Question One **12 marks** **(Start a new booklet)** **Marks**

a) Factorise $45x^2 - 80y^2$ **2**

b) Solve for x: $|2x+1| = 3$ **2**

c) If α and β are the roots of $3x^2 - 5x + 1 = 0$, find the value $\frac{1}{\alpha} + \frac{1}{\beta}$ **2**

d) Express $x - \frac{1}{x}$ as an exact value in its simplest form if $x = 1 - \sqrt{5}$ **2**

e) Simplify $(x+2)^2 - (x+3)(x-3)$ **2**

f) Evaluate $\lim_{x \to 2} \frac{x^2 - 6x + 8}{x^2 - 4}$ **2**

THGS Year 12 Trial Examination 2 Unit Advanced 2010

Question Two **12 Marks** **(Start a new booklet)** **Marks**

a) The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$

i) Find the co-ordinates of the midpoint of BC **1**

ii) Find the equation of the line passing through C which is parallel to AB **2**

iii) Prove that ABC is a right-angled triangle. **2**

b) The line L has equation $y + 2x = 12$ and the curve C has equation $y = x^2 - 4x + 9$

i) By completing the square, state the co-ordinates of the minimum point of the curve C. **2**

ii) Find the co-ordinates of the point of intersection of L and C. **3**

c) The line AB has equation $3x + 2y = -21$. Find the perpendicular distance from the origin to the line AB, in surd form. **2**

Question Three **12 Marks** **(Start a new booklet)** **Marks**

a) Differentiate the following functions:

i) $y = (7 - 3x)^6$ **2**

ii) $y = x \tan x$ **2**

iii) $y = \dfrac{x}{\sin 2x}$ **2**

b) Evaluate the following integrals

i) $\displaystyle\int_1^4 \sqrt{x}\, dx$ **2**

ii) $\displaystyle\int_0^2 e^{3x}\, dx$ **2**

c) Find $\displaystyle\int \frac{6x}{x^2 + 3}\, dx$ **2**

Question Four **12 Marks** **(Start a new booklet)** **Marks**

a) In an arithmetic sequence, the sixth term is 13 and the tenth term is 1:

i) Find the first term and the common difference. **2**

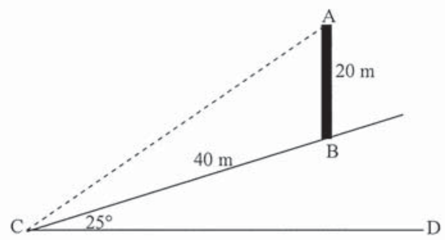
ii) Find the sum of the first twenty terms. **2**

b) A container holds 50 litres of oil. A pump withdraws 10 litres on the first stroke and 7.5 litres on the second stroke. On each future stroke, the pump withdraws ¾ of the amount of the previous stroke:

i) show that the container will never be emptied **1**

ii) find how much oil will finally remain in the container **2**

c)



A 20 metre high vertical mast AB is placed 40 metres from the base C of a slope inclined at 25° to the horizontal. A wire support AC is used to keep the mast in position:

i) Explain why angle $ABC = 115°$ **1**

ii) Show that the length of the wire AC is 51·7 m (to 1 decimal place) **2**

iii) Hence find the angle which the wire AC makes with the slope CB. Answer to the nearest degree. **2**

Question Five **12 Marks** **(Start a new booklet)** **Marks**

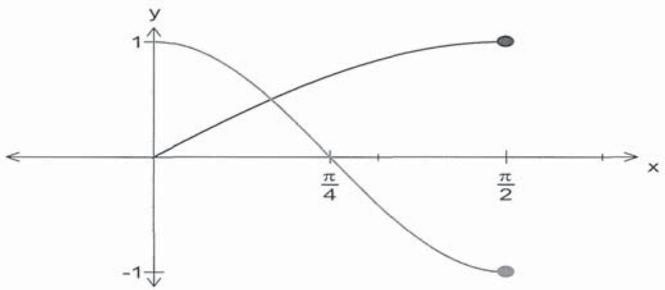
a) i) Copy and complete this table for $f(x) = x\,e^x$, giving values to 2 decimal places. **1**

x	0	1	2
$f(x)$			

ii) Use Simpson's rule to estimate the value of $\int_0^2 x\,e^x\,dx$ **2**

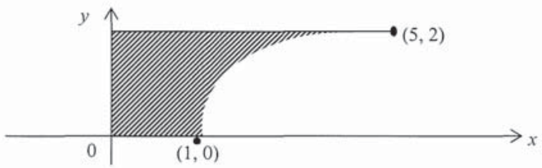
b) The diagram shows the graphs of $y = \sin x$ and $y = \cos 2x$ for $0 \le x \le \frac{\pi}{2}$.

The graphs intersect at $A\left(\frac{\pi}{6}, \frac{1}{2}\right)$



Find the area of the shaded region. **4**

c) The diagram shows the graph of $y = \sqrt{x-1}$ between (1, 0) and (5, 2).



The shaded region is rotated about the y-axis. Find the volume of the solid formed. **5**

Question Six **12 Marks** **(Start a new booklet)** **Marks**

a) Solve for x: $3^x \times 9^{x+1} = \frac{1}{3}$ **2**

b) In the diagram $ABCD$ is a straight line, and E lies on CF.
$BF = EF$, $\angle BFE = 44°$, $\angle DCE = 146°$ and $\angle CBE = x°$

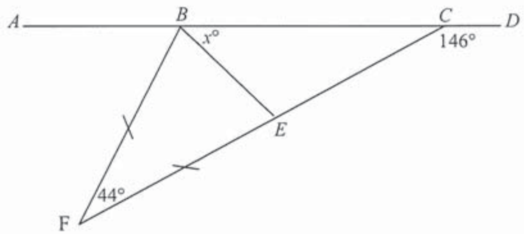


Diagram not to scale

i) Find the value of x giving reasons. **3**

ii) State why $BE = EC$. **1**

c)

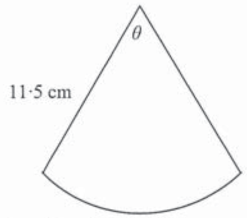


Diagram not to scale

The radius of a sector of a circle is 11·5 cm, and its perimeter is 36·8 cm:

i) Find the size of the angle θ to the nearest degree **2**

ii) Find the area of the sector. **1**

Question Six continued **Marks**

d)

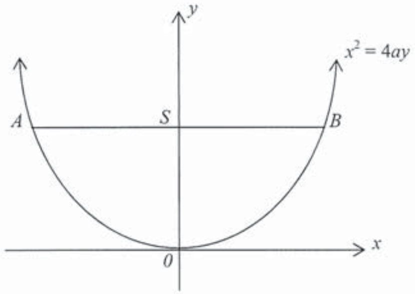


Diagram not to scale

The diagram shows the graph at the parabola $x^2 = 4ay$, with focus S, and AB is the latus rectum (that is, the focal chord parallel to the x-axis of the parabola).

Prove that the length of the latus rectum is 4a units. **3**

Question Seven on the next page

Question Seven **12 Marks** **(Start a new booklet)** **Marks**

a) A particle moves in a straight line. At time t seconds, its distance is x metres from a fixed point 0 and its velocity is give by the equation

$$v = 4t - 3t^2$$

Initially the particle is at $x = 3$

i) Find the position of the particle when $t = 2$ **3**

ii) Find the velocity of the particle when the acceleration is zero **2**

iii) Find the acceleration of the particle when the particle becomes instantaneously at rest during the motion. **2**

b) The rate of flow of water into a large container is given by:

$$\frac{dv}{dt} = \frac{30}{t+1}$$

where v is in litres and t is in minutes.

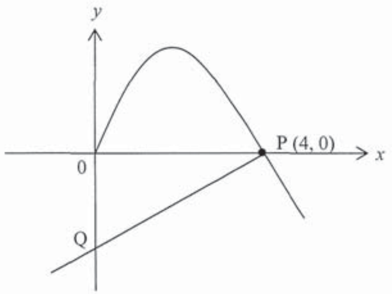
Initially, there are 40 litres of water in the container.

i) Find the volume of water in the container after 4 minutes. **3**

ii) How long does it take for the container to hold 160 litres. **2**

Question Eight **12 marks** **(Start a new booklet)** **Marks**

a) A curve $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$ defined for $x \geq 0$, crosses the x-axis at the point $P(4, 0)$ the normal to the curve at P meets the y-axis at the point Q, as shown in the diagram below:



i) Show that the gradient of the curve at $P(4, 0)$ is -2 **1**

ii) Find the area of triangle OPQ **3**

iii) Find the total area of the region bounded by the curve and the lines PQ and QO. **2**

b) A particle moves in a straight line with a constant acceleration of 2m/sec^2 towards the right. It is initially at rest, 2 metres to the right of the origin:

i) Find an expression for the velocity after t seconds **1**

ii) Find the velocity after 3 seconds **1**

iii) When is the velocity 10m/sec? **1**

iv) Find an expression for the displacement after t seconds. **1**

v) When is the particle 6 metres to the right of the origin. **1**

vi) How far does the particle move in the third second. **1**

Question Nine **12 Marks** **(Start a new booklet)** **Marks**

a) On the birth of Fernando, his parents decide to set up an investment fund for him. They agree to deposit \$500 on the 18th September 2008 (the date of birth) and \$500 on every subsequent birthday up to and including his 21st birthday. Interest is being paid at the rate 6% p.a. to be compounded:

i) How much will the first \$500 deposit mature to after 21 years? **2**

ii) How much will Fernando receive on his 21st birthday? **2**

b) Find the volume of the solid of revolution formed by rotating the curve $y = e^x + e^{-x}$ about the x-axis between $x = -1$ and $x = 1$, leaving your answer in exact form: **4**

c) i) Find the x co-ordinate of the point on the graph of $y = 5 - 14x - 2x^2$ where the tangent is parallel to the line $2x + y - 3 = 0$ **2**

ii) Hence find the equation of this tangent from part (i) **2**

Question Ten **12 Marks** **(Start a new booklet)** **Marks**

Consider the function $f(x) = \frac{e^x}{x}$

i) What is the domain of $f(x)$? **1**

ii) The first derivative of $f(x)$ is $f'(x) = \frac{xe^x - e^x}{x^2}$
Show that the second derivative can be written as:
$f''(x) = \frac{e^x\left[(x-1)^2 + 1\right]}{x^3}$ **2**

iii) Find the co-ordinates of the stationary point and determine its nature **3**

iv) Show that there are no points of inflexion **1**

v) For what values of x is the curve concave up and concave down? **2**

vi) Discus briefly what happens to $f(x)$ when $x \to \infty$ **1**

vii) Sketch the graph of $y = f(x)$ **2**

END OF TRIAL EXAMINATION

YEAR 12 2unit ADVANCED Trial Sol^n 2010

Question 1

a $45x^2 - 80y^2$
$5(9x^2 - 16y^2)$

$5\left[(3x+4y)(3x-4y)\right]$

b) $|2x+1| = 3$

$\pm(2x+1) = 3$

$2x+1 = 3$
$2x = 2$
$x = 1$ check ✓

or

$-2x - 1 = 3$
$-2x = 4$
$2x = -4$ ✓
$x = -2$ check

c) $3x^2 - 5x + 1 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta}$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}$

$\alpha+\beta = \frac{5}{3}$ $\alpha\beta = \frac{1}{3}$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{5}{3}}{\frac{1}{3}}$

$\frac{5}{3} \times \frac{3}{1} = 5$

d) $x - \frac{1}{x}$ $x = 1 - \sqrt{5}$

$\Rightarrow 1-\sqrt{5} - \frac{1}{1-\sqrt{5}} \times \frac{1+\sqrt{5}}{1+\sqrt{5}}$

$\Rightarrow 1-\sqrt{5} - \frac{(1+\sqrt{5})}{-4}$

$\Rightarrow \frac{5-3\sqrt{5}}{4}$

e) $(x+2)^2 - (x+3)(x-3)$

$\Rightarrow (x^2+4x+4) - (x^2-9)$

$\Rightarrow 4x + 13$

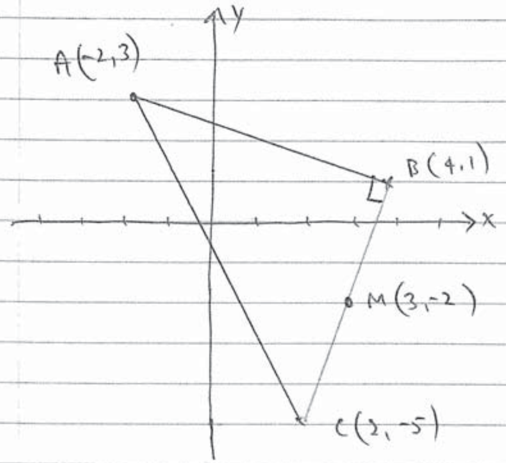
f) $\lim_{x\to 2} \frac{x^2 - 6x + 8}{x^2 - 4}$

$\lim_{x\to 2} \Rightarrow \frac{(x-4)(x-2)}{(x+2)(x-2)}$

$\lim_{x\to 2} \frac{x-4}{x+2} = -\frac{1}{2}$

Question 2

a)



i) Midpoint BC

$M\left[\frac{4+2}{2}, \frac{1+-5}{2}\right]$

$M(3,-2)$

ii) $A(-2,3)$ $B(4,1)$

$m(AB) = \frac{3-1}{-2-4}$

$= \frac{2}{-6}$

$m(AB) = -\frac{1}{3}$

$m = -\frac{1}{3}$ $C(2,-5)$

$y+5 = -\frac{1}{3}(x-2)$ $\times 3$

$3y+15 = -1(x-2)$

$3y+15 = -x+2$

$\boxed{x+3y+13=0}$

iii) $m_1(AB) = -\frac{1}{3}$

$m(BC)$ $B(4,1)$ $C(2,-5)$

$m(BC) = \frac{1--5}{4-2}$

$m_2 = \frac{6}{2} = 3$

$m_1 \times m_2 = -1$

$-\frac{1}{3} \times 3 = -1$

$\therefore AB \perp BC$

b) $x^2-4x+9=0$

i) $x^2-4x=-9$

$x^2-4x+(-2)^2 = -9+(-2)^2$

$(x-2)^2 = -5$

$(x-2)^2+5 = y$

Vertex $(2,5)$ Min TP.

ii) $y = x^2-4x+9$ $y = 12-2x$.

$12-2x = x^2-4x+9$

$0 = x^2-2x-3$

$(x+1)(x-3)$

$x=-1$ $x=3$

$y=14$ $y=6$

$(3,6)$ $(-1,14)$

c) $3x+2y=-21$ Origin$(0,0)$

$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{21}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{21\sqrt{13}}{13}$

Question 3

i) $y=(7-3x)^6$

$y' = 6(7-3x)^5 \times -3$

$y' = -18(7-3x)^5$

ii) $y = x\tan x$.

$y = u \cdot v$

$u=x$ $v=\tan x$

$u'=1$ $v'=\sec^2 x$.

$y' = x\cdot\sec^2 x + \tan x \cdot 1$

$y' = x\sec^2 x + \tan x$.

$y = \frac{x}{\sin 2x}$

$y = \frac{u}{v}$

$u = x \quad v = \sin 2x$

$u' = 1 \quad v' = 2\cos 2x$

$y' = \frac{\sin 2x \cdot 1 - x \cdot 2\cos 2x}{(\sin 2x)^2}$

$\Rightarrow y' = \frac{\sin 2x - 2x\cos 2x}{\sin^2 2x}$

b) i) $\int_1^4 \sqrt{x}\, dx$

$\int_1^4 x^{1/2}\, dx$

$\left[\frac{2}{3} x^{3/2}\right]_1^4$

$\Rightarrow \left(\frac{2}{3}\sqrt{4^3}\right) - \frac{2}{3}\sqrt{1^3}$

$\Rightarrow \left(\frac{2}{3} \times 8\right) - \frac{2}{3}$

$\frac{16}{3} - \frac{2}{3} = \frac{14}{3}$

ii) $\int_0^2 e^{3x}\, dx$

$y = e^{3x}$

$y' = 3e^{3x}$

$\int 3e^{3x}\, dx = e^{3x}$

$\Rightarrow \left[\frac{1}{3} e^{3x}\right]_0^2$

$\frac{1}{3}(e^6 - e^0)$

$\frac{1}{3}(e^6 - 1)$

iii) $\int \frac{6x}{x^2+3}\, dx$

$\Rightarrow 3\log(x^2+3) + C$

Question 4

a) $T_6 = 13 \quad T_{10} = 1$

i) $a + 5d = 13 \quad a + 9d = 1$

Solve

$a + 9d = 1$

$a + 5d = 13$

$4d = -12$

$d = -3$

Sub to find a

$a + 5d = 13$

$a - 15 = 13$

$\boxed{a = 28} \quad \boxed{d = -3}$

ii) Sum of first 20 terms.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$a = 28$

$n = 20$

$d = -3$

$S_{20} = 10[56 + (19)\times -3]$

$= 10[56 - 57]$

$S_{20} = -10$

b) $S_n = 10 + 7.5 + \ldots$

$S_n = \frac{a}{1-r}$

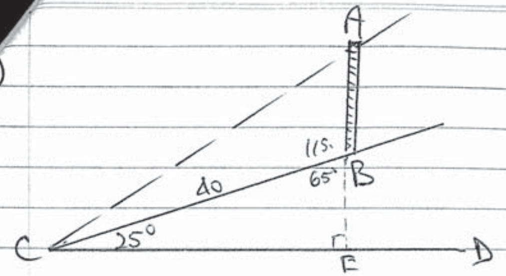
$= \frac{10}{1 - \frac{3}{4}}$

$S_\infty = \frac{10}{\frac{1}{4}} = 40$

limiting sum is 40

$\therefore$ 10 litres remain.

)

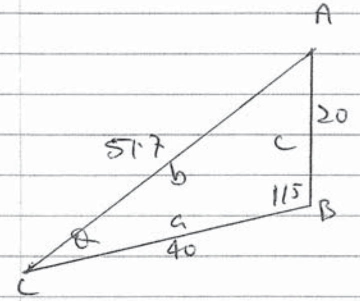


i) $\angle ABC = 115^\circ$

$\angle CBE = 65$

$\therefore \angle ABC = 115^\circ$ (Supp $\angle$)

ii)



$b^2 = a^2 + c^2 - 2ac\cos B^\circ$

$b^2 = 40^2 + 20^2 - 2 \times 40 \times 20 \cos 115^\circ$

$b^2 = 2000 + 676$

$b = \sqrt{2676} = 51.7$ m.

iii) $\dfrac{b}{\sin B} = \dfrac{c}{\sin C^\circ}$

$\dfrac{51.7}{\sin 115} = \dfrac{20}{\sin\theta}$

$51.7 \sin\theta = 20 \sin 115^\circ$

$\sin\theta = \dfrac{20 \sin 115^\circ}{51.7}$

$\sin\theta = 0.365739007$

$\theta = 21^\circ$

Question 5

a)

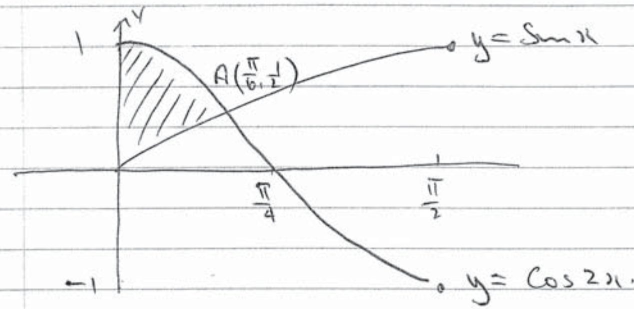
x	0	1	2
$f(x)$	0	2.72	14.78

$f(x) = x e^x$

$\int_0^2 x e^x \, dx \doteq \frac{1}{3}\left[0 + 4 \times 2.72 + 14.78\right]$

$\doteq 8.55$ (2 dec pl)

b)



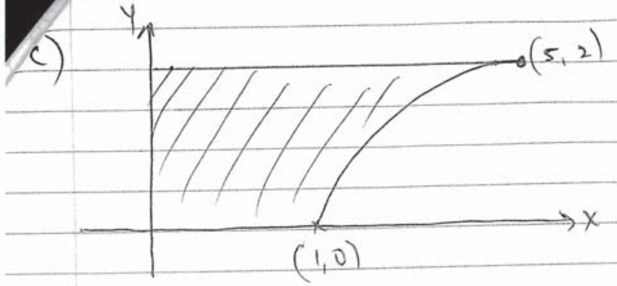
$A = \int_0^{\frac{\pi}{6}} (\cos 2x - \sin x)\, dx$

$= \left[\frac{1}{2}\sin 2x + \cos x\right]_0^{\frac{\pi}{6}}$

$\Rightarrow \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) - (0 + 1)$

$\Rightarrow \dfrac{3\sqrt{3}}{4} - 1$ units^2

c)



$y = \sqrt{x-1}$

$y = \sqrt{x-1}$

$y^2 = x-1$

$x = y^2+1$ Make x-subject {Rotated about y}

$V = \pi \int_0^2 (y^2+1)^2 \, dy$

$V = \pi \int_0^2 (y^4+2y^2+1) \, dy$

$= \pi \left[\frac{1}{5}y^5 + \frac{2}{3}y^3 + y \right]_0^2$

$= \pi \left[\frac{32}{5} + \frac{16}{3} + 2 \right]$

$V = \frac{206\pi}{15}$ unit^3

Question 6

a) $3^x \times 9^{x+1} = \frac{1}{3}$

$3^x \times (3^2)^{x+1} = 3^{-1}$

$3^x \times 3^{2x+2} = 3^{-1}$

$3^{3x+2} = 3^{-1}$

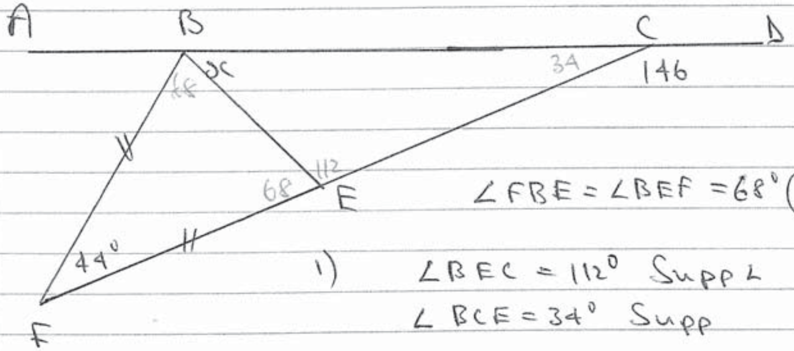
equate indices

$3x+2 = -1$

$3x = -3$

$\boxed{x = -1}$

b)



$\angle FBE = \angle BEF = 68°$ (isos Δ)

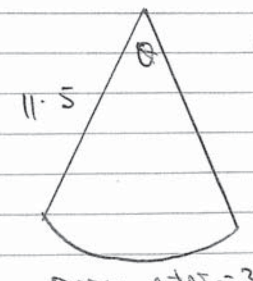
i) $\angle BEC = 112°$ Supp $\angle$

$\angle BCE = 34°$ Supp

$\therefore \angle EBC = x = 34°$ (Sum Δ)

ii) Base angles $= 34°$ $\therefore$ isos.

c)



perimeter = 36.8

i) $P = r\theta + 2r$

$36.8 = 11.5\theta + 23$

$11.5\theta = 13.8$

$\theta = \frac{13.8}{11.5}$

$\theta = 1.2$ Radians

$1.2 \times \frac{180}{\pi}$

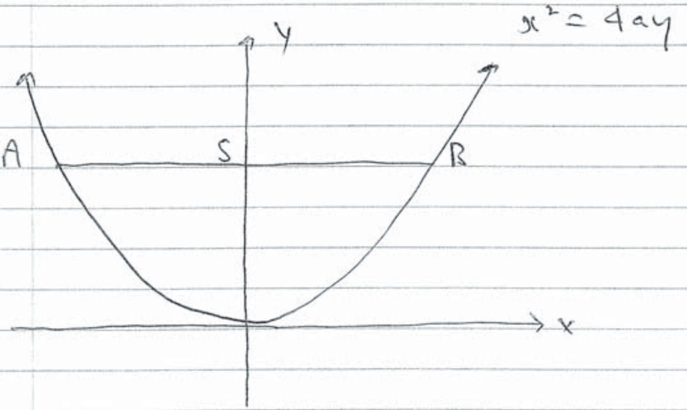
$= 69°$

Area of Sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 11.5^2 \times 1.2$

$A = 79.35 \text{ cm}^2$

d)



Prove $AB = 4a$ unit

Focus $S(0,a)$ $\therefore AB \Rightarrow y = a$

Substitute $y = a$ into $x^2 = 4ay$

$\Rightarrow x^2 = 4a \cdot a$

$x^2 = 4a^2$

$x = \sqrt{4a^2}$

$x = \pm 2a$

$= 4a$ units

$\boxed{\text{Question 7}}$

a) $V = 4t - 3t^2$

i) initially particle is at $x=3$

ie at $t=0$ $x=3$

$V = 4t - 3t^2$

$x = \int 4t - 3t^2 \, dt$

$x = \frac{4t^2}{2} - \frac{3t^3}{3} + c$

$x = 2t^2 - t^3 + c$

at $t=0$ $x=3$

$\therefore 3 = 0 + c$ $\therefore c = 3$

$x = 2t^2 - t^3 + 3$

at $t = 2$

$x = 8 - 8 + 3$ $\boxed{x = 3 \text{ m}}$

ii) $V = 4t - 3t^2$

$a = \frac{dV}{dt} = 4 - 6t$

at $a = 0$

$0 = 4 - 6t$

$6t = 4$

$t = \frac{4}{6}$

$t = \frac{2}{3}$ secs.

$V = 4t - 3t^2$ at $t = \frac{2}{3}$

$V = 4\left(\frac{2}{3}\right) - 3\left(\frac{2}{3}\right)^2$

$= \frac{8}{3} - 3\left(\frac{4}{9}\right)$

$= \frac{8}{3} - \frac{12}{9}$

$\frac{24}{9} - \frac{12}{9}$

$= \frac{12}{9}$

$= \frac{4}{3}$ or $1\frac{1}{3}$ m/sec.

i) $\frac{dV}{dt} = \frac{30}{t+1}$

$V = \int \frac{30}{t+1}\,dt$

$V = 30\ln(t+1) + c$

at $t=0$ $V=40$

$40 = 30\ln 1 + c$

$\therefore c = 40$

$\boxed{V = 30\ln(t+1) + 40}$

find Volume at $t = 4$.

$V = 30\ln(5) + 40$

$V = 88.3$ litres.

b) i) $\int \frac{30}{t+1}\,dt$

$V = 30\ln(t+1) + c$

at $t=0$ $V=40$

$40 = 30\ln 1 + C$

$\therefore c = 40$

$V = 30\ln(t+1) + 40$

ii) $160 = 30\ln(t+1) + 40$

$120 = 30\log_e(t+1)$

$4 = \log_e(t+1)$

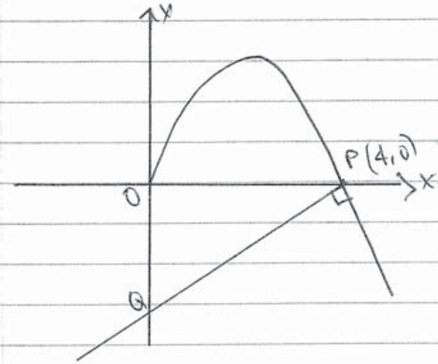
$\log_a y = x \rightarrow y = a^x$

$\log_e(t+1) = 4$

$t+1 = e^4$

$\boxed{t = e^4 - 1}$

Question 8

a) 

$y = 4x^{1/2} - x^{3/2}$

$dy/dx = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{1/2}$

$dy/dx = \frac{2}{x^{1/2}} - \frac{3}{2}x^{1/2}$.

at $x = 4$.

$m = \frac{2}{2} - \frac{3}{2} \times 2$

$m = 1 - 3$ $\boxed{m = -2}$

ii) Area of triangle OPQ

$m_{PQ} = \frac{1}{2}$ $P(4,0)$

$y - 0 = \frac{1}{2}(x - 4)$

$2y = 1(x - 4)$

$2y = x - 4$

$-x + 2y = -4$

$x - 2y = 4$

at $x = 0$ cut y-axis

$\therefore -2y = 4$

$y = -2$

$Q(0, -2)$

$A = \frac{1}{2}bh$.

$= \frac{1}{2} \times 2 \times 4$

$= 4\ u^2$

iii) Find area bounded by curve and x
lines PQ and QO

Region bounded by curve and x-axis.

$A = \int_0^4 4x^{1/2} - x^{3/2}\,dx$

$= \frac{2}{3} \cdot 4x^{3/2} - \frac{2}{5}x^{5/2}$

$= \left[\frac{8}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^4$

$= \frac{8 \times 8}{3} - \frac{64}{5}$

$= \frac{128}{15}$ units

Area Region $= \frac{128}{15} + 4$

$= 12\frac{8}{15}$ unit^2.

b) $a = 2\,m/sec^2$, it is initially at rest 2m to right of origin

i)
$a = 2$
$v = \int 2\,dt$
$v = 2t + c$

at $t=0$ $v=0$

$\boxed{v = 2t}$

ii) $v = 2t$ at $t=3$

$v = 6\,m/sec$

iii) When is velocity $10\,m/sec$

$v = 2t$
$10 = 2t$

$t = 5$ secs.

iv) displacement

$v = 2t$

$x = \int 2t\,dt$

$x = \frac{2t^2}{2}$

$\boxed{x = t^2 + c}$

at $t=0$ $x=2$

$\therefore c = 2$

$x = t^2 + 2$

v) $x = t^2 + 2$
$6 = t^2 + 2$

$t^2 = 4$ $\therefore t = 2$ sec.

vi) How far in 3rd second.

$x = t^2 + 2$ $t=2$ $x=6$

$x = t^2 + 2$ $t=3$ $x=11$

$\therefore$ 5 metres

Question 9

a) i) $A = 500(1.06)^{21}$

$= \$1699.78$

ii) $\dfrac{500 \times 1.06(1.06^{21}-1)}{1.06-1}$

$= \$21,196.15 + \$500$

$= \$26,696.15$

b) $y = e^x + e^{-x}$

$V = \pi \int_{-1}^{1} (e^x - e^{-x})^2 \, dy$

$= \pi \int_{-1}^{1} e^{2x} + 2 + e^{-2x} \, dx$

$= \pi \left[\frac{1}{2} e^{2x} + 2x - \frac{1}{2} e^{-2x} \right]_{-1}^{1}$

$= \pi \left[\frac{1}{2} e^2 + 2 - \frac{1}{2} e^{-2} \right] - \left[\frac{1}{2} e^{-2} - 2 - \frac{1}{2} e^2 \right]$

$= \pi \left[e^2 + 4 + e^{-2} \right]$

$= \pi (11.53)$

$= 36.2 \, u^3$

c) $y = 5 - 14x - 2x^2$

$y' = -14 - 4x$

$-2 = -14 - 4x$

$4x = -12$

$\boxed{x = -3}$

Sub to find y

$y = 29$

$(-3, 29)$

$y - 29 = -2(x+3)$

$y - 29 = -2x - 6$

$\boxed{2x + y - 23 = 0}$

Question 10

i) Domain $x \neq 0$ all real

ii) $f'(x) = \dfrac{xe^x - e^x}{x^2}$

$f''(x) = \dfrac{x^2 \left[e^x + xe^x - e^x \right] - \left[xe^x - e^x \right] 2x}{x^4}$

$= \dfrac{x^3 e^x - 2x^2 e^x + 2xe^x}{x^4}$

$= \dfrac{xe^x (x^2 - 2x + 2)}{x^4}$

$f''(x) = \dfrac{e^x \left[(x-1)^2 + 1 \right]}{x^3}$

iii) Stationary Point $f'(x) = 0$

$\therefore xe^x - e^x = 0$

$e^x (x-1) = 0$

$\therefore$ at $x = 1$

Sub to find y

$y = \dfrac{e^x}{x}$ at $x = 1$

$y = e$

Nature $f''(x)$

$f''(1) = \dfrac{e^1 \times 1}{1} > 0$

$\therefore$ Min TP $(1, e)$

iv) No pts of inflexion $f''(x) = 0$

ie $(x-1)^2 + 1 > 0$ for all values of x

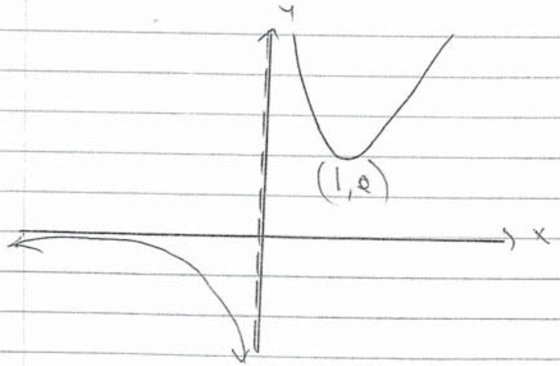
ie $f''(x) \neq 0$ $\therefore$ No pt inflexion

v) Concave up $f''(x) > 0$ occurs for $x > 0$

Concave down $f''(x) < 0$ occurs for $x < 0$

vi) $\lim_{x \to \infty} \frac{e^x}{x} \Rightarrow 0$ tends to ∞ for large values of x.

vii



END OF SOLUTIONS