HORNSBY GIRLS' HIGH SCHOOL

2010
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time- 5 minutes
- Working Time – 3 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (120)

- Attempt Questions 1– 10
- All questions are of equal value

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Total Marks – 120

Attempt Questions 1-10
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your student number and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Evaluate $\log_e 2 + \pi^2 - 1$ correct to two decimal places. **2**

(b) Express $\dfrac{1}{5-\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a, b are rational numbers. **2**

(c) Solve $\dfrac{3}{|x-1|} < 2$ and graph the solution on a number line. **2**

(d) Find the limiting sum of the geometric series **2**

$$6 - 4 + 2\frac{2}{3} - \ldots.$$

(e) Solve $27^{1-x} = 81^x \times 9^{-x}$. **2**

(f) Find the equation of the line that passes through the point $(-1, 3)$ and is perpendicular to the line joining $(-3, 6)$ and $(-1, 2)$. **2**

Question 2 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Find the derivative of $x^2 \cos x$. **2**

(b) Find $\int \frac{x^2+3}{x^4}dx$ **2**

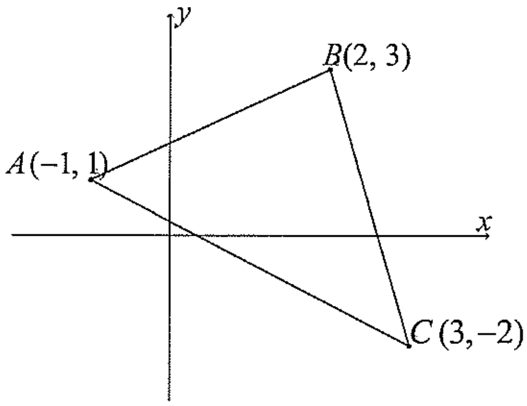
(c) (i) Find the derivative of $(10x+1)^4$ in simplest form. **2**

(ii) Hence evaluate $\int_0^1 20(10x+1)^3 dx$. **3**

(d) Find the equation of the normal to the curve $y = \frac{x^2}{e^x}$ at the point where $x = 0$. **3**

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The points $A(-1, 1)$, $B(2, 3)$ and $C(3, -2)$ form the vertices of a triangle as shown.



NOT TO SCALE

(i) Find the length of AC. **1**

(ii) Show that the equations of AC is $3x + 4y - 1 = 0$ **2**

(iii) Determine the shortest distance from B to AC. **2**

(iv) Calculate the area of ΔABC. **1**

(v) Find the angle that AC makes with the positive direction of the x-axis. **1**

(b) During the first week of January a salesperson made \$15 000 worth of sales. In the second week of January she made \$18 000 worth of sales and each week thereafter she continued to increase her sales by \$3 000.

(i) Write a formula for how much she made in sales in the n^{th} week. **1**

(ii) How much did she make in sales in the last week of December (Use 52 weeks in the year). **1**

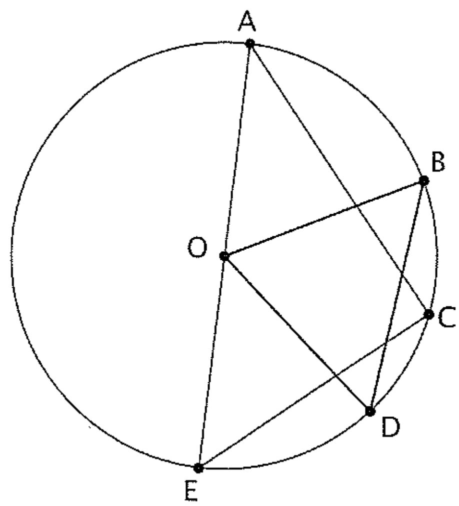
(iii) How much did she have in total sales for the year? **1**

(iv) During which week of the year did her total sales exceed the \$1 million? **2**

Question 4 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) If $\sin A = \dfrac{12}{13}$ and A is acute, find the exact value of $\cot A$. **2**

(b)



NOT TO SCALE

In the diagram above, AC has length $4\sqrt{3}$ cm, CE has length 4cm, and $\angle ACE = \dfrac{\pi}{2}$.

(i) Show that the length of the radius is $4cm$. **2**

(ii) The ratio of the area of sector BOD to the area of the circle is $1:8$. Find the area of the sector BOD. **2**

(iii) Hence, or otherwise, find the length of arc BD. **2**

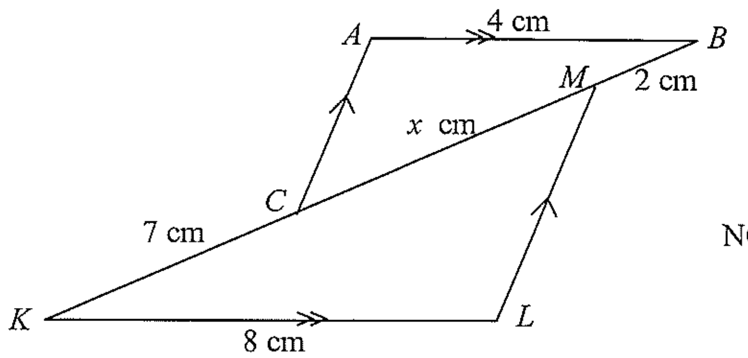
(iv) Find the exact area of the minor segment BCD. **2**

(c) Find the focus and equation of the directrix for the parabola **2**

$$(y-3)^2 = 2(x+1).$$

Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a)



NOT TO SCALE

In the diagram $AB \parallel KL$ and $AC \parallel ML$, $AB = 4$ cm, $MB = 2$ cm, KC=7cm and KL=8cm.

(i) Show ΔABC is similar to ΔKLM. **3**

(ii) Find the length of CM. **2**

(b) A particle is moving along a straight line. Its displacement from a fixed point on the line at time t seconds is given by:

$x = 4t^3 + 3t^2 - 18t + 1, \quad t \geq 0$ where x is in metres.

(i) Find the velocity, v, in terms of t. **1**

(ii) Find the acceleration, a, in terms of t. **1**

(iii) At what time(s) does the particle come to rest? **2**

(iv) Where does the particle come to rest? **1**

(v) How far does the particle travel in the first 2 seconds? **2**

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Solve the following equation for x: **2**

$$10^{2x} - 5.10^{x} + 4 = 0$$

(b) Consider the function $f(x) = x^3 - 3x^2 - 9x + 27$.

(i) Find the coordinates of the points where the curve $y = f(x)$ intercepts with the axes. **2**

(ii) Find the coordinates of the stationary points and determine their nature. **4**

(iii) Find the coordinates of the point/s of inflexion. **1**

(iv) Sketch the graph of $y = f(x)$, indicating clearly the intercepts, stationary points and points of inflexion. **3**

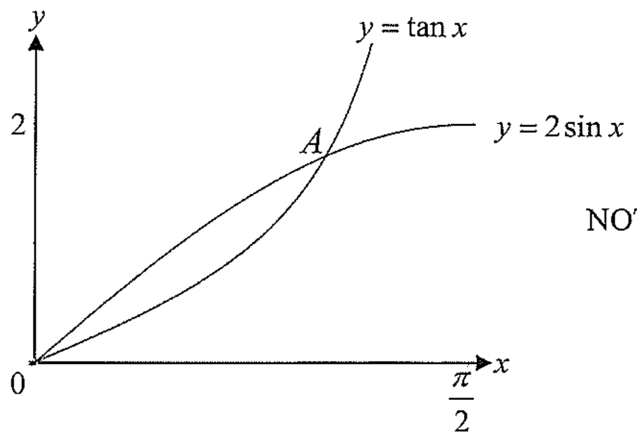
Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) (i) For what values of m does the line $y = m(x+1)$ have no points of intersection with the parabola $y = 2x^2$? **3**

(ii) Hence, or otherwise, find the equations of the two tangents to the parabola $y = 2x^2$ which pass through the point $(-1, 0)$. **2**

(b) Sketch the function $y = 2\sin 3x$, for $0 \le x \le \pi$. Hence, or otherwise, determine the values of x in this domain for which $2\sin 3x \ge 1$. **3**

(c)



The diagram shows the curves $y = \tan x$ and $y = 2\sin x$ for $0 \le x \le \frac{\pi}{2}$.

(i) Show the coordinates of A are $\left(\frac{\pi}{3}, \sqrt{3}\right)$. **1**

(ii) Show that $\frac{d}{dx}\left[\ln(\cos x)\right] = -\tan x$. **1**

(iii) Hence find the area between $y = \tan x$ and $y = 2\sin x$ for $0 \le x \le \frac{\pi}{2}$. **2**

Question 8 (12 marks) Use a SEPARATE sheet of paper. **Marks**

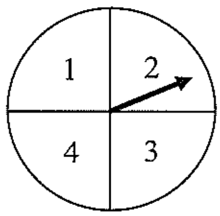
(a) A Year 12 Biology class tested to see how much bacteria was present in a variety of food samples at their school canteen.
It is known that after t hours the number of bacteria (N) present in a particular type of food is given by the formula $N = Ae^{kt}$.

(i) If initially there were 20 000 bacteria present, calculate the value of A. **1**

(ii) After three hours, there were 45 000 bacteria present. Calculate the value of k correct to 2 decimal places. **2**

(iii) How long would it take for the initial number of bacteria to triple in quantity? **2**

(b) Sean and Peter used the spinner shown below to play a game.

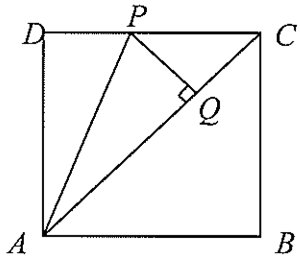


Sean spun the spinner twice and added the results of the two spins to get his score. Peter then took his turn and spun the spinner twice, adding the results of his two spins to get his score. The player with the highest score won the game

(i) What is the probability that Sean scored a 6 in the game? **1**

(ii) Sean's score was 6. What is the probability that Peter won the game? **1**

(c) In the diagram, $ABCD$ is a square. PA bisects $\angle DAC$.
Q is the foot of the perpendicular from P to AC.



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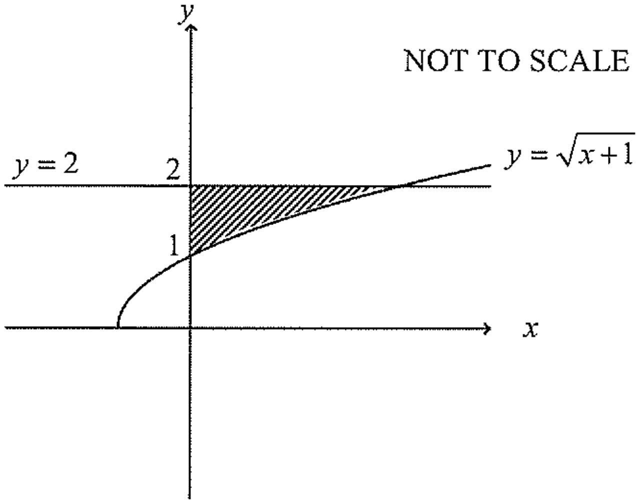
Prove that:

(i) ΔADP is congruent to ΔAQP. **3**

(ii) $QC = DP$. **2**

Question 9 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a)



The shaded region in the diagram is bounded by the curve $y=\sqrt{x+1}$, the y-axis and the line $y=2$. **3**

Find the volume of the solid formed when the shaded region is rotated about the x-axis.

(b) A school soccer team has a probability of 0.7 of losing or drawing any match and a probability of 0.3 of winning any match.

(i) Find the probability of the team winning at least one of three consecutive matches. **2**

(ii) What is the least number of consecutive matches the team must play to be 90% certain it will win at least one match? **2**

(c) Lisa borrows \$20 000 at 3% per quarter reducible interest. She pays the loan off over 5 years by paying quarterly repayments of \$ R. Let \$ A_n by the amount of money Lisa still owes after the nth repayment.

(i) Write an expression for A_1. **1**

(ii) Show that $A_n = 20000\times 1.03^n - R(1.03^{n-1}+...+1.03^2+1.03+1)$ **2**

(iii) Hence find the value of R. **2**

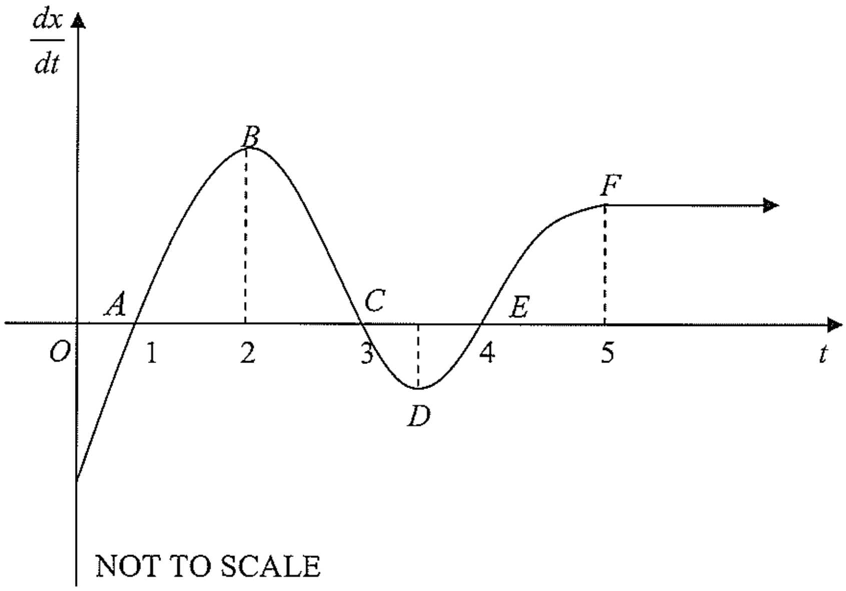
Question 10 (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) An object is moving along the x axis.

The graph below shows the velocity, $\frac{dx}{dt}$, of the object as a function of time, t.

The coordinates of the points on the graph are $A\ (1, 0)$, $B\ (2, 8)$, $C\ (3, 0)$, $D(3.5, -3)$, $E\ (4, 0)$ and $F\ (5, 6)$.

The velocity is constant for $t \geq 5$.



(i) The object is initially at the origin. During which time(s) is the displacement of the object decreasing ? **1**

(ii) Draw a sketch of the graph representing the acceleration of the particle. **2**

(iii) Using Simpson's rule, estimate the distance travelled between $t = 1$ and $t = 3$. **2**

(iv) Sketch the displacement, x, as a function of time. **2**

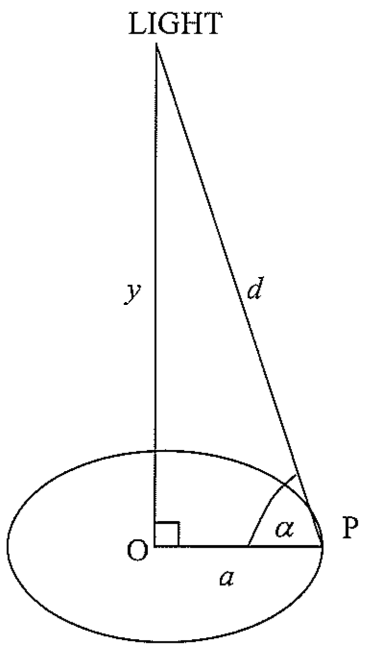
Question 10 continues over page

Question 10 (continued)

(b) A light is to be placed over the centre of a circle, radius a units.

The intensity, I, of the light varies as the sine of the angle, α, at which the rays strike the illuminated surface, divided by the square of the distance, d, from the light.

i.e. $I = \dfrac{k\sin\alpha}{d^2}$ where k is a constant.



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(i) Show that $I = \dfrac{ky}{\left(y^2 + a^2\right)^{\frac{3}{2}}}$ **2**

(ii) Find the best height for a light to be placed over the centre of a circle so as to provide the maximum illumination to the circumference. **3**

End of paper

Q1. (12 MARKS)

a) 9.56275 (2M)

[10.17 using $\log_{10}$ instead of $\ln$]

b) $\frac{1}{5-\sqrt{7}} \times \frac{5+\sqrt{7}}{5+\sqrt{7}} = \frac{5+\sqrt{7}}{25-7}$

$= \frac{5+\sqrt{7}}{18}$ $\therefore a = \frac{5}{18}, b = \frac{1}{18}$ (2M)

c) $\frac{3}{|x-1|} < 2$

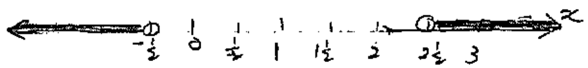
$3 < 2|x-1|$

$|x-1| > \frac{3}{2}$

either $x-1 > \frac{3}{2}$ or $-(x-1) > \frac{3}{2}$

$x > 2\frac{1}{2}$ $\quad x-1 < -\frac{3}{2}$

$x < -\frac{1}{2}$ (2M)



d) $a = 6$, $r = -\frac{2}{3}$

$S_\infty = \frac{6}{1+\frac{2}{3}}$

$= \frac{18}{5}$ or $\underline{3.6}$ (2M)

e) $27^{1-x} = 81^x \times 9^{-x}$

$3^{3-3x} = 3^{4x} \times 3^{-2x}$

$3-3x = 4x-2x$

$5x = 3$

$x = \frac{3}{5}$ or $\underline{0.6}$ (2M)

f) $m = \frac{6-2}{-3+1}$

$= -2$

$\therefore \perp m_2 = \frac{1}{2}$ since $m_1 m_2 = -1$ (2M)

Required line is given by: $\frac{y-3}{x+1} = \frac{1}{2}$

$y = \frac{1}{2}x + 3\frac{1}{2}$ OR $x - 2y + 7 = 0$

Q2. (12 MARKS)

a) $\frac{d}{dx} x^2 \cos x = 2x\cos x - x^2 \sin x$ (2M)

b) $\int (x^{-2} + 3x^{-4})\,dx = -x^{-1} - x^{-3} (+c)$ (2M)

$= -\frac{1}{x} - \frac{1}{x^3} (+c)$

c) (i) $4(10x+1)^3 \times 10 = 40(10x+1)^3$ (2M)

(ii) $\frac{1}{2}\int_0^1 40(10x+1)^3\,dx = \frac{1}{2}\left[(10x+1)^4\right]_0^1$ (3M)

$= \frac{1}{2}\{(10\times 1+1)^4 - 1^4\}$

$= 7320$

d) $\frac{dy}{dx} = \frac{e^x \times 2x - x^2 e^x}{e^{2x}}$ (3M)

$= \frac{e^x(2x-x^2)}{e^{2x}}$

$= xe^{-x}(2-x)$

When $x = 0$, $y = 0$, $\frac{dy}{dx} = 0$ $\therefore$ Tangent is a Horizontal line.

$\therefore$ The Normal is a Vertical line: $x = 0$

(12 MARKS)

a) (i) $d_{AC} = \sqrt{(1+2)^2 + (-1-3)^2}$ (1M)

$= 5$

(ii) $m_{AC} = \frac{-2-1}{3+1}$ (2M)

$= -\frac{3}{4}$

Equation AC: $y - 1 = \frac{-3}{4}(x+1)$

$4y - 4 = -3x - 3$

$3x + 4y - 1 = 0$ (as required)

(iii) $d_{\perp} = \frac{|3\times 2 + 4\times 3 - 1|}{\sqrt{3^2+4^2}}$ (2M)

$= \frac{17}{5}$ or $3\frac{2}{5}$ or 3.4

(iv) $A = \frac{1}{2} AC \times \frac{17}{5}$ (1M)

$= \frac{1}{2} \times 5 \times \frac{17}{5}$

$= 8\frac{1}{2}$ area is $8\frac{1}{2}$ square units.

(v) $\tan\theta = m_{AC} = \frac{-3}{4}$

$\theta = 180° - \alpha$ where acute angle $\alpha \doteqdot 36.869°$ $= 36°52'$

$\doteqdot 143.130°$

$= 143°8'$ (to nearest minute) (1M)

Q3 b) (i) $T_n = 15000 + (n-1)3000$ (1M)

$= 12000 + 3000n$

(ii) $T_{52} = 12000 + 3000 \times 52$

$= 168000$ She made \$168 000 in sales. (1M)

(iii) $S_{52} = \frac{52}{2}(15000 + 168000)$

$= 4\,758\,000$ Total sales were \$4758000 (1M)

(iv) $\frac{n}{2}(15000 + 12000 + 3000n) > 1\,000\,000$

$27000n + 3000n^2 > 2000000$

$3n^2 + 27n - 2000 > 0$

Consider: $3n^2 + 27n - 2000 = 0$

$n = \frac{-27 \pm \sqrt{27^2 + 4\times 3\times 2000}}{6}$

$\doteqdot 21.709\ldots$ since $n > 0$ (2M)

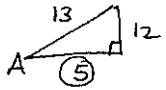
$\therefore n > 21.7$

$\therefore$ During 22nd week sales would exceed the \$1 million.

NB $\begin{cases} n = 21 & S_{21} = 945000 \\ n = 22 & S_{22} = 1023000. \end{cases}$

Q4. (12 MARKS)

a)



$\therefore \cot A = \frac{5}{12}$ (2M)

b) (i) Radius is half of AE. ($\frac{\pi}{2} = 90°$) (2M)

$AE^2 = (4\sqrt{3})^2 + 4^2$ (By Pythagoras' Thm)

$AE^2 = 64$

$AE = 8$, since a length.

$\therefore$ Radius is $\frac{1}{2}AE = 4$ cm. (as required).

(ii) Area of Sector $BOD = \frac{1}{8}\pi(4^2)$ (2M)

$= 2\pi$

(iii) $l = 4 \times \frac{1}{8} \times 2\pi$ ($\theta^c = \frac{\pi}{4}$) (2M)

$= \pi$

(iv) $A_{\text{minor segment}} = A_{\text{sector}} - A_{\triangle BOD}$

$= 2\pi - \frac{1}{2}\times 4^2 \sin\frac{\pi}{4}^c$ (2M)

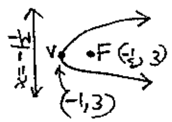
$= 2\pi - \frac{8\sqrt{2}}{2}$

$= 2\pi - 4\sqrt{2}$, Exact area is $(2\pi - 4\sqrt{2})\text{cm}^2$

c) Vertex $(-1, 3)$

$4a = 2$

$a = \frac{1}{2}$



$\therefore$ Focus is $(-\frac{1}{2}, 3)$

Directrix is $x = -1\frac{1}{2}$ (2M)

Q5 (12 MARKS)

a) (i) In ΔABC, ΔLKM (3M)

$\angle ABC = \angle LKM$ (alternate angles, $AB \parallel KL$)

$\angle ACB = \angle LMK$ (alternate angles, $AC \parallel ML$)

$\angle A = \angle L$ (3rd angle)

$\therefore \Delta ABC \parallel\!\mid \Delta LKM$ (equiangular)

(ii) $\frac{4}{8} = \frac{x+2}{x+7}$ (ratio of matching sides in similar triangles) (2M)

$4x + 28 = 8x + 16$

$4x = 12$

$x = 3$ $\therefore$ CM = 3cm

b) (i) $v = 12t^2 + 6t - 18$ (1M)

(ii) $a = 24t + 6$ (1M)

(iii) At rest when $v = 0$ i.e. $6(2t^2 + t - 3) = 0$

$6(2t+3)(t-1) = 0$

$\therefore t = 1$ since $t \geq 0$. (2M)

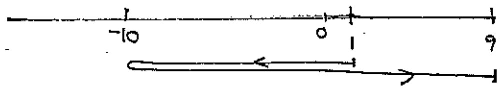
(iv) When $t = 1$, $x = 4 + 3 - 18 + 1$

$= -10$ Particle is 10 metres to the left of O. (1M)

(v) In first 2 seconds the particle travels from 1 ($t=0$) to -10 ($t=1$) (at rest) to 9 ($t=2$)

$t = 2$, $x = 4\times2^3 + 3\times2^2 - 18\times2 + 1$

$= 9$



$\therefore$ It travels 30 metres. (2M)

OR By integration $d = \left|\int_0^1 v\,dt\right| + \int_1^2 v\,dt$ (since $v = 0$ at $t = 1$ see (iii))

Q6 (12 MARKS)

a) (i) P(Scoring a 6) = $\frac{3}{16}$ (1M)

(ii) P(Peter scored 7 or 8 to win the game) = $\frac{3}{16}$ (1M)

	1	2	3	4
1	2	3	4	5
2	3	4	5	(6)
3	4	5	(6)	7
4	5	(6)	7	8

b) $f(x) = x^3 - 3x^2 - 9x + 27$

(i) $0 = x^3 - 3x^2 - 9x + 27$

$x^2(x-3) - 9(x-3) = 0$

$(x^2 - 9)(x-3) = 0$

$(x+3)(x-3)^2 = 0$

$x = -3$ or 3.

Coordinates are $(-3, 0)$ $(3, 0)$ (2M)

(ii) $f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x+1)(x-3)$

Stationary points occur when $f'(x) = 0$

When $x = -1$ or 3.

$f''(x) = 6x - 6$

$f''(-1) = -12 < 0$ $\therefore$ MAX. turning point at $(-1, 32)$

$f''(3) = 12 > 0$ $\therefore$ MIN. turning point at $(3, 0)$ (4M)

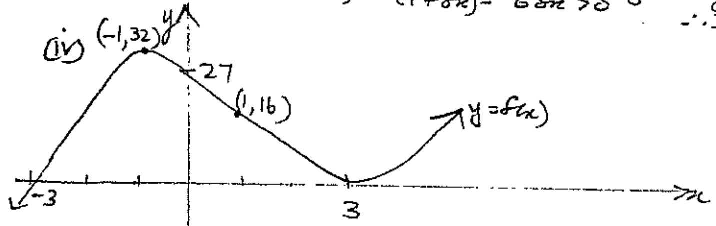
(iii) Possible Inflexion point when $6(x-1) = 0$

$x = 1$

$x = 1 - \delta x$, $f''(1 - \delta x) = -6\delta x < 0$
$x = 1 + \delta x$, $f''(1 + \delta x) = 6\delta x > 0$ } Change in concavity. (1M)

$\therefore$ Inflexion Point at $(1, 16)$

(iv)



(3

Q7 c) (i) A is where $\tan x = 2\sin x$

$$\frac{\sin x}{\cos x} = 2\sin x \quad , \quad \cos x \neq 0 \qquad (1M)$$

$$\sin x - 2\sin x\cos x = 0$$

$$\sin x(1 - 2\cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = 0 \qquad x = \frac{\pi}{3} \quad , \quad \tan\frac{\pi}{3} = \sqrt{3}$$

O is $(0,0)$ $\therefore A$ is $\left(\frac{\pi}{3}, \sqrt{3}\right)$ (as required)

(ii) $$\frac{d}{dx}\ln(\cos x) = \frac{\frac{d}{dx}(\cos x)}{\cos x}$$

$$= \frac{-\sin x}{\cos x} \qquad (1M)$$

$$= -\tan x \quad \text{(as required)}$$

(iii) $$A = \int_0^{\frac{\pi}{3}} (2\sin x - \tan x)\,dx$$, point of intersection when $x = \frac{\pi}{3}$

$$= \left[-2\cos x + \ln(\cos x)\right]_0^{\frac{\pi}{3}}$$

$$= -2\cos\frac{\pi}{3} + \ln\left(\cos\frac{\pi}{3}\right) - \left(-2\cos 0 + \ln(\cos 0)\right)$$

$$= -2\times\frac{1}{2} + \ln\frac{1}{2} + 2\times 1 - \ln 1 \qquad (2M)$$

$$= 1 + \ln\frac{1}{2} \quad \text{or} \quad \ln\frac{1}{2} = \ln 1 - \ln 2 = -\ln 2$$

$$= 1 - \ln 2$$

Q7 (12 MARKS)

a) (i) Points of intersection given by: $m(x+1) = 2x^2$ (3M)

$$2x^2 - mx - m = 0$$

For NO points of intersection $\Delta < 0$

$$m^2 - 4\times 2\times(-m) < 0$$

$$m^2 + 8m < 0$$

$$m(m+8) < 0$$

$$-8 < m < 0$$

Check: $m = -4$

$-4\times 4 = -16 < 0$ ✓

(ii) Lines through $(-1,0)$ given by: $y - 0 = m(x+1)$ (2M)

$y = m(x+1)$ (from (i))

For tangents $\Delta = 0 \therefore m = 0, -8$

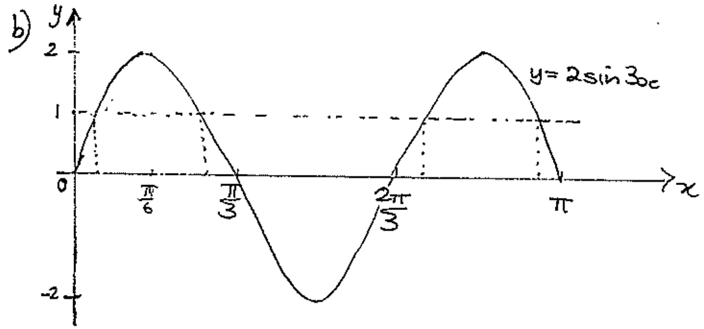
$\therefore$ When $m = 0$ $\quad y = 0$

" $\quad m = -8$ $\quad y = -8(x+1)$

$y = -8x - 8$ or $8x + y + 8 = 0$

The two tangents are $y = 0$ and $y = -8x - 8$.

b)



(3M)

$$2\sin 3x \geq 1$$

$$\sin 3x \geq \frac{1}{2}$$

Consider: $\sin 3x = \frac{1}{2}$

$$3x = \frac{\pi}{6}$$

$$x = \frac{\pi}{18}$$

$\therefore$ For $2\sin 3x \geq 1$

$$\frac{\pi}{18} \leq x \leq \frac{\pi}{3} - \frac{\pi}{18} \quad , \quad \frac{2\pi}{3} + \frac{\pi}{18} \leq x \leq \pi - \frac{\pi}{18}$$

$$\frac{\pi}{18} \leq x \leq \frac{5\pi}{18} \quad , \quad \frac{13\pi}{18} \leq x \leq \frac{17\pi}{18}$$

Q8 (12 MARKS)

a) (i) $N = Ae^{kt}$

$20000 = Ae^{0}$

$A = 20000$ (1M)

(ii) $45000 = 20000\,e^{3k}$

$e^{3k} = \frac{9}{4}$

$3k = \ln\frac{9}{4}$

$k = \frac{1}{3}\ln\frac{9}{4}$

$\doteqdot 0.27031\ldots$

$= 0.27$ (2 decimal places) (2M)

(iii) $3A = Ae^{kt}$

$3 = e^{kt}$

$kt = \ln 3$

$t = \frac{\ln 3}{k}$ (2M)

$\doteqdot 4.064\ldots$ $\quad \frac{\ln 3}{0.27} \doteqdot 4.0689\ldots$

∴ It would take approx. 4.1 hours to triple in quantity.

b) Let $u = 10^{x}$, $u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$u = 4$, $u = 1$

$10^{x} = 4$, $10^{x} = 1$

$x = \log_{10} 4$ or $x = 0$ (2M)

c) (i) In ΔADP, ΔAQP,

AP is common,

$\angle ADP = \angle AQP$ (both 90°, angle in a square)

$\angle DAP = \angle QAP$ (PA bisects $\angle DAC$, given)

∴ $\Delta ADP \equiv \Delta AQP$ (AAS) (3M)

(ii) $DP = PQ$ (matching sides in congruent triangles) (2M)

In PQAD, $\angle DPQ = 360° - (2\times 90° + 45°)$ (angle sum of quadrilateral, angles in a square, diagonal in a square bisects angle in a square)

$= 135°$

In ΔPCQ, $\angle QPC = 45°$ (straight angle)

$= \angle PCQ$ (angle in square bisector by diagonal)

∴ ΔPCQ is isosceles (2 angles equal)

∴ $PQ = QC$ (angles opposite equal sides in a triangle)

∴ $QC = DP$ (equals to equals are equal).

Q9 (12 MARKS)

a) $V = \pi\int_0^3 (2^2 - y^2)\,dx$ $\quad$ when $y = 2$, $2 = \sqrt{x+1}$, $4 = x+1$, $x = 3$

$= \pi\int_0^3 \left(4 - (\sqrt{x+1})^2\right)dx$

$= \pi\int_0^3 \left(4 - (x+1)\right)dx$

$= \pi\int_0^3 (3 - x)\,dx$ (3M)

$= \pi\left[3x - \frac{x^2}{2}\right]_0^3$

$= \pi\left[9 - \frac{9}{2}\right]$

$= \frac{9\pi}{2}$ $\quad$ Volume is $\frac{9\pi}{2}$ units^3.

b) (i) P(winning at least one match) $= 1 - P$(winning none) (2M)

$= 1 - (0.7)^3$ ie losing/drawing all

$= 1 - 0.343$

$= 0.657$

(ii) $1 - (0.7)^n \geq 0.9$ $\quad$ OR By solving: $1 - (0.7)^n = 0.9$

$0.1 \geq 0.7^n$

$\ln 0.1 \geq n\log 0.7$

$n \geq \frac{\ln 0.1}{\ln 0.7}$ (NB $\log 0.7 < 0$) (2M)

$n \geq 6.455\ldots$ $\quad$ $n=6$ 88.2% ; $n=7$ 91.8%

$n = 7$ ∴ Least number of consecutive matches is 7.

c) (i) 5 years = 20 quarters, $A_1 = 20000(1.03) - R$ (1M)

(ii) $A_2 = A_1 \times (1.03) - R$

$= 20000(1.03)^2 - 1.03R - R$

$A_3 = 20000(1.03)^3 - 1.03^2R - 1.03R - R$ (2M)

$= 20000(1.03)^3 - R(1.03^2 + 1.03 + 1)$

$A_n = 20000(1.03)^n - R(1.03^{n-1} + \ldots + 1.03^2 + 1.03 + 1)$ (as required)

(iii) Loan repaid after 20 payments, ie $A_{20} = 0$

$0 = 20000(1.03)^{20} - R(1.03^{19} + 1.03^{18} + \ldots + 1.03^2 + 1.03 + 1)$

Geometric Series $a = 1$, $r = 1.03$, $n = 20$ in reverse order.

$R\left(\frac{1 - 1.03^{20}}{-0.03}\right) = 20000(1.03)^{20}$ (2M)

$R = 20000(1.03)^{20} \times \frac{-0.03}{1 - 1.03^{20}}$

$\doteqdot 1344.3147$

Q10. (12 MARKS)

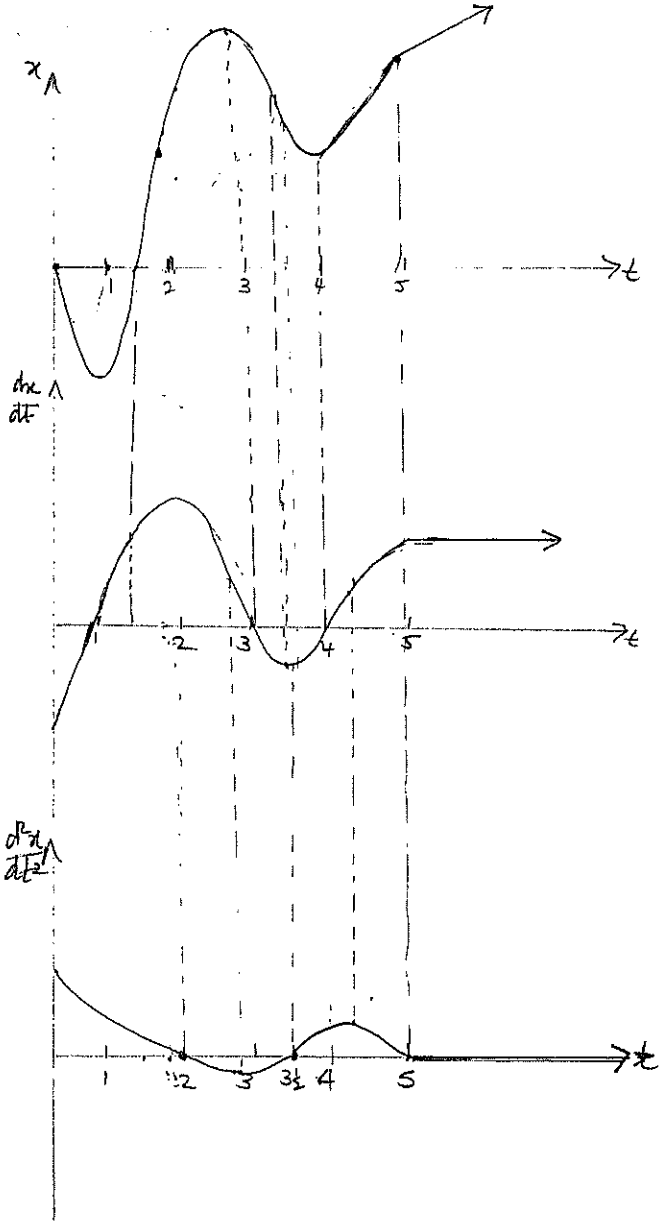
a) (i) Decreasing when $\frac{dx}{dt} < 0$, ie $0 < t < 1$, $3 < t < 4$ (1M)

(ii) (iii) See sketches.

(iv) Distance travelled $\doteqdot \frac{3-1}{6}\{f(0) + 4f(1) + f(3)\}$

$= \frac{1}{3}(0 + 4 \times 8 + 0)$

$= 10\frac{2}{3}$ units. (2M)



(2M)

(2M)

contd.

Q10 b)

(i) $I = \frac{k \sin\alpha}{d^2}$ $\quad d = \sqrt{y^2+a^2}$, $\sin\alpha = \frac{y}{d}$ (2M)

$= k \cdot \frac{y}{d^3}$

$= \frac{ky}{(\sqrt{y^2+a^2})^3}$

$= \frac{ky}{(y^2+a^2)^{\frac{3}{2}}}$

(ii) $\frac{dI}{dy} = \frac{k(y^2+a^2)^{\frac{3}{2}} - \frac{3}{2}ky(y^2+a^2)^{\frac{1}{2}} \times 2y}{(y^2+a^2)^3}$ (3M)

$= \frac{k(y^2+a^2)^{\frac{1}{2}}[y^2+a^2 - 3y^2]}{(y^2+a^2)^3}$

$= \frac{k(a^2-2y^2)}{(y^2+a^2)^{\frac{5}{2}}}$

Stationary points occur when $\frac{dI}{dy} = 0$

ie. $k(a^2-2y^2) = 0$

$y^2 = \frac{a^2}{2}$

$y = \frac{a\sqrt{2}}{2}$ since $y > 0$ (a height).

Test for maximum:

y	$\frac{a\sqrt{2}}{2}^{-} \doteqdot 0.6a$	$\frac{a\sqrt{2}}{2}$	$\frac{a\sqrt{2}}{2}^{+} \doteqdot 0.8a$
I'	0.28 > 0 +	0	−0.28 < 0 −

$\therefore$ + / MAX \ −

MAX. height is $\frac{a\sqrt{2}}{2}$ units.