Question 1 (12 marks)

(a) Factorise the expression $2x^2 - 7x - 15$. **2**

(b) Fully factorise $x^3 + 27$. **2**

(c) Find the values of x for which $|x+2| \geq 3$. **2**

(d) Find a and b such that $\left(2-\sqrt{3}\right)^2 = a + b\sqrt{3}.$ **2**

(e) Differentiate $y = 3x^3 - \dfrac{4}{x}$, with respect to x. **2**

(f) Express in simplest surd form the expression $2\sqrt{27} + \sqrt{12} - \sqrt{75}$. **2**

Question 2 (12 marks) Start a new booklet

(a) Solve for x: $\dfrac{x+2}{3} - \dfrac{x-3}{4} = 2.$ **2**

(b) Differentiate the following with respect to x:

(i) $y = 2x^3 \log x.$ **2**

(ii) $y = (3 + \sin x)^4.$ **2**

(c) Find:

(i) $\int 4e^{3x} dx.$ **1**

(ii) $\int \dfrac{x+1}{x^2+2x} dx.$ **2**

(d) Solve $8^{2x-1} = 4\sqrt{8}$ **3**

Question 3 (12 marks) Start a new booklet

(a) Find the equation of the normal to the curve $y = \sqrt{x+2}$ at the point (2, 2). **4**

(b) For the geometric series $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - ....$, find:

(i) the common ratio (r). **1**

(ii) the limiting sum. **2**

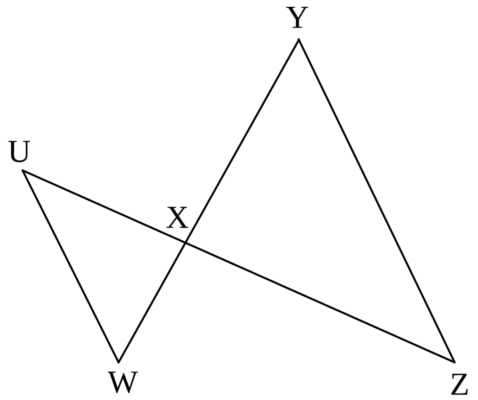
(c) Evaluate $\int_0^{\frac{\pi}{4}} \cos x \, dx$. **2**

(d) Solve $\log_{10}(x+3) - \log_{10} x = 1$. **3**

Question 4 (12 marks) Start a new booklet

(a) Given that $\dfrac{dy}{dx} = 12x + 2$, and that there exists a stationary point at (1, 2), find the equation of the curve. **2**

(b)



NOT DRAWN TO SCALE

The lines *UZ* and *WY* intersect at X.

Given that $UW \parallel YZ$. *UW* = 10cm, *WX* = 9 cm, *UX* = 8cm and *XZ* = 12cm:

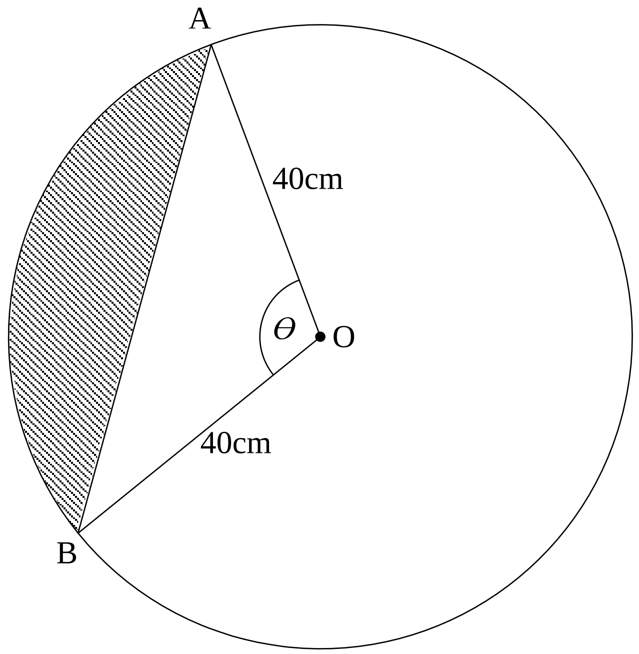
(i) Draw a neat sketch of this figure, and mark all information it. **1**

(ii) Prove that $\Delta UWX \;|||\; \Delta XYZ$. **2**

(iii) Hence, or otherwise, find the length of *ZY*. **2**

Question 4 continues on page 5

(c) A sector AOB is to be cut from a circle of radius 40cm so that the area of the sector is 2400cm^2.



(i) Find the size of the angle (θ) at the centre of the sector AOB, giving your answer in radians. **2**

(ii) Find the length of the arc cut off by the sector AOB. **1**

(iii) Find the area of the shaded segment correct to the nearest cm^2. **2**

End of Question 4

Question 5 (12 marks) Start a new booklet

(a) The quadratic equation $x^2+(k-3)x+k=0$, has real roots. Find all possible values for k. **3**

(b) A parabola has the equation $2y=x^2-4x+6$.

(i) Rewrite the equation in the form $(x-h)^2=4a(y-k)$ by completing the square. **1**

(ii) State the coordinates of the vertex. **1**

(ii) Find the focal length. **1**

(c) Consider the function $y=1+3x-x^3$, for $-2\le x\le 3$.

(i) Find all stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **1**

(iii) Sketch the curve for $-2\le x\le 3$. **1**

(iv) What is the minimum value for the curve over the stated domain? **1**

Question 6 (12 marks) Start a new booklet

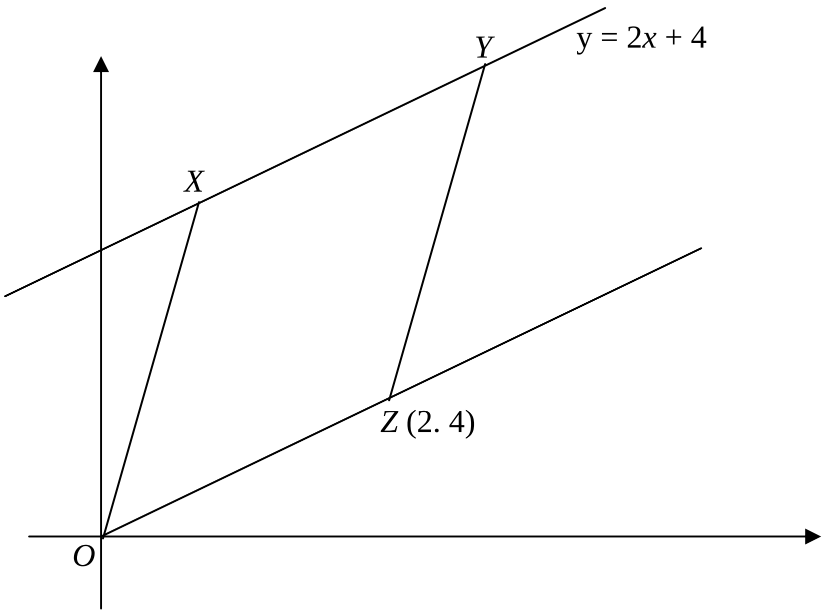
(a) The sum of the first n terms of an AP is given by $S_n = \dfrac{n(3n+1)}{2}$. Find:

(i) S_1 and S_2. **2**

(ii) the first two terms of the AP. (T_1 and T_2). **2**

(iii) an expression for the "n"th term. **1**

(b)



Copy this diagram into your booklet.

(i) Show that XY is parallel to OZ **2**

(ii) Show why $\angle XYO = \angle YOZ$. **1**

(iii) Prove that $OXYZ$ is a parallelogram if OY divides OXYZ into 2 congruent triangles. **1**

(c) Find the volume generated when the curve $y = x^4$, between the origin and the point (2, 16), is rotated **about the *y*–axis**. Give your answer in exact form. **3**

Question 7 (12 marks) Start a new booklet

(a) Find the domain and range of $y = \dfrac{1}{\sqrt{4 - x^2}}$. **2**

(b) Solve $4\cos^2\theta - 6 = 6\sin\theta$, *for* $0^\circ \le \theta \le 2\pi$. **3**

(c) (i) Draw on the same diagram : **2**

$y = \sin 2x$ and $y = \dfrac{1}{3}x$, for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

(ii) Use the graph to find approximate solutions for the equation **2**

$\sin 2x = \dfrac{1}{3}x$.

(d) Find the area enclosed by the curve $y = \dfrac{x}{x^2 + 1}$ and the x- axis, **3**
from x = 1 to x = 3. Give your answer in exact form.

Question 8 (12 marks) Start a new booklet

(a) The mass of a radioactive substance is given by $M = 10e^{-kt}$.

(i) What is the initial mass? **1**

(ii) If, after 100 years, the mass has reduced to 5 kg, find the value of 'k', correct to 4 decimal places. **2**

(iii) How many years would it take to reduce to 8kg? Give answer to the nearest year. **3**

(iv) What mass would remain after 1 000 years? Give answer to nearest gram. **2**

(b) A particle moves in a straight line. At time "t" seconds, its displacement in "x" metres, from a fixed point O, is given by $x = 3 - \cos 2t \quad for \quad 0 \le t \le \pi$.

(i) Find an expression for the velocity (v) of the particle. **1**

(ii) Sketch a graph of 'v' relative to time. **1**

(iii) Hence, or otherwise, find when the particle is at rest. **2**

Question 9 (12 marks) Start a new booklet

(a) The points P and Q have coordinates (4, –1) and (1, 3) respectively.

(i) The line $\boldsymbol{k}$ has the equation $x + 2y - 2 = 0$. Verify that P lies on this line. **1**

(ii) Draw a neat sketch showing P, Q and the line $\boldsymbol{k}$. **1**

(iii) Find the perpendicular distance of Q from the line $\boldsymbol{k}$, giving your answer in simplest surd form. **1**

(b) Jane borrows \$30 000 from a building society, with interest at 11.5%pa. It is to be repaid in equal yearly instalments.

(i) (α) How much must be repaid each year if the loan is to be repaid over 30 years? **2**

(β) using the repayment calculated in part (α), how much will be outstanding at the end of 18 years? **1**

(ii) If the loan is repaid at \$4500 per year, when will it be paid off? **3**

(c) A particle moves in a straight line so that at time t seconds, its distance x cm from a fixed point O is given by $x = 4\log_e(1+t) - 2t \geq 0, \quad t \geq 0$.

(i) Find the initial position. **1**

(ii) Find the initial velocity. **1**

(iii) Find an expression for the acceleration in terms of 't', and show that it is always negative. **1**

Question 10 (12 marks) Start a new booklet

(a) Jane has a bottle containing 200 millilitres of water. She pours more water in for 20 seconds until it is full. During this time, the flow rate (R) of the water is given by $R = 4(20 - t)$ millilitres/sec.

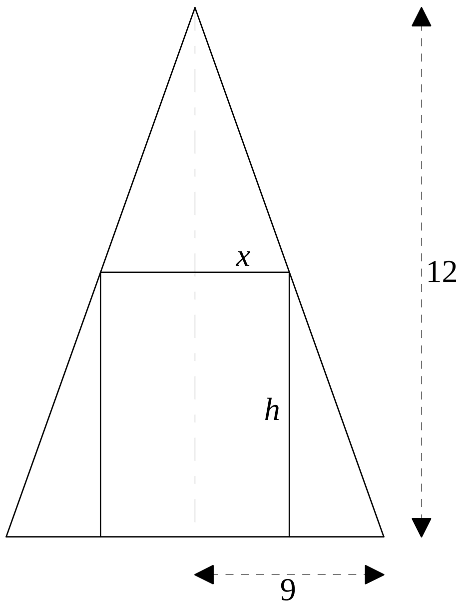
(i) Find an expression for the volume after 't' seconds, where $t \leq 20$. **2**

(ii) How many millilitres are in the bottle when it is full. **1**

(iii) How long was it before the bottle was half full? Give your answer in exact form. **2**

Question 10 continues on page 12

(b) The diagram below is a two dimensional representation of a cylinder of radius x cm, and height h cm, inscribed in a right circular cone of radius 9cm and height 12cm.



(i) Using similar triangles, express h in terms of x. **2**

(ii) Show that the volume of the cylinder is given by $V = \frac{4\pi}{3}\left(9x^2 - x^3\right)$. **2**

(iii) Find the value of x for which this volume is a maximum. *You must provide a reason why your calculation results in a maximum volume.* **3**

End of Paper