When is the level of interest in the lesson decreasing most rapidly, i.e. when is $\dfrac{dI}{dt}$ at its minimum value? **3**

(e) Find the location of any points of inflection on the curve $I(t) = 5te^{-2t}$. **1**

(f) Using your findings from (a) – (e) above make a neat sketch of $I(t)$ for $0 < t < 2$. Your sketch should include all important features. **2**

QUESTION 10 (12 marks)

(a) According to one cosmological theory, there were equal amounts of uranium isotopes ^{235}U and ^{238}U at the creation of the universe in the "big bang".

At present there are 137.7 ^{238}U atoms for each ^{235}U atom. Using the half-lives

- 4.51 billion years for ^{238}U,
- 0.71 billion years for ^{235}U,

and assuming exponential decay, calculate the age of the universe. **4**

(b) Kippsy the mathematical kangaroo always hops (i.e. jumps) according to mathematical rules. One day, Kippsy decides to go hopping according to the following rules:

- The length of odd number hops (1st, 3rd, 5th hop etc.), in metres, is given by the arithmetic series $t_n = 4 - (n-1)$, where n =1, 3, 5, ... is an odd number;
- The length of even number hops (2nd, 4th, 6th hop etc.), in metres, is given by the geometric series $T_N = \frac{192}{63}\left(\frac{1}{2}\right)^{\frac{N-2}{2}}$, where N =2, 4, 6, ... is an even number;
- If the length of a hop is negative according to the relevant series, Kippsy hops the prescribed distance *backwards*.

(i) Write down the first term and common difference for the series t_n. **1**

(ii) Find the total distance hopped by Kippsy (in both directions) after the first 16 hops? **3**

(iii) Find where Kippsy is relative to her starting point after 12 hops. **3**

(iv) Describe the pattern of Kippsy's hops after she has been hopping for a long time. **1**

End of paper

Yr 12 TRIAL 2-UNIT 2004

Question 1.

(a) $3x - 6 - (4 - 2x)$
$= 3x - 6 - 4 + 2x$
$= 5x - 10$

(b) $\sqrt{27} - \sqrt{3} + \sqrt{18}$
$= 3\sqrt{3} - \sqrt{3} + 3\sqrt{2}$
$= 2\sqrt{3} + 3\sqrt{2}$

(c) $|x - 1| = 2x - 1$

$x - 1 = 2x - 1$
$0 = x$
$x = 0$

$-x + 1 = 2x - 1$
$2 = 3x$
$x = 2/3$

(d) $n = 0.69696969$ (1)
$100n = 69.696969$ (2)
(2) − (1)

$99n = 69$
$n = \frac{69}{99}$

(e) $3x - y = 5$ (1) × 3
$5x + 3y = -8$ (2)
$9x - 3y = 15$ (3)
(2) + (3)
$14x = 7$
$x = 1/2$
Subst $x = 1/2$ into (1)
$3(1/2) - y = 5$
$-y = 3.5$
$y = -3.5$

(f) $\frac{\sqrt{5}}{2 - \sqrt{5}} \times \frac{2 + \sqrt{5}}{2 + \sqrt{5}}$

$= \frac{2\sqrt{5} + 5}{4 - 5}$

$= \frac{2\sqrt{5} + 5}{-1}$

$= -2\sqrt{5} - 5$

$\therefore a = -5 \quad b = -2$

Question 2.

$A(5,3)$ (x_1, y_1) $B(-2,5)$ $C(4,-3)$ (x_2, y_2)

(a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(1) $d = \sqrt{(4 - 5)^2 + (-3 - 3)^2}$
$d = \sqrt{(-1)^2 + (-6)^2}$
$d = \sqrt{1 + 36} = \sqrt{37}$ (1)

(b) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-3 - 3}{4 - 5}$ (1)

$m = \frac{-6}{-1}$

$m = 6$ (1)

(c) $y - y_1 = m(x - x_1)$
(1) $y - 3 = 6(x - 5)$
$y - 3 = 6x - 30$
(1) $y = 6x - 27$ or $6x - y - 27 = 0$

(d) $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = \left| \frac{6(-2) + (-1)(5) + (-27)}{\sqrt{6^2 + (-1)^2}} \right|$ (1)

$= \left| \frac{-44}{\sqrt{37}} \right| \approx 7.23$ or $\frac{44}{\sqrt{37}}$ (1)

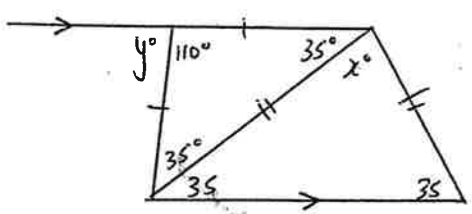
(e) $A = \frac{1}{2}bh$
$A = \frac{1}{2} \times \sqrt{37} \times \frac{44}{\sqrt{37}}$ (1)
$A = 22\,u^2$ (1)

(f) $\text{mid} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{5 + 4}{2}, \frac{3 - 3}{2}\right)$
$= \left(\frac{9}{2}, 0\right)$ (1)
$= (4.5, 0)$

$y - y_1 = m(x - x_1)$
$y - 0 = -\frac{1}{6}(x - 4.5)$
$y = -\frac{1}{6}x + \frac{3}{4}$ (1)

Question 3

(a)



$x = 180 - 35 - 35$

$x = 110°$ (1)

$y = 180 - 110°$

$y = 70°$ (2)

(b) (i) (1) $\angle JMN = \angle JKL$ (corres $\angle$ in ∥) A

$\angle JNM = \angle JLK$ (" " ") A

$\therefore \triangle JMN$ ||| $\triangle JKL$ (AAA) (1)

(ii) $\dfrac{9}{12} = \dfrac{8-x}{8}$ (1)

$72 = 12(8-x)$

$72 = 96 - 12x$

$12x = 24$

$x = 2$ (1)

(c) (i) vertex $= (-3, 0)$ (1)

(ii) focal Length = 2 units (1)

(iii) focus $= (-3, 2)$ (2)

(iv) directrix $y = -2$ (1)

Question 4

(a) $l = r\theta$

$l = 3 \times \dfrac{60\pi}{180}$ (1)

$l = \pi$ (1)

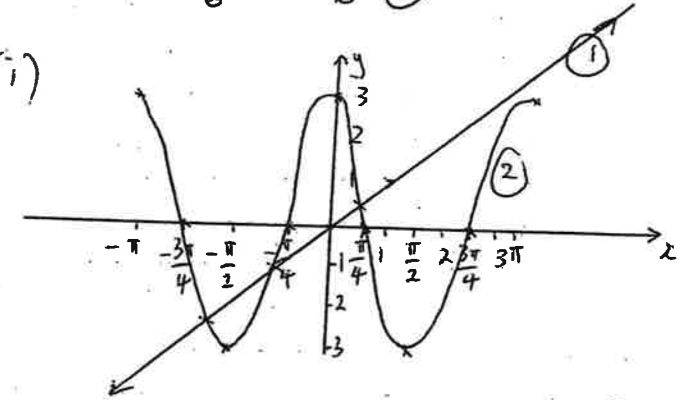
(b) $2\cos x = -\sqrt{3}$

$\cos x = -\dfrac{\sqrt{3}}{2}$ (1)

$x = 150°, 210°$ (1)

$x = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$ (1)

(c) (i)



(ii) 3 solutions

(d) $y = x \sin x$ $u = x$ $v = \sin x$

$u' = 1$ $v' = \cos x$

$y' = x\cos x + \sin x$ (1)

$y' = \pi\cos\pi + \sin\pi$

$y' = -\pi$ $y'_{perp} = \dfrac{1}{\pi}$ (1)

$y - 0 = \dfrac{1}{\pi}(x - \pi) \Rightarrow y = \dfrac{x}{\pi} - 1$ (1)

Question 5

(a) $3^{3x-2} = 11$

$\log_e 3^{3x-2} = \log_e 11$

$(3x-2)\log_e 3 = \log 11$ (1)

$3x-2 = \dfrac{\log 11}{\log 3}$

$3x = \dfrac{\log 11}{\log 3} + 2$

$x = 1.39$ (1)

(b) $y = \log_e\left(\dfrac{2x+1}{3x-1}\right)$

$y = \log_e(2x+1) - \log_e(3x-1)$ (1)

$y' = \dfrac{2}{2x+1} - \dfrac{3}{3x-1}$ (1)

(d) (i) $\int \dfrac{1}{x} + \dfrac{1}{x^2}\,dx$

$= \log_e x - \dfrac{1}{x} + c$ (1)

(ii) $\int \dfrac{4x}{x^2+1}\,dx$

$= 2\int \dfrac{2x}{x^2+1}\,dx$ (1)

$= 2\log_e(x^2+1) + c$ (1)

(e) $y = x^2 \ln x$ $\quad u = x^2 \quad v = \ln x$

$u' = 2x \quad v' = \dfrac{1}{x}$

$y' = 2x.\ln x + x^2.\dfrac{1}{x}$ (1)

$y' = 2x\ln x + x$ (1)

(f) $\dfrac{2}{x+4} + \dfrac{3}{x-4}$

$= \dfrac{2(x-4) + 3(x+4)}{x^2-16}$ (1)

$= \dfrac{2x-8+3x+12}{x^2-16}$

$= \dfrac{5x+4}{x^2-16}$ (1)

$\therefore \int \dfrac{5x+4}{x^2-16} = \int \dfrac{2}{x+4} + \dfrac{3}{x-4}$ (1)

$= 2\log_e(x+4) + 3\log_e(x-4)$

Question 6

(a) $\int x^6 - 2x^4 \, dx$

$= \frac{x^7}{7} - \frac{2x^5}{5} + c$ (2)

(b) $\int_{-1}^{2} \frac{x^4 + 3x^3 + x^2}{x} \, dx$

$\frac{x^4}{x} + \frac{3x^3}{x} + \frac{x^2}{x}$ (1)

$\int_{-1}^{2} x^3 + 3x^2 + x \, dx$

$= \left[\frac{x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$

$= \left[\frac{2^4}{4} + 2^3 + \frac{2^2}{2}\right] - \left[\frac{(-1)^4}{4} + (-1)^3 + \frac{(-1)^2}{2}\right]$ (1)

$= 14 - \left[\frac{1}{4}\right]$

$= 14\frac{1}{4}$ (1)

(c) $\int \sqrt{(3-x)} \, dx$

$\int (3-x)^{1/2} \, dx$ (1)

$= \frac{(3-x)^{3/2}}{-1 \times 3/2} + c$

$= -\frac{2\sqrt{(3-x)^3}}{3} + c$ (1)

(d) Trape $\doteqdot \frac{h}{2}\left[(1.011 + 1.559) + 2(1.179 + 1.322 + 1.447)\right]$

$h = \frac{3-1}{4}$ $h = \frac{1}{2}$

$A \doteqdot \frac{1}{4}\left[10.466\right]$ (1) (1)

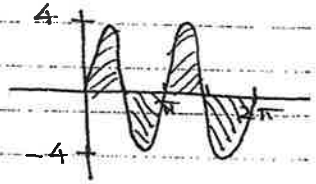
$A \doteqdot 2.6165 \, u^2$ (1)

(e) (1) No, Area between $\frac{1}{2}$ and 1 will be negative $\therefore$ you need to integrate (1) $\left|\int_{1/2}^{1} f(x)\right| + \int_{1}^{2} f(x)\,dx$

Trial HSC – 2004

Solutions & Marking Scheme

7) a) i) $T = \frac{2\pi}{2} = \pi$. (1)



ii) $A = 2 \times 2 \times \int_0^{\pi/2} 4\sin 2x \, dx$

$= 16 \int_0^{\pi/2} \sin 2x \, dx$ (1)

$= 16 \left[-\frac{1}{2}\cos 2x \right]_0^{\pi/2}$

$= 16 \left[\frac{-1}{2}\cos\pi + \frac{1}{2}\cos 0 \right]$

$= 16.$ (1)

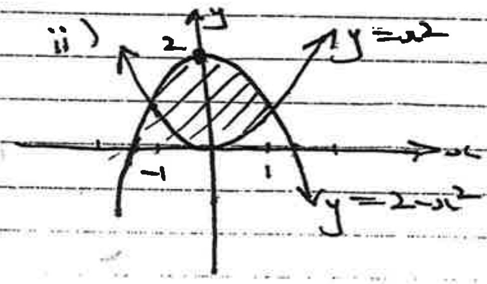
b) i) $y = x^2 = 2 - x^2 \Rightarrow 2x^2 = 2$

$x^2 = 1.$

(1) $x = \pm 1.$ (1)

Points: $(1,1)$ & $(-1,1)$.

ii)



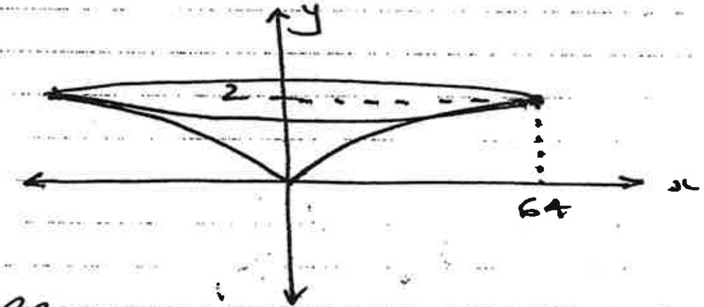
iii) $A = 2\int_0^1 [2 - x^2 - x^2]\,dx$

$= 2\int_0^1 (2 - 2x^2)\,dx$ } $a = 0$, $b = 1$ (1); $s = 2$, $t = 2$ (1)

$= 2\left[2x - \frac{2x^3}{3}\right]_0^1$ (1)

$= 2\left[2 - \frac{2}{3}\right] = \frac{8}{3}$ units². (1)

c)



$2y = x^{1/3} \Rightarrow 8y^3 = x$

$64y^6 = x^2$ (1)

$V = \pi \int y^2\,dx$ OR $\pi \int x^2\,dy$. (1)

Here, $V = \pi \int_0^2 x^2\,dy$

$= \pi \int_0^2 64y^6\,dy$ (1) $= \frac{8192\pi}{7}$ (1)

$$= \pi \left[\frac{64 y^7}{7} \right]_0^2 = \frac{\pi \cdot 64 \cdot 128}{7}$$

$$= \frac{8192\pi}{7}. \quad (1)$$

(8) a) $T_n = a + (n-1)d.$

$T_3 = a + 2d = 14 \quad (1)$

$T_7 = a + 6d = 30 \quad (2)$ } (1)

$(2) - (1) \Rightarrow 4d = 16 \Rightarrow \boxed{d = 4}$ (1)

$(1) \Rightarrow a + 8 = 14 \Rightarrow \boxed{a = 6}$ (1)

b) i) $r = \frac{4(\pi - 3)}{-2} = -\frac{8(\pi-3)^2}{4(\pi-3)} = \ldots$

$r = -2(\pi - 3) \approx -0.283$

Since $|r| < 1$, G.P. has a limiting sum. (1)

ii) $S_\infty = \frac{a}{1-r}$ (1)

$$= \frac{-2}{1 - (-2(\pi-3))} = \frac{-2}{1 + 2(\pi - 3)}$$

$$= [illegible] = \frac{-2}{2\pi - 5} = S_\infty$$

(1) $\approx -1.56.$

[illegible]

(8) c) $\dot{x} = t^3 - 7t - 7$

i) $x = \int \left(\frac{dx}{dt}\right) dt$

$= \int (t^3 - 7t - 7)\, dt$ } (1)

$= \frac{t^4}{4} - \frac{7t^2}{2} - 7t + c.$

$x(0) = -6 \Rightarrow c = -6.$

$x(t) = \frac{t^4}{4} - \frac{7t^2}{2} - 7t - 6.$ (1)

ii) $\ddot{x} = \frac{d}{dt}(\dot{x}) = 3t^2 - 7$

$\ddot{x}(3) = 3 \cdot 3^2 - 7 = 20 \text{ ms}^{-2}.$ (1)

iii) $x(3) = \frac{3^4}{4} - \frac{7 \cdot 3^2}{2} - 7 \cdot 3 - 6 = -\frac{153}{4}$ m (1)

$= -38.25$ m

$\dot{x}(3) = 3^3 - 21 - 7 = -1 \text{ ms}^{-1}.$ (1)

iv) Have $x < 0$, $v < 0$, $a > 0$ so particle is to left of origin, moving left and accelerating, i.e. velocity becoming less negative. (1)

(9) a) $I = 5te^{-2t}$

$$\frac{dI}{dt} = 5t \cdot -2e^{-2t} + 5e^{-2t}$$
$$= 5e^{-2t}(-2t+1)$$
$$= 5e^{-2t}(1-2t) \quad (1)$$

b) $$\frac{d^2I}{dt^2} = 5e^{-2t} \cdot -2 + (1-2t) \cdot -10e^{-2t}$$
$$= -10e^{-2t} - 10e^{-2t} + 20te^{-2t} \quad (1)$$
$$= 20te^{-2t} - 20e^{-2t} \quad (1)$$
$$= 20e^{-2t}(t-1).$$

c) Max. interest when $I' = 0$. (1)

$\Rightarrow 5e^{-2t}(1-2t) = 0 \Rightarrow t = \frac{1}{2}$. (1)

Concavity check. $I''(\frac{1}{2}) = 20e^{-1}(-\frac{1}{2}) < 0$. (1)

So, max. interest when $t = \frac{1}{2}$, i.e. 30 mins into lesson

d) Need minimum value for I', i.e. $I'' = 0$ (1)

$I'' = 0 \Rightarrow t = 1$. (1)

Check for concavity. (1) $I''(1+\varepsilon) > 0$, $I''(1-\varepsilon) < 0$ -ve _/ +ve

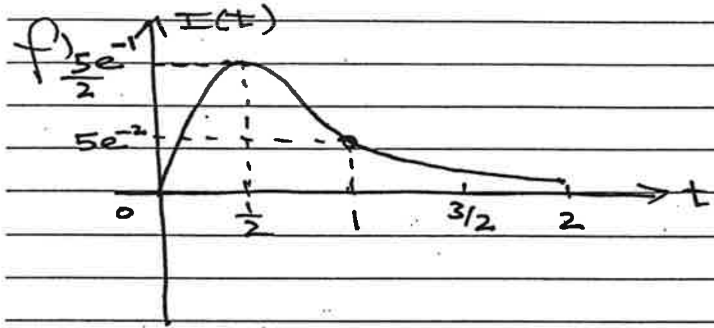
So, min. value for I'' when $t = 1$ hr.
Interest decreasing most rapidly after 1 hr.

e) Point of inflection when $I'' = 0$, i.e. $t = 1$.

Check for concavity.

t	$1-\varepsilon$	1	$1+\varepsilon$
I''	<0	0	>0

So, point of inf. at $t = 1$. (1)



(2)

(10) a) For ^{238}U, assume $A_{238}(t) = A_0 e^{-Kt}$

For ^{235}U, assume $A_{235}(t) = B_0 e^{-Lt}$, where t is time in billions of years.

Know $A_{238}(4.51) = \frac{A_0}{2} = A_0 e^{-4.51K}$

$$\Rightarrow e^{-4.51K} = \frac{1}{2} \Rightarrow K = -\frac{1}{4.51}\ln(\tfrac{1}{2}).$$

Similarly, $L = -\frac{1}{0.71}\ln(\frac{1}{2})$

$K \approx 0.154$ (1)
$L \approx 0.976$ (1)

Now, have $\frac{A_{238}(t)}{A_{235}(t)} = \frac{A_0}{B_0} e^{(-K+L)t} = 137.7$ (1)

But $A_0 = B_0$, so $e^{-(K-L)t} = 137.7$

$\Rightarrow t = -\frac{\ln(137.7)}{K-L} \approx 5.99$. (1)

So. age of universe approx. 6 bill. yrs.

(10) b) i) $t_n = 4 - (n-1)$, $n = 1, 3, 5, \ldots$

Series is $4 + 2 + 0 - 2 - 4 - \ldots$

$a = 4$, $d = -2$. (1)

ii) 16 hops ⇒ 8 from each series.

Odds: $4 + 2 + 0 - 2 - 4 - 6 - 8 - 10 = -24$

Distance $= 4 + 2 + 0 + 2 + 4 + 6 + 8 + 10 = 36$m (1)

Evens: $T_N = \frac{192}{63}\left(\frac{1}{2}\right)^{\frac{N-2}{2}}$, $N = 2, 4, 6, \ldots$

Series is $\frac{192}{63} + \frac{192}{63}\left(\frac{1}{2}\right) + \frac{192}{63}\left(\frac{1}{2}\right)^2 + \ldots$

$a = \frac{192}{63}$, $r = \frac{1}{2}$, $N = 8$. (1)

$S_8 = \frac{192}{63}\left(1 - \frac{1}{2}^8\right) \Big/ \left(1 - \frac{1}{2}\right) \approx 6.07$m. (1)

Total distance hopped $= 36 + 6.07 = 42.07$m.

iii) Displacement after 12 hops $= S_6 \text{ (AP)} + S_6 \text{ (GP)}$

From (ii), $S_6 = -6$ (1)

$S_6 = \frac{192}{63}\left(1 - \frac{1}{2}^6\right) \Big/ \left(1 - \frac{1}{2}\right) = 6$. (1)

So, total displacement $= -6 + 6 = 0$,

i.e. Kippsy back at her starting point. (1)

iv) As $N \to \infty$, $T_N \to 0$ so Kippsy hopping backwards every 2nd hop, at ever increasing distances. (1)