STUDENT NAME: _____

TEACHER: _____

Founded 1982

THE HILLS GRAMMAR SCHOOL

TASK 4 Trial Examination 2015

YEAR 12

MATHEMATICS

Time Allowed: Three hours (plus five minutes reading time)

Weighting: 40%

Instructions:

- Approved calculators may be used
- Attempt all questions
- Start all questions on a new sheet of paper
- The marks for each question are indicated on the examination
- Show all necessary working

MCQ	Question 11	Question 12	Question 13	Question 14	Question 15	Question 16	TOTAL
/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1 What is the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures?

(A) 0.040

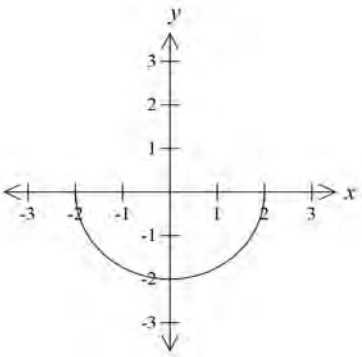
(B) 0.0410

(C) 0.0409

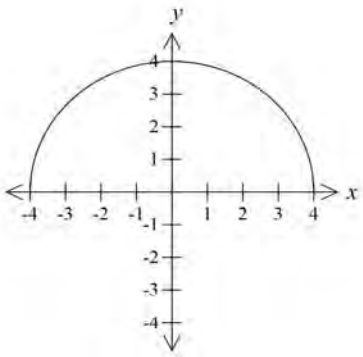
(D) 0.041

2 Which graph best represents $y = \sqrt{4 - x^2}$?

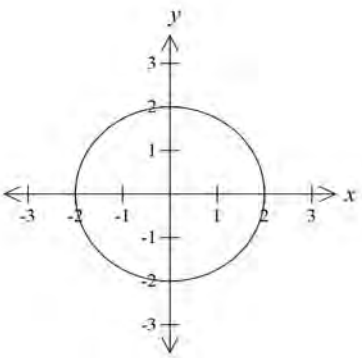
(A)



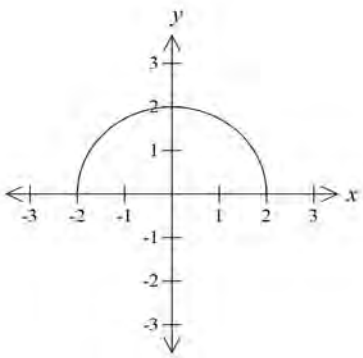
(B)



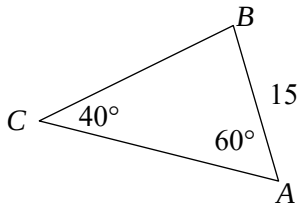
(C)



(D)



3 What is the correct expression for AC in triangle ABC?



(A) $\dfrac{15\sin 80^\circ}{\sin 40^\circ}$

(B) $\dfrac{15\sin 80^\circ}{\sin 60^\circ}$

(C) $\dfrac{15\sin 40^\circ}{\sin 60^\circ}$

(D) $\dfrac{\sin 40^\circ}{15\sin 80^\circ}$

4 The line $6x - ky = 2$ passes through the point $(3,2)$. What is the value of k?

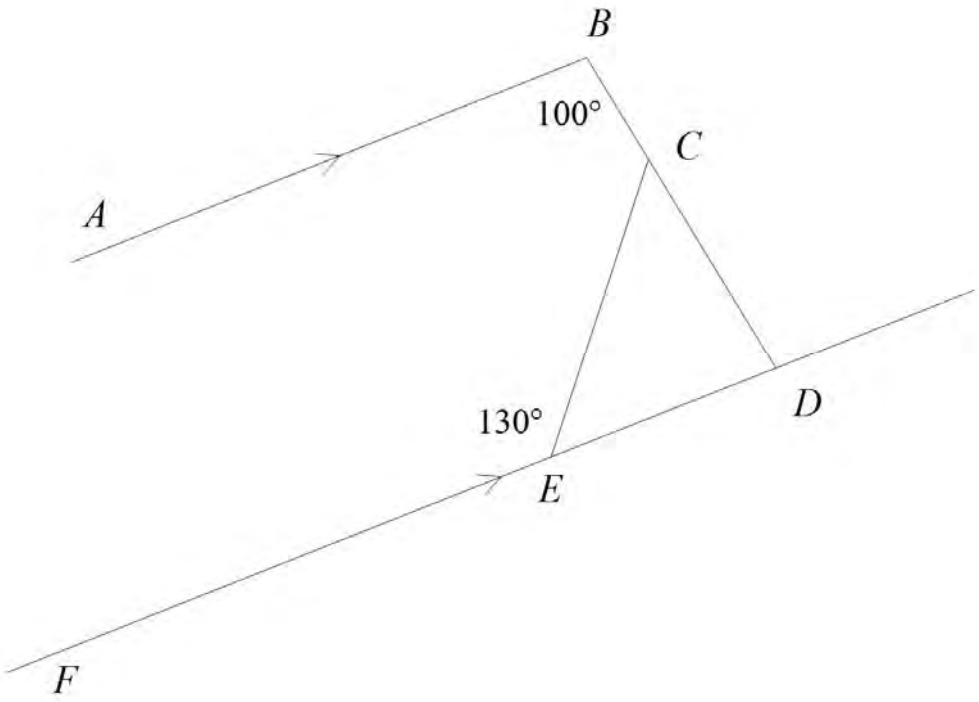
(A) $\dfrac{10}{3}$

(B) $-\dfrac{10}{3}$

(C) -8

(D) 8

5 In the diagram below, AB is parallel to FD, $\angle ABC = 100^\circ$ and $\angle CEF = 130^\circ$



What is the value of $\angle BCE$?

(A) 100°

(B) 110°

(C) 120°

(D) 130°

6 What is the value of $f'(3)$ if $f(x) = 3x - x^3$?

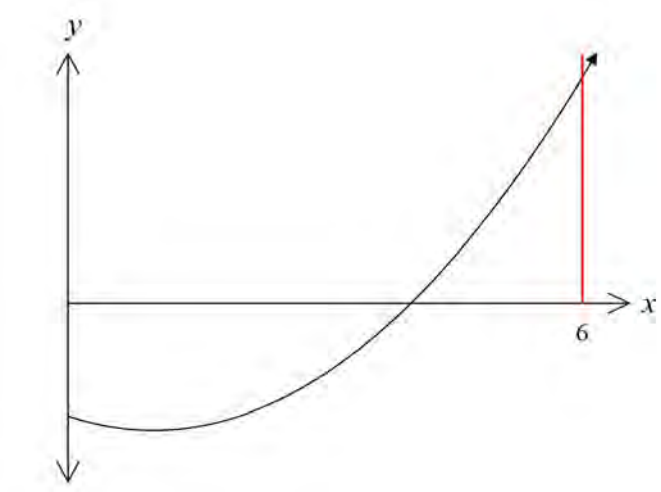
(A) $f'(3) = -18$

(B) $f'(3) = -24$

(C) $f'(3) = 0$

(D) $f'(3) = 9$

7 The diagram below shows part of the graph of $y = x^2 - 2x - 8$.



What is the correct expression for the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

(A) $A = \int_0^5 x^2 - 2x - 8dx + \left| \int_5^6 x^2 - 2x - 8dx \right|$

(B) $A = \int_0^4 x^2 - 2x - 8dx + \left| \int_4^6 x^2 - 2x - 8dx \right|$

(C) $A = \left| \int_0^5 x^2 - 2x - 8dx \right| + \int_5^6 x^2 - 2x - 8dx$

(D) $A = \left| \int_0^4 x^2 - 2x - 8dx \right| + \int_4^6 x^2 - 2x - 8dx$

8 What is the solution to the inequation $x^2 + 4x + 3 \ge 0$?

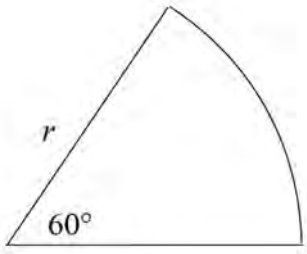
(A) $x \le 1$ or $x \ge 3$

(B) $x \le -1$ or $x \ge -3$

(C) $x \ge 1$ or $x \le 3$

(D) $x \ge -1$ or $x \le -3$

9 The sector below has an area of 10π square units.



Not to scale

What is the value of r?

(A) $\sqrt{60}$

(B) $\sqrt{60\pi}$

(C) $\sqrt{\dfrac{\pi}{3}}$

(D) $\sqrt{\dfrac{1}{3}}$

10 An infinite geometric series has a first term of 8 and a limiting sum of 12. What is the common ratio?

(A) $\dfrac{1}{6}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{2}$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Question 11-16, your responses should include relevant mathematical reasoning and /or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Find the natural domain of the function $f(x) = \sqrt{3-x}$. **1**

(b) Find $\int x^2 + 1 dx$ **1**

(c) Solve $x^2 - x - 1 = 0$ writing your answer in simplest surd form. **2**

(d) Differentiate $y = \sqrt{9 - 2x^3}$. **2**

(e) Evaluate $\int_0^3 x^2 - 3 dx$. **2**

(f) Evaluate $\lim_{x \to 3} \frac{5x-15}{x^2+4x-21}$. **2**

(g) Fully simplify $(3\sqrt{5} - 2\sqrt{3})^2$. **2**

(h) Simplify $\frac{x-3y}{x^3 y} \div \frac{3y-x}{xy^3}$. **3**

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) If $f'(x) = 6x^2 + 5x - 1$ and $f(-1) = 5$, find an expression for $f(x)$. **2**

(b) (i) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$. **1**

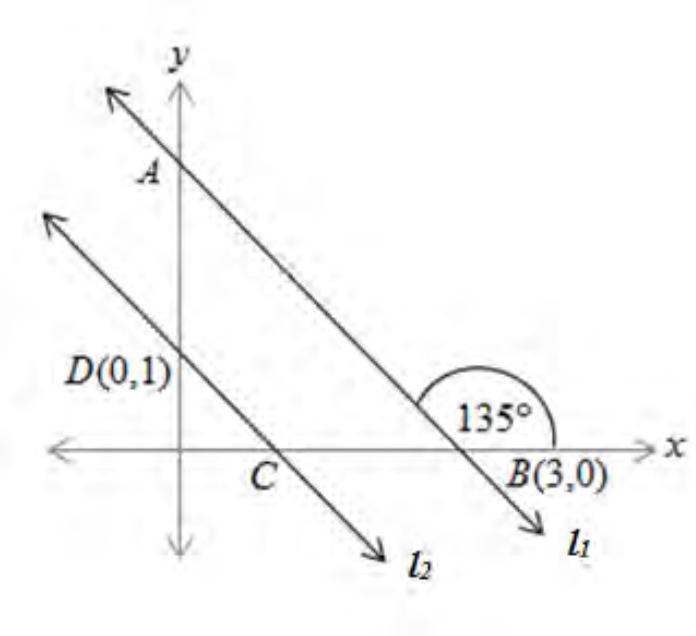
(ii) Hence, or otherwise, evaluate $\int_1^{e^3} \ln(x)\,dx$. Leave your answer in exact form. **2**

(c) A particle moves so that its displacement from the origin is given by $x = -t^2 + 7t + 8$ (where x is displacement in metres and t is time in seconds).

(i) Show that the initial displacement of the particle is 8 metres. **1**

(ii) At what time will the particle be at the origin? **2**

(d) The line l_1 makes an angle of 135° at the point B (3,0). It cuts the y-axis at A.
The line l_2 is parallel to the line l_1. Its y-intercept is D (0,1) and its x-intercept is C.



(i) Show that the equation of the line l_1 is $x + y - 3 = 0$ **2**

(ii) Hence, or otherwise, find the equation of the line l_2. **1**

(iii) Find the perpendicular distance between the lines l_1 and l_2. **2**

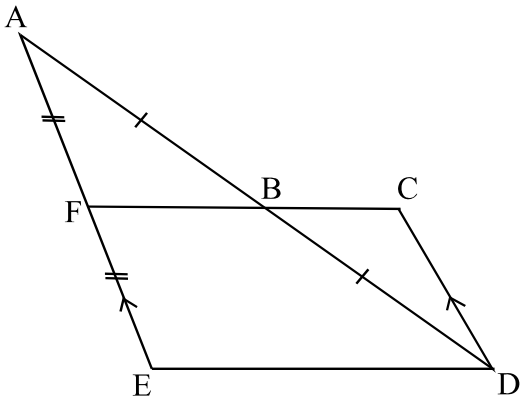
(iv) Find the area of the quadrilateral $ABCD$. **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) What is the derivative of $\frac{e^x}{x^2}$? **2**

(b) In the diagram, the line *FC* bisects *AE* at *F* and *AD* at *B*. The line *AE* is parallel to *CD*.



NOT TO SCALE

(i) Explain why $ED = 2BF$. **1**

(ii) Prove that $\Delta ABF \equiv \Delta DBC$. **3**

(c) A population of a country grows over time according to $P = P_0 e^{kt}$. The population grew from 350 000 in 2001 to 460 000 in 2005. The rate of population growth is proportional to the population size.

(i) Find the growth rate per year, correct to 3 decimal places. **2**

(ii) Find the population of the country in 2015, correct to the nearest person. **1**

(iii) Calculate the rate of change of the population in 2021. **2**

(d) The area enclosed between the curve $y = 4 - x^2$ and the line $y = 4 - 2x$ is rotated about the x-axis.

(i) Sketch the region between the two graphs. **2**

(ii) Find the volume of the solid generated between these two graphs, leaving your answer in terms of π. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Let α and β be roots of the equation $x^2 - 8x + 5 = 0$. Find the value of $\alpha^2 + \beta^2$. **2**

(b) For the function $y = xe^{2x}$,

(i) Find the stationary point and determine its nature. **3**

(ii) Find any points of inflection. **3**

(iii) Sketch the function in the domain $-3 \le x \le 0.5$. **2**

(c) Madison is learning to drive. Her first lesson is 10 minutes long. Her second lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson.

(i) How long will Madison's fifteenth lesson be? **1**

(ii) How many minutes of lessons will Madison have completed after her fifteenth lesson? **2**

(iii) During which lesson will Madison have completed a total of 1150 minutes of driving lessons? **2**

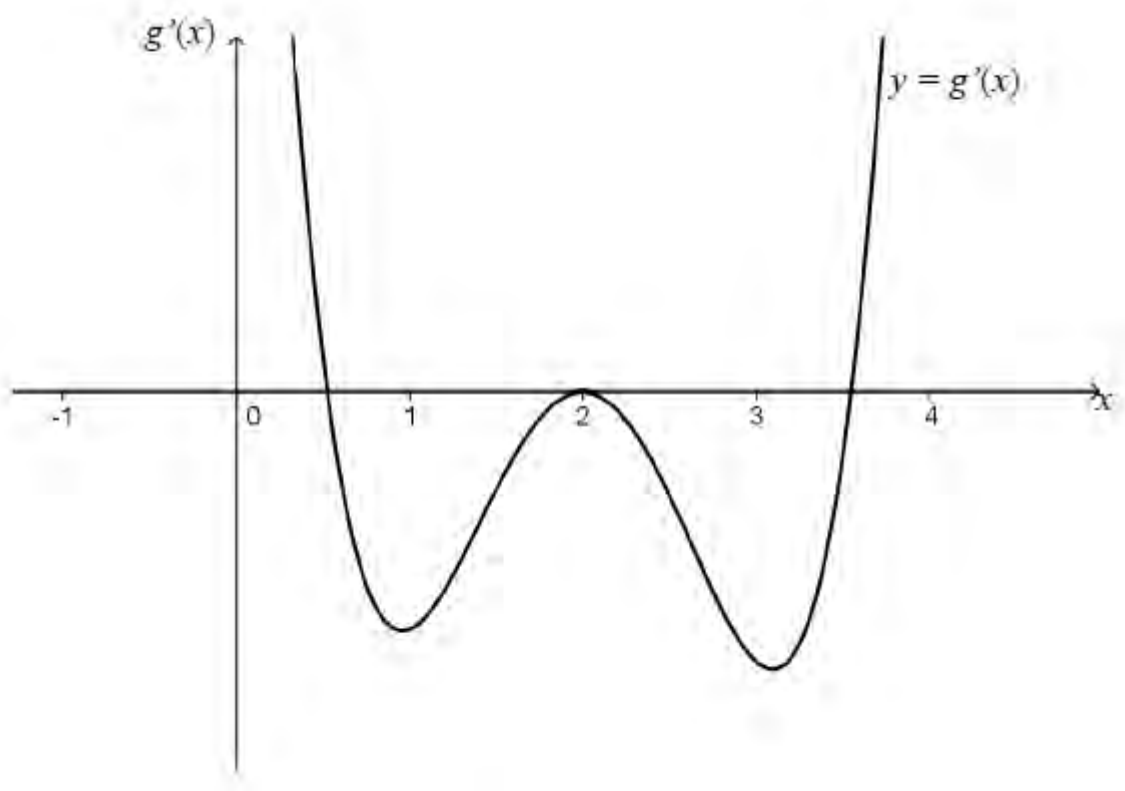
End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) What are the exact solutions to the equation $e^{6x} - 7e^{3x} + 6 = 0$? **3**

(b) Find the equation of the parabola which has its vertex at (2,0) and its directrix is given by $x = 5$. **2**

(c) Shown below is a graph of the derivative function $y = g'(x)$.



(i) If the function $y = g(x)$ were to be drawn using information from the graph above, what feature would exist on the graph at $x = 2$? Justify your answer. **2**

(ii) In your answer booklet, draw a neat sketch of a possible function for $y = g(x)$ given that $g(0) = 0$. **2**

(d) Alex borrowed \$60 000 to buy a small business. He was charged 6% p.a. compounding monthly on the balance owing and he repaid the loan plus interest in equal monthly repayment over 5 years.

(i) Show that Alex owed \$ $(60\,300 - M)$ immediately after making his first monthly repayment of \$$M$. **1**

(ii) Show that Alex owed \$ $[\ 60\,000(1.005)^3 - M\,(1.005^2 + 1.005 + 1)\ ]$ immediately after he made three monthly repayments. **2**

(iii) Calculate his monthly repayment, \$$M$ to the nearest five cents. **2**

(iv) Calculate the total amount of interest paid. **1**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

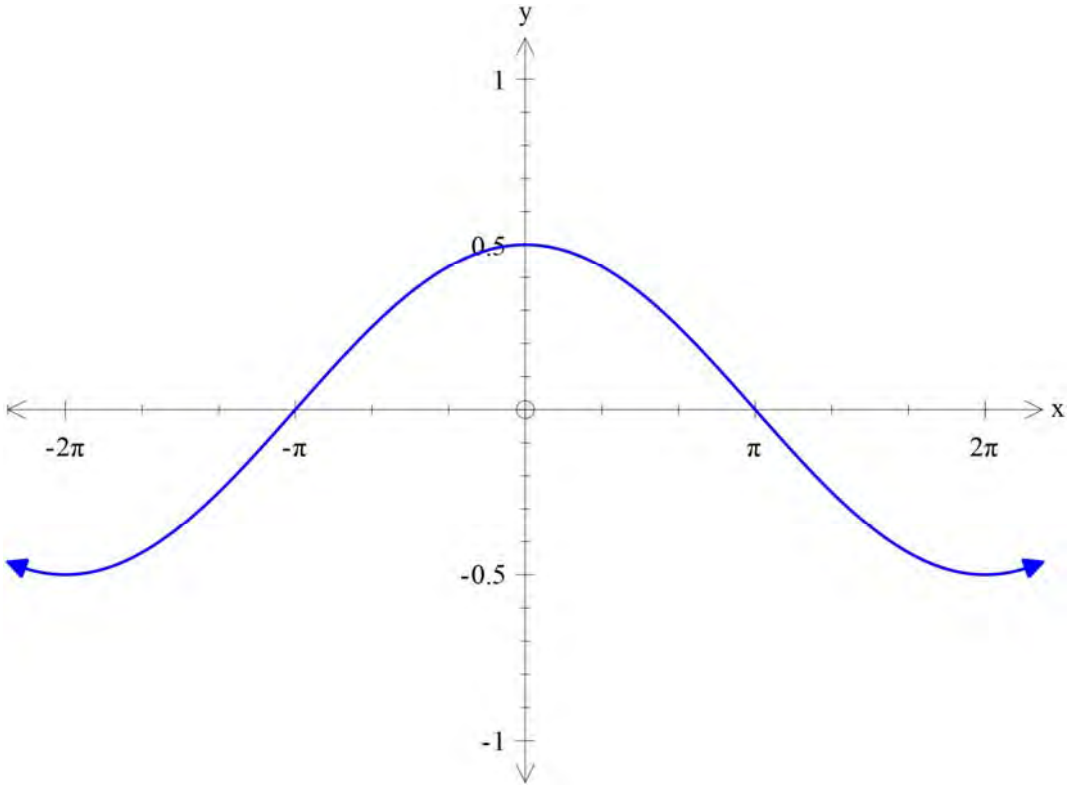
(a) For the curve $y = \ln(x-2)$,

(i) Sketch the curve. **1**

(ii) State its domain and range. **2**

(b) Sketch the function $y = -3\sin\frac{x}{2}$ in the domain $-2\pi \le x \le 4\pi$ **2**

(c) For the function $y = \frac{1}{2}\cos\frac{x}{2}$ below,



(i) Find the area between the curve and the lines $x = \frac{\pi}{2}$ and $x = 2\pi$. **3**

(ii) Use one application of Simpson's Rule to find an approximation for the area between the curve and the lines $x = -\pi$ and $x = \pi$. **2**

(iii) What is the percentage error of your approximation from (ii) compared with the actual area between the curve and the lines $x = -\pi$ and $x = \pi$? **1**

(d) The area enclosed by the curve $y = \sqrt{r^2 - x^2}$ is rotated about the x-axis.

(i) What is the name given to the solid that is generated? **1**

(ii) Explain why the volume of the solid of revolution between $x = -r$ and $x = r$ is **twice** the integral $\pi \int_0^r (r^2 - x^2)dx$. **1**

(iii) Show that the volume of the solid formed is $\frac{4}{3}\pi r^3$ **2**

End of paper

ANSWER SHEET FOR MULTIPLE CHOICE SECTION

Student Exam number:_____
Teacher: _____

1. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

2. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

3. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

4. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

5. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

6. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

7. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

8. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

9. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

10. **A** ⬭ **B** ⬭ **C** ⬭ **D** ⬭

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments

Multiple Choice

1) ans = 0.04099...
= 0.0410 (3sf) → (B)

2) (D)

3) $\frac{AC}{\sin 80} = \frac{15}{\sin 40}$

$AC = \frac{15 \sin 80}{\sin 40}$ → (A)

4) $6(3) - (2)k = 2$

$-2k = -16$

$k = 8$ → (D)

5) (D)

6) $f'(x) = 3 - 3x^2$

$f'(3) = 3 - 3(3)^2$

$= 3 - 27$

$= -24$ → (B)

7) $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

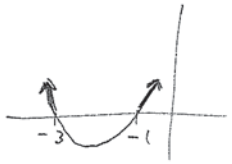
$x = 4, -2$ → (D)

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments

8) $x^2 + 4x + 3 \geq 0$

$(x+1)(x+3) \geq 0$



$x \leq -3, x \geq -1$ → (D)

9) $A = \frac{1}{2} r^2 \theta$

$10\pi = \frac{1}{2} r^2 \frac{\pi}{3}$

$\frac{r^2}{2} = 30$

$= 60$

$r = \sqrt{60}$ → (A)

10) $S_\infty = \frac{a}{1-r}$

$12 = \frac{8}{1-r}$

$12 - 12r = 8$

$-12r = -4$

$r = \frac{1}{3}$ → (C)

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
Question 11	
a) $f(x) = \sqrt{3-x}$	some students failed to realise you cannot take the square root of a negative. i.e $3-x \geq 0$.
$3 - x \geq 0$	
$-x \geq -3$	
$x \leq 3$ /1	
b) $\int x^2 + 1 \, dx$	
$= \frac{x^3}{3} + x + C$ /1	
c) $x^2 - x - 1 = 0$	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
$= \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$ 1	good.
$= \frac{1 \pm \sqrt{5}}{2}$ 1 /2	
d) $y = \sqrt{9 - 2x^3} = (9 - 2x^3)^{\frac{1}{2}}$ 1	
$y' = \frac{1}{2}(9-2x^3)^{-\frac{1}{2}} \cdot -6x^2$	
$= -3x^2 (9-2x^3)^{-\frac{1}{2}}$ 1	good
or $= -\frac{3x^2}{\sqrt{9-2x^3}}$ /2	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
e) $\int_0^3 x^2 - 3 \, dx$	
$= \left[\frac{x^3}{3} - 3x\right]_0^3$ 1	
$= \frac{(3)^3}{3} - 3(3) - 0$	good.
$= 9 - 9$	
$= 0$ 1 /2	
f) $\lim_{x \to 3} \frac{5x - 15}{x^2 + 4x - 21}$	many failed to first factorise then sub in $x = 3$.
$= \lim_{x \to 3} \frac{5(x-3)}{(x+7)(x-3)}$	
$= \frac{5}{3+7}$	
$= \frac{1}{2}$ /2	
g) $(3\sqrt{5} - 2\sqrt{3})^2$	good.
$= (3\sqrt{5})^2 + 2 \times (3\sqrt{5})(-2\sqrt{3}) + (-2\sqrt{3})^2$ 1	
$= 45 - 12\sqrt{15} + 12$	
$= 57 - 12\sqrt{15}$ 1 /2	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
h) $\frac{x-3y}{x^3y} \div \frac{3y-x}{xy^3}$	
$= \frac{x-3y}{x^3y} \times \frac{xy^3}{3y-x}$	
$= \frac{x-3y}{x^2} \times \frac{y^2}{-(x-3y)}$	some failed to take out −1 here in order to cancel.
$= -\frac{y^2}{x^2}$	
Question 12	
a) $f'(x) = 6x^2 + 5x - 1$	
$f(x) = \frac{2}{1}\frac{6}{3}x^3 + \frac{5x^2}{2} - x + C$	answered ok by most.
$5 = 2(-1)^3 + \frac{5(-1)^2}{2} - (-1) + C$	
$C = \frac{7}{2}$ 2	
$\therefore f(x) = 2x^3 + \frac{5x^2}{2} - x + \frac{7}{2}$	
b) i) $\frac{d}{dx}(x\ln x - x) = u'v + v'u - 1$	many failed to use product rule
$= \ln x + \frac{1}{x} \times x - 1$	
$= \ln x + 1 - 1$ 1	
$= \ln x$ as req'd.	
ii) $\int_1^{e^3} \ln(x)\,dx = [x\ln x - x]_1^{e^3}$	students could not relate part (i) to part (ii)
$= (e^3 \ln e^3 - e^3) - (\ln 1 - 1)$	
$= 3e^3 - e^3 - -1$ 2	
$= 2e^3 + 1$	

uggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
c) i) initial displacement is x when $t=0$	well answered.
$\therefore x = -(0)^2 + 7(0) + 8$ 1	
$= 8$ m	
ii) i.e. find t when $x=0$	
$-t^2 + 7t + 8 = 0$	many failed to state $t \geq 0$.
$-(t^2 - 7t - 8) = 0$	
$(t-8)(t+1) = 0$ 2.	
$t = 8, -1 \quad (t \geq 0)$	
$\therefore$ at the origin when $t = 8$ seconds.	
d) i) $m_{l_1} = \tan\theta$	co-ord geom very poorly answered, students failed to recall formulae + relate grad of ll lines.
$= \tan 135$ 2	
$= -1$	
l_1: $y - y_1 = m(x - x_1)$	
$y - 0 = -1(x-3)$	
$y = -x + 3$	
$x + y - 3 = 0$	
ii) $m_{l_1} = m_{l_2} = -1$	
l_2: $y - y_1 = m(x - x_1)$	
$y - 1 = -1(x - 0)$	
$y - 1 = -x$	
$x + y - 1 = 0$	
(or $y = -x + 1$)	

gested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
iii) $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ using $D(0,1)$ and l_1. $= \frac{\lvert 1 - 3 \rvert}{\sqrt{1^2+1^2}}$ 2 $= \frac{2}{\sqrt{2}}$ $= \sqrt{2}$ units	Poor use of correct formula
iv) $A = \frac{1}{2}h(a+b)$ $d_{DC} = \sqrt{2}$ $d_{AB} = \sqrt{18} = 3\sqrt{2}$ $\therefore A = \frac{1}{2} \times \sqrt{2}(\sqrt{2} + 3\sqrt{2})$ $= \sqrt{2}(\sqrt{2} + 3\sqrt{2}) \div 2$ $= \sqrt{2} \times 4\sqrt{2} \div 2$ 2 $= 8 \div 2$ $= 4$ units^2	Answered OK with exception of a few who failed to use correct formula

Suggested Solutions, Marking Scheme and Markers' comments

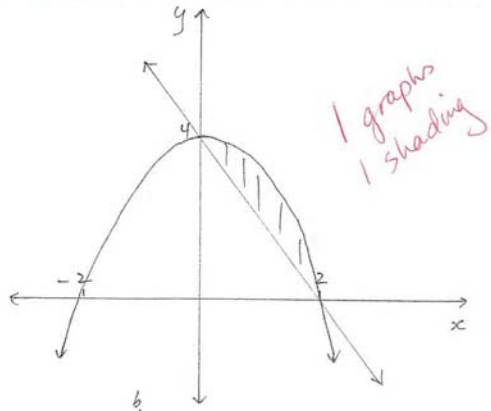
Suggested solution(s)	comments
Question 13 a) $\frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{u'v - v'u}{v^2}$ $= \frac{e^x x^2 - 2x e^x}{x^4}$ | $= \frac{e^x x(x-2)}{x^4}$ $= \frac{e^x(x-2)}{x^3}$ |	Several students wrote this as a product, but were unsuccessful in the derivative of a product ⇒ learn the rules.
b) i) $\triangle AFB \equiv \triangle AED$ (equiangular) $\frac{AF}{AE} = \frac{AB}{AD} = \frac{1}{2}$ | $\therefore ED = 2BF$	Other versions accepted.
ii) A: $\angle ABF = \angle CBD$ (vert. opp. $\angle$'s) | A: $\angle AFB = \angle BCD$ (alt. $\angle$'s equal) | S: $AB = BD$ (given) | $\therefore \triangle ABF \equiv \triangle DBC$ (AAS) (or SAS, $FB = BC$)	And other versions

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
c) i) $P = P_0 e^{kt}$ $P_0 = 350{,}000$. when $t=4$, $P = 460{,}000$ $460{,}000 = 350{,}000 e^{4k}$ $e^{4k} = \frac{46}{35}$ | $\log_e e^{4k} = \log_e \left(\frac{46}{35}\right)$ $4k = \ln \frac{46}{35}$ $k = \ln \frac{46}{35} \div 4$ $= 0.068$ (3 d.p.) |	Usually well done, although some students used $t=5$.
ii) i.e. find P when $t = 14$ $P = 350{,}000 e^{14k}$ $= 910{,}924.32\ldots$ $= 910{,}924$ (nearest person) |	ECF
iii) i.e. find $\frac{dP}{dt}$ when $t = 20$ $\frac{dP}{dt} = kP$ $= k P_0 e^{kt}$ $= k\, 350{,}000 e^{20k}$ | $= 93{,}774$ people / yr |	ECF.

accept 92729 people for using $k = 0.068$.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
d) i)  | graphs | shading	
ii) $V = \pi \int_a^b y^2\, dx$ $= \pi \int_0^2 (4-x^2)^2\, dx - \pi \int_0^2 (4-2x)^2\, dx$ | $= \pi \int_0^2 16 - 8x^2 + x^4 - (16 - 16x + 4x^2)\, dx$ $= \pi \int_0^2 16 - 8x^2 + x^4 - 16 + 16x - 4x^2\, dx$ $= \pi \int_0^2 x^4 - 12x^2 + 16x\, dx$ $= \pi \left[\frac{x^5}{5} - \frac{12x^3}{3_1} + \frac{16x^2}{2_1}\right]_0^2$ ✓ $= \pi \left(\frac{(2)^5}{5} - 4(2)^3 + 8(2)^2\right)$ $= \frac{32\pi}{5}$ units^3 |	Expansions were badly done and compounded errors.

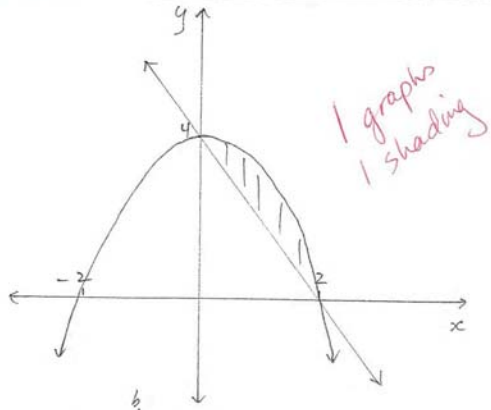
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
Question 14	
a) $x^2 - 8x + 5 = 0$	
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	Mostly well done, but students must be able to find $\alpha+\beta$ and $\alpha\beta$.
$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$	
$= \left(-\frac{-8}{1}\right)^2 - 2\left(\frac{5}{1}\right)$ |	
$= 64 - 10$	
$= 54$ |	
b) $y = xe^{2x}$	
i) $y' = u'v + v'u$	
$= e^{2x} + 2e^{2x}x$	
$= e^{2x}(1+2x)$ |	This part was reasonably well done
SP's exist when $y' = 0$.	
$e^{2x}(1+2x) = 0$	
$e^{2x} = 0$ or $1+2x = 0$	
↓ (no soln.) $x = -\frac{1}{2}$ |	Errors here with the negative being omitted

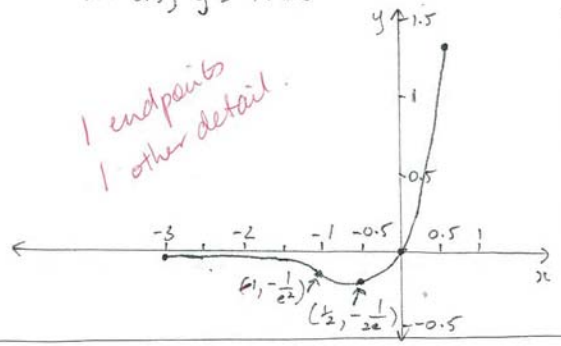
x	-0.6	-0.5	-0.4
y'	-0.04	0	$+0.09$
	\	—	/

when $x = -\frac{1}{2}$, $y = -\frac{1}{2}e^{2x-\frac{1}{2}}$	
$= -\frac{1}{2e}$ |	Some students did not find the y value or determine max/min value.
$\therefore$ minimum turning point at $\left(\frac{1}{2}, -\frac{1}{2e}\right)$	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
d) i)  | graphs | shading	
ii) $V = \pi \int_a^b y^2\,dx$ |	
$= \pi \int_0^2 (4-x^2)^2\,dx - \pi \int_0^2 (4-2x)^2\,dx$	Expansions were badly done and compounded errors.
$= \pi \int_0^2 16 - 8x^2 + x^4 - (16 - 16x + 4x^2)\,dx$	
$= \pi \int_0^2 16 - 8x^2 + x^4 - 16 + 16x - 4x^2\,dx$	
$= \pi \int_0^2 x^4 - 12x^2 + 16x\,dx$	
$= \pi \left[\frac{x^5}{5} - \frac{12x^3}{3} + \frac{16x^2}{2}\right]_0^2$	
$= \pi \left(\frac{(2)^5}{5} - 4(2)^3 + 8(2)^2\right)$	
$= \frac{32\pi}{5}$ units^3 |	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
ii) $y' = e^{2x}(1+2x)$ $y'' = u'v + v'u$ $= 2e^{2x}(1+2x) + 2e^{2x}$ $= 2e^{2x}(1+2x+1)$ $= 2e^{2x}(2+2x)$ $= 4e^{2x}(1+x)$	— some very bad errors here.
POI's exist when $y''=0$ and concavity changes. |	— change in concavity went unchecked.
$4e^{2x}(1+x) = 0$ $4e^{2x} = 0$ or $1+x=0$ ↓ no soln. $x = -1$ |	Minus sign left out.
x: −1.1 | −1 | −0.9 y'': −0.04 | 0 | 0.07	
when $x=-1$, $y = -e^{-2} = -\frac{1}{e^2}$ |	Watch transcription of −ve
∴ POI exists at $(-1, -\frac{1}{e^2})$	
iii) when: $x=-3$, $y=-0.007$ $x=0.5$, $y=1.36$	Not well answered generally
	| endpoints | other detail.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
c) 10, 15, 20, ...	
i) AP with $a=10$, $d=5$ $T_{15} = a + (n-1)d$ $= 10 + 5(15-1)$ $= 80$ mins. |	Good
ii) i.e. find S_{15} $S_n = \frac{n}{2}(a+l)$ $S_{15} = \frac{15}{2}(10+80)$ | $= 675$ min $\rightarrow$ 11h 15min |	Good.
iii) i.e. find n when $S_n = 1150$ $S_n = \frac{n}{2}(2a + (n-1)d)$ $1150 = \frac{n}{2}(20 + 5(n-1))$ $= \frac{n}{2}(20 + 5n - 5)$ $= \frac{n}{2}(15 + 5n)$ | $2300 = 15n + 5n^2$ $5n^2 + 15n - 2300 = 0$ | $n^2 + 3n - 460 = 0$ $(n+23)(n-20) = 0$ ↓ ∴ $n = 20$ (i.e. the 20th lesson)	Error in question: 1150 is in minutes, not hours. Marking was awarded for this version of working and also for students working on merit (ie correct units worked through as best as possible.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments

Question 15

a) $e^{2x} - 7e^{3x} + 6 = 0$

let $u = e^{3x}$

$u^2 - 7u + 6 = 0$

$(u-6)(u-1) = 0$

$u = 1, 6$ (3)

$\therefore e^{3x} = 1$, $e^{3x} = 6$

$\ln e^{3x} = \ln 1$ $\ln e^{3x} = \ln 6$

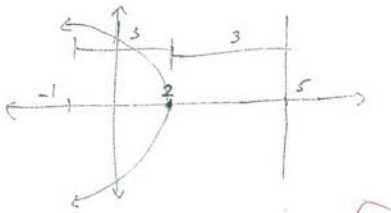
$3x = \ln 1$ $3x = \ln 6$

$x = \frac{\ln 1}{3}$ $x = \frac{\ln 6}{3}$

$= 0$

Many Students got to 1 + 6 but couldn't go further.

b)



$V = (2, 0)$

$a = 3$ (2)

$\therefore (y-0)^2 = -4 \times 3 (x-2)$

$y^2 = -12(x-2)$

few mixed $x^2 = 4a(y-k)$!

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments

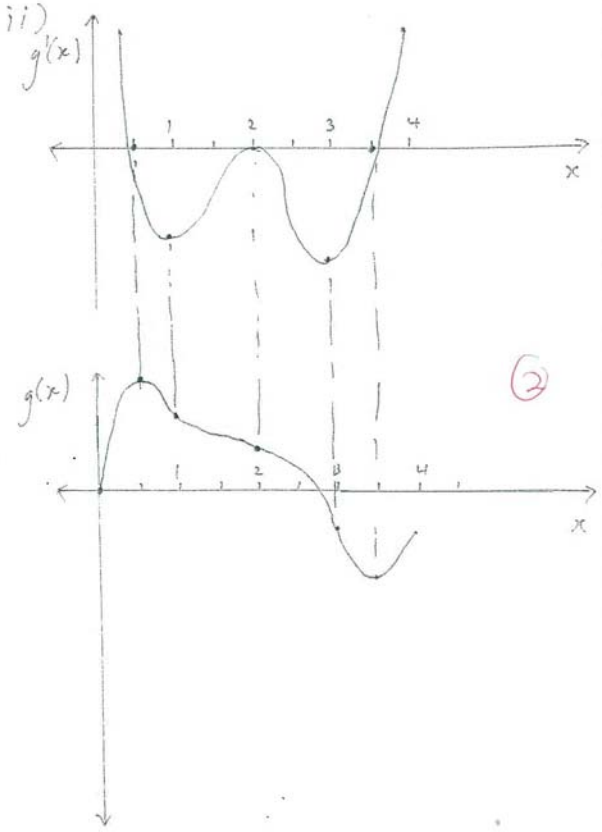
c) i)

y	1.9	2	2.1
$g'(x)$	−	0	−

$\therefore$ horizontal point of inflection at $x = 2$ since $g'(2) = 0$ and $g'(x) < 0$ on either side of $x = 2$. (2)

Many stated SP, no follow through

ii)



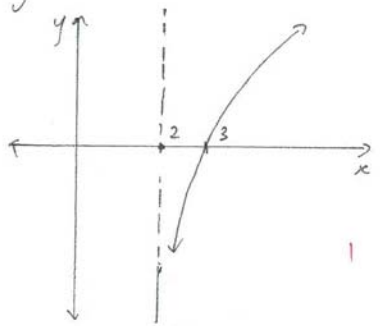
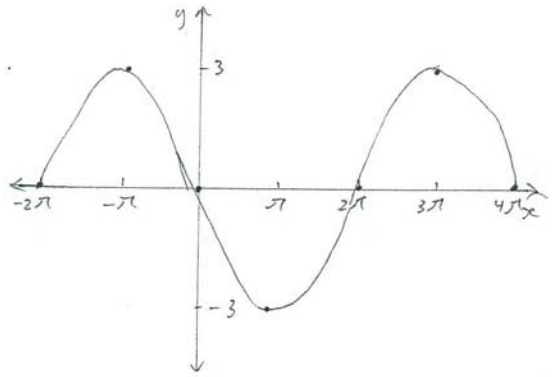
(2)

Very poorly answered as Student failed to interpret $g'(x)$ to $f(x)$ esp point $x =$

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
d) $r = 0.06 \div 12 = 0.005$	
i) $A_1 = P(1+r)^{\wedge} - M$	✓
$= 60000(1.005)^1 - M$	
$= \$(60300 - M)$ 1	
ii) $A_3 = (P+I) - (R+I)$	
$P + I = 60000(1.005)^3$	
2	
$R + I = M + M(1.005)^1 + M(1.005)^2$	✓
$= M(1.005^2 + 1.005 + 1)$	
$\therefore A_3 = \$\left(60000(1.005)^3 - M(1.005^2 + 1.005 + 1)\right)$	
iii) $A_{60} = 0$	
$0 = 60000(1.005)^{60} - \frac{M(1.005^{60}-1)}{0.005}$	
$M(1.005^{60}-1) = 60000(1.005)^{60} \times 0.005$	most ans v. well.
2	
$M = \$1159.95$	
iv) Interest $= (1159.95 \times 60) - 60000$	poorly answered.
$= \$9597.$ 1	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
Question 16	
a) $y = \ln(x-2)$	
i)  1	Some failed to show the asymptote at $x=2$.
/1	
ii) domain: $x > 2$ 1	✓ done well. 'ecf' marks awarded.
range: all real y 1	
/2	
b) $y = -3\sin\frac{x}{2}$, $-2\pi \le x \le 4\pi$	
amp $= -3$, period $= \frac{2\pi}{b} = \frac{2\pi}{1/2} = 4\pi$	
	Some used +3 as the amplitude or failed to realise period was 4π.
/2	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
c) $y = \frac{1}{2}\cos\frac{x}{2}$	
i) $A = \int_{\pi/2}^{\pi} \frac{1}{2}\cos\frac{x}{2}\,dx + \left| \int_{\pi}^{2\pi} \frac{1}{2}\cos\frac{x}{2}\,dx \right|$ /1	- Some calculated the integral: $\int_{\pi/2}^{2\pi}$ for which 2 ecf marks was awarded if evaluated correctly.
$= \left[\sin\frac{x}{2}\right]_{\pi/2}^{\pi} + \left| \left[\sin\frac{x}{2}\right]_{\pi}^{2\pi} \right|$	
$= \left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right) + \left|\left(\sin\pi - \sin\frac{\pi}{2}\right)\right|$ /1	
$= \left(1 - \frac{1}{\sqrt{2}}\right) + \left|(0-1)\right|$	
$= 1 - \frac{1}{\sqrt{2}} + 1$	
$= 2 - \frac{1}{\sqrt{2}}$	
OR $= \frac{4-\sqrt{2}}{2}$ units^2 /1 /3	
ii) $A \doteq \frac{\pi}{3}\left(\frac{1}{2}\cos-\frac{\pi}{2} + 4\times\frac{1}{2}\cos\frac{0}{2} + \frac{1}{2}\cos\frac{\pi}{2}\right)$ /1	- Some failed to use $\frac{1}{3}\pi$ as the width of the strip.
$\doteq \frac{\pi}{3}(0+2+0)$	
$\doteq \frac{2\pi}{3}$ units^2 /1 /2	
iii) $\frac{\frac{2\pi}{3} - 2}{2} \times 100 = \underline{4.7\%}$ (1dp) /1	- ecf mark awarded if above area used.

i) 1.29. ii) 2.09

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
d) i) Sphere /1	Some said circle.
ii) $y = \sqrt{r^2 - x^2}$ is even / symmetrical about y-axis	Needed to state the curve is even.
$\int_0^r$ produces hemi-sphere	
$\int_{-r}^{r}$ produces a sphere. /1	
iii) $\pi\int_{-r}^{r} (r^2 - x^2)\,dx$	
$= \pi\left[r^2x - \frac{x^3}{3}\right]_{-r}^{r}$ /1	The variable here is x not r.
$= \pi\left(r^3 - \frac{r^3}{3} - \left(-r^3 - \frac{-r^3}{3}\right)\right)$	i.e $\int r^2 \neq \frac{r^3}{3}$
$= \pi\left(\frac{2}{3}r^3 - -\frac{2}{3}r^3\right)$	
$= \pi\left(\frac{4}{3}r^3\right)$	
$= \frac{4}{3}\pi r^3$ as req'd. /1 /2	