Hornsby Girls' High School
Trial Examination
1999
Mathematics
2 Unit

Time Allowed 3 Hours
(Plus 5 Mins Reading Time)

Directions to Candidates

- Attempt all Questions
- All questions are of equal value
- All necessary working should be shown. Marks may be deducted for careless or badly arranged work
- Standard Integrals are printed on the last page
- Approved calculators may be used
- Each question must be started on a new page

QUESTION 1. **Start a new page** Marks

(a) Factorise : $16x^2 - 9$ 1

(b) Convert $\frac{3\pi}{5}$ radians to degrees 1

(c) Find a primitive of : $3 - 2x^2$ 2

(d) When the Goods and Services Tax (GST) is introduced, some items will be exempt while others will incur a tax of 10%. The local supermarket is advertising a service, 2

"Find out what you will pay under a GST".

Meg takes her trolley to Johnny, the checkout operator, and he tells her that her bill is $285. If the GST is imposed it will rise to $296.40.
What percentage of Meg's bill is GST exempt?

(e) Given $f(x) = 1 - x^3$, find the value of a if $f(a) = 65$. 2

(f) Find the values of a and b if $\frac{1}{2\sqrt{3} - 1} = a + b\sqrt{3}$ 2

(g) If $\frac{a}{b} = \frac{4}{3}$ and $a + b = 28$, find the values of a and b.

QUESTION 2. **Start a new page**

Marks

(a) Differentiate: 4

(i) $(x^2+5)^{-3}$

(ii) $x^2 \tan x$

(iii) $\dfrac{\log_e x}{x}$

(b) Find $\int \frac{x^2}{x^3-2}\,dx$ 2

(c) Evaluate : 2

(i) $\int_0^{\frac{\pi}{4}} \cos 2x\, dx$

(ii) $\int_0^{\ln 3} e^x dx$

4

(d) Points A (1, 2), B (4, 7) and C (3, 8) form a triangle.

Draw the triangle in your Writing Booklet.

(i) Calculate the length of AB as a surd.

(ii) Determine the equation of AB in general form.

(iii) Find the perpendicular distance of the point C from the line AB as a surd.

(iv) Hence, or otherwise, find the area of triangle ABC.

QUESTION 3. **Start a new page**

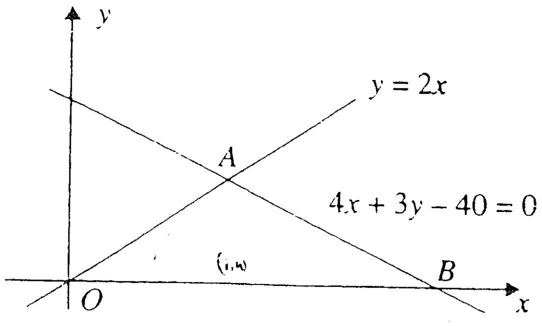
Mark

(a) If α and β are the roots of $5x^2 - 2x - 3 = 0$, find the value of: 3

(i) $\alpha + \beta$.

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$.

(b) 6



NOT TO SCALE

In the diagram, OA is the line $y = 2x$, and AB is the line $4x + 3y - 40 = 0$.

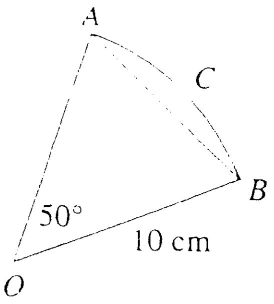
Copy or trace the diagram and answer the following.

(i) Find the coordinates of A.

(ii) Calculate the angle of inclination of the line AB to the nearest degree.

(iii) Write down the three inequalities satisfied by the points that lie inside ΔOAB.

(c) 3



NOT TO SCALE

The diagram shows a sector $OACB$ with sector angle 50° and radius 10 centimetres.

(i) Convert 50° to radian measure.

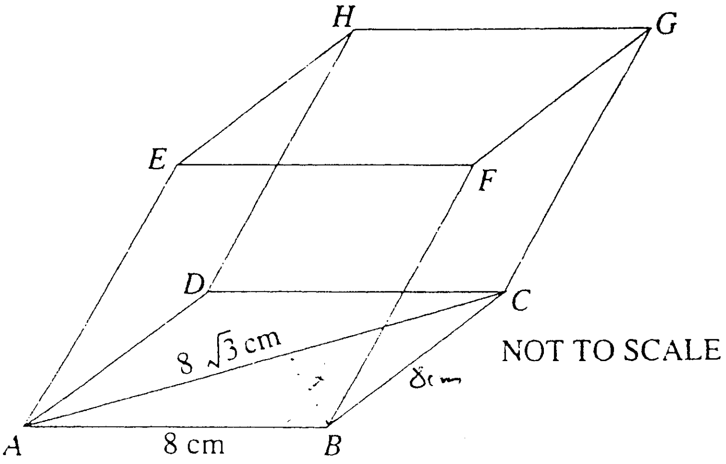
(ii) Calculate, correct to one decimal place, the area of segment ABC.

QUESTION 4. **Start a new page**

Marks

(a) Simplify $\frac{\log x^3 - \log x}{\log x^2 + \log x}$. 2

(b) 5



ABCDEFGH represents a calcite crystal with six equal rhombic faces. In each rhombic face, the longer diagonal is $8\sqrt{3}$ cm, and the sides are 8 centimetres long.

(i) Find the size of the angle *ABC* in the rhombic face *ABCD*.

(ii) Calculate the surface area of the crystal.

(c) Consider the function $y = 1 + \sin 2x$. 5

(i) State the period and the range of the function.

(ii) Sketch the graph of $y = 1 + \sin 2x$ for $0 \leq x \leq 2\pi$.

(iii) Write down the number of solutions to the equation $\sin 2x = -1$ for $0 \leq x \leq 2\pi$. (Do not solve the equation.)

QUESTION 5. **Start a new page**

Mark

(a) Find the exact value of $\sec\frac{\pi}{4} + \cot\frac{\pi}{6}$. 2

(b) Four metal disks, numbered 1, 2, 3 and 4 are placed in a bag. Two disks are selected at random and placed together on a tabletop to form a two-digit number. 4

(i) By using a tree digram, or otherwise, find how many two digit numbers which can be formed.

(ii) Find the probability that the number formed is 21.

(iii) Determine the probability that the number formed is divisible by 3.

(c) Consider the arithmetic series 47 + 41 + 35 + ... 6

(i) State the common difference.

(ii) If T_n is the nth term of the series, find the smallest n for which $T_n < 0$.

(iii) If S_n is the sum of n terms of the series, find the smallest n for which $S_n < 0$.

QUESTION 6. **Start a new page**

Marks

(a) A bottle of vintage wine cost \$375 when first released. After t years its value, \$$V$, is given by $V = 375e^{0.05t}$. **3**

(i) Find the value of the bottle of wine after 10 years, correct to the nearest dollar.

(ii) Find how many years it takes for the value of the wine to increase to \$1000 per bottle. Give the answer to the nearest one tenth of a year.

(b) Consider the curve $y = e^{2x}(1-x)$. **9**

(i) Find the y-intercept.

(ii) Determine where the curve crosses the x-axis.

(iii) Find the one stationary point, and determine its nature.

(iv) Discuss the behaviour of the curve as $x \to +\infty$.

(v) Discuss the behaviour of the curve as $x \to -\infty$.

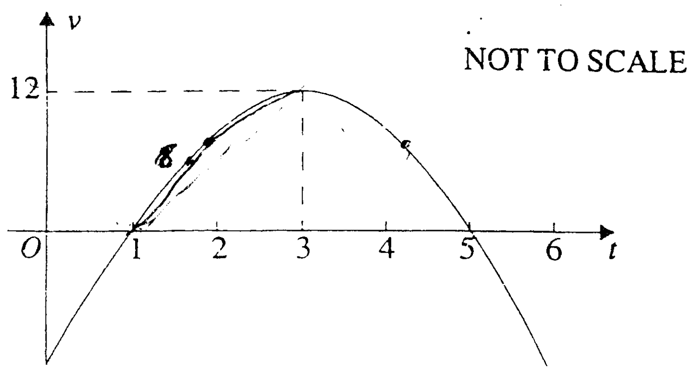
(vi) Sketch the curve.

QUESTION 7.

Start a new page

(a) A particle is moving in a straight line. 8

At time t seconds its velocity is v metres per second. The diagram shows the graph of v as a function of t.



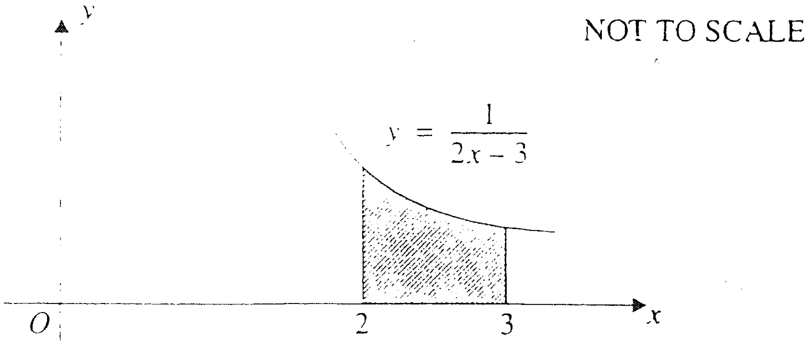
(i) At what times does the particle change direction?

(ii) Over what period of time does the particle have positive acceleration?

(iii) Use Simpson's rule with 3 function values to determine the distance travelled between $t = 1$ and $t = 5$.

(iv) If the equation of the above velocity graph is $v = -3t^2 + 18t - 15$, find the distance travelled in the first 6 seconds.

(b) 4



The diagram shows part of the graph of the function $y = \dfrac{1}{2x-3}$.

The shaded region is bounded by the curve, the x-axis, and the lines $x = 2$ and $x = 3$. The region is rotated around the x-axis to form a solid.

Find the exact volume of the solid in its simplest form.

QUESTION 8. **Start a new page.**

Marks

(a) Two dice are thrown and the difference of the numbers showing is noted. 3

(i) By using a table of outcomes, or otherwise, calculate the probability of getting a difference of four.

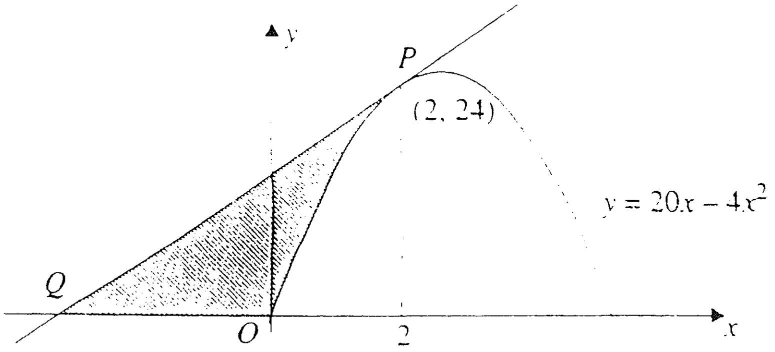
(ii) If two pairs of dice are thrown what is the probability they both give a difference of four?

(b) For the quadratic expression $mx^2 - 6x - 1$, 3

(i) find the discriminant

(ii) show that there are no values of m for which the expression is positive definite.

(c) 6



In the graph, a tangent is drawn at P (2, 24) on the parabola $y = 20x - 4x^2$. The tangent intersects the x-axis at Q.

(i) Show that the equation of the tangent is $4x - y + 16 = 0$.

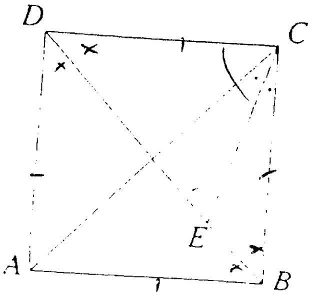
(ii) Find the coordinates of Q.

(iii) Find the area of the shaded region POQ.

QUESTION 9. **Start a new page**

(a) If the first three terms of a geometric series are $\frac{a+b}{a-b} + m + \frac{a-b}{a+b}$, find the possible values of m.

(b)

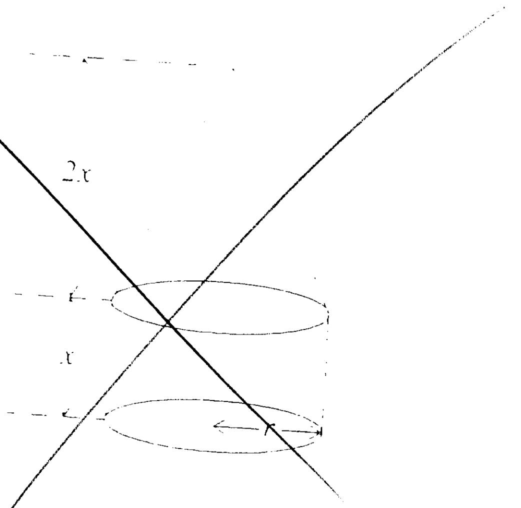


In the diagram $ABCD$ is a square with diagonals AC and BD. The point E lies on DB, and the interval CE bisects $\angle ACB$.

(i) Prove that $\angle DCE = \angle DEC$.

(ii) Hence show $DE = DA$.

(c) A grain silo has a cylindrical shaped wall and a cone shaped roof as in the diagram. Let the radius of the base of the silo be r metres, the height of the cylinder be x metres and the height of the cone be $2x$ metres.



(i) Show that if the length of the slant side of the cone is 20 metres, then $r^2 = 20^2 - 4x^2$.

(ii) Show that the volume, V, of the silo is given by $V = \frac{20}{3}\pi(100x - x^3)$.

(iii) Find the exact height of the silo so that it holds the maximum amount of grain.

QUESTION 10. **Start a new page**

Marks

(a) If $9^{2n-7} = 27^{2n-5}$, find n. **2**

(b) A sum of \$2000 is invested into a retirement account and interest is compounded, at a rate of 8% per annum, every six months. 6

(i) If no further deposit is made into the account, how much is in the account at the end of 20 years?

(ii) Now suppose that at the beginning of the second year, and at the beginning of each subsequent year, a further \$500 is deposited into the account.

How much will be in the account at the end of 20 years?

(c) Consider the function $f(x) = \pi x - \cos \pi x$. 4

(i) Find $f''\left(\frac{1}{2}\right)$.

(ii) Show that the point $\left(\frac{1}{2}, \frac{\pi}{2}\right)$ is a point of inflexion on the graph of $y = f(x)$.

2U Trial - HGHS

Question 1

a) $16x^2-9$ (1)

$=(4x-3)(4x+3)$

b) $\frac{3\pi}{5}\times\frac{180}{\pi}=108°$ (1)

c) $\int 3-2x^2$

$=3x-\frac{2x^3}{3}+C$ (2)

d) $\left.\begin{array}{l}x+y=285\\ x+1\cdot1y=296-40\end{array}\right\}\begin{array}{l}1\\2\end{array}$

$y=11\cdot4$

$y=\$114$

$x=\$171$

OR

10% = \$11-40 of taxed component

∴ \$114-00 non-taxed

\$171

%

$\frac{171}{285}=60\%$ exempt. (2)

e) $f(x)=1-x^3$

$f(a)=1-a^3=65$

$-a^3=64$

$a^3=64$

$a=-4$ (2)

f) $\frac{1}{2\sqrt{3}-1}\cdot\frac{2\sqrt{3}+1}{2\sqrt{3}+1}$

$=\frac{2\sqrt{3}+1}{12-1}$

$=\frac{2\sqrt{3}+1}{11}=\frac{1}{11}+\frac{2}{11}\sqrt{3}$

$\therefore a=\frac{1}{11}\quad b=\frac{2}{11}$ (2)

g) $\frac{a}{b}=\frac{4}{3}$ $\quad a+b=28$ (2)

$a=\frac{4b}{3}$ (1)

$\therefore \frac{4b}{3}+b=28$

$4b+3b=84$

$7b=84$ (2)

Question 2

if -2 $\frac{1}{2}$

a) i) $(x^2+5)^{-3}$

$\frac{dy}{dx}=-3(x^2+5)^{-4}2x$

$=\frac{-6x}{(x^2+5)^4}$ ①

ii) $y=x^2\tan x$ ①

$\frac{dy}{dx}=x^2\sec^2x+\tan x.2x.$

$=\frac{x^2}{\cos^2x}+\frac{2x\sin x}{\cos x}$

$=\frac{x}{\cos x}\left(\frac{x}{\cos x}+2\sin x\right)$

iv) $y=\frac{\log_e x}{x}$

$\frac{dy}{dx}=\frac{x.\frac{1}{x}-\ln x.1}{x^2}$

$=\frac{1-\ln x}{x^2}$ ②

b) $\int\frac{x^2}{x^3-2}dx$

$=\frac{1}{3}\int\frac{3x^2}{x^3-2}dx$

$=\frac{1}{3}\ln(x^3-2)+C$ ②

c) i) $\int_0^{\frac{\pi}{4}}\cos 2x\,dx$

$=\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$

$=\frac{1}{2}\sin\frac{\pi}{2}-\frac{1}{2}\sin 0$

$=\frac{1}{2}$ ①

ii) $\int_0^{\ln 3}e^x dx$

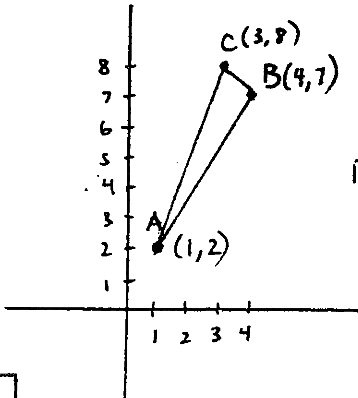
$=\left[e^x\right]_0^{\ln 3}$

$=e^{\ln 3}-e^0$

$=3-1$

$=2$ ①

d)



i) $AB=\sqrt{5^2+3}$

$=\sqrt{34}$

ii) $\frac{y-7}{x-4}=\frac{2-7}{1-4}$

$\frac{y-7}{x-4}=\frac{-5}{-3}$

$-3y+21=-5x+2$

$5x-3y+1=0$ ①

iii) $(3,8)$ $\quad 5x-3y+1=0$

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}=\frac{|5.3-3.8+1|}{\sqrt{25+9}}$

$=\frac{|-8|}{\sqrt{34}}=\frac{8\sqrt{34}}{34}=$

iv) $A=\frac{1}{2}\times\sqrt{34}\times\frac{4\sqrt{34}}{17}=4u^2$ ①

Question 3

a) $5x^2-2x-3=0$

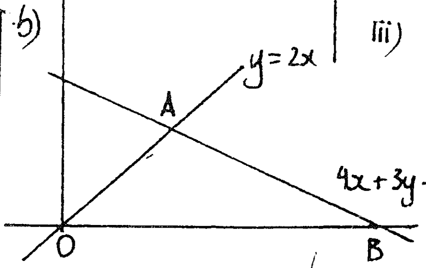
i) $\alpha+\beta=\frac{2}{5}$ ①

ii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{\frac{2}{5}}{\frac{-3}{5}}$

$=-\frac{2}{3}$ ②

b)



i) $\left.\begin{array}{l}y=2x\\ 4x+3y-40=0\end{array}\right\}$ (1) (2)

$4x+6x-40=0$

$10x-40=0$

$10x=40$

$x=4\quad y=8$

$A(4,8)$

ii) $4x-3y-40=0$

$y=0\quad 4x=40\quad x=10$

$B(10,0)$

AB $\quad m=\frac{8}{-6}=-\frac{4}{3}$

$\tan\theta=-\frac{4}{3}$ (2

$\theta=127°$ (with + direction of x-ax

iii) $y>0$

$y<2x$

$4x+3y-40$ (2

c) i) $50°=\frac{5\pi}{18}$ ①

ii) $A=\frac{1}{2}r^2(\theta-\sin\theta)$

$=\frac{1}{2}.100.\left(\frac{5\pi}{18}-\sin\frac{5}{1}\right.$

$=5\cdot3\text{ cm}^2$ (2)

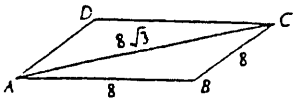
QUESTION 4.

(a) $\dfrac{\log x^3 - \log x}{\log x^2 + \log x} = \dfrac{3\log x - \log x}{2\log x + \log x}$

$= \dfrac{2\log x}{3\log x}$

$= \dfrac{2}{3}$ Aw. 2

(b)



(i) $\cos B = \dfrac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$

$= \dfrac{8^2 + 8^2 - (8\sqrt{3})^2}{2 \times 8 \times 8}$

$= \dfrac{64 + 64 - 192}{128}$

$= -\dfrac{1}{2}$

$\therefore \angle ABC = 120°$ Aw. 2

(ii) Surface area $= 6 \times \text{area}(ABCD)$

$= 6 \times \left(2 \times \frac{1}{2}(AB)(BC)\sin 120°\right)$

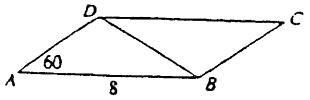
$= 6 \times (8 \times 8 \times \sin 120°)$

$= 6 \times 8 \times 8 \times \dfrac{\sqrt{3}}{2}$

$= 192\sqrt{3}$ cm^2 Aw. 3

or 333 cm^2 (3 sig. fig.)

) Alternate Solution:



Triangle ABD is equilateral

$\therefore BD = 8$ cms.

$\therefore$ surface area $= 6 \times \left(\frac{1}{2} \times AC \times BD\right)$

$= 6 \times \left(\frac{1}{2} \times 8\sqrt{3} \times 8\right)$

$= 192\sqrt{3}$ cm^2

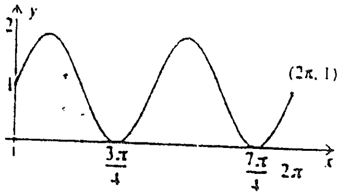
c) $y = 1 + \sin 2x$

(i) Period $= \dfrac{2\pi}{2}$

$= \pi$ units

Range: $0 \leq y \leq 2$ Aw. 2

(ii)



Aw. 2

(iii) For the graph $y = 1 + \sin 2x$ in part (ii) there are two x intercepts for $0 \leq x \leq 2\pi$.

When $y = 0$: $\sin 2x + 1 = 0$

$\therefore \sin 2x = -1$ has two solutions. Aw. 1

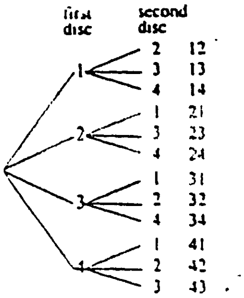
QUESTION 5.

(a) $\sec\dfrac{\pi}{4} + \cot\dfrac{\pi}{6} = \dfrac{1}{\cos\frac{\pi}{4}} + \dfrac{1}{\tan\frac{\pi}{6}}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}} + \dfrac{1}{\frac{1}{\sqrt{3}}}$

$= \sqrt{2} + \sqrt{3}$

(b) (i)



Number of 2-digit numbers = 12. Aw. 2

(ii) P(21 is formed) $= \dfrac{1}{12}$ Aw. 1

(iii) Numbers divisible by 3 are 12, 21, 24, 42

P(number is divisible by 3) $= \dfrac{4}{12}$

$= \dfrac{1}{3}$ Aw. 1

(c) (i) $d = T_2 - T_1$

$= 41 - 47$

$\therefore d = -6$ Aw. 1

(ii) $T_n < 0$

$a + (n-1)d < 0$

$47 + (n-1)(-6) < 0$

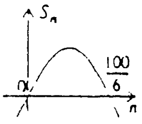
$47 - 6n + 6 < 0$

$6n > 53$

$n > 8\frac{5}{6}$

$\therefore$ smallest n is 9. Aw. 2

(iii) $S_n < 0$



$\frac{n}{2}[2a + (n-1)d] < 0$

$\frac{n}{2}[94 - 6(n-1)] < 0$

$n(100 - 6n) < 0$

$n < 0$ No solution

$n > \dfrac{100}{6}$

$\therefore$ smallest n is 17. Aw. 3

QUESTION 6.

(a) $V = 375e^{0.05t}$

(i) $V = 375 \times e^{0.05 \times 10}$

$= 618.27$

$\therefore$ value is \$618 (to the nearest dollar). Aw. 1

(ii) $1000 = 375e^{0.05t}$

$\dfrac{1000}{375} = e^{0.05t}$

$\ln\left(\dfrac{1000}{375}\right) = 0.05t$

$t = 19.616\ldots$

$t = 19.6$ years, correct to 1 dec. place Aw. 2

(b) $y = e^{2x}(1 - x)$

(i) To find y-intercept, let $x = 0$

$y = e^{2 \times 0}(1 - 0)$

$= 1 \times 1$

$= 1$

$\therefore y$ intercept is 1. Aw. 1

(ii) To find where curve crosses x-axis, let $y = 0$

$e^{2x}(1 - x) = 0$

Now $e^{2x} > 0$ for all x

$\therefore 1 - x = 0$

$x = 1$

$\therefore$ crosses x-axis at $x = 1$. Aw. 1

(iii) $\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{dy}{dx} = e^{2x} \times (-1) + (1 - x) \times 2e^{2x}$

$= -e^{2x} + 2e^{2x} - 2xe^{2x}$

$= e^{2x} - 2xe^{2x}$

$= e^{2x}(1 - 2x)$

To find stationary point, let $\dfrac{dy}{dx} = 0$.

$e^{2x}(1 - 2x) = 0$

$1 - 2x = 0$ (N.B. $e^{2x} \neq 0$)

$x = \dfrac{1}{2}$

When $x = \dfrac{1}{2}$, $y = \dfrac{e}{2}$

$\therefore$ stationary point is $\left(\frac{1}{2}, \frac{e}{2}\right)$.

Now at $x = 0$, $\dfrac{dy}{dx} = e^0 \times (1 - 0)$

$= 1$

> 0

and at $x = 1$, $\dfrac{dy}{dx} = e^2(1 - 2 \times 1)$

$= -e^2 < 0$

$\therefore \left(\frac{1}{2}, \frac{e}{2}\right)$ is a maximum turning point (/ ‾‾ \ ).

Aw. 4

(iv) $y = e^{2x}(1 - x)$

As $x \to \infty$, $e^{2x} \to \infty$ and $1 - x < 0$.

$\therefore e^{2x}(1 - x) \to -\infty$ Aw. 1

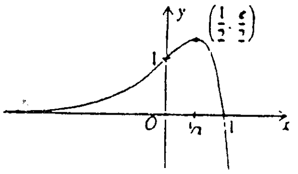
That is, the curve approaches $-\infty$.

(v) As $x \to -\infty$, $e^{2x} \to 0$ and $1 - x > 0$.

$\therefore e^{2x}(1 - x) \to 0$. Aw. 1

That is, the curve approaches the x-axis.

(vi)



Aw. 1

QUESTION 7.

(a) (i) Particle changes direction at $t = 1$ and $t = 5$ seconds. Aw.

(ii) Particle has positive acceleration for $0 < t < 3$. Aw. 1

(iii) Simpson's Rule:

Distance $= \dfrac{b-a}{6}\left(f(a) + 4f\left(\dfrac{a+b}{2}\right) + f(b)\right)$

$= \dfrac{5-1}{6}(0 + 4 \times 12 + 0)$

$= \dfrac{4}{6} \times 48$

$= 32$ m Aw. 2

(iv) Distance travelled in first second

$= \left|\int_0^1 (-3t^2 + 18t - 15)\ dt\right|$

$= \left|[-t^3 + 9t^2 - 15t]_0^1\right|$

$= |(-1 + 9 - 15) - 0|$

$= 7$ m

Distance travelled between $t = 1$ and $t = 5$ is 32 m from (i).

[Note: Simpson's rule gives exact area under parabolic curves]

Distance travelled between $t = 5$ and $t = 6$ is 7 m (by symmetry)

$\therefore$ total distance $= 7 + 32 + 7$

$= 46$ m Aw. 4

Alternate Solution:

Distance

$$= \left|\int_0^1 (-3t^2 + 18t - 15)dt\right| + \int_1^5 (-3t^2 + 18t - 15)dt$$

$$+ \left|\int_5^6 (-3t^2 + 18t - 15)dt\right|$$

$$= \left|[-t^3 + 9t^2 - 15t]_0^1\right| + [-t^3 + 9t^2 - 15t]_1^5$$

$$+ \left|[-t^3 + 9t^2 - 15t]_5^6\right|$$

$$= |(-1 + 9 - 15) - 0|$$

$$+ [(-125 + 225 - 75) - (-1 + 9 - 15)]$$

$$+ |(-216 + 324 - 90) - (-125 + 225 - 75)|$$

$$= 7 + 32 + 7$$

$$= 46 \text{ m}$$

(b) Volume $= \pi\int_c^d y^2 dx$

$$= \pi\int_2^3 \left(\frac{1}{2x-3}\right)^2 dx$$ Aw. 1

$$= \pi\int_2^3 (2x-3)^{-2} dx$$

$$= \pi\left[-\frac{1}{2}(2x-3)^{-1}\right]_2^3$$ Aw. 1

$$= \pi\left[\frac{-1}{2(2x-3)}\right]_2^3$$

$$= \pi\left[\frac{-1}{2(6-3)} - \frac{-1}{2(4-3)}\right]$$ Aw. 1

$$= \pi\left[-\frac{1}{6} + \frac{1}{2}\right]$$

$$= \frac{\pi}{3} \text{ cubic units}$$ Aw. 1

QUESTION 8.

(a) (i) Table of differences.

Die 1

		1	2	3	4	5	6
Die 2	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

P(difference of 4) $= \frac{4}{36}$

$= \frac{1}{9}$ Aw. 2

(ii) P(both give difference of 4) $= \frac{1}{9} \times \frac{1}{9}$

$= \frac{1}{81}$ Aw. 1

(b) (i) $mx^2 - 6x - 1$

Discriminant $= \Delta$

$= b^2 - 4ac$

$= (-6)^2 - 4 \times m \times (-1)$

$= 36 + 4m$ Aw. 1

(ii) For expression to be positive definite, we require $m > 0$ and $\Delta < 0$. Aw. 1

$\Delta = 36 + 4m < 0$

$4m < -36$

$m < -9$

$\therefore$ if $\Delta < 0$, then $m < 0$ Aw. 1

(c) (i) $y = 20x - 4x^2$

$\frac{dy}{dx} = 20 - 8x$

At $x = 2$, $\frac{dy}{dx} = 20 - 8 \times 2$

$= 4$

Equation of tangent given by

$y - y_1 = m(x - x_1)$

$y - 24 = 4(x - 2)$

$y - 24 = 4x - 8$

$\therefore 4x - y + 16 = 0$ Aw. 1

QUESTION 9.

(a) (i) Geometric series is $\frac{a+b}{a-b} + m + \frac{a-b}{a+b}$.

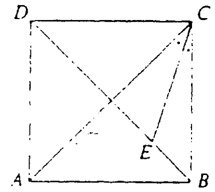
$$\therefore \frac{m}{\frac{a+b}{a-b}} = \frac{\frac{a-b}{a+b}}{m}$$

$$\therefore m^2 = \frac{a+b}{a-b} \times \frac{a-b}{a+b}$$

$$= 1$$

$$\therefore m = \pm 1$$ Aw. 2

(b)



(i) Aim: to prove $\angle DCE = \angle DEC$

$ABCD$ is a square (given)

$\therefore \angle DCA = \angle ACB$

$= 45°$ (Diagonal AC bisects $\angle DCB$)

$\therefore \angle ACE = 22\frac{1}{2}°$ (Given CE bisects $\angle ACB$)

$\therefore \angle DCE = 77\frac{1}{2}°$ ($\angle DCA + \angle ACE$)

$\angle DEC = \angle DBC + \angle BCE$ (in any triangle, exterior angle = sum of interior opposites)

$= 45° + 22\frac{1}{2}°$ (same reason as above)

$= 77\frac{1}{2}°$

$\therefore \angle DCE = \angle DEC$ Aw. 3

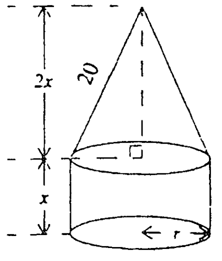
(ii) From (i), ΔDEC is isosceles

$\therefore DC = DE$ (sides opposite equal $\angle$s DCE, DEC)

But $DC = DA$ (sides of square) Aw. 1

$\therefore DE = DA$

(c)



(i) From diagram

$r^2 + (2x)^2 = 20^2$

$r^2 + 4x^2 = 20^2$

$r^2 = 20^2 - 4x^2$ Aw. 1

∴ volume of silo: $V = \pi r^2 x + \frac{1}{3}\pi r^2(2x)$

$= \frac{5}{3}\pi r^2 x$

$= \frac{5}{3}\pi x(400 - 4x^2)$

$= \frac{20}{3}\pi(100x - x^3)$

Aw. 1

(iii) $\frac{dV}{dx} = \frac{20}{3}\pi(100 - 3x^2)$

$\frac{d^2V}{dx^2} = \frac{20}{3}\pi(-6x)$

$= -40\pi x$

To find stationary point, let $\frac{dV}{dx} = 0$.

$\frac{20\pi}{3}(100 - 3x^2) = 0$

$3x^2 = 100$

$x^2 = \frac{100}{3}$

$\therefore x = \frac{10}{\sqrt{3}} \quad (x > 0)$

$\frac{d^2V}{dx^2} < 0$ for $x = +\frac{10}{\sqrt{3}}$

∴ maximum volume occurs where $x = \frac{10}{\sqrt{3}}$

∴ required height $= 3 \times \frac{10}{\sqrt{3}}$

$= 10\sqrt{3}$ metres Aw. 4

QUESTION 10.

(a) $9^{2n-7} = 27^{2n-5}$

$(3^2)^{2n-7} = (3^3)^{2n-5}$

$3^{4n-14} = 3^{6n-15}$

$\therefore 4n - 14 = 6n - 15$

$1 = 2n$

$n = \frac{1}{2}$ Aw. 2

(b) (i) $A = P\left(1 + \frac{r}{100}\right)^n$

$= 2000\left(1 + \frac{4}{100}\right)^{40}$ Aw. 1

$= 2000(1.04)^{40}$

$= \$9602.04$ Aw. 1

(ii) Amount $= 500 \times 1.04^{38} + 500 \times 1.04^{36} + 500 \times 1.04^{34} + \ldots + 500 \times 1.04^2$ Aw. 1

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\therefore$ amount $= \frac{500 \times 1.04^2((1.04^2)^{19} - 1)}{(1.04)^2 - 1}$ Aw.1

$= \$22790.57$ Aw. 1

∴ amount in account at end of 20 years

$= \$9602.04 + \$22790.57$

$= \$32392.61$ Aw. 1

$f''(0.4) = \pi^2\cos(0.4\pi)$

$\approx 3.049\ldots$

> 0

$f''(0.6) = \pi^2\cos(0.6\pi)$

$\approx -3.049\ldots$

< 0

$\therefore \left(\frac{1}{2}, \frac{\pi}{2}\right)$ is a point of inflection on $y = f(x)$.

(c) $f(x) = \pi x - \cos(\pi x)$

(i) $f'(x) = \pi + \pi\sin(\pi x)$

$f''(x) = \pi^2\cos(\pi x)$

$f''\left(\frac{1}{2}\right) = \pi^2\cos\left(\frac{\pi}{2}\right)$

$= 0$ Aw. 1

(ii) $f\left(\frac{1}{2}\right) = \frac{\pi}{2} - \cos\frac{\pi}{2}$

$= \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$

$\therefore \left(\frac{1}{2}, \frac{\pi}{2}\right)$ lies on $y = f(x)$. Aw. 1

From (i), $f''\left(\frac{1}{2}\right) = 0$

If δ is some small positive value

$f''\left(\frac{1}{2} - \delta\right) = \pi^2\cos\left[\pi\left(\frac{1}{2} - \delta\right)\right]$

$= \pi^2\cos\left(\frac{\pi}{2} - \pi\delta\right)$

$> 0 \quad \left(\frac{\pi}{2} - \pi\delta \text{ is in 1st quadrant}\right)$

Aw. 1

$f''\left(\frac{1}{2} + \delta\right) = \pi^2\cos\left[\pi\left(\frac{1}{2} + \delta\right)\right]$

$= \pi^2\cos\left(\frac{\pi}{2} + \pi\delta\right)$

$< 0 \quad \left(\frac{\pi}{2} + \pi\delta \text{ is in 2nd quadrant}\right)$

∴ concavity changes about $x = \frac{1}{2}$. Aw. 1

$\therefore \left(\frac{1}{2}, \frac{\pi}{2}\right)$ is a point of inflection on $y = f(x)$.

(ii) To find Q, let $y = 0$ in equation of tangent.

$4x - 0 + 16 = 0$

$x = -4$

∴ coordinates of Q are $(-4, 0)$ Aw. 1

(iii) Area of triangle $= \frac{1}{2} \times 6$

$= 72$ squ. ts

Area under curve $= \int_0^2 (20x - 4x^2)\ dx$

$= \left[10x^2 - \frac{4}{3}x^3\right]_0^2$

$= \left(40 - \frac{32}{3}\right) - 0$

$= 29\frac{1}{3}$ square units

∴ area of region $POQ = 72 - 29\frac{1}{3}$

$= 42\frac{2}{3}$ square units

Aw. 4