HORNSBY GIRLS HIGH SCHOOL

by mendavy

MATHEMATICS

YEAR 12 TRIAL EXAMINATION

2001

Time Allowed – 3 hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

a) Attempt all 10 questions.
b) Start each question on a new page.
c) All questions are of equal value.
d) All necessary working should be shown. Marks may be deducted for careless or badly arranged work.
e) Board-approved calculators may be used.

Question 1

a) Evaluate $\left[\dfrac{\sqrt{3.12+6.9}}{5.03-2.9}\right]^3$ correct to two decimal places **1**

b) Evaluate $2|-2|^2 - 2^0$ **1**

c) Rationalise the denominator of $\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ **2**

d) Express $3x^2 + 27x + 60$ as the product of three factors **2**

e) Solve $6-(x-4)=x$ **2**

f) The value of a new car decreased by 12% or \$1500 in one year. What was the original value of the car ? **2**

g) If $v^2 = u^2 + 2as$ find all possible values of u when $v = 35$, $a = 9.8$ and $s = 25$ (correct one decimal place). **2**

Question 2 (Start a new page)

(a) Differentiate 5

i) $8x^5 - 7x^{-5}$

ii) $\sin 5x$

iii) $\dfrac{2x}{\log_e 2x}$

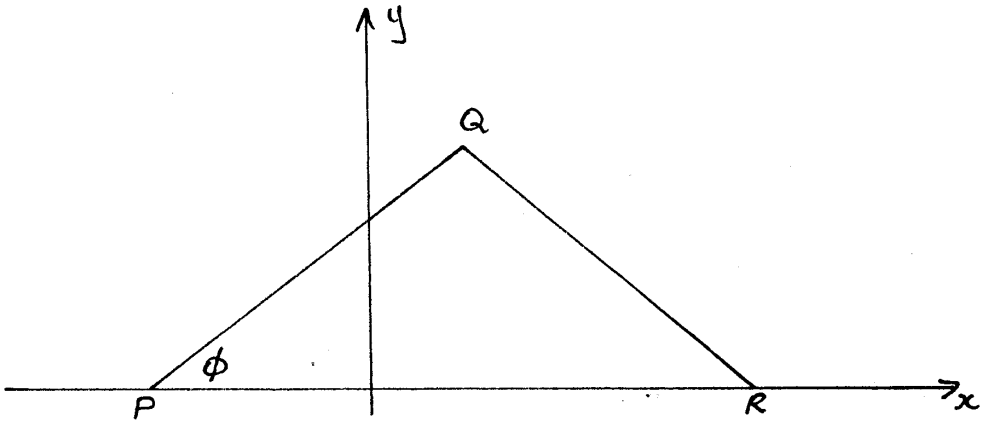
(b) Evaluate 5

i) $\int_0^5 4e^{2x}dx$

ii) $\int_{-1}^{1} (2x+7)^4 dx$

(c) Find $\int \frac{5y}{y^2+8} dy$ 2

Question 3 (Start a new page)



In the diagram P, Q and R have coordinates (-5,0), (1,7) and (7,0) and $\angle QPR = \phi$.

a) Find the equation of the line PQ in general form 2

b) Find the mid-point, M, of the interval QR 1

c) Find the distance PQ 2

d) Find the perpendicular distance from the point M to the line passing through the points P and Q 2

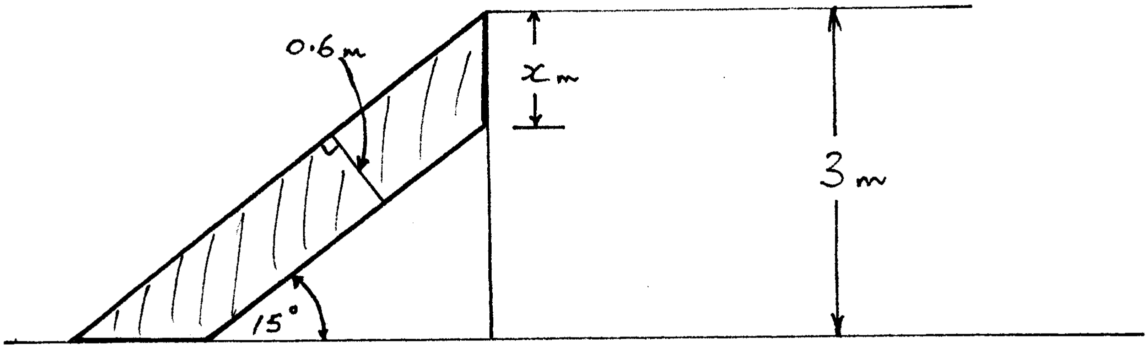
e) Find the exact area of the triangle PQM 2

f) Find the value of $\tan\phi$ 2

g) Prove that the point (-2,4) does not lie on the interval PQ 1

Question 4 (Start a new page)

A) The diagram shows a ramp, inclined at 15 degrees to the horizontal, that a builder is to build to allow cars to drive up a level, 3 metres high, in a car park. The cross-section of the ramp is in the shape of a trapezium and has been shaded on the diagram. The thickness of the ramp is 0.6 metres.



a) calculate: **4**

i) The value of, x , on the diagram (correct to 3 decimal places)

ii) Show that the area of the cross-section of the ramp is 6.2 m^2 (correct to 1 decimal place)

b) The ramp is 5 metres wide and is to be made of concrete. What volume of concrete will be used to make this ramp ? **1**

B) The population of a small country town grows slowly at a rate proportional to its current population. The population exactly two years ago was 10 000 and is now 10 200.

Find

a) The growth constant of the population **2**

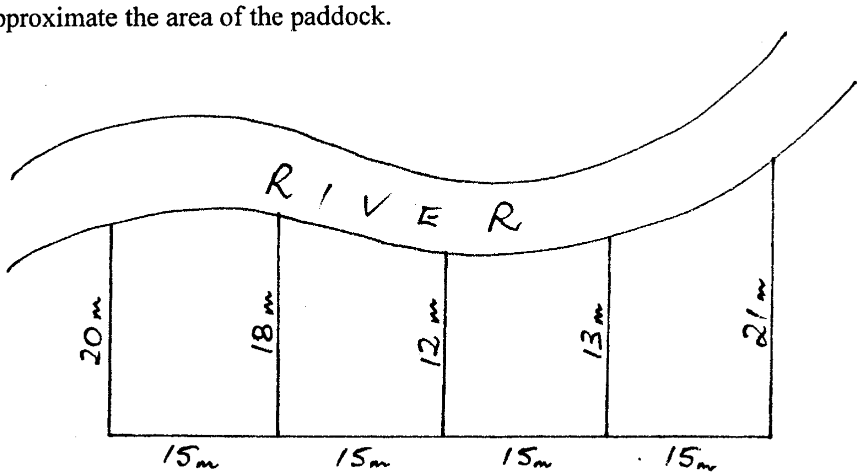
b) The number of years, from now, that it will take the population to exceed 11 500 **2**

c) The population of the town in 5 years from today **1**

C) Solve $8^x = 4^{x-1}$ **2**

Question 5 (Start a new page)

a) The diagram below (not drawn to scale) shows a paddock with one side bounded by a river. Use Simpson's Rule with the five function values shown on the diagram to approximate the area of the paddock. **3**



b) When Jill was born her mother deposited \$180 into a Trust Account earning 12% p.a. interest compounded annually. She continued to deposit \$180 into this account each time Jill had a birthday. The last payment was made on Jill's sixteenth birthday. Calculate the total amount in the account on Jill's twenty-fifth birthday. **4**

c) Consider this arithmetic series $3 + 8 + 13 + 18 + \dots\dots\dots + 488$

i) How many terms are in this series **1**

ii) Find the sum of all the terms in this series **2**

d) For a particular series the sum to n terms is given by $S_n = 2^n + n^2$.
What would be the tenth term of this series ? **2**

Question 6 (Start a new page)

a) A particle starts from O and moves along a straight line so that after t seconds its distance from O is x cm, where

$$x = 6t - \frac{t^3}{2}.$$

i) After how many seconds does it return to O and what is its velocity at that time ? **2**

ii) What is its distance from O when its velocity is zero and what is its acceleration here ? **2**

iii) What is its average velocity during the first two seconds of the motion ? **2**

b) The continuous curve corresponding to the function $y = f(x)$ has the following properties in the closed interval $a \le x \le b$

$$f(x) > 0, \quad f'(x) < 0, \quad f''(x) > 0$$

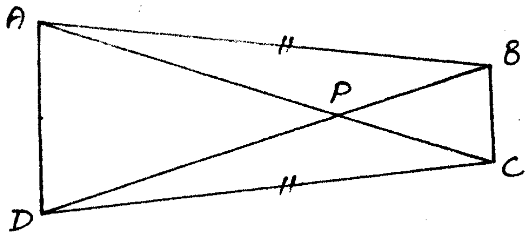
i) Sketch neatly a curve satisfying these conditions. **2**

ii) State the least value of $f(x)$ in this interval **1**

c) Find the stationary points and any points of inflexion of $y = x^3 - x^2 - x - 1$. **3**

Question 7 (Start a new page)

a) ABCD is a quadrilateral in which AB = DC and $\angle BAC = \angle BDC$. **5**
P is the point of intersection of the diagonals.



Prove that:

i) $\Delta APB \equiv \Delta DPC$

ii) ΔPBC is isosceles

b) A parabola has equation $y^2 - 6y + 25 = 8x$. Express this in the form **7**

$(y-p)^2 = 4a(x-q)$ and hence find:

i) The coordinates of its vertex

ii) The equation of its axis of symmetry

iii) Its focal length

iv) The coordinates of its focus

v) The equation of its directrix

Question 8 (Start a new page)

a) A athlete knows that she has a 20% chance of winning the 100 metre sprint event and a 30% chance of winning the 200 metres sprint. If she competes in both events at an athletics carnival what is the probability that she will: **4**

i) Win both events

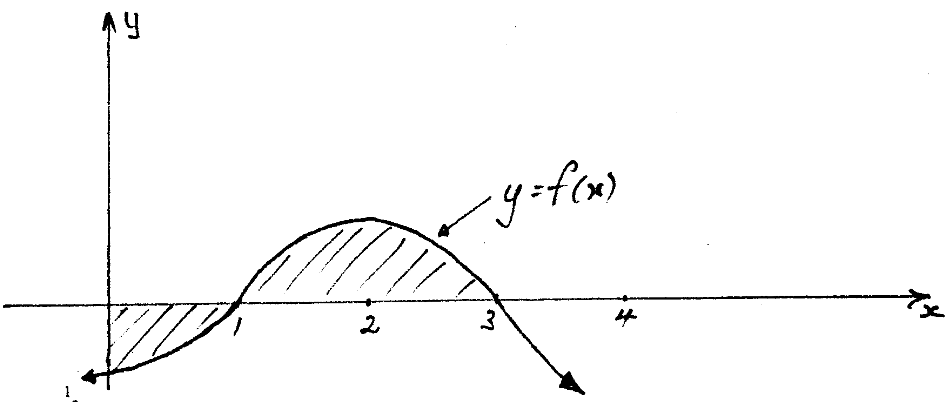
ii) Not win both events

iii) Win one event only

iv) Win at least one event

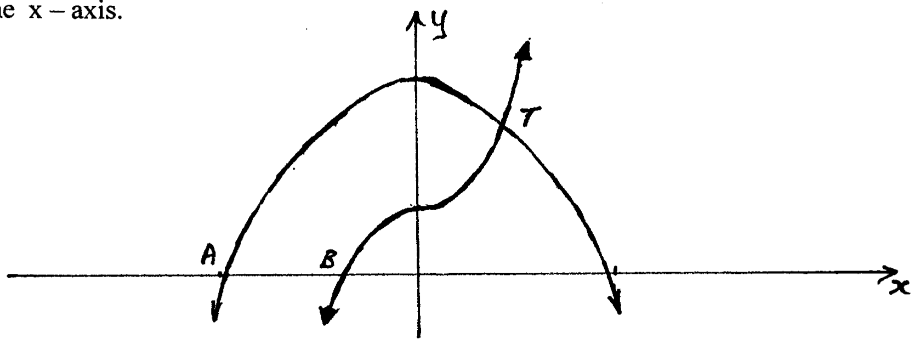
b) In the diagram below $\int_1^3 f(x)dx = 5$ and the area of the shaded region is 7 units^2. **1**

3



Evaluate $\int_0^1 f(x)dx$

c) The diagram below shows the shaded region enclosed by the curves $y = 3 - x^2$, $y = x^3 + 1$ and the x – axis.



i) Find the coordinates of the points A and B. **2**

ii) Show that the point (T) of intersection between the curves is (1,2) **2**

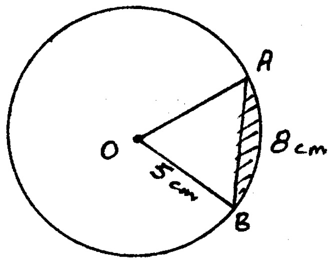
iii) Hence find the area of the shaded region in exact form. **3**

Question 9 (Start a new page)

a) In the circle with center O, arc AB is 8cm and the radius OB is 5cm. Find: **3**

i) The size of angle AOB, giving your answer in radians.

ii) The area of the shaded segment, giving your answer correct to 2 decimal places.



b) The area under the curve $y = 1 - e^{-x}$, above the x– axis and between $x = 0$ and $x = 1$, is rotated about the x – axis. Prove that the volume of the generated solid is **3**

$$\frac{\pi}{2}\left(4e^{-1} - e^{-2} - 1\right) \text{ cubic units}$$

c) Sharon was driving a car in an off-road rally competition. From the start, S, Sharon drove 55 km due east to, A. At A, she proceeded on a bearing of 055^0 for 100 km to B. At B, she changed course to a bearing of 130^0 and continued in this direction until she reached the finish at C. (C is due east of A).

i) Draw a diagram representing all this information on your answer sheet. **1**

ii) Show that angle ACB = 40^0. **2**

iii) Find the distance from B to C. Give your answer to the nearest kilometre. **2**

iv) It took Sharon 24 minutes to travel from the start to the finish. What was her average speed in km/h ? **1**

Question 10 (Start a new page)

a) A mechanic borrows \$P to buy new equipment for his business. The interest is compounded monthly at the rate of 18% p.a. The mechanic intends to repay the loan by making repayments of \$M per month (at the end of each month).

i) Write an expression for the amount owed by the mechanic at the end of the first month ? **1**

ii) Write an expression for the amount owed at the end of N months. **2**

iii) If the mechanic had borrowed \$40 000 calculate the amount of the monthly repayment (\$M) if he wishes to repay the loan in 5 years. **3**

b) The mass, m grams, of a raindrop falling for, t, seconds in a humid cloud, is increasing at a rate

$$\frac{dm}{dt} \text{ where } \frac{dm}{dt} = \frac{1}{100}\left[t + \frac{t^2}{10}\right] \ gs^{-1}$$

i) If the initial mass of the raindrop is zero, what is the mass of the raindrop after 20 seconds ? **2**

ii) If the raindrop started as a smoke particle of mass 0.001 g, how much heavier would it be after 20 seconds than the raindrop in part (i) ? **1**

c) Observe that:

$$\begin{aligned} 1 &= 1 \\ 3x &= x + 2x \\ 5x^2 &= x^2 + 2x^2 + 2x^2 \\ 7x^3 &= x^3 + 2x^3 + 2x^3 + 2x^3 \\ 9x^4 &= x^4 + 2x^4 + 2x^4 + 2x^4 + 2x^4 \end{aligned}$$

By studying the above arrangement, or otherwise, find in simplest algebraic form, an expression for the limiting sum of the series

$$1 + 3x + 5x^2 + 7x^3 + 9x^4 + \dots\dots\dots + (2n-1)x^{n-1} + \dots\dots\dots$$ **3**

END OF PAPER

YEAR 12 TRIAL 2U 2001

1)
a) $3.28(21829)$

b) 7

c) $\frac{\sqrt5-\sqrt2}{\sqrt5+\sqrt2}\times\frac{\sqrt5-\sqrt2}{\sqrt5-\sqrt2}$

$= \frac{5-2\sqrt{10}+2}{3}$

$= \frac{7-2\sqrt{10}}{3}$

d) $3x^2+27x+60$

$= 3(x^2+9x+20)$

$= 3(x+5)(x+4)$

e) $6-(x-4)=x$

$6-x+4=x$

$10=2x$

$x=5$

f) $12\% = \$1500$

$1\% = \$125$

$100\% = \$12500$

g) $v^2=u^2+2as$

$35^2=u^2+2\times9.8\times25$

$u^2 = 735$

$u = \pm27.1(10883)$

2)
a) i) $40x^4+35x^{-6}$

ii) $5\cos5x$

iii) $\frac{\ln2x\cdot2-2x\cdot\frac{1}{x}}{(\ln2x)^2}$

$= \frac{2\ln2x-1}{(\ln2x)^2}$

or $= \frac{\ln4x^2-1}{(\ln2x)^2}$

b) i) $\left[2e^{2x}\right]_0^5$

$= 2e^{10}-2e^0$

$= 2e^{10}-2$ or $2(e^{10}-1)$

or 44050.932

ii) $\left[\frac{1}{10}(2x+7)^5\right]_{-1}^{1} = \frac{1}{10}(9^5-5^5)$

$= 5592.4$

c) $\int\frac{5y}{y^2+8}\,dy = \frac52\int\frac{2y}{y^2+8}\,dy$

$= \frac52\ln(y^2+8)+C$

3)
a) $\frac{y-7}{x-1} = \frac{0-7}{-5-1}$

$-6y+42=-7x+7$

$7x-6y+35=0$

b) $x=\frac{1+7}{2}$ $y=\frac{7+0}{2}$

$=4$ $=3\frac12$

$\therefore (4,3\frac12)$

c) $d^2=6^2+7^2$

$d^2=36+42$

$d^2=78$

$d=\sqrt{78}$

$=8.8(317609)$

d) $d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\frac{|7\times4-6\times3\frac12+35|}{\sqrt{7^2+6^2}}$

$=\frac{28-21+35}{\sqrt{78}}$

$=\frac{42}{\sqrt{78}}$

$=\frac{7\sqrt{78}}{13} = 4.7(555635)$

e) $A=\frac12\times\sqrt{78}\times\frac{42}{\sqrt{78}}$

$=21$

f) $\tan\theta=\frac76$

g) $7x-6y+35=0$

$7\times-2-6\times4+35=0$

$-14-24+35=0$

$-3\neq0$

$\therefore$ not on line.

4)
A) a) i) $\cos15^\circ=\frac{0.6}{x}$

$x=\frac{0.6}{\cos15}$

$=0.62(11657)$

ii) $\sin15^\circ=\frac{3}{H}$

$H=\frac{3}{\sin15}=11.5911$

$\sin15^\circ=\frac{2.38}{h}$

$h=\frac{2.38}{\sin15^\circ}=9.1956$

$\therefore A=\frac12(11.5911+9.1956)\times0.6$

$=10.39335\times0.6$

$=6.236$

$=6.2$

b) $V=6.2\times5$

$=31\ m^3$ $(31.0\to31.2)$

B) a) $P=P_0e^{kt}$

$10200=10000e^{2k}$

$e^{2k}=1.02$

$2k=\ln1.02$

$k=0.0099$

b) $11500=10200e^{0.0099t}$

$0.0099t=\ln\left(\frac{11500}{10200}\right)$

$t=12.1$ years

$\therefore$ 13 years.

c) $P=10200e^{5\times0.0099}$

$=10717.605$

$\therefore$ 10717 or 10718

c) $8^x=4^{x-1}$

$2^{3x}=2^{2x-2}$

$3x=2x-2$

$x=-2$

5)
a) $A=\frac{30}{6}\left[20+4\times18+12\right]+\frac{30}{6}\left[12+4\times13+21\right]$

$=5\times104+5\times85$

$=945\ (m^2)$

b) $A=180(1.12)^{25}+\dots\dots+180(1.12)^9$

$=180\left(\frac{1.12^9(1.12^{17}-1)}{1.12-1}\right)$

$=\$24400.49$

c) i) $a=3$ $d=5$

$T_n=a+(n-1)d$

$488=3+(n-1)5$

$485=5n-5$

$5n=490$

$n=98$

ii) $S_n=\frac n2(2a+(n-1)d)$

$S_{98}=\frac{98}{2}(2\times3+97\times5)$

$=24059$

d) $S_{10}=2^{10}+10^2 = 1124$

$S_9=2^9+9^2=593$

$\therefore T_{10}=1124-593$

$=531$

6) i) $x=6t-\frac{t^3}{2}$

$\frac{dx}{dt}=6-\frac{3t^2}{2}$

when $x=0$ $6t-\frac{t^3}{2}=0$

$12t-t^3=0$

$t(12-t^2)=0$

$\therefore t=0$ or $\sqrt{12}$

when $x=\sqrt{12}$ $V=6-\frac{3\times12}{2}$

$=-12$

$\therefore$ returns when $t=\sqrt{12}$ $v=-12$.

ii) $v = 6 - \frac{3t^2}{2}$

when $v = 0$, $\frac{3t^2}{2} = 6$

$3t^2 = 12$

$t^2 = 4$

$t = 2$

when $t = 2$, $x = 6\times 2 - \frac{2^3}{2}$

$= 8$

$a = -3t$

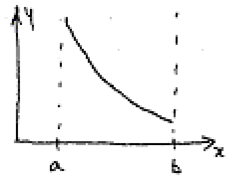
when $t = 2$, $a = -6$

iii) $S = \frac{D}{T}$

$S = \frac{8}{2}$

$= 4$ cm/s

b) i)



ii) $f(b)$

c) $y = x^3 - x^2 - x - 1$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

$\frac{d^2y}{dx^2} = 6x - 2$

stat pt when $\frac{dy}{dx} = 0$

i.e. $3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$3x = -1$ $\quad x = 1$

$x = -\frac{1}{3}$

when $x = 1$, $y = 1 - 1 - 1 - 1$

$= -2$

$x = -\frac{1}{3}$, $y = (-\frac{1}{3})^3 - (\frac{1}{3})^2 + \frac{1}{3} - 1$

$= -\frac{22}{27}$

pt inflexion when $\frac{d^2y}{dx^2} = 0$

i.e. $6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

when $x = \frac{1}{3}$, $y = (\frac{1}{3})^3 - (\frac{1}{3})^2 - \frac{1}{3} - 1$

$= -1\frac{11}{27}$

$\therefore$ stat pts $(1, -2)$ + $(-\frac{1}{3}, -\frac{22}{27})$

inflexion pt $(\frac{1}{3}, -1\frac{11}{27})$

7) a) i) In $\triangle$'s APB + DPC

AB = DC (given)

$\angle BAC = \angle BDC$ (given)

$\angle APB = \angle DPC$ (vert opp)

$\therefore \triangle APB \equiv \triangle DPC$ (AAS)

ii) In $\triangle$PBC

PB = PC corresponding sides of congruent $\triangle$s.

b) $y^2 - 6y + 25 = 8x$

$y^2 - 6y + 9 = 8x - 16$

$(y-3)^2 = 8(x-2)$

i) $(2, 3)$

ii) $y = 3$

iii) 2

iv) $(4, 3)$

v) $x = 0$

8) a) i) $\cdot 2 \times \cdot 3 = 0.06$ $\quad$ 6%

ii) $\cdot 8 \times \cdot 7 = 0.56$ $\quad$ 56%

iii) $\cdot 2 \times \cdot 7 + \cdot 8 \times \cdot 3 = \cdot 14 + \cdot 24$

$= 0.38$ $\quad$ 38%

iv) $1 - 0.56 = 0.44$ $\quad$ 44%

b) -2.

c) i) when $y = 0$

$3 - x^2 = 0$

$x = \pm\sqrt{3}$

$\therefore A = (-\sqrt{3}, 0)$

$x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

$\therefore B = (-1, 0)$

ii) $3 - x^2 = x^3 + 1$

$x^3 + x^2 - 2 = 0$

when $x = 1$

$1 + 1 - 2 = 0$

$0 = 0$

TRUE $\therefore x = 1$ is a soln.

when $x = 1$ $\quad y = 3 - 1^2$

$= 2$

$\therefore (1, 2)$ is pt T

iii) $A = \int_{-\sqrt{3}}^{1} (3 - x^2 - x^3 - 1)\,dx$

$= \int_{-\sqrt{3}}^{1} 2 - x^2 - x^3 \; dx$

$= \left[2x - \frac{x^3}{3} - \frac{x^4}{4}\right]_{-\sqrt{3}}^{1}$

$= (2 - \frac{1}{3} - \frac{1}{4}) - (-2\sqrt{3} + \frac{3\sqrt{3}}{3} - \frac{9}{4})$

$= 2 - \frac{1}{3} - \frac{1}{4} + \sqrt{3} + \frac{9}{4}$

$= \frac{11}{3} + \sqrt{3}$ or $3\frac{2}{3} + \sqrt{3}$

$= (5.3987175)$

9) a) i) $l = r\theta$

$8 = 5\theta$

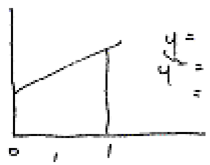
$\theta = \frac{8}{5}$

ii) $A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}\times 25\times\frac{8}{5} - \frac{1}{2}\times 25\times\sin\frac{8}{5}$

$= 7.51$ $\quad (7.50533)$

b)



$y = 1 - e^{-x}$

$y^2 = (1 - e^{-x})^2$

$= 1 - 2e^{-x} + e^{-2x}$

$V = \pi\int_0^1 y^2\,dx$

$= \pi\int_0^1 1 - 2e^{-x} + e^{-2x}\; dx$

$= \pi\left[x + 2e^{-x} - \frac{1}{2}e^{-2x}\right]_0^1$

$= \pi\left[1 + 2e^{-1} - \frac{1}{2}e^{-2} - 0 - 2 + \frac{1}{2}\right]$

$= \pi(-\frac{1}{2} - \frac{1}{2}e^{-2} + 2e^{-1})$

$= \frac{\pi}{2}(4e^{-1} - e^{-2} - 1)$

c) i)

ii) $\angle CBX = 130 - 90$

$= 40$

$\angle ACB = \angle CBX = 40$ (alt $\angle$'s BX ∥ AC)

iii) $\frac{BC}{\sin 35} = \frac{100}{\sin 40°}$

$BC = \frac{100\sin 35°}{\sin 40°}$

$= 89.(2\;32653)$

iv) $S = \frac{89 + 100 + 55}{24/60}$

$= 610$ (km/h)

10) a) i) $A_1 = 1.015P - M$

ii) $A_2 = 1.015^2P - 1.015M - M$

$A_3 = 1.015^3P - 1.015^2M - 1.015M - M$

$\vdots$

$A_n = 1.015^nP - M(1.015^{n-1} + 1.015^{n-2} + \dots + 1.015^1 + 1.015^0)$

iii) $1.015^{60} \times 40000 - M\left(\frac{1(1.015^{60} - 1)}{1.015 - 1}\right) = 0$

$M = \frac{1.015^{60} \times 40\,000 \times 0.015}{1.015^{60} - 1}$

$= \$1015.74 \quad (\$1015 - \$1016)$

b) i) $\frac{dm}{dt} = \frac{1}{100}\left[t + \frac{t^2}{10}\right]$

$m = \frac{1}{100}\left[\frac{t^2}{2} + \frac{t^3}{30}\right] + C$

when $t = 0$, $m = 0$ $\therefore C = 0$

$\therefore$ when $t = 20$

$m = \frac{1}{100}\left[\frac{400}{2} + \frac{8000}{30}\right]$

$= 4\frac{2}{3}$ g or 4.7 g or $4.\dot{6}$ g

ii) when $t = 0$, $M = 0.001$ $\therefore C = 0.001$

$\therefore M = \frac{1}{100}\left[\frac{t^2}{2} + \frac{t^3}{30}\right] + 0.001$

$\therefore$ 0.001 g

c) $S = \frac{1}{1-x} + \frac{2x}{1-x} + \frac{2x^2}{1-x} + \dots$

$S = \frac{1}{1-x}(1 + 2x + 2x^2 + \dots)$

$= \frac{1}{1-x}\left(1 + \frac{2x}{1-x}\right)$

$= \frac{1}{1-x} + \frac{2x}{(1-x)^2}$

$= \frac{1 - x + 2x}{(1-x)^2} = \frac{1+x}{(1-x)^2}$